Research Article
Unified Finite Horizon $H_{\infty}$ Fusion Filtering for Networked Dynamical System

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This paper addresses the $H_{\infty}$ fusion filtering problem for networked dynamical systems, where measurements may arrive at fusion center in four different scenes and the fusion center could receive none, one, or multiple measurements in a fusion period. A unified $H_{\infty}$ performance criterion function, which is suitable for different measurement arrival scenes, is designed for the filtering process of networked dynamical systems. Then, the $H_{\infty}$ performance criterion function is described as an indefinite quadratic inequality and solved by a novel noise projection method in Krein space. On this basis, a unified finite horizon $H_{\infty}$ filtering method is proposed for networked dynamical systems. Simulation results are provided to illustrate the correctness and the effectiveness of the theoretical analysis.

1. Introduction

The filtering methods are widely utilized in the fields of signal processing and automatic control for dynamical systems. With the development of computer and information technology, researchers begin to pay more and more attention on networked dynamical systems, such as the open channel networks and networked control systems [1–3]. However, it is inevitable that the measurement data is transmitted in the networked dynamical systems with different time delay.

In networked dynamical systems, the targets of interest are (remotely) observed by various sensors. The sampled measurements may arrive at information processing centers (especially refer to fusion filters, as in this paper) in different scenes through the transport network. Scene 1: the measurement arrives at the fusion filter in time, which is abbreviated to “ITM” in this paper. Scene 2: the measurement which arrives at the fusion filter with some time delay, but still in the sampled sequence, is abbreviated to “ISDM.” Scene 3: the delay measurement arrives at the fusion filter out of the sampled sequence, which is abbreviated to “OOSM.” Scene 4: the sampled measurement is missing in the transmitting process, which is also named as “packet dropout” (abbreviated to “PD”). The traditional filters are mainly proposed for systems with all measurements in Scene 1, such as Kalman filter, $H_{\infty}$ filter. For the system with measurements arriving at fusion filter in other scenes, several effective filtering methods have also been proposed, recently.

(1) For systems with measurement in Scene 2, some novel filtering methods are proposed based on Kalman filter [4, 5]. And the developed $H_{\infty}$ filtering approaches are also deduced for this kind of systems with bounded energy noises [6, 7].

(2) For systems with measurement in Scene 3, several OOSM filtering problems are investigated with the help of such technologies as nonstandard smoothing [8], Kalman filter with measurement weighted summation [9, 10], and reorganized innovation [11, 12].

(3) For systems with measurement in Scene 4, several filtering approaches are developed based on the traditional Kalman filter [13–15] or $H_{\infty}$ filter [16], based on different descriptions of the packet dropout phenomenon, such as the Markovian jump approach [13] and the binary Bernoulli distribution approach [14–16].
Although some recent approaches have considered the systems in multiple measurement arrival scenes [17, 18], most results of them are deducted for the system with single sensor. Few papers address the filtering problems for the networked dynamical system with multiple sensors. In [19, 20], the fusion filtering methods for networked multisensor systems are deduced based on Kalman filter, which requires the system noise to satisfy zero mean Gaussian distribution with known variance, which, however, is usually not available. To the best of the author’s knowledge, the filtering problem for the networked multisensor system with unknown statistic noises has not been fully investigated and still remains challenging.

Motivated by the above discussion, a unified finite horizon $H_{\infty}$ filtering method is proposed for the networked dynamical system in this paper in which four different kinds of measurement arrival scenes are dealt with in a unified manner. Because of the complex arrival scenes of networked measurements, the fusion filter for the networked dynamical system could receive none, one, or multiple measurements in a fusion period. The $H_{\infty}$ filtering algorithm should be deduced to achieve a $H_{\infty}$ performance criterion function. In the traditional $H_{\infty}$ performance criterion function, an ideal assumption condition is that the measurement sampling time is the same as the measurement arrival time and the filtering time. However, in networked dynamical systems, the arrival time of a networked measurement mostly is not equal to its sampling time. And the fusion filter deals with the sampled measurement at its arrival time, rather than its sampling time. However, in networked dynamical systems, the arrival time of measurements, which, however, is not this case in traditional $H_{\infty}$ filtering. Thus, the sampling time of measurements, which, however, is not this case in traditional $H_{\infty}$ filtering.

According to the traditional finite horizon $H_{\infty}$ filter for single-sensor system, for a given scalar $\gamma > 0$, $\bar{z}(k | k)$ can be obtained as an approximation of $z(k)$ based on the received measurements $\{y(i) | i = 1, 2, \ldots, k\}$ to guarantee the following $H_{\infty}$ performance criterion function:

\[
\sup_{w \in l_{2}[1,N]} \left( \frac{1}{N} \sum_{i=1}^{k} e_{i}^{T} e_{i}(i) \right) \times \left( \frac{1}{N} \sum_{i=1}^{k} w^{T}(i) w(i) + \sum_{i=1}^{k} w^{T}(i, i-1) w(i, i-1) \right) \times \left( \frac{1}{N} \sum_{i=1}^{k} w^{T}(i, i-1) w(i, i-1) \right)^{-1} < \gamma^{2},
\]

where $e_{i}(k) = z(k) - \bar{z}(k | k)$, $\bar{x}_{0} = x(0)$, and $\bar{x}_{0}$ is an initial estimate of $x(0)$. $P_{0}$ is a given positive definite matrix with compatible dimension.

Remark 1. In the above performance criterion function, the filtering time of the fusion filter is the same as the sampling time of measurements, which, however, is not this case in networked dynamical systems.

In a fusion period, the networked measurement could arrive at the fusion filter in four different scenes considered in this paper; namely, the measurement could be ITM, ISDM, or OOSM. The unified filter for networked dynamical systems would deal with various measurement arrival scenes. The filtering time of networked measurement is its arrival time, rather than its sampling time. This means that the performance criterion function shown in (2) cannot be directly extended to the unified filtering process of networked dynamical systems. In the next section, a novel performance criterion function is built for various measurement arrival scenes in the networked dynamical system, firstly. On this
basis, a unified finite horizon $H_{\infty}$ fusion filtering method is deduced.

3. Unified Finite Horizon $H_{\infty}$ Fusion Filtering for Networked Dynamical System

3.1. Performance Criterion Function. Let $\kappa(i)$ be a counter, which counts for the number of the measurements received by the fusion filter in the fusion period $[i, i+1)$. At the start of the period, $\kappa(i) = 0$. Whenever a measurement arrives at the fusion filter, $\kappa(i) = \kappa(i) + 1$. Denote the $j$th measurement received by the fusion filter in the period $[i, i+1)$ as $y_{\alpha_{j}}(\beta_{j})$, in which the notations $j, \alpha_{j}, \beta_{j}$ mean that $y_{\alpha_{j}}(\beta_{j})$ is sampled by sensor $\alpha_{j}$ at the sampled time $\beta_{j}$. Here $\beta_{j} \leq i$, $\alpha_{j} \leq N$, and $j \leq iN$ all are positive integers.

According to the measurement arrival scenes, for a given $\gamma > 0$, a novel finite horizon $H_{\infty}$ fusion filtering performance criterion function could be given as follows in the fusion period $[k, k+1)$ to obtain $\tilde{z}(k \mid k)$ based on the received measurement space $[y_{\alpha_{i}}(\beta_{i}) | 1 \leq i \leq k$ and $\kappa(i) \neq 0, j = 1, \ldots, \kappa(i)]$. Consider

$$
\sup_{w \in \mathcal{L}_{1}, [1, N]} \left( \sum_{\kappa(i) \neq 0}^{\kappa(k)} \sum_{j=1}^{\kappa(i)} \left[ w_{z_{j}}(i) e_{z_{j}}(i) \right]^T \right) \times \left( \sum_{\kappa(i) \neq 0}^{\kappa(k)} \sum_{j=1}^{\kappa(i)} \left[ v_{\alpha_{j}}(\beta_{j}) \right]^T v_{\alpha_{j}}(\beta_{j}) \right) \right. \\
+ \sum_{i=1}^{k} w_{z_{j}}^T(i) w_{z_{j}}(i, i+1) + \sum_{i=1}^{\gamma} \left( \gamma_{0}^{-1} \right) < \gamma^2,
$$

where $e_{z_{j}}(i) = \tilde{z}_{j}(i) - z(i)$, $\tilde{z}_{j}(i)$ is the $j$th estimate of $z(i)$, updated with the measurement $y_{\alpha_{j}}(\beta_{j})$.

Remark 2. The performance criterion function shown in (3) can be utilized for the finite horizon $H_{\infty}$ fusion filtering processes of the dynamical system with measurements which could arrive at fusion filter in the aforementioned four kinds of arrival scenes.

(1) When none measurement arrives at the fusion filter in the period $[k, k+1)$, $\kappa(k) = 0$.

(2) When a measurement firstly arrives at the fusion filter in $[k, k+1)$, $\kappa(k) = 1$. If $\beta_{1} = k$, the measurement is an ITM. If $\beta_{1} < k$, this measurement is a delay measurement (an ISDM or a OOSM) sampled at $\beta_{1}$ by sensor $\alpha_{1}$.

(3) When a second (third, ... , etc.) measurement arrives in $[k, k+1)$, then let $\kappa(k) = \kappa(k) + 1$. The new measurement would also be an ITM or a delay measurement.

3.2. Unified Finite Horizon $H_{\infty}$ Filter Design. The performance criterion function shown in (3) can also be described as the following indefinite quadratic inequality:

$$
\begin{align*}
J_{\kappa(k)} &= \sum_{\kappa(i) \neq 0}^{\kappa(k)} \sum_{j=1}^{\kappa(i)} \left[ v_{\alpha_{j}}(\beta_{j}) \right]^T v_{\alpha_{j}}(\beta_{j}) - \gamma^{-2} e_{z_{j}}^T(i) e_{z_{j}}(i) \\
+ \sum_{i=1}^{k} w_{z_{j}}^T(i, i-1) w_{z_{j}}(i, i-1) + \sum_{i=1}^{\gamma} \left( \gamma_{0}^{-1} \right) > 0,
\end{align*}
$$

Remark 3. The quadratic form $J_{\kappa(k)}$ satisfies the indefinite quadratic inequality above if and only if (1) $J_{\kappa(k)}$ has a stationary point, (2) the value of $J_{\kappa(k)}$ at the stationary point is a minimum, and (3) the minimum is positive.

The stationary point of an indefinite quadratic form in Hilbert space corresponds to a projection in Krein space which is obtained to obtain the stationary point of $J_{\kappa(k)}$ in this paper. A Krein space state-space model associated with the system shown in (1) is introduced as

$$
\tilde{x}(i) = F(i, i-1) \tilde{x}(i-1) + \tilde{w}(i, i-1), \quad i = 1, \ldots, k,
$$

where $\tilde{w}(i) = \beta_{1} = k$, $\kappa(i) \neq 0$, $0 < j \leq \kappa(i)$,

$$
\begin{align*}
\forall_{\alpha_{j}}(\beta_{j}) &= H_{\alpha_{j}}(\beta_{j}) \tilde{x}(\beta_{j}) + \forall_{\alpha_{i}}(\beta_{j}) \tilde{x}(\beta_{j}), \\
1 \leq i \leq k, \quad \kappa(i) \neq 0, \quad 0 < j \leq \kappa(i),
\end{align*}
$$

with

$$
\begin{align*}
\begin{bmatrix}
\begin{bmatrix} \mathbf{x}(0) - \tilde{\mathbf{x}} \end{bmatrix} \\
\begin{bmatrix} \mathbf{w}(j_{1}, j_{1}-1) \\
\forall_{\alpha_{j_{1}}}(\beta_{j_{1}}) \\
\forall_{\alpha_{j_{2}}}(\beta_{j_{2}}) \\
\mathbf{v}_{z_{j_{1}}}(l_{1}) \\
\mathbf{v}_{z_{j_{2}}}(l_{2})
\end{bmatrix}
\end{bmatrix} & = \\
\begin{bmatrix}
P_{0} & 0 & 0 & 0 & 0 \\
0 & P_{0} & 0 & 0 & 0 \\
0 & 0 & P_{0} & 0 & 0 \\
0 & 0 & 0 & P_{0} & 0 \\
0 & 0 & 0 & 0 & P_{0}
\end{bmatrix}^{-1} & = \begin{bmatrix}
\begin{bmatrix} \mathbf{x}(0) - \tilde{\mathbf{x}} \end{bmatrix} \\
\begin{bmatrix} \mathbf{w}(j_{2}, j_{2}-1) \\
\forall_{\alpha_{j_{2}}}(\beta_{j_{2}}) \end{bmatrix} \\
\end{bmatrix}
\end{align*}
$$

Denote $\mathbf{W}(k) := [\mathbf{w}(1, 0), \ldots, \mathbf{w}(k, k-1)]^{T}$, $\tilde{\mathbf{x}}(k) := [\mathbf{x}(0), \mathbf{w}(k, k-1)]^{T}$, the stationary point of the indefinite quadratic form shown in (4) corresponds to the projection of $\tilde{\mathbf{x}}(k)$ into the Krein subspace $\Gamma_{\alpha_{k}}(k)$ spanned by $[\forall_{\alpha_{k}}(\beta_{j}), \tilde{x}_{j}(i) | 1 \leq i \leq k$ and $\kappa(i) \neq 0, j = 1, \ldots, \kappa(i)]$. 


According to the Krein space state equation shown in (5), we have \( x(i - 1) = F(i - 1, i)(x(i) - \hat{w}(i - 1)) \), and thus
\[
\begin{align*}
\tilde{y}^*_j(i - 1) &= H^*_j(i) x(i) + \nu^*_j(i)
\end{align*}
\]
is an orthogonal basis of \( \Gamma_{\kappa(k)}(k) \). The projection of \( \tilde{x}(k) \) into \( \Gamma_{\kappa(k)}(k) \) is given by
\[
\tilde{x}_{\kappa(k)}(k | k) = \sum_{\kappa(i) \neq 0}^{k} \tilde{x}(k), \tilde{e}_{\nu_{yz,j}(i)}(i) \left( R^*_{\nu_{yz,j}(i)}(i) \right)^{-1} \tilde{e}_{\nu_{yz,j}(i)}(i).
\]
The projection of \( \tilde{x}(k) \) into \( \Gamma_{\kappa(k)}(k) \) is
\[
\tilde{x}_{\kappa(k)}(k | k) = \sum_{\kappa(i) \neq 0}^{k} \left( \tilde{x}(k), \tilde{e}_{\nu_{yz,j}(i)}(i) \right) \left( R^*_{\nu_{yz,j}(i)}(i) \right)^{-1} \tilde{e}_{\nu_{yz,j}(i)}(i)
\]
and the noise projection \( \tilde{w}(i, \beta_j | i - 1) \) in (12) is given by
\[
\tilde{w}(i, \beta_j | i - 1) = \tilde{w}_{\kappa(i - 1)}(i, \beta_j, i - 1)
\]
in which
\[
\tilde{w}_{\kappa(i - 1)}(i, \beta_j, i - 1) = \tilde{x}_{\nu_{yz,j}(i)}(i, \beta_j, i - 1).
\]
Let \( \tilde{e}_{\nu_{yz,j}(i)}(i) := \left( \tilde{e}_{\nu_{yz,j}(i)}(i), \tilde{e}_{\nu_{yz,j}(i)}(i) \right) \), \( \hat{R}_{\nu_{yz,j}(i)}^{(i)} := \left( \hat{R}_{\nu_{yz,j}(i)}^{(*)} \right)^{-1} \) is the projection of \( \tilde{e}_{\nu_{yz,j}(i)}(i) \) into \( \Gamma_{\nu_{yz,j}(i)}(i) \).

\[\tilde{e}_{\nu_{yz,j}(i)}(i) := \left( \tilde{e}_{\nu_{yz,j}(i)}(i), \tilde{e}_{\nu_{yz,j}(i)}(i) \right) = \left( \hat{R}_{\nu_{yz,j}(i)}^{(*)} \right)^{-1} \]

It is obvious that \( \tilde{e}_{\nu_{yz,j}(i)}(i) \perp \Gamma_{\nu_{yz,j}(i)}(i) \), and \( \tilde{e}_{\nu_{yz,j}(i)}(i) \perp \Gamma_{\nu_{yz,j}(i)}(i) \). The projection of \( \tilde{x}(k) \) into \( \Gamma_{\nu_{yz,j}(i)}(i) \) is given by
\[
\tilde{x}_{\nu_{yz,j}(i)}(i | i - 1) = F(i, i - 1) \tilde{x}_{\nu_{yz,j}(i)}(i, \beta_j, i - 1).
\]
where
\[
K_\alpha(k) := \left( \mathbf{x}(k) \right)^T \mathbf{y}_{(x,z)}(k) \left( \mathbf{R}_{y,z,y}(k) \right)^{-1}
\]
\[
\mathbf{R}_{y,z,y}(k) := \mathbf{R}_{x,y,z}(k) \left( \mathbf{R}_{z,x,y}(k) \right)^{-1},
\]
\[
\mathbf{P}_{xw,y}(k) \left( k, \beta_{k}^x(k) \mid k - 1 \right) = \left( \mathbf{x}(k) \right)^T \mathbf{w}_{e}(k) \left( k, \beta_{k}^e(k) \mid k - 1 \right)
\]
\[
\mathbf{R}_{w,u}(k) \left( k, \beta_{k}^w(k) \mid k - 1 \right) + \mathbf{R}_{u,y}(k) \left( k, \beta_{k}^u(k) \mid k - 1 \right)
\]
\[
\mathbf{R}_{u,y}(k) \left( k, \beta_{k}^u(k) \mid k - 1 \right)
\]
\[
\mathbf{P}_{xw,y}(k) \left( k, \beta_{k}^x(k) \mid k - 1 \right)
\]
\[
\mathbf{R}_{w,u}(k) \left( k, \beta_{k}^w(k) \mid k - 1 \right)
\]
\[
\mathbf{R}_{u,y}(k) \left( k, \beta_{k}^u(k) \mid k - 1 \right)
\]
\[
\mathbf{P}_{xw,y}(k) \left( k, \beta_{k}^x(k) \mid k - 1 \right)
\]
\[
\mathbf{R}_{w,u}(k) \left( k, \beta_{k}^w(k) \mid k - 1 \right)
\]
\[
\mathbf{R}_{u,y}(k) \left( k, \beta_{k}^u(k) \mid k - 1 \right)
\]
\[
\mathbf{P}_{xw,y}(k) \left( k, \beta_{k}^x(k) \mid k - 1 \right)
\]
\[
\mathbf{R}_{w,u}(k) \left( k, \beta_{k}^w(k) \mid k - 1 \right)
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\[
\mathbf{R}_{u,y}(k) \left( k, \beta_{k}^u(k) \mid k - 1 \right)
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\[
\mathbf{P}_{xw,y}(k) \left( k, \beta_{k}^x(k) \mid k - 1 \right)
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\[
\mathbf{R}_{w,u}(k) \left( k, \beta_{k}^w(k) \mid k - 1 \right)
\]
\[
\mathbf{R}_{u,y}(k) \left( k, \beta_{k}^u(k) \mid k - 1 \right)
\]
\[
\mathbf{P}_{xw,y}(k) \left( k, \beta_{k}^x(k) \mid k - 1 \right)
\]
\[
\mathbf{R}_{w,u}(k) \left( k, \beta_{k}^w(k) \mid k - 1 \right)
\]
\[
\mathbf{R}_{u,y}(k) \left( k, \beta_{k}^u(k) \mid k - 1 \right)
\]
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\mathbf{P}_{xw,y}(k) \left( k, \beta_{k}^x(k) \mid k - 1 \right)
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\mathbf{R}_{w,u}(k) \left( k, \beta_{k}^w(k) \mid k - 1 \right)
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\[
\mathbf{R}_{u,y}(k) \left( k, \beta_{k}^u(k) \mid k - 1 \right)
\]
\[
\mathbf{P}_{xw,y}(k) \left( k, \beta_{k}^x(k) \mid k - 1 \right)
\]
\[
\mathbf{R}_{w,u}(k) \left( k, \beta_{k}^w(k) \mid k - 1 \right)
\]
\[
\mathbf{R}_{u,y}(k) \left( k, \beta_{k}^u(k) \mid k - 1 \right)
\]
\[
\mathbf{P}_{xw,y}(k) \left( k, \beta_{k}^x(k) \mid k - 1 \right)
\]
\[
\mathbf{R}_{w,u}(k) \left( k, \beta_{k}^w(k) \mid k - 1 \right)
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\[
\mathbf{R}_{u,y}(k) \left( k, \beta_{k}^u(k) \mid k - 1 \right)
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\mathbf{P}_{xw,y}(k) \left( k, \beta_{k}^x(k) \mid k - 1 \right)
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\mathbf{R}_{w,u}(k) \left( k, \beta_{k}^w(k) \mid k - 1 \right)
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\mathbf{R}_{u,y}(k) \left( k, \beta_{k}^u(k) \mid k - 1 \right)
\]
\[
\mathbf{P}_{xw,y}(k) \left( k, \beta_{k}^x(k) \mid k - 1 \right)
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\mathbf{R}_{w,u}(k) \left( k, \beta_{k}^w(k) \mid k - 1 \right)
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\mathbf{R}_{u,y}(k) \left( k, \beta_{k}^u(k) \mid k - 1 \right)
\]
\[
\mathbf{P}_{xw,y}(k) \left( k, \beta_{k}^x(k) \mid k - 1 \right)
\]
\[
\mathbf{R}_{w,u}(k) \left( k, \beta_{k}^w(k) \mid k - 1 \right)
\]
\[
\mathbf{R}_{u,y}(k) \left( k, \beta_{k}^u(k) \mid k - 1 \right)
\]
\[
\mathbf{P}_{xw,y}(k) \left( k, \beta_{k}^x(k) \mid k - 1 \right)
\]
\[
\mathbf{R}_{w,u}(k) \left( k, \beta_{k}^w(k) \mid k - 1 \right)
\]
\[
\mathbf{R}_{u,y}(k) \left( k, \beta_{k}^u(k) \mid k - 1 \right)
\]
\[
\mathbf{P}_{xw,y}(k) \left( k, \beta_{k}^x(k) \mid k - 1 \right)
\]
\[
\mathbf{R}_{w,u}(k) \left( k, \beta_{k}^w(k) \mid k - 1 \right)
\]
\[
\mathbf{R}_{u,y}(k) \left( k, \beta_{k}^u(k) \mid k - 1 \right)
\]
\[
\mathbf{P}_{xw,y}(k) \left( k, \beta_{k}^x(k) \mid k - 1 \right)
\]
\[
\mathbf{R}_{w,u}(k) \left( k, \beta_{k}^w(k) \mid k - 1 \right)
\]
\[
\mathbf{R}_{u,y}(k) \left( k, \beta_{k}^u(k) \mid k - 1 \right)
\]
\[ P_{\text{wu}(k)}(k, \beta_k^*(k) | k - 1) := \langle w(k, \beta_k^*(k) | k - 1) \rangle = \langle w(k, \beta_k^*(k)), \hat{w}_{\beta_k^*(k)}(k - 1) \rangle = \langle w(k, \beta_k^*(k)), \hat{w}_{\beta_k^*(k)}(k - 1) \rangle \]

\[ \times \left( R_{yz,x(k-1)}^*(k) \right)^{-1} R_{yz,x(k-1)}^*(k, \beta_k^*(k), k) \]

\[ = P_{\text{wu}(k-1)}(k, \beta_k^*(k) | k - 1) + \sum_{i=1}^{k} F(k, i) F^T(k, i), \quad \beta_k^*(k) = k \]

\[ \text{and} \]

\[ Q(k, \beta_k^*(k)) = \begin{cases} 0, & \text{if } \sum_{i=1}^{k} F(k, i) F^T(k, i) = \beta_k^*(k) \leq k \end{cases} \] (19)

The projection of \( \bar{x}(k) \) in (14) corresponds to a stationary point of the indefinite quadratic form \( J_{\kappa(k)} \) in (4), and the value of \( J_{\kappa(k)} \) at this stationary point is

\[ J'_{\kappa}(k) = \sum_{i=1}^{\kappa} (e^*_{yz}(i))^T R_{yz,j}^*(i) e^*_{yz}(i) \]

\[ = J'_{\kappa(k-1)}(k) + (e^*_{yz}(k))^T R_{yz,x(k)}^*(k) \]

\[ \times e^*_{yz,x(k)}(k), (20) \]
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Letting \( \hat{z}^*_\nu(k | k) = \tilde{z}_\nu(k | k - 1) - R_{yx,\nu}(k) R_{yx,\nu}^{-1}(k) e^*_{yx,\nu}(k | k - 1) \), the last term in (20) is

\[
\begin{align*}
  \mathbf{e}^*_{yx,\nu}(k | k) &= \begin{pmatrix} \mathbf{R}^*_{yx,\nu}(k | k) \end{pmatrix}^{-1} \mathbf{e}^*_{yx,\nu}(k | k - 1) \\
  &= \begin{pmatrix} \mathbf{e}^*_{yx,\nu}(k | k - 1) \end{pmatrix}^T R_{yx,\nu}(k) \mathbf{e}^*_{yx,\nu}(k | k - 1) \\
  &\quad + \begin{pmatrix} \tilde{z}_{\nu}(k | k) - \hat{z}^*_{\nu}(k | k) \end{pmatrix}^T \\
  &\quad \times \begin{pmatrix} R_{xx,\nu}(k) - R_{yx,\nu}(k) R_{yx,\nu}^{-1}(k) R_{yx,\nu}(k) \end{pmatrix}^{-1} \\
  &\quad \times \begin{pmatrix} \tilde{z}_{\nu}(k | k) - \hat{z}^*_{\nu}(k | k) \end{pmatrix}.
\end{align*}
\]

(24)

According to Lemma 12 in [21], \( J^*_{\nu}(k) \) is the minimum of \( J_{\nu}(k) \) if and only if \( \mathbf{R}^*_{yx,\nu}(k) \) and \( \mathbf{R}^*_{yx,\nu}(k) \) have the same inertia. Considering the block triangular factorization of \( \mathbf{R}^*_{yx,\nu}(k) \) as shown in (23), the minimum condition can also be given by

\[
\begin{align*}
  R_{xx,\nu}(k) - R_{yx,\nu}(k) R_{yx,\nu}^{-1}(k) R_{yx,\nu}(k) \leq 0.
\end{align*}
\]

(25)

Therefore, a choice of \( \tilde{z}_{\nu}(k | k) \) to guarantee \( J^*_{\nu}(k) > 0 \) is \( \tilde{z}_{\nu}(k | k) = \hat{z}^*_{\nu}(k | k) \), and the minimum of \( J_{\nu}(k) \) is

\[
\begin{align*}
  J_{\nu}(k) &= J^*_{\nu}(k) + \mathbf{e}^*_{yx,\nu}(k | k - 1) \\
  &\quad \times \begin{pmatrix} R_{yx,\nu}(k) \end{pmatrix}^{-1} \mathbf{e}^*_{yx,\nu}(k | k - 1).
\end{align*}
\]

(26)

Then, the estimation of the signal to be estimated is

\[
\begin{align*}
  \tilde{z}(k | k) &= L(k) \tilde{x}_\nu(k | k).
\end{align*}
\]

(27)

in which

\[
\begin{align*}
  \tilde{x}_\nu(k | k) &= \hat{x}_\nu(k | k - 1) + K_{\nu}(k) \mathbf{e}^*_{yx,\nu}(k | k - 1), \\
  K_{\nu}(k) &= R_{yx,\nu}(k) R_{yx,\nu}^{-1}(k).
\end{align*}
\]

(28)

The parameters \( R_{yx,\nu}(k) \) and \( R_{yx,\nu}^{-1}(k) \) in (29) can be obtained by iterating the equations in (18).

In summary, the unified finite horizon \( H_\infty \) fusion filtering algorithm is given by

\[
\begin{align*}
  \tilde{z}(k | k) &= L(k) \tilde{x}_\nu(k | k) \\
  \tilde{x}_\nu(k | k) &= \hat{x}_\nu(k | k - 1) + K_{\nu}(k) \mathbf{e}^*_{yx,\nu}(k | k - 1) \\
  \tilde{x}_\nu(k | k) &= F(k, k - 1) \tilde{x}_\nu(k - 1 | k - 1),
\end{align*}
\]

(29)

in which

\[
\begin{align*}
  \tilde{x}_\nu(k | k) &= \tilde{x}_\nu(k | k) \\
  e_{\nu}(k | k - 1) &= F(k, k - 1) \tilde{x}_\nu(k - 1 | k - 1), \\
  K_{\nu}(k) &= H_{\nu}(k) P_{\nu}(k) H_{\nu}^T(k) + I^{-1}, \\
  e_{\nu}(k | k - 1) &= e_{\nu}(k | k - 1) - \tilde{x}_\nu(k | k - 1) + H_{\nu}(k) \tilde{x}_\nu(k | k - 1).
\end{align*}
\]

(30)
The corresponding Riccati equations are
\[
P_1(k) = F(k, k-1) \times (P_N(k-1) - R_{y_j z_j}(k) R_{y_j z_j}^{-1}(k) \times R^T_{y_j z_j N}(k-1)) \times F^T(k, k-1) + I
\]
\[
P_{l+1}(k) = P_l(k) - R_{y_j z_j}(k) R_{y_j z_j}^{-1}(k) R^T_{y_j z_j}(k),
\]
\[
R_{y_j z_j}(k) = P_l(k) \left[ (H_l(k))^T L^T(k) \right] \times \left[ (H_l(k))^T L^T(k) \right] + \left[ \begin{array}{cc} I & 0 \\ 0 & -\gamma^2 I \end{array} \right].
\]

4. Simulation

In order to illustrate the viability and the effectiveness of the proposed method, the discrete dynamical system as shown in (1) is considered in this section, in which \( F(k, k-1) = \left[ \begin{array}{cc} 1 & 1 \\ 0 & 1 \end{array} \right] \), \( H_i = [1, 1], (i = 1, 2), L = [0.1, 0], \gamma = 1.5 \). Moreover, the initial value is selected as \( x_0 = [1, 5] \), and \( P_0 = \left[ \begin{array}{cc} 1 & 1/2 \\ 1/2 & 2 \end{array} \right] \).

The measurement arrival scenes are designed as follows. All the measurements are sampled in time. For Sensor 1, all the measurements arrive at the fusion filter in time. For Sensor 2, the measurements sampled at the moments with indexes modulo 3 equal to 1 or 2 arrive at the fusion filter with one-step delay, and other measurements arrive in time, as shown in Figure 1.

In this simulation, two simulation results are compared. The first one is the result of the proposed method in the above measurement arrival scene, which is for short marked as “Algorithm 1” in this section. The other one is the simulation result of the sequential fusion filtering method in the scene that all the measurements arrive in time, as shown in Remark 4, which is marked as “Algorithm 2.”

According to the simulation results shown in Table 1, Figures 1 and 2, the following performances of the proposed algorithm are illustrated.

1. The proposed algorithm could deal with different kinds of arrived measurements: ITMs (the measurements sampled by Sensor 1 and the ones sampled by Sensor 2 at the sampled moments with indexes divided by 3 exactly), ISDMs (the measurements sampled by Sensor 2 at the sampled moments with indexes modulo 3 equal to 1), and OOSMs (the measurements sampled by Sensor 2 at the sampled moments with indexes modulo 3 equal to 2).

2. For Algorithm 1, at the fusion time with the indexes divided by 3 exactly, the delay measurements can all arrive at the fusion center. At this fusion time, the fusion filtering results of Algorithm 1 are better than the ones of Algorithm 2. The mean absolute estimation errors at these fusion times of Algorithm 1 are 0.3787, while the one of Algorithm 2 is 0.3832. It is because more amount of information is applied to update the estimate at this fusion time in Algorithm 1 (Figure 3).

3. It is implied that the proposed algorithm could deal with the delay measurements effectively.

5. Conclusion

In this paper, a unified finite horizon \( H_\infty \) filtering method is proposed for general networked dynamical systems, the fusion filter of which could receive none, one, or multiple measurements in a fusion period. According to the complex arrival scenes of networked measurements, a novel \( H_\infty \) performance criterion function is built to restrain the \( H_\infty \) filtering process. Based on the projection method in
Krein space, a novel $H_{\infty}$ filtering method is proposed to uniformly deal with various delay measurements and ITMs in the centralized fusion frame. Otherwise, there are several interesting future directions along the line of this work:

(1) how to deal with the networked measurements in the distributed fusion frame in a uniform manner,

(2) how to deal with various quantified delay measurements for networked multisensor systems.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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