Research Article

Adaptive Filtering Backstepping for Ships Steering Control without Velocity Measurements and with Input Constraints

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We consider the problem of course tracking for ships with uncertainties and unknown external disturbances, in the presence of input magnitude and rate saturation. The combination of approximation-based adaptive technique and radial basis function (RBF) neural network allows us to handle the unknown disturbances from the environment and uncertain ship dynamics. By employing the adaptive filtering backstepping, the full-state feedback controller is first derived. Then the output feedback controller is designed with the unmeasurable state estimated by using a high-gain observer. In order to cope with the input constraints, an auxiliary system is introduced to the output feedback controller, and the semiglobal uniform boundedness of the modified control solution is verified. Simulation results are presented for the course tracking of a cargo ship, which are demonstrative of the excellent performance of the proposed controller.

1. Introduction

Due to the presence of highly nonlinear dynamics of ships and time-varying environment disturbances, it is a great challenge to control ships to track the desired course. In recent years, sophisticated ship autopilots have been proposed based on advanced control engineering concepts, such as backstepping [1], model reference adaptive control [2], feedback linearization [3, 4], and so on. In [1], two configurations of nonlinear controllers using backstepping were designed for the ship course control system. An autopilot was designed based on the internal model control approach in [2]. In [3], the technique of H_∞ I/O linearization was proposed for the nonlinear ship course-keeping control problem. Reference [4] proposed Lyapunov and Hurwitz based control approaches for an input-output linearization applied nonlinear vessel steering system. However, these model-based methods may not be applicable since they are generally useful only when dealing with systems with explicit knowledge dynamics, which are often difficult to obtain.

To overcome the limitations of model-based controllers, we employ approximation-based control techniques to compensate the unknown disturbances from the external environment and uncertain ship dynamics, so that the proposed control algorithm can be easily applied to different ship autopilots. Some good results for ship course control in the literature have been presented by using neural networks or fuzzy logic systems to approximate the unknown terms. Adaptive fuzzy H_∞ control was proposed for the ship steering problem in [5], where the unknown functions were approximated by using fuzzy logic systems. In [6], neural networks trained under back propagation were used for intelligent controlling of ships with the optimal guidance task. A direct adaptive RBF neural network control algorithm was presented for a class of ship course with uncertain discrete-time nonlinear systems, and the RBF network was used to emulate the desired feedback control and approximate the unknown function in [7].

Since it is convenient to construct the Lyapunov function, backstepping methods have attracted many attentions from

researchers [1, 8, 9]. However, there are two main disadvantages for the conventional backstepping approach: (a) the knowledge of the accurate dynamic model is needed; (b) the complex computing is caused due to the high-order derivatives of the virtual control. Using the approximation-based techniques, the first shortcoming can be addressed. Some adaptive backstepping methods for the ship course control problem have been proposed [8, 9]. To deal with the second one, the dynamic surface control (DSC) was introduced in [10]. A simpler algorithm using DSC for ship control system was proposed in [11]. In [12], a novel command filtered backstepping method was provided, which included a second-order filter and avoided computing the derivatives of the virtual control. By the further study, Dong et al. in [13] researched the adaptive command filtered backstepping, in which the unknown terms in the model were considered.

From a practical perspective, the velocities of ships are usually difficult to be measured directly and likely to be corrupted by noises if the location differentiation is used. Hence, the output feedback controller should be developed to replace the full-state feedback controller. Thus, the observer needs to be designed during the control design. A passive nonlinear observer was designed for dynamic positioning ships [14]. Furthermore, an augmented passive observer, with an adaptive wave filter, was developed in [15]. However, these observer designs are all based on the ship dynamics. In this paper, an output feedback controller is derived with the unmeasurable state estimated by using a high-gain observer, which is simple to design and does not require the ship models. On the other hand, the actuator saturation commonly exists in many practical control systems. There have been few practical studies on the ship control design with input constraints considered. In [16], an adaptive steering control design for uncertain ship dynamics subject to input constraints was presented based on the antiwindup technique. To cope with input constraints for ships, the authors of [17] proposed an adaptive neural control of ship course autopilot with input saturation. Recently, we have considered the input constraints during the control design for surface ships [18], with an auxiliary design system introduced. However, in these papers, only the magnitude saturation is contained, whereas in the autopilot system, the rate saturation of rudder is similarly important as the magnitude saturation. In addition, researchers in [2] considered both the magnitude saturation and rate saturation in the control design, but using a model-based control approach.

The main contribution of this paper is an adaptive course controller designed by using the filtering backstepping and RBF neural network for ships subject to unknown external disturbances, model uncertainties, and input constraints, with only the heading angle measurable. The combination of approximation-based adaptive technique and RBF neural network is used to estimate the unknown disturbances from the environment and uncertain ship dynamics. Based on the full-state feedback controller, which is first designed by using adaptive filtering backstepping, an output feedback controller is derived via certainty equivalence principle, with the unmeasurable state estimated by a high-gain observer. To cope with the magnitude saturation and rate saturation

FIGURE 1: The reference coordinate frames of the ship motion.

of the rudder, a constrained controller is obtained by introducing an auxiliary system to the unconstrained controller. For both the unconstrained and constrained controllers, the semiglobal uniform boundedness of all closed-loop error signals is guaranteed through Lyapunov analysis.

The remainder of this paper is organized as follows. Section 2 formulates the course-tracking task for ships with input constraints. Section 3 presents the design of the adaptive filtering backstepping controller by using the unconstrained full-state feedback control, unconstrained output feedback control with a high-gain observer, and constrained output feedback control. In Section 4, simulation studies are shown to demonstrate the effectiveness of the proposed control scheme. Finally, conclusions are made in the last section.

2. Problem Formulation

2.1. Ship Autopilot Model with Input Constraints. Consider the commonly used nonlinear model of the ship autopilot as [1]

$$T\ddot{\psi} + H(\dot{\psi}) + \Delta H(\psi, \dot{\psi}) = K\delta + w(t), \quad (1)$$

where ψ and δ denote the heading and rudder angle of the ship, respectively (see Figure 1), $H(\dot{\psi})$ is the nonlinear maneuvering characteristic which is unknown and continuous here, $\Delta H(\psi, \dot{\psi})$ denotes the model uncertainties, and $w(t)$ is the external disturbance which is unknown and bounded. K is the gain constant, and T is the time constant.

In practical applications, the dynamic model of the ship is complemented by the model of the steering gear, and the control input signal passes the steering gear firstly. Hence, it is necessary to take account of the rudder dynamic while designing the controller. Generally, the steering gear dynamic characteristic can be described by the following equation [1]:

$$T_R \dot{\delta} + \delta = K_R \delta_R, \quad (2)$$

where δ_R is the input of the steering machine, K_R is the gain constant, and T_R is the time constant.

Moreover, the steering machine is highly nonlinear. During the ship steering control, the dominating nonlinearities

are the magnitude and rate saturation of the rudder angle. Considering the presence of input constraints, the following nonlinear model of the control input saturation is presented [19]:

$$\dot{\delta}_R = \text{sat}_2(\text{sat}_1(\delta_c) - \delta_R), \quad (3)$$

where the saturation functions $\text{sat}_1(\cdot)$ and $\text{sat}_2(\cdot)$ denote the magnitude saturation and rate saturation, respectively, which are defined as

$$\text{sat}_i(\cdot) = \text{sign}(\cdot) \times \min(|\cdot|, \beta_i), \quad \beta_i > 0 \quad (4)$$

with $\beta_1 = \delta_{\max}$ and $\beta_2 = \dot{\delta}_{\max}$.

The model of the steering machine with saturation nonlinearities is schematically shown in Figure 2, where the signal δ_c denotes the output of the controller.

Define the state variables as $x_1 = \psi$, $x_2 = \dot{\psi}$, $x_3 = K\delta/T$ and the control variables as $v = \delta_R$, $u = \delta_c$; then considering the input constraints, the state-space model of the autopilot with the rudder dynamic characteristic is derived as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3 + f(x_1, x_2, t), \\ \dot{x}_3 &= a_1 x_3 + a_2 v, \\ \dot{v} &= \text{sat}_2(\text{sat}_1(u) - v) \end{aligned} \quad (5)$$

with $a_1 = -1/T_R$ and $a_2 = KK_R/TT_R$, where $f(x_1, x_2, t) = [\omega(t) - H(x_2) - \Delta H(x_1, x_2)]/T$ denote the unknown disturbances from the environment and uncertain dynamics.

Lemma 1 (see [20]). *For bounded initial conditions, if there exists a C^1 continuous and positive definite Lyapunov function $V(x)$ satisfying $\rho_1(\|x\|) \leq V(x) \leq \rho_2(\|x\|)$, such that $\dot{V}(x) \leq -\mu V(x) + v$, where $\rho_1, \rho_2 : R^n \rightarrow R$ are class K functions and v is a positive constant, then the solution $x(t)$ is uniformly bounded.*

In practice, it is not easy to measure the angle velocity of ships directly, and the angle velocity is likely to be influenced by noises if the differentiation of the heading angle is used. Hence, the control objective of this paper is to design the control law u without angle velocity measurement and with input constraints, while the output follows a desired heading $\psi_d(t)$, such that the resulting closed-loop system is stable in the sense of semiglobal uniform boundedness.

Assumption 2. For all $t \geq 0$, $\psi_d(t)$ and $\dot{\psi}_d(t)$ are continuous and bounded.

Remark 3. Assumption 2 requires the desired signal to be sufficiently smooth to avoid the actuator saturation induced by sudden jumps of the tracking error due to discontinuous command inputs.

2.2. RBF Neural Network Approximator. In order to deal with the uncertain parts, the neural network is usually used to estimate the uncertain model. RBF neural network, which has

FIGURE 2: The model of the steering machine with input saturations.

FIGURE 3: The general structure of RBF neural network.

been proved to approximate any continuous function with arbitrary precision, is commonly adopted in the modeling and controlling of nonlinear systems. Therefore, in this paper, the unknown function $f(x_1, x_2, t)$ will be estimated online by using the RBF neural network.

Figure 3 shows the general structure of RBF network with m inputs, n nodes, and a single output. It is a single hidden layer feed-forward network, and the mapping from input to output layer is nonlinear while the mapping from hidden layer to output layer is linear. This property can speed up the learning process and avoid the local minimization problem.

Letting $X = [x_1, x_2, \dots, x_m]^T \in R^m$ be the input vector and let $H = [h_1, h_2, \dots, h_n]^T \in R^n$ be the radial basis vector of the network, where h_j is Gaussian function,

$$h_j(X) = \exp\left(-\frac{\|X - c_j\|^2}{2b_j^2}\right), \quad j = 1, 2, \dots, n, \quad (6)$$

where $c_j = [c_{j1}, c_{j2}, \dots, c_{jm}]^T$ is the central vector of the j th node, and $b_j > 0$ is the basis width parameter of the j th node.

Then, using the RBF network, the continuous function $f(X) : R^m \rightarrow R$ can be represented as follows:

$$f(X) = \theta^T H(X) + \varepsilon(X), \quad (7)$$

where $\theta \in R^n$ is the weight vector and $\varepsilon(X)$ is the approximation error which is bounded over the compact set, that is, $\varepsilon(X) \leq \bar{\varepsilon}$, $\forall X \in \Omega_X$, where $\bar{\varepsilon}$ is an unknown positive constant.

According to the universal approximation property [7], the continuous function $f(X)$ can be smoothly approximated over a compact set $\Omega_X \subset R^m$ to arbitrary any degree of accuracy as

$$f(X) = \theta^{*T} H(X) + \varepsilon^*(X), \quad (8)$$

where θ^* denotes the ideal constant weight vector and $\varepsilon^*(X)$ is the approximation error as $\theta = \theta^*$. The ideal weight vector

θ^* is defined as the value of θ that minimizes $|\varepsilon(X)|$ for all $X \in \Omega_X \subset \mathbb{R}^m$; that is,

$$\theta^* := \arg \min_{\theta \in \mathbb{R}^n} \left\{ \sup_{X \in \Omega_X} |f(X) - \theta^T H(X)| \right\}. \quad (9)$$

3. Adaptive Control Design

In this paper, we employ approximation-based adaptive filtering backstepping of the ship dynamics (5). Unlike the conventional backstepping, the filtering backstepping avoids the tedious algebra related to computing the command signal derivatives [12]. Since the function $f(x_1, x_2, t)$ is not known exactly in practice, the RBF network is used to approximate the unknown function $f(x_1, x_2, t)$ online during the controller design. Unconstrained full-state feedback controller will be derived first. Based on this, an unconstrained output feedback controller will be subsequently designed via certainty equivalence principle [21], with the unmeasurable state estimated by a high-gain observer. At last, the output feedback controller with input constraints will be obtained with some modifications.

3.1. Unconstrained Adaptive Full-State Feedback Control. In the design of the unconstrained controller, the input constraints of the ship are not considered. Thus, the signals δ_c and δ_R in Figure 2 are equal; that is, $\delta_c = \delta_R$; then in this subsection, v is the control law to be designed, and the controlled plant consists of the first three equations of (5).

Step 1. Define the tracking error variables as

$$z_1 = x_1 - \psi_d, \quad z_2 = x_2 - \alpha_1, \quad z_3 = x_3 - \alpha_2, \quad (10)$$

where α_1 and α_2 are the virtual controls.

According to the standard backstepping method, the virtual control α_1 can be easily derived as

$$\alpha_1 = -k_1 z_1 + \dot{\psi}_d. \quad (11)$$

In order to avoid computing derivatives of the virtual control in the next step, a filter (designed later) is introduced. The input of the filter, called pseudocontrol, needs to be designed, and the outputs of the filter are α_1 and $\dot{\alpha}_1$, which will be used in the following steps.

The pseudocontrol signal $\bar{\alpha}_1$ is designed as

$$\bar{\alpha}_1 = -k_1 z_1 + \dot{\psi}_d. \quad (12)$$

Step 2. Using the standard backstepping method, we can obtain the virtual control as

$$\alpha_2 = -k_2 z_2 - \hat{f}(x_1, x_2, t) + \dot{\alpha}_1 - z_1. \quad (13)$$

For the filtering backstepping, define pseudocontrol signal $\bar{\alpha}_2$ as

$$\bar{\alpha}_2 = -k_2 z_2 - \hat{f}(x_1, x_2, t) + \dot{\alpha}_1 - \xi_1, \quad (14)$$

where ξ_1 is the compensated tracking error and $\hat{f}(x_1, x_2, t)$ denotes the estimate of the unknown function by utilizing the RBF network; then, from (7) and (8), we have

$$f(x_1, x_2, t) = \theta^{*T} H(X) + \varepsilon(X), \quad (15)$$

$$\hat{f}(x_1, x_2, t) = \hat{\theta}^T H(X), \quad (16)$$

where $X = [x_1, x_2]^T$ are the input variables to the RBF network, and $\varepsilon(X)$ is the approximation error.

Substituting (16) into (14), (14) can be rewritten as

$$\bar{\alpha}_2 = -k_2 z_2 - \hat{\theta}^T H(X) + \dot{\alpha}_1 - \xi_1. \quad (17)$$

Define the compensated tracking error ξ_1 as

$$\xi_1 = z_1 - \sigma_1, \quad (18)$$

where the dynamic of the signal σ_1 is defined as

$$\dot{\sigma}_1 = -k_1 \sigma_1 + (\alpha_1 - \bar{\alpha}_1) + \sigma_2 \quad (19)$$

with $\sigma_1(0) = 0$ and σ_2 defined as

$$\dot{\sigma}_2 = -k_2 \sigma_2 + (\alpha_2 - \bar{\alpha}_2), \quad \sigma_2(0) = 0. \quad (20)$$

Step 3. Similar to Step 2, define the compensated tracking error signals ξ_2 and ξ_3 as

$$\xi_2 = z_2 - \sigma_2, \quad \xi_3 = z_3. \quad (21)$$

Then, the control law and the adaptive law are designed as

$$v = \frac{1}{a_2} (-k_3 z_3 - a_1 x_3 + \dot{\alpha}_2 - \xi_2), \quad (22)$$

$$\dot{\hat{\theta}} = \Gamma (-\gamma \hat{\theta} + H(X) \xi_2), \quad (23)$$

where $\gamma > 0$ and Γ is a positive definite matrix.

As mentioned above, the command filter is introduced to compute α_i and $\dot{\alpha}_i$ ($i = 1, 2$) without differentiations. Define the state-space model of the command filter as [12]

$$\begin{bmatrix} \dot{p}_{i,1} \\ \dot{p}_{i,2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} p_{i,1} \\ p_{i,2} \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix} \bar{\alpha}_i, \quad (24)$$

where $\bar{\alpha}_i$ are the input variables, $\alpha_i = p_{i,1}$ and $\dot{\alpha}_i = p_{i,2}$ are the outputs of the filter, and the filter initial conditions are $p_{i,1}(0) = \bar{\alpha}_i(0)$ and $p_{i,2}(0) = 0$. The filter design parameters are $\omega_n > 0$ and $\zeta \in (0, 1]$, and the designer would select an appropriate ω_n so that α_i and $\dot{\alpha}_i$ will accurately track $\bar{\alpha}_i$ and $\dot{\bar{\alpha}}_i$, respectively [10].

3.1.1. Stability Analysis. Firstly, we analyze the properties of the compensated tracking errors ξ_i ($i = 1, 2, 3$) and the neural parameter estimation error $\bar{\theta} = \theta^* - \hat{\theta}$ by considering the following Lyapunov function candidate:

$$V = \frac{1}{2} (\xi_1^2 + \xi_2^2 + \xi_3^2) + \frac{1}{2} \bar{\theta}^T \Gamma^{-1} \bar{\theta}. \quad (25)$$

Differentiating the tracking errors yields

$$\begin{aligned}\dot{z}_1 &= \dot{x}_1 - \dot{\psi}_d = -k_1 z_1 + z_2 + (\alpha_1 - \bar{\alpha}_1), \\ \dot{z}_2 &= \dot{x}_2 - \dot{\alpha}_1 = z_3 + \alpha_2 + f(x_1, x_2, t) - \dot{\alpha}_1 \\ &= -k_2 z_2 + z_3 - \xi_1 + (\alpha_2 - \bar{\alpha}_2) + \bar{\theta}^T H(X) + \varepsilon(X), \\ \dot{z}_3 &= \dot{x}_3 - \dot{\alpha}_2 = a_1 x_3 + a_2 v - \dot{\alpha}_2 = -k_3 z_3 - \xi_2.\end{aligned}\quad (26)$$

The dynamics of the compensated tracking errors are derived as follows:

$$\begin{aligned}\dot{\xi}_1 &= \dot{z}_1 - \dot{\sigma}_1 = -k_1 \xi_1 + \xi_2, \\ \dot{\xi}_2 &= \dot{z}_2 - \dot{\sigma}_2 = -k_2 \xi_2 + \xi_3 - \xi_1 + \bar{\theta}^T H(X) + \varepsilon(X), \\ \dot{\xi}_3 &= \dot{z}_3 = -k_3 \xi_3 - \xi_2.\end{aligned}\quad (27)$$

Then, the derivative of V is obtained as

$$\begin{aligned}\dot{V} &= \xi_1 \dot{\xi}_1 + \xi_2 \dot{\xi}_2 + \xi_3 \dot{\xi}_3 + \bar{\theta}^T \Gamma^{-1} \dot{\bar{\theta}} \\ &= \xi_1 (-k_1 \xi_1 + \xi_2) \\ &\quad + \xi_2 (-k_2 \xi_2 + \xi_3 - \xi_1 + \bar{\theta}^T H(X) + \varepsilon(X)) \\ &\quad + \xi_3 (-k_3 \xi_3 - \xi_2) - \bar{\theta}^T (-\gamma \hat{\theta} + H(X) \xi_2) \\ &= -k_1 \xi_1^2 - k_2 \xi_2^2 - k_3 \xi_3^2 + \xi_2 \varepsilon(X) + \bar{\theta}^T \gamma \hat{\theta}.\end{aligned}\quad (28)$$

Noting the following facts:

$$\begin{aligned}\xi_2 \varepsilon(X) &\leq \xi_2^2 + \frac{\|\bar{\varepsilon}\|^2}{4}, \\ \bar{\theta}^T \gamma \hat{\theta} &= \bar{\theta}^T \gamma \theta^* - \bar{\theta}^T \gamma \tilde{\theta} \leq -\left(\frac{\gamma}{2}\right) \bar{\theta}^T \gamma \tilde{\theta} + \frac{\gamma \|\theta^*\|^2}{2},\end{aligned}\quad (29)$$

it can be shown that

$$\begin{aligned}\dot{V} &\leq -k_1 \xi_1^2 - (k_2 - 1) \xi_2^2 - k_3 \xi_3^2 - \frac{\gamma \lambda_{\min}(\Gamma) \bar{\theta}^T \Gamma^{-1} \tilde{\theta}}{2} \\ &\quad + \bar{\theta}^T \gamma \hat{\theta} + \frac{\|\bar{\varepsilon}\|^2}{4} + \frac{\gamma \|\theta^*\|^2}{2} \\ &\leq -\mu V + v,\end{aligned}\quad (30)$$

where μ and v are positive constants defined by

$$\begin{aligned}\mu &= 2 \min \left\{ k_1, k_2 - 1, k_3, \frac{\gamma \lambda_{\min}(\Gamma)}{2} \right\}, \\ v &= \frac{\|\bar{\varepsilon}\|^2}{4} + \frac{\gamma \|\theta^*\|^2}{2}.\end{aligned}\quad (31)$$

From (30) and Lemma 1, it is clear that the compensated tracking errors ξ_i and the neural parameter estimation error $\tilde{\theta}$ are semiglobally uniformly ultimately bounded.

Secondly, the properties of the tracking errors z_i ($i = 1, 2, 3$) are addressed. From Theorem 2 in [12] we have that

$\alpha_i - \bar{\alpha}_i = O(1/\omega_n)$ and $\dot{\alpha}_i - \dot{\bar{\alpha}}_i = O(1/\omega_n)$. That is, by increasing the command filter parameter ω_n , α_i and $\dot{\alpha}_i$ can be made arbitrarily close to $\bar{\alpha}_i$ and $\dot{\bar{\alpha}}_i$, respectively. Then, from the differential equations (19) and (20), we can see that the signals σ_i will converge to an arbitrarily small neighborhood of zero. Finally, according to (18) and (21) and the convergence of ξ_i , the tracking error signals z_i are bounded.

3.2. Unconstrained Adaptive Output Feedback Control. As mentioned in the previous section, the angle velocity of the ship is unmeasurable in this paper. Therefore, the full-state feedback is not available in practice. In this section, we tackle the output feedback problem for ships by utilizing high-gain observers.

Lemma 4 (see [22]). *Consider the following linear system:*

$$\begin{aligned}\beta \dot{\pi}_i &= \pi_{i+1}, \quad i = 1, 2, \dots, n-1, \\ \beta \dot{\pi}_n &= -\lambda_1 \pi_n - \lambda_2 \pi_{n-1} - \dots - \lambda_{n-1} \pi_2 - \pi_1 + y(t),\end{aligned}\quad (32)$$

where β is a small positive constant, and $\lambda_1, \dots, \lambda_n$ are chosen such that the polynomial $s^n + \lambda_1 s^{n-1} + \dots + \lambda_{n-1} s + 1$ is Hurwitz. Suppose that the system output $y(t)$ and its first n derivatives are bounded, so that $|y(t)| < Y_k$; then the following properties hold.

(1) Consider the following:

$$\frac{\pi_k}{\beta^{k-1}} - y^{(k-1)} = -\beta \varsigma^{(k)}, \quad k = 1, 2, \dots, n, \quad (33)$$

where $\varsigma := \pi_n + \lambda_1 \pi_{n-1} + \dots + \lambda_{n-1} \pi_1$ and $\varsigma^{(k)}$ denotes the k th derivative of ς .

(2) There exist positive constants t^* and g_k (independent of β) such that $|\varsigma^{(k)}| \leq g_k$ for all $t > t^*$.

According to Lemma 4, we can see that π_{k+1}/β^k asymptotically converges to $y^{(k)}$ with a small time constant. Therefore, we can use π_{k+1}/β^k to estimate the output derivatives up to the n th order.

Generally, the heading of ships can be measured by using the compass, and the measured values are commonly in the range of $[0^\circ, 360^\circ]$. In the ship control system, the measured heading angles are translated to the range of $[-180^\circ, +180^\circ]$; that is, $x_1 \in [-180^\circ, +180^\circ]$. Then, using Lemma 4, let $n = 2$ and $y(t) = x_1$; the estimate of the unmeasurable state x_2 can be expressed as

$$\hat{x}_2 = \frac{\pi_2}{\beta}, \quad (34)$$

where π_2 can be computed via the following model:

$$\begin{aligned}\beta \dot{\pi}_1 &= \pi_2, \\ \beta \dot{\pi}_2 &= -\lambda_1 \pi_2 - \pi_1 + x_1.\end{aligned}\quad (35)$$

Now, we revisit the control law (22) and adaptive law (23) for the full-state feedback case. Based on the certainty

equivalence principle, we modify them by replacing the unmeasurable variables with their estimates, such that

$$v = \frac{1}{a_2} \left(-k_3 z_3 - a_1 x_3 + \dot{\alpha}_2 - \hat{\xi}_2 \right), \quad (36)$$

$$\dot{\hat{\theta}} = \Gamma \left(-\gamma \hat{\theta} + H(\hat{X}) \hat{\xi}_2 \right), \quad (37)$$

where $\hat{\xi}_2 = \hat{z}_2 - \sigma_2 = (\hat{x}_2 - \alpha_1) - \sigma_2$ and $\hat{X} = [x_1, \hat{x}_2]^T$.

Moreover, (17) is changed as

$$\bar{\alpha}_2 = -k_2 \hat{z}_2 - \bar{\theta}^T H(\hat{X}) + \dot{\alpha}_1 - \xi_1. \quad (38)$$

Denote $\tilde{x}_2 = x_2 - \hat{x}_2$. Invoking (10), (18), (20), and (36)–(38) and using the property $H(X) - H(\hat{X}) = eH_t$, where H_t is a bounded vector function and e is a small positive constant [23], the derivatives of z_i and ξ_i are derived as

$$\dot{z}_1 = -k_1 z_1 + (\alpha_1 - \bar{\alpha}_1) + \hat{z}_2 + \tilde{x}_2, \quad (39)$$

$$\begin{aligned} \dot{z}_2 = & -k_2 \hat{z}_2 + z_3 - \xi_1 + (\alpha_2 - \bar{\alpha}_2) + \bar{\theta}^T H(\hat{X}) \\ & + \theta^{*T} eH_t + \varepsilon(X), \end{aligned} \quad (40)$$

$$\dot{z}_3 = -k_3 z_3 - \hat{\xi}_2, \quad (41)$$

$$\dot{\xi}_1 = \dot{z}_1 - \dot{\sigma}_1 = -k_1 \xi_1 + \hat{\xi}_2 + \tilde{x}_2, \quad (42)$$

$$\begin{aligned} \dot{\xi}_2 = \dot{z}_2 - \dot{\sigma}_2 = & -k_2 \hat{\xi}_2 + \xi_3 - \xi_1 + \bar{\theta}^T H(\hat{X}) \\ & + \theta^{*T} eH_t + \varepsilon(X), \end{aligned} \quad (43)$$

$$\dot{\xi}_3 = \dot{z}_3 = -k_3 \xi_3 - \hat{\xi}_2. \quad (44)$$

Then taking the time derivative of V yields

$$\begin{aligned} \dot{V} = & \xi_1 \dot{\xi}_1 + \xi_2 \dot{\xi}_2 + \xi_3 \dot{\xi}_3 + \bar{\theta}^T \Gamma^{-1} \dot{\bar{\theta}} \\ = & \xi_1 \left(-k_1 \xi_1 + \hat{\xi}_2 + \tilde{x}_2 \right) + \xi_3 \left(-k_3 \xi_3 - \hat{\xi}_2 \right) \\ & + \xi_2 \left(-k_2 \hat{\xi}_2 + \xi_3 - \xi_1 + \bar{\theta}^T H(\hat{X}) + \theta^{*T} eH_t + \varepsilon(X) \right) \\ & - \bar{\theta}^T \left(-\gamma \hat{\theta} + H(\hat{X}) \hat{\xi}_2 \right) \\ = & -k_1 \xi_1^2 - k_2 \xi_2^2 - k_3 \xi_3^2 + k_2 \xi_2 \tilde{x}_2 + \xi_3 \tilde{x}_2 \\ & + \bar{\theta}^T H(\hat{X}) \tilde{x}_2 + \xi_2 \theta^{*T} eH_t + \xi_2 \varepsilon + \gamma \bar{\theta}^T \hat{\theta}. \end{aligned} \quad (45)$$

Due to the boundedness of the basis function, we assume that $\|H(\hat{X})\| \leq D$, where D is a positive constant. Then, using the techniques of inequalities, we have

$$k_2 \xi_2 \tilde{x}_2 \leq 0.5 \xi_2^2 + 0.5 k_2^2 \tilde{x}_2^2,$$

$$\xi_3 \tilde{x}_2 \leq 0.05 \xi_3^2 + 5 \tilde{x}_2^2,$$

$$\bar{\theta}^T H(\hat{X}) \tilde{x}_2 \leq \left(\frac{\gamma}{4} \right) \bar{\theta}^T \bar{\theta} + \left(\frac{D^2}{\gamma} \right) \tilde{x}_2^2,$$

$$\xi_2 \theta^{*T} eH_t \leq 0.5 \xi_2^2 + 0.5 e^2 \|H_t\|^2 \|\theta^*\|^2,$$

$$\xi_2 \varepsilon \leq 0.5 \xi_2^2 + 0.5 \|\varepsilon\|^2,$$

$$\gamma \bar{\theta}^T \hat{\theta} = \gamma \bar{\theta}^T \theta^* - \gamma \bar{\theta}^T \tilde{\theta} \leq -\left(\frac{\gamma}{2} \right) \bar{\theta}^T \tilde{\theta} + \left(\frac{\gamma}{2} \right) \|\theta^*\|^2. \quad (46)$$

Denoting $V_{\text{obs}} = 0.5 \chi^T \chi$, where $\chi = [\chi_1, \chi_2]^T$, χ_1 and χ_2 are the estimation errors of the high-gain observer. Substituting the inequalities (46) into (45) yields

$$\begin{aligned} \dot{V} \leq & -k_1 \xi_1^2 - (k_2 - 1.5) \xi_2^2 - (k_3 - 0.05) \xi_3^2 - \left(\frac{\gamma}{2} \right) \bar{\theta}^T \tilde{\theta} \\ & + \left(k_2^2 + 10 + \frac{2D^2}{\gamma} \right) V_{\text{obs}} + \left(0.5 e^2 \|H_t\|^2 + \frac{\gamma}{2} \right) \|\theta^*\|^2 \\ & + 0.5 \|\varepsilon\|^2. \end{aligned} \quad (47)$$

According to the second property of Lemma 4, we have

$$V_{\text{obs}} \leq 0.5 \beta^2 (g_1^2 + g_2^2). \quad (48)$$

Finally, the derivative of V is derived as

$$\begin{aligned} \dot{V} \leq & -k_1 \xi_1^2 - (k_2 - 1.5) \xi_2^2 - (k_3 - 0.05) \xi_3^2 - \left(\frac{\gamma}{4} \right) \bar{\theta}^T \tilde{\theta} \\ & + \left(0.5 k_2^2 + 5 + \frac{D^2}{\gamma} \right) \beta^2 (g_1^2 + g_2^2) \\ & + \left(0.5 e^2 \|H_t\|^2 + \frac{\gamma}{2} \right) \|\theta^*\|^2 + 0.5 \|\varepsilon\|^2 \\ \leq & -\mu V + v, \end{aligned} \quad (49)$$

where

$$\begin{aligned} \mu = & 2 \min \left\{ k_1, k_2 - 1.5, k_3 - 0.05, \frac{\gamma \lambda_{\min}(\Gamma)}{4} \right\}, \\ v = & \left(0.5 k_2^2 + 5 + \frac{D^2}{\gamma} \right) \beta^2 (g_1^2 + g_2^2) \\ & + \left(0.5 e^2 \|H_t\|^2 + \frac{\gamma}{2} \right) \|\theta^*\|^2 + 0.5 \|\varepsilon\|^2. \end{aligned} \quad (50)$$

To ensure that $\mu > 0$, the control parameters are chosen to satisfy the following conditions:

$$k_1 > 0, \quad k_2 > 1.5, \quad k_3 > 0.05. \quad (51)$$

That is, the error signals, including the compensated tracking errors ξ_i and the neural parameter estimation error $\tilde{\theta}$, are still semiglobally uniformly ultimately bounded, with the high-gain observer used.

Similar to the analysis in the last section, the tracking error signals z_i ($i = 1, 2, 3$) in the output feedback control system are also bounded.

3.3. Constrained Adaptive Output Feedback Control. In the design of the constrained controller, the input constraints, including the magnitude saturation and rate saturation of the rudder, are considered. The control law (36) does not assure the constraints, and the control input may exceed the magnitude or rate limitation of the actuator. The saturated control input may cause an aggressive adaptation in the adaptive law (37). This can make the ship course unstable. In this section, the signal u becomes the control law to be designed, and the fourth equation of (5) is contained in the control design.

In order to tackle the input saturation problem, a second-order nonlinear filter with limiters is introduced between the signals u and v . Figure 4 shows the visual description for the second-order filter, and the state-space model of the filter is defined as [24]

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} 2\zeta_0\omega_0 \left\{ \text{sat}_2 \left[\frac{\omega_0^2}{2\zeta_0\omega_0} (\text{sat}_1(u) - y_1) \right] - y_2 \right\} \\ y_2 \end{bmatrix}, \quad (52)$$

where ω_0 and ζ_0 are the natural frequency and damping ratio of the filter, y_1 is the filtered output of the input signal u , and y_2 is the derivative of y_1 .

It is obvious from Figure 4 that v is the filtered output of u ; that is, $v = y_1$. Hence, the control law u is designed as the form of (36):

$$u = \frac{1}{a_2} (-k_3 z_3 - a_1 x_3 + \dot{\alpha}_2 - \hat{\xi}_2). \quad (53)$$

And the derivative of z_3 becomes

$$\dot{z}_3 = \dot{x}_3 - \dot{\alpha}_2 = -k_3 z_3 - \hat{\xi}_2 + a_2 (v - u). \quad (54)$$

Meanwhile, the compensated tracking error ξ_3 defined in (20) is modified as

$$\xi_3 = z_3 - \sigma_3, \quad (55)$$

where the signal σ_3 is defined as:

$$\dot{\sigma}_3 = -k_3 \sigma_3 + a_2 (v - u) \quad (56)$$

with $\sigma_3(0) = 0$.

Then we can obtain the derivative of ξ_3 as

$$\dot{\xi}_3 = \dot{z}_3 - \dot{\sigma}_3 = -k_3 \xi_3 - \hat{\xi}_2. \quad (57)$$

It is clear that the derivative of V is equal to (45) and satisfies the inequality (49) finally. Hence, the error signals, including ξ_i ($i = 1, 2, 3$) and the neural parameter estimation error $\bar{\theta}$, are still semiglobally uniformly ultimately bounded with the nonlinear filter (52) introduced. Note that the definition of ξ_3 here is different from that in the last subsection.

Furthermore, among the dynamics of z_i , only the dynamic of z_3 is changed (see (41) and (55)). Due to the property of the filter, we have $v - u = O(1/\omega_0)$; hence, from (56), we can see that σ_3 will be close to zero arbitrarily. Then, according to (55), z_3 is bounded. That is, the tracking error signals z_i ($i = 1, 2, 3$) are all uniformly bounded.

FIGURE 4: Schematic structure of the second-order filter.

4. Case Study

In this section, the simulation on a cargo ship is carried out by using Matlab/Simulink. The parameters of the ship autopilot model are $T = 107.3$ s and $K = 0.185$ s⁻¹, with the nonlinear function $H(\dot{\psi}) = \dot{\psi} + \dot{\psi}^3$ and the model uncertainty $\Delta H(\psi, \dot{\psi}) = \sin[(\psi - \psi_d) + (\dot{\psi} - \dot{\psi}_d)]$. The parameters of the rudder dynamic satisfy $T_R/K_R = 2$. Without loss of generality, we define the external disturbance as

$$w(t) = 1 + 0.5 \sin(0.05t) + 0.8 \cos(0.05t). \quad (58)$$

The control objective is to track, using only the measurable signal $\psi(t)$, the desired course $\psi_d(t) = 30^\circ \sin(0.01t)$, in the presence of the maximum rudder $\delta_{\max} = 40^\circ$ and maximum rate of rudder $\dot{\delta}_{\max} = 3.5^\circ/\text{s}$. The initial heading of the ship is $x_1(0) = 20^\circ$.

The RBF neural network approximator with 10 nodes and 2 inputs is used, where $X = [x_1, \hat{x}_2]$ are the inputs. The parameters of the basis function are $c = 10 \times \text{ones}(2, 10)$ and $b = 10 \times \text{ones}(10, 1)$, where $\text{ones}(m, n)$ denotes a $m \times n$ matrix with all the elements being one. Choose the parameters of the weight updating law (37) as $\gamma = 6.86$ with Γ being an identity matrix, and the initial weight is set as $0.1 \times \text{ones}(10, 1)$.

The high-gain observer is designed according to (35) with $\beta = 0.15$ and $\lambda_1 = 2$, while the control law is based on (12), (38), and (53) with the filters designed as (24) and (52). We choose $k_1 = 0.03$, $k_2 = 10$, and $k_3 = 0.06$. The parameters of the filter (24) for both $\bar{\alpha}_1$ and $\bar{\alpha}_2$ are $\omega_n = 10$ and $\zeta = 1$, while the parameters of the filter (52) are $\omega_0 = 1$ and $\zeta_0 = 6$.

To illustrate the effectiveness of the proposed scheme, we compare the tracking performance of the proposed controller with a PID controller, as well as a nonadaptive backstepping controller, as shown in Figure 5. The PID control law is

$$u_{\text{PID}} = -k_p z_1 - k_D (\hat{x}_2 - \dot{\psi}_d) - k_I \int_0^t z_1(s) ds, \quad (59)$$

where z_1 and $\hat{x}_2$ can be obtained from (10) and (34), respectively, and the PID parameters are selected as $k_p = 0.1$, $k_I = 100$, and $k_D = 0.05$ through lots of simulations. The nonadaptive backstepping adopts the same parameters as the proposed controller. From Figure 5, it can be shown that, using the PID control, both the steady-state and transient performances are less satisfactory, whereas, using the proposed method, the performances are best. Moreover, the proposed controller has faster decay of tracking error and lower steady-state value than the nonadaptive backstepping, and the reason is that the bounded unknown function $f(x_1, x_2, t)$ is compensated by using the RBF network. In contrast, for the nonadaptive backstepping, one can estimate

FIGURE 5: Comparison of course-tracking performance.

FIGURE 6: Control input using the proposed controller.

a conservative bound of $f(x_1, x_2, t)$ and augment a robust term to dominate the disturbance, but this approach may lead to an overlarge control input if the bound of the disturbance is overestimated.

Besides, the proposed method avoids computing the derivatives of the virtual controls and consequently simplifies the backstepping implementation, so that it is beneficial from the practical application viewpoint.

Figure 6 shows the curve of control input by using the proposed method. In general, better tracking precision needs more control effort and may cause oscillations of the control. But as seen from Figure 6, using the proposed control, there are only a few slight oscillations at 50~100 s, and in the rest of the tracking process, the rudder changes smoothly. In addition, since the input constraints are considered in the control design, the magnitude and rate of the rudder are bounded within the maximum values.

Figure 7 shows the small time convergence of the high-gain observer estimate to the output derivative. Within about 1.5 s, the estimate peaks at its saturation value and then converges rapidly to the actual derivative of the rudder

FIGURE 7: Observer error.

FIGURE 8: Norm of the RBF network weights.

angle. After that, the estimate error remains in a small neighborhood of zero. The norm of the approximation weight is fast bounded as seen from Figure 8.

5. Conclusions

In this paper, the problem of course tracking for ships with uncertainties and unknown external disturbances has been investigated. During the control design, both the magnitude and rate saturation of the input signal are considered. An adaptive full-state feedback controller using RBF network and filtering backstepping is first derived, and the final controller is obtained by using a high-gain observer to estimate the unmeasurable angle velocity and introducing an auxiliary system to compensate the input saturations. Stability analysis is made to verify the semiglobal uniform boundedness of the final closed-loop system. The excellent performance of the aforementioned control scheme is validated through simulation tests for the course tracking of a cargo ship.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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