Research Article

Application of Fuzzy Optimization to Production-Distribution Planning in Supply Chain Management

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A production-distribution model has been developed that not only allocates the limited available resources and equipment to produce the products over the time periods, but also determines the economical distributors for dispatching the products to the distribution centers or retailers. The model minimizes production, inventory holding, backordering, and transportation cost while considering the time value of money. Since uncertainty is an inevitable issue of any real-world production system, then to provide a realistic model, the concept of fuzzy sets has been applied in the proposed mathematical modeling. To illustrate and show the feasibility and validity of the model, a real case analysis, which is pertaining to a mineral water bottling production factory, has been used. The case has been solved using a three-step solution approach developed in this study. The results show the feasibility and validity of the mathematical model, and also the solution procedure.

1. Introduction

Nowadays, with the globalization and the evolution of newer global marketplace over the time, manufacturing companies have been forced to find methods to design and operate efficient supply chains to meet customer demands and maximize the profit of uncertain business environment [1, 2]. A typical supply chain may consist of a number of suppliers and a plant, where processing of the relevant materials adds value to what have been received from suppliers and delivers the products to one or more distributors to dispatch them to several distribution centers (DC) or retailers. In such a supply chain, transportation cost may vary from one distributor to another due to variations in skills, labor cost, or different vehicle types, and equipment that is being used to do the transport. Hence, a significant interest has arisen in production-distribution planning decision (PDPD) of supply chains.

Production planning is a process, which involves allocating available equipment and resources over a time period to perform a series of required tasks in order to manufacture finished products, according to a specified schedule [3, 4], while distribution planning includes methods for determining the ways to get the materials and products from the delivery points to the consuming points in supply chains [5]. In general, optimization of supply chain decision in isolation keeps the firms away from gaining maximum possible effectiveness [6–8]. Therefore, great efforts have been made to optimize production and distribution planning problems simultaneously. Several researchers have investigated the advantages of integrated production-distribution planning in different manufacturing environments. For instance, Park indicated the effectiveness of integrating production and distribution planning in multiplant, multi-item, multiperiod, and multiretailer environment [9]. Safaei et al. demonstrated...
the overall cost reduction resulting from the integration in a multiproduct, multisite, multiperiod production-distribution network [10]. Amorim et al. studied the effects of concurrent optimization of production and distribution decisions for perishable products [11].

A comprehensive review on integrated production-distribution models and techniques is conducted by Fahimnia et al. According to the review, some prominent characteristics that influence the complexity of models are as follows: single stage or multistage supply network, single or multiple objective functions, single or multiple products, single or multiple plants or distribution centers, and single or multiperiod [5]. This section is not intended to review or classify the literature but to direct readers to some of the most relevant and most recent studies in the area of integrated production and distribution planning. Reviewing these and other related studies reveals that most of the integrated production-distribution models in the literature are deterministic. However, one of the realities of current production systems is that the goals and other relevant inputs such as available resources, market demand, and production rates are not deterministic. Hence, conventional mathematical models that assume the input parameters are deterministic and crisp cannot handle the uncertainty of real manufacturing systems. In such a situation, fuzzy and also stochastic mathematical modeling can be applied to cope with the decision making under uncertainties of manufacturing environments [12–14].

Most recently, fuzzy programming as one of the methods that is able to take into account uncertainty of manufacturing systems has been applied in integrated production-distribution planning problems. Lin and Liang developed a fuzzy multiobjective linear programming (FMOLP) model for an aggregate production planning problem [15]. Then, Wang and Liang developed a possibilistic linear programming (PLP) approach to solve a multiproduct and multiperiod aggregate production planning [16]. Aliev et al. developed a multiproduct and multiperiod mathematical model in a supply chain to integrate the production and distribution decisions simultaneously. They developed a genetic algorithm-based method to solve the model [17]. However, although there are more studies that have considered fuzzy aggregate production planning in a supply chain [6, 18, 19], to the best of the authors’ knowledge there is not any previous study that considers the production planning and distributor selection in a supply chain simultaneously.

The aim of this study is to develop a production-distribution mathematical model that not only determines the production planning of the system, but also selects the best contractors for dispatching the products to the distribution centers or retailers. Moreover, since some data from real-world manufacturing environments are unobtainable or imprecise, in order to provide a more realistic mathematical model, fuzzy set theory has been applied in this study. The remainder of this paper is organized as follows. Section 2 is dedicated to the problem description. In Section 3, the mathematical model will be presented. Then, Section 4 provides a discussion regarding the solution procedure. In Section 5, a real case from a mineral water bottling factory will be illustrated to show the feasibility and validity of the proposed mathematical model. Finally, in Section 6 this study will be concluded.

2. Problem Description

This study assumes that there is a mineral water bottling factory that produces different kinds of bottled water. The main product of this factory, which is 1.5-liter bottles of mineral water, is dispatched to several distribution centers by several companies as distributors to satisfy imprecise demand over a medium planning horizon. On the other hand, the factory capacity and also labor levels are imprecise and fuzzy due to unobtainable or incomplete available records. This study aims to develop a fuzzy mathematical model that not only determines the optimal production plan for the system, but also selects the economical contract that provides the minimum transportation cost for dispatching the products to the distribution centers of the supply chain in an uncertain environment.

The assumptions of the proposed fuzzy mathematical model are as follows.

1. Triangular distribution pattern is applied to represent all fuzzy numbers.
2. The objective function is considered to be fuzzy with imprecise aspiration level.
3. The nonincreasing continuous linear membership function is used to specify the decision maker's satisfaction degree.
4. The minimum operator is applied to aggregate the fuzzy sets.
5. The demand of each distribution center over a special time period might be satisfied or backordered, although the backorders have to be fulfilled next period until the end of the planning horizon.
6. Each distribution center for each time period has a minimum requirement of supply that should be satisfied.

Assumption 1 addresses the application of triangular fuzzy numbers to represent imprecise data. Simplicity and effectiveness of triangular fuzzy numbers enhance the computational efficiency and also facilitate data acquisition [19–22]. Assumption 2 is to state the fuzziness of the objective function. Assumption 3 is used to mention that according to the satisfaction degree of the decision maker (DM) linear membership function in comparison with other membership functions is preferred. Assumption 4 is to state that the concept of fuzzy decision-making of Bellman and Zadeh, together with the minimum operator of Zimmermann is used to aggregate all fuzzy sets in this study [23, 24]. Assumption 5 shows that a portion of demand is allowed to be backordered, but it has to be fulfilled in the next time periods of the running of manufacturing system. Finally, assumption 6 is to ensure that each distribution center in each period at least will receive a minimum amount of products to supply to the retailers.
3. Mathematical Model Development

The main aim of this study is to provide a fuzzy mathematical model that not only determines the aggregate production plans in a midterm period, but also selects the economical contracts that dispatch the products of supply chain to distribution centers or retailers. The notations that are used in the proposed mathematical model are as follows.

3.1. Index Sets

\[ j: \text{Index for destinations } j = 1, \ldots, J \]
\[ n: \text{Index for time periods } n = 1, \ldots, N \]
\[ k: \text{Index for distributors } k = 1, \ldots, K \]  

3.2. Decision Variables

\[ Q_n: \text{Production volume in period } n \text{ (boxes)} \]
\[ I_n: \text{Inventory level in period } n \text{ (boxes)} \]
\[ B_n: \text{Backorder level in period } n \text{ (boxes)} \]
\[ T_{jkn}: \text{Number of transported products to destination } j \text{ by distributor } k \text{ in period } n \text{ (boxes)} \]
\[ Y_k = \begin{cases} 
1 & \text{If distributor } k \text{ dispatches the products to the DCs or retailers.} \\
0 & \text{Otherwise.} 
\end{cases} \]  

3.3. Parameters

\[ CP_n: \text{Production cost per box in period } n \text{ ($/box)} \]
\[ CL_n: \text{Inventory-holding cost per box in period } n \text{ ($/box)} \]
\[ CB_n: \text{Backordering cost per box in period } n \text{ ($/box)} \]
\[ CT_{jkn}: \text{Per-box transportation cost to destination } j \text{ in period } n \text{ by distributor } k \text{ ($/box)} \]
\[ FC_{jkn}: \text{Fixed transportation cost to destination } j \text{ in period } n \text{ by distributor } k \text{ ($)}} \]
\[ D_n: \text{Demand of product in period } n \text{ (box)} \]
\[ R_{jn}: \text{Minimum supply of product for destination } j \text{ in period } n \text{ (box)} \]
\[ MARR: \text{Minimum attractive rate of return as the escalating factor for the costs ()} \%
\[ U: \text{Required machine-hour to produce one product (machine-hour/box)} \]
\[ F_n: \text{Maximum factory capacity in period } n \text{ (machine-hour)} \]
\[ V: \text{Required man-hour to produce one product (man-hour/box)} \]
\[ M_n: \text{Maximum manpower available in period } n \text{ (man-hour)} \]
\[ S: \text{Required warehouse volume per box of product (m}^2\text{/box)} \]
\[ AW_n: \text{Maximum available warehouse volume in period } n \text{ (m}^2\text{).} \]

3.4. Fuzzy Mathematical Model

\[
\text{Min } z \equiv \sum_{n=1}^{N} \left( CP_n \cdot Q_n + CL_n \cdot I_n + CB_n \cdot B_n \right. \\
\left. + \sum_{k=1}^{K} \sum_{j=1}^{J} Y_k \cdot \left( CT_{jkn} \cdot T_{jkn} + FC_{jkn} \right) \right) (1 + MARR)^n \\
\text{subject to}
\]
\[
I_{n-1} + Q_n - I_n = \sum_{k=1}^{K} \sum_{j=1}^{J} \left( Y_k \cdot T_{jkn} \right) \forall n, (3)
\]
\[
D_n \leq B_{n-1} + Q_n - I_n + B_n = D_n \forall n, (4)
\]
\[
Q_n \geq B_{n-1} \forall n, (5)
\]
\[
\sum_{k=1}^{K} \left( Y_k \cdot T_{jkn} \right) \geq R_{jn} \forall j, \forall n, (6)
\]
\[
U \cdot Q_n \leq F_n \forall n, (7)
\]
\[
V \cdot Q_n \leq M_n \forall n, (8)
\]
\[
\sum_{k=1}^{K} \sum_{j=1}^{J} Y_k \cdot \left( S \cdot T_{jkn} \right) \leq AW_n \forall n, (9)
\]
\[
\sum_{k=1}^{K} Y_k = 1, (10)
\]
\[
Q_n, O_n, I_n, B_n, T_{jkn} \geq 0 \forall n, Y_k = \{0, 1\}. (11)
\]

The first part of the fuzzy mathematical model, in (2), is the objective function. It minimizes the total cost of the system which includes production, inventory, backorder, and transportation cost. Equation (3) indicates that the amounts of products that can be transported in each time period are those products that have been produced in the same period plus the amount of products that have been stocked from the previous period minus the amount of products that will be maintained in the warehouse for the next time period. Based on the fifth assumption of this study, which declares that the demand for each time period might be satisfied or backordered, (4) has been developed. Moreover, based on the same assumption, (5) implies that the amount of products, which are produced in each period, at least should satisfy...
the backorder of the previous time period. Equation (6) is based on the sixth assumption. It tries to satisfy the minimum supply requirement for each distribution center in every time period. Equations (7), (8), and (9) are orderly related to the normal limitation in the capacity of machine-hour, man-hour, and the available warehouse volume of the factory. Equation (10) shows that the decision maker only looks for one contractor to dispatch the products to distribution centers or retailer. Finally, set of (11) shows the nonnegativity and kinds of decision variables that have been used in the proposed fuzzy mathematical model.

4. Solution Procedure

The fuzzy mathematical model that has been developed in this study is flexible in the value of the objective function and also has vagueness in some constraints. Hence, the fuzzy model has to be converted into an equivalent crisp one in order to be solved by ordinary methods. For this purpose, a three-step procedure has been proposed as following.

(1) In this study, it has been assumed that a triangular distribution pattern is adopted for all the fuzzy data. Hence, based on the level of \( \alpha \)-cut, all the imprecise constraints are converted into crisp ones using the weighted averaging method.

(2) The fuzzy objective function, based on the decision's maker satisfaction degree, is treated.

(3) The auxiliary variable \( L \) is introduced at first. After that the original mathematical model is transformed into an equivalent ordinary mathematical model by using minimum operator. Then, the ordinary linear programming (LP) model is solved by conventional methods.

4.1. Treatment of Fuzzy Constraints. In solving fuzzy mathematical models, if there is any vagueness in the constraints, it should be treated. In this study, the volume of demand, the factory capacity level, and also the level of manpower available in each of the planning periods, because of the vague nature of these data, have been considered to be fuzzy with triangular fuzzy distribution. The reason of using triangular fuzzy numbers for the data is its flexibility and simplicity in performing the fuzzy arithmetic operations [19, 22]. These fuzzy numbers, as it can be seen from the proposed mathematical model, are devoted to the right-hand side of the constraints (4), (7), and (8). In this study, in order to convert these fuzzy numbers into crisp ones, the weighted average method has been used [19, 25]. This method, in addition to the level of \( \alpha \)-cut, which is the minimal acceptable possibility level of occurrence for the data, uses the three prominent components of every triangular distribution pattern that are the most pessimistic \( (p) \), most likely \( (m) \), and most optimistic value \( (o) \) for the set of available data. Figure 1 shows the distribution of demand \( (D) \) in each period, which has the triangular distribution pattern.

In this study, in order to convert the fuzzy demand into a crisp number, the weighted average method has been used.

After defuzzification, the corresponding expression for the fuzzy equation (4) is as follows:

\[
I_{n-1} - B_{n-1} + Q_n - I_n + B_n = w_1 \cdot D_{n,p}^p + w_2 \cdot D_{n,m}^m + w_3 \cdot D_{n,o}^o \quad \forall n. \tag{12}
\]

In (12), \( w_1, w_2, \) and \( w_3 \) are the corresponding weights for the most pessimistic, the most likely, and the most optimistic values for demand, while the summation of these three weights is equal to one \( (w_1 + w_2 + w_3 = 1) \). In addition, based on the concept of the most likely values, which has been proposed by Lai and Hwang, this study has set \( w_1, w_2, \) and \( w_3 \) as \( (w_2 = 4/6) \) and \( (w_1 = w_3 = 1/6) \) [26]. Similarly, two other fuzzy constraints of (7) and (8) can be converted into crisp expressions by using the weighted average method as follows:

\[
U \cdot Q_n \leq w_1 \cdot M_{n,p}^p + w_2 \cdot M_{n,m}^m + w_3 \cdot M_{n,o}^o \quad \forall n, \tag{13}
\]

\[
V \cdot Q_n \leq w_1 \cdot M_{n,p}^p + w_2 \cdot M_{n,m}^m + w_3 \cdot M_{n,o}^o \quad \forall n.
\]

4.2. Treating the Objective Function. When the objective function of a mathematical model is fuzzy, it should be transformed to an auxiliary crisp one in order to be proceeded for solving the model. The most important part of this treatment is to find an appropriate membership function for this conversion. In this study, it has been assumed that according to the satisfaction degree of the decision maker, linear membership function is the most suitable function for this transformation. The process of defuzzification in this study is based on the concept of fuzzy decision making of Bellman and Zadeh together with the fuzzy programming method of Zimmermann [23, 24]. In this regard, at first the negative ideal solution (NIS) and positive ideal solution (PIS) for the fuzzy objective function will be introduced as follows:

\[
Z_{\text{PIS}} = \text{Min} z \quad Z_{\text{NIS}} = \text{Max} z. \tag{14}
\]

Then, the membership function, which according to the satisfaction degree of the decision maker is supposed to be nonincreasing continuous linear, will be defined for the fuzzy
objective function. The formulation that has been used in this study for treating the objective function is as follows:

\[
f(z) = \begin{cases} 
1 & \text{if } Z_{\text{NIS}} - Z 
\leq Z_{\text{PIS}} 
\leq Z 
< Z_{\text{NIS}} \leq Z 
\end{cases}
\]  
(15)

Figure 2 depicts a schematic diagram of a nonincreasing linear membership function.

4.3. Developing an Auxiliary Mathematical Model. In solving fuzzy programming models, after the defuzzification of constraints and objective functions, in order to solve the fuzzy mathematical model by ordinary methods, it should be converted into an auxiliary crisp model. In this study, the minimum operator has been used. The auxiliary mathematical model of this study can be constructed as follows:

\[
\text{Max } L \\
\text{s.t. } L \leq f(z), \\
\text{Equations } (3), (5) - (6), (9) - (10), (12) - (13), \\
Q_n, O_n, I_n, B_n, T_{jkn}, L \geq 0 \forall n, Y_k = \{0, 1\}.
\]  
(16)

5. Model Implementation

In this section, the developed fuzzy mathematical model will be applied for a mineral water bottling factory that is located in Bardsir, Kerman, Iran. The main product of the factory is 1.5-liter bottles of mineral water, and each dozen of bottles is packed in one box. (i) The main product of the factory is 1.5-liter bottles of mineral water, and each dozen of bottles is packed in one box. (ii) The planning horizon of the factory is three months, June, July, and August.
Table 6: Results for the mineral water bottling production company (box).

<table>
<thead>
<tr>
<th>Period</th>
<th>Q</th>
<th>I</th>
<th>B</th>
<th>Rafsanjan</th>
<th>Sirjan</th>
<th>Kerman</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>k=1</td>
<td>k=2</td>
<td>k=1</td>
</tr>
<tr>
<td>June</td>
<td>12800</td>
<td>800</td>
<td>0</td>
<td>2000</td>
<td>—</td>
<td>2500</td>
</tr>
<tr>
<td>July</td>
<td>10900</td>
<td>0</td>
<td>300</td>
<td>1500</td>
<td>—</td>
<td>2000</td>
</tr>
<tr>
<td>August</td>
<td>13000</td>
<td>0</td>
<td>0</td>
<td>1000</td>
<td>—</td>
<td>2000</td>
</tr>
</tbody>
</table>

Objective function: \( L = 100\% \) \( Z = 70170.19 \)

Table 7: Sensitivity analysis for the \( \alpha \)-cut level.

<table>
<thead>
<tr>
<th>( \alpha ) level</th>
<th>Period</th>
<th>Q</th>
<th>I</th>
<th>B</th>
<th>Rafsanjan</th>
<th>Sirjan</th>
<th>Kerman</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>k=1</td>
<td>k=2</td>
<td>k=1</td>
</tr>
<tr>
<td>( \alpha = 0.3 )</td>
<td>June</td>
<td>12786</td>
<td>766</td>
<td>0</td>
<td>2000</td>
<td>—</td>
<td>2500</td>
</tr>
<tr>
<td></td>
<td>July</td>
<td>10988</td>
<td>0</td>
<td>246</td>
<td>1500</td>
<td>—</td>
<td>2000</td>
</tr>
<tr>
<td></td>
<td>August</td>
<td>12986</td>
<td>0</td>
<td>0</td>
<td>1000</td>
<td>—</td>
<td>2000</td>
</tr>
</tbody>
</table>

Objective function: \( L = 100\% \) \( Z = 70283.98 \)

| \( \alpha = 0.7 \)  | June   | 12826 | 866 | 0  | 2000 | — | 2500 | — | 7960 | — |
|        | July   | 10728 | 0  | 406 | 1500 | — | 2000 | — | 8094 | — |
|        | August | 13026 | 0  | 0  | 1000 | — | 2000 | — | 10026| — |

Objective function: \( L = 100\% \) \( Z = 69943.24 \)

(iii) The boxes of 1.5-liter bottles of mineral water are distributed to three distribution centers.

(iv) There are two candidates as the distributors for the products of the factory.

(v) The initial inventory for the main product of the factory is 500 boxes while the inventory for the end period (August) is zero.

(vi) The initial backorder and also the end backordering volume in the third period (August) are equal to zero.

(vii) According to the previous time studies, production of each box of the main product utilizes 0.015 machine-hours of the capacity of the factory.

(viii) The required manpower to produce one box of the product equals 0.09 man-hours.

(ix) The dimension of each box of the main product is \( 31 \times 27 \times 37 \text{ cm}^3 \). Hence, it has been considered that every 32 boxes of the product can be stored in 1 m\(^3\) of the space of the warehouse. The dimension of the warehouse equals \( 15 \times 20 \text{ m}^2 \) and the boxes are stored to 1.5 m height. Hence, the volume of the warehouse equals 450 m\(^3\).

(x) Minimum attractive rate of return as the escalating factor for the costs is equal to 8%.

(xi) The value for \( \alpha \) level, which is the minimal possibility level for accepting the membership, for all the fuzzy and imprecise numbers is considered to be 0.5.

5.2. Computational Results and Discussions. In this section, on the basis of the solution procedure that has been developed in this study, at first, the fuzzy constraints are treated using ((12)-(13)). Then, it is the turn to treat the fuzzy objective function, by introducing the PIS, NIS, and related linear membership function using (15). Finally, the auxiliary mathematical model is developed based on (16). This model can be solved by using conventional methods. In this study, Lingo Optimization Software has been applied to solve the model. The amounts of \( Z_{\text{PIS}} \) and \( Z_{\text{NIS}} \), after solving the ordinary mathematical programming model at \( \alpha = 0.5 \), are equal to \( Z_{\text{PIS}} = 70170.19 \) and \( Z_{\text{NIS}} = 73842.91 \). The results of the mathematical model for the case have been summarized in Table 6.

After solving the model, in order to provide more information on the results, the effects of variation of \( \alpha \)-cut on the results of the problem have been investigated. For this purpose, sensitivity analysis of the parameter of \( \alpha \)-cut has been conducted by solving the mathematical model for \( \alpha = 0.3, 0.7 \). The results are presented in Table 7.

According to the results and sensitivity analysis, several managerial implications will be risen as follows.

(i) The mathematical model not only provides an aggregate production planning for the production system, but also selects the best contractors for transporting the products to the distribution centers or retailers.

(ii) By application of fuzzy set theory, the model is capable of handling the imprecise nature of data, and providing a better imitation of the real-world uncertain environments.
(iii) The feasibility of the results implies the validity of the proposed mathematical model and also the solution approach.

(iv) As from the results it can be observed that the value of $L$ (the level of the satisfaction of the decision maker) is equal to 100%. It is an indicator for the full satisfaction of the decision maker with the results.

(v) As the results of sensitivity analysis indicate, the degree of satisfaction of the decision maker, which is shown by the value of $L$, does not have any changes against the variation of the $\alpha$-cut level, whereas the optimal solution of the problem changes.

6. Conclusion

In today's competitive and uncertain business environment, designing and operating efficient supply chains are crucial for every industry to meet customer demands and maximize the profit. Separate optimization of supply chain keeps the firms away from gaining maximum possible effectiveness. Hence, a mathematical model for a production-distribution problem was developed in this study that not only allocates the limited resources to the production of the products, but also determines the best contractors for dispatching the products to the distribution centers or retailers. Additionally, in order to make the model more realistic, because some data of real-world production systems are imprecise or unobtainable, the fuzzy set theory was applied to the mathematical model. For illustration and verification of the proposed fuzzy mathematical model, data from a mineral water bottling factory was applied to the model and solved by a three-step solution approach that also was developed in this study. Solving the real case by the solution method and finding feasible solutions for the model show the feasibility and validity of the proposed fuzzy mathematical model and also the developed solution procedure. The results show the full satisfaction of the decision maker with the results. Moreover, it is shown that, even by varying the level of $\alpha$-cut, there is not any change in the level of satisfaction of the decision maker.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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