Research Article

Filtering Based Recursive Least Squares Algorithm for Multi-Input Multioutput Hammerstein Models

Ziyun Wang, Yan Wang, and Zhicheng Ji

Key Laboratory of Advanced Process Control for Light Industry (Ministry of Education), Jiangnan University, Wuxi 214122, China

Correspondence should be addressed to Yan Wang; yanwangjn@163.com

Received 28 June 2014; Revised 7 September 2014; Accepted 25 September 2014; Published 16 October 2014

Academic Editor: Haranath Kar

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This paper considers the parameter estimation problem for Hammerstein multi-input multioutput finite impulse response (FIR-MA) systems. Filtered by the noise transfer function, the FIR-MA model is transformed into a controlled autoregressive model. The key-term variable separation principle is used to derive a data filtering based recursive least squares algorithm. The numerical examples confirm that the proposed algorithm can estimate parameters more accurately and has a higher computational efficiency compared with the recursive least squares algorithm.

1. Introduction

Parameter estimation is an important approach to model dynamical systems and has been widely used in estimating the parameters for nonlinear systems [1–3], deriving system identification methods [4–7], identifying state-space models [8, 9], and developing solutions for matrix equations [10–13]. For example, Dehghan and Hajarian discussed several solution methods for different matrix equations [14–16]. In the area of system control and modeling, Shi and Fang developed a Kalman filter based identification for systems with randomly missing measurements [17], gave output feedback stabilization [18], and presented a robust mixed $H_2/H_\infty$ control of networked control systems [19].

The least squares algorithm is a fundamental method [20–22] and many methods such as the iterative algorithm [23, 24] and the gradient algorithm [25] are widely used in the parameter estimation. In the field of Hammerstein system identification, several methods have been developed [26]. For example, a least squares based iterative algorithm and an auxiliary model based recursive least squares algorithm have been presented, respectively, for Hammerstein nonlinear ARMAX systems and Hammerstein output error systems [27, 28]; a Newton recursive algorithm and a Newton iterative algorithm for Hammerstein controlled autoregressive systems are presented in [29].

Consider a multi-input multioutput (MIMO) Hammerstein finite impulse response (FIR) system depicted by

$$y(t) = B(z)u(t) + N(z)v(t),$$

where $u(t) := [u_1(t), u_2(t), \ldots, u_r(t)]^T \in \mathbb{R}^r$ is the nonlinear system input vector with zero mean and unit variances, $y(t) \in \mathbb{R}^m$ is the measurement of $x(t) := B(z)u(t)$ but is corrupted by $w(t) := N(z)v(t)$, $v(t) \in \mathbb{R}^m$ is the white noise vector with zero mean, and $B(z)$ and $N(z)$ are polynomials in the unit backward shift operator $z^{-1}$ [$z^{-1}y(t) = y(t-1)$]:

$$B(z) := I_{ncr} + \sum_{j=1}^{nb} B_j z^{-j}, \quad B_j \in \mathbb{R}^{ncr},$$

$$N(z) := 1 + \sum_{j=1}^{nd} d_j z^{-j}, \quad d_j \in \mathbb{R}^1.$$
The nonlinear block in the Hammerstein model is a linear combination of the known basis $f := (f_1, f_2, \ldots, f_r)$:

$$
\mathbf{u}(t) = f(\mathbf{u}(t)) := [f_1(u_1(t)), f_2(u_2(t)), \ldots, f_r(u_r(t))]^T \in \mathbb{R}^r,
$$

where the superscript $T$ denotes the matrix transpose. The function $f_i(u_i(t))$ in (3) is a nonlinear function of a known basis $(y_1, y_2, \ldots, y_n)$:

$$
f_i(u_i(t)) := \sum_{j=1}^{n_i} c_j y_j(u_i(t)),
$$

where the coefficients $c := (c_1, c_2, \ldots, c_{n_i})$ are unknown. Substituting (4) into (3) yields

$$
\mathbf{u}(t) = \sum_{i=1}^{n_i} c_i y_i(\mathbf{u}(t)),
$$

$$
y_i(\mathbf{u}(t)) := [y_1(u_1(t)), y_1(u_2(t)), \ldots, y_1(u_r(t))]^T \in \mathbb{R}^r.
$$

Assume that $\mathbf{u}(t) = \mathbf{0}$, $\mathbf{v}(t) = \mathbf{0}$, and $\mathbf{v}(t)$ is known for $t \leq 0$, and the orders $n_1, n_2$, and $n_3$ are known but can be obtained by trial and error. In general, the orders of the Hammerstein model should be large when the nonlinear system is used in prediction; otherwise, the orders should be small if the system is applied for control.

The objective of this paper is to estimate the unknown parameter matrices: $B_1, c_1, d_1$ from the available input-output data $[\mathbf{u}(t), y(t)]$ of the multivariable Hammerstein finite impulse response moving average (FIR-MA) models [30].

Recently, the filtering idea has received much attention [31-33]. Xiao and Yue studied input nonlinear dynamical adjustment models and presented a recursive generalized least squares algorithm and a filtering least squares algorithm by replacing the unknow terms in the information vectors with their estimates [34]. The overparameterization method in [34] leads to a redundant estimated product of the nonlinear systems and requires extra computation. Differing from the work in [30, 34, 35], this paper discusses the estimation problem of the MIMO Hammerstein systems using the data filtering idea and transfers the FIR-MA system to controlled autoregressive model by means of the key-term variable separate principle in [36-38]. The proposed algorithm used in this paper can extend to study parameter estimation problems of dual-rate/multirate sampled systems [39-42] and other linear or nonlinear systems [43-46].

Briefly, the rest of this paper is recognized as follows. Section 2 discusses a recursive least squares algorithm for the Hammerstein systems. Section 3 presents a filtering based recursive least squares algorithm by transferring an FIR-MA model to a controlled autoregressive model. Section 4 provides an illustrative example. Finally, some concluding remarks are offered in Section 5.

2. The MRLS Algorithm

For comparison, the MRLS algorithm is listed in Section 2. Here we introduce some notations. $t$ represents the current time in this paper and "$A := X$" or "$X := A$" means that "$A$ is defined as $X$". The symbol $I_{m \times n}$ represents an identity matrix of size $r$ followed by a null matrix of the last $m - r$ rows when $m \geq r$ and vice versa. The norm of a matrix (or a column vector) $X$ is defined by $\|X\| := \text{tr}(XX^T)$. $\otimes$ denotes the Kronecker product or direct product: if $A = [a_{ij}] \in \mathbb{R}^{m \times n}$, $B = [b_{ij}] \in \mathbb{R}^{p \times q}$, then $A \otimes B := [a_{ij}B] \in \mathbb{R}^{mp \times nq}$; $\text{col}[X]$ is supposed to be the vector formed by the column of the matrix $X$: if $X = [x_1, x_2, \ldots, x_n] \in \mathbb{R}^{m \times n}$, then $\text{col}[X] := [x_1^T, x_2^T, \ldots, x_n^T]^T \in \mathbb{R}^{mn}$.

From (1)-(5), the intermediate variables $\mathbf{x}(t)$ and $\mathbf{w}(t)$ and output of the system $\mathbf{y}(t)$ can be expressed as

$$
\mathbf{x}(t) = \left[ I_{m \times r} + \sum_{i=1}^{n_1} B_i z_i^{-1} \right] \mathbf{u}(t),
$$

$$
\mathbf{w}(t) = \left[ 1 + \sum_{i=1}^{n_2} d_i z_i^{-1} \right] \mathbf{v}(t),
$$

$$
\mathbf{y}(t) = \sum_{i=1}^{n_3} \left[ y_i(\mathbf{u}(t)) \right] + \sum_{i=1}^{n_3} B_i \mathbf{u}(t - i) + \sum_{i=1}^{n_3} d_i \mathbf{v}(t - i) + \mathbf{v}(t).
$$

Note that the subscripts (Roman) $s$ and $n$ denote the first letters of "system" and "noise" for distinguishing the types of the unknown parameter vectors or matrices, respectively. Define the parameter matrix $\Theta_1$, the parameter vectors $\Theta_2, \Theta_3$, and the information vectors $\Phi_1(t), \Phi_2(t)$, and $\Phi_3(t)$ as

$$
\Theta_1 := [B_1, B_2, \ldots, B_{n_1}] \in \mathbb{R}^{m \times (m_n)},
$$

$$
\Theta_2 := [c_1, c_2, \ldots, c_{n_2}] \in \mathbb{R}^{n_1},
$$

$$
\Theta_3 := [d_1, d_2, \ldots, d_{n_3}] \in \mathbb{R}^{n_2},
$$

$$
\Phi_1(t) := \left[ \mathbf{u}(t - 1), \mathbf{u}(t - 2), \ldots, \mathbf{u}(t - n_3) \right]^T \in \mathbb{R}^{n_3},
$$

$$
\Phi_2(t) := [\eta_1(t), \eta_2(t), \ldots, \eta_{n_2}(t)] \in \mathbb{R}^{m \times n_2},
$$

$$
\Phi_3(t) := [y_i(\mathbf{u}(t)) \mathbf{0}] \in \mathbb{R}^{n_3},
$$

Then, we have

$$
\mathbf{x}(t) = \Theta_1^T \Phi_1(t) + \Phi_2(t) \Theta_2,
$$

$$
\mathbf{w}(t) = \Phi_1(t) \Theta_3 + \mathbf{v}(t),
$$

$$
\mathbf{y}(t) = \mathbf{x}(t) + \mathbf{w}(t)
$$

$$
= \Theta_1^T \Phi_1(t) + \Phi_2(t) \Theta_2 + \Phi_3(t) \Theta_3 + \mathbf{v}(t).
$$
Distinguished from the hierarchical identification methods, we reparameterize the model in (7) by using the Kronecker product to get a parameter matrix $\mathbf{g}$ and by gathering the input information vectors $\varphi_i(t)$ and $\varphi_n(t)$ and output information matrix $\varphi_n(t)$ into one information matrix $\mathbf{P}(t)$ to obtain an information matrix $\mathbf{Q}(t)$ as follows:

$$\mathbf{g} := \begin{bmatrix} \theta_1^T \\ \theta_2^T \\ \vdots \\ \theta_n^T \end{bmatrix} \in \mathbb{R}^{n_0}, \quad n_0 := mn_{n_b} + n_c + n_{d_t},$$

$$\mathbf{Q}(t) := \begin{bmatrix} \varphi_1^T(t) \\ \varphi_n^T(t) \end{bmatrix} \in \mathbb{R}^{m \times (mn_{n_b} + n_c)},$$

$$\varphi_i(t) := \begin{bmatrix} \varphi_i^T(t) \\ \mathbf{I}_m \end{bmatrix} \in \mathbb{R}^{m \times n_0}, \quad i = 1, 2, \ldots, n_c,$$

$$\varphi_n(t) := \begin{bmatrix} \varphi_n^T(t) \\ \mathbf{I}_m \end{bmatrix} \in \mathbb{R}^{m \times (mn_{n_b} + n_c)}.$$

Thus, we obtain

$$y(t) = \mathbf{Q}(t) \mathbf{g} + v(t). \quad (11)$$

Equation (11) is the identification model of the multivariable Hammerstein FIR-MA system. Defining and minimizing the cost function

$$J(\mathbf{g}) := \|y(t) - \mathbf{Q}(t) \mathbf{g}\|^2$$

and using the least squares search principle, we obtain the following recursive least squares algorithm [35] to obtain parameter estimates $\hat{\mathbf{g}}(t)$:

$$\hat{\mathbf{g}}(t) = \hat{\mathbf{g}}(t - 1) + \mathbf{L}(t) \left[ y(t) - \mathbf{Q}(t) \hat{\mathbf{g}}(t - 1) \right], \quad (13)$$

$$\mathbf{L}(t) = \mathbf{P}(t - 1) \mathbf{Q}^T(t) \left[ \mathbf{I}_m + \mathbf{P}(t - 1) \mathbf{Q}^T(t) \right]^{-1}, \quad (14)$$

$$\mathbf{P}(t) = \left[ \mathbf{I}_{n_0} - \mathbf{L}(t) \mathbf{Q}(t) \right] \mathbf{P}(t - 1), \quad \mathbf{P}(0) = \mathbf{P}_0 \mathbf{I}_{n_0}. \quad (15)$$

Since the information matrix $\mathbf{Q}(t)$ in (13) contains the unknown intermediate variables $\tilde{u}(t - i)$ and the unmeasurable terms $v(t - i)$, the recursive algorithm in (13)–(15) cannot compute the parameter estimate $\hat{\mathbf{g}}(t)$. The solution here is replacing the unknown intermediate variables $\tilde{u}(t - i)$ and the unmeasurable terms $v(t - i)$ in $\mathbf{Q}(t)$ with the variable estimates (or the outputs of the auxiliary model) $\tilde{u}(t - i)$ and the estimates $\tilde{v}(t - i)$ based on the auxiliary model identification idea. The replaced information matrices are defined as

$$\tilde{\mathbf{Q}}(t) := \begin{bmatrix} \tilde{\varphi}_1(t) \\ \tilde{\varphi}_n(t) \end{bmatrix} \in \mathbb{R}^{m \times n_0},$$

$$\tilde{\varphi}_i(t) := \begin{bmatrix} \tilde{\varphi}_1^T(t) \otimes \mathbf{I}_m \\ \varphi_n^T(t) \end{bmatrix} \in \mathbb{R}^{m \times (mn_{n_b} + n_c)},$$

$$\tilde{\varphi}_1(t) := \begin{bmatrix} \tilde{u}^T(t - 1) \\ \tilde{u}^T(t - 2) \\ \vdots \\ \tilde{u}^T(t - n_{b}) \end{bmatrix} \in \mathbb{R}^{n_{b}},$$

$$\tilde{\varphi}_n(t) := \begin{bmatrix} \tilde{v}(t - 1) \\ \tilde{v}(t - 2) \\ \vdots \\ \tilde{v}(t - n_{d}) \end{bmatrix} \in \mathbb{R}^{n_{d}}.$$

Define the parameter estimation matrices

$$\hat{\mathbf{g}}(t) := \begin{bmatrix} \hat{\vartheta}_1(t) \\ \hat{\vartheta}_n(t) \end{bmatrix} \in \mathbb{R}^{n_0},$$

$$\hat{\vartheta}_i(t) := \begin{bmatrix} \vartheta_i^T(t) \end{bmatrix} \in \mathbb{R}^{mn_{n_b} + n_c},$$

$$\hat{\vartheta}_1(t) := \begin{bmatrix} \tilde{B}_1(t), \tilde{B}_2(t), \ldots, \tilde{B}_{n_{b}}(t) \end{bmatrix} \in \mathbb{R}^{m \times (mn_{b})},$$

$$\hat{\vartheta}_2(t) := \begin{bmatrix} \tilde{c}_1(t), \tilde{c}_2(t), \ldots, \tilde{c}_{n_{c}}(t) \end{bmatrix} \in \mathbb{R}^{n_{c}},$$

$$\hat{\vartheta}_n(t) := \begin{bmatrix} \tilde{a}_1(t), \tilde{a}_2(t), \ldots, \tilde{a}_{n_{d}}(t) \end{bmatrix} \in \mathbb{R}^{n_{d}}.$$

By replacing the parameters $c_i$ ($i = 1, 2, \ldots, n_c$) in (4) with $\tilde{c}(t)$, the output of the proposed auxiliary model $\tilde{u}(t)$ is given by

$$\tilde{u}(t) = \sum_{i=1}^{n_c} \tilde{c}_i(t) \gamma_i(u(t))$$

$$= \tilde{c}_1(t) \gamma_1(u(t)) + \tilde{c}_2(t) \gamma_2(u(t))$$

$$+ \cdots + \tilde{c}_{n_{c}}(t) \gamma_{n_{c}}(u(t)). \quad (18)$$

From (11), we obtain $v(t) = y(t) - \mathbf{Q}(t) \mathbf{g}$. Replacing $\mathbf{Q}(t)$ and $\mathbf{g}$ with $\tilde{\mathbf{Q}}(t)$ and $\hat{\mathbf{g}}(t - 1)$, the residual $\tilde{v}(t)$ can be written as $\tilde{v}(t) = y(t) - \tilde{\mathbf{Q}}(t) \hat{\mathbf{g}}(t - 1)$.

To summarize, we conclude the following recursive least squares algorithm for multivariable Hammerstein FIR-MA models (the MRLS algorithm for short):

$$\tilde{\mathbf{g}}(t) = \tilde{\mathbf{g}}(t - 1) + \mathbf{L}(t) \left[ y(t) - \tilde{\mathbf{Q}}(t) \hat{\mathbf{g}}(t - 1) \right],$$

$$\mathbf{L}(t) = \mathbf{P}(t - 1) \tilde{\mathbf{Q}}^T(t) \left[ \mathbf{I}_m + \mathbf{P}(t - 1) \tilde{\mathbf{Q}}^T(t) \right]^{-1},$$

$$\mathbf{P}(t) = \left[ \mathbf{I}_{n_0} - \mathbf{L}(t) \tilde{\mathbf{Q}}(t) \right] \mathbf{P}(t - 1),$$

$$\tilde{\vartheta}_1(t) := \begin{bmatrix} \tilde{u}^T(t - 1) \\ \tilde{u}^T(t - 2) \\ \vdots \\ \tilde{u}^T(t - n_{b}) \end{bmatrix} \in \mathbb{R}^{n_{b}},$$

$$\tilde{\vartheta}_2(t) := \begin{bmatrix} \tilde{\gamma}_1(u(t)) \\ \mathbf{0} \end{bmatrix},$$

$$\tilde{\vartheta}_n(t) := \begin{bmatrix} \tilde{\varphi}_1^T(t) \otimes \mathbf{I}_m, \varphi_n^T(t) \end{bmatrix} \in \mathbb{R}^{m \times (mn_{n_b} + n_c)}.$$
\[ \tilde{\Theta}(t) = \begin{bmatrix} \text{col} \left[ \tilde{\Theta}_T^T(t) \right] \\
\tilde{\Theta}_1(t) \\
\tilde{\Theta}_2(t) \\
\vdots \\
\tilde{\Theta}_n(t) \end{bmatrix}, \]

\[ \tilde{\Theta}_1^T(t) = \begin{bmatrix} \tilde{B}_1(t), \tilde{B}_2(t), \ldots, \tilde{B}_{n_b}(t) \end{bmatrix}, \]

\[ \tilde{\Theta}_2(t) = \begin{bmatrix} \tilde{c}_1(t), \tilde{c}_2(t), \ldots, \tilde{c}_{n_b}(t) \end{bmatrix}^T, \]

\[ \tilde{\Theta}_n(t) = \begin{bmatrix} \tilde{d}_1(t), \tilde{d}_2(t), \ldots, \tilde{d}_{n_b}(t) \end{bmatrix}^T. \] (19)

### 3. The F-MRLS Algorithm

The convergence rate of the MRLS algorithm in Section 2 is slow because the noise information intermediate variables \( w(t) \) contain unmeasurable time-delay noise \( v(t) \). The solution here is to present a filtering based recursive least squares algorithm (the F-MRLS algorithm) for the multivariable Hammerstein models by filtering the rational function algorithm (the F-MRLS algorithm) for the multivariable autoregressive (CAR) model. Multiplying both sides of (1) by \( N^{-1}(z) \) yields

\[ N^{-1}(z) y(t) = B(z) N^{-1}(z) \tilde{u}(t) + v(t), \] (20)

or

\[ y_f(t) = B(z) \tilde{u}_f(t) + v(t), \] (21)

where

\[ \tilde{u}_f(t) := \frac{1}{N(z)} \tilde{u}(t) = \frac{1}{N(z)} \sum_{i=1}^{n_b} \tilde{c}_i(t) y_i(u(t)) = c_1 U_1(t) + c_2 U_2(t) + \cdots + c_{n_b} U_{n_b}(t), \]

\[ y_f(t) := \frac{1}{N(z)} y(t) = [1 - N(z)] y_f(t) + y(t) = y(t) - \frac{\sum_{i=1}^{n_b} d_i(t) y_f(t-i)}{N(z)} \] (22)

\[ U_i(t) := \frac{1}{N(z)} y_i(u(t)) \in \mathbb{R}^r, \quad i = 1, 2, \ldots, n_b, \]

\[ \zeta_j(t) := \begin{bmatrix} U_j(t) \\
0 \end{bmatrix} \in \mathbb{R}^{m}, \quad j = 1, 2, \ldots, n_b. \]

Thus, (21) can be rewritten as

\[ y_f(t) = \left[ I_{mrx} + \sum_{i=1}^{n_b} B_i z^{-i} \right] \tilde{u}_f(t) + v(t) = \left[ \tilde{u}_f(t) \right] + B_1 \tilde{u}_f(t-1) + B_2 \tilde{u}_f(t-2) + \cdots + B_{n_b} \tilde{u}_f(t-n_b) + v(t), \] (23)

Define the filtered information matrices:

\[ \Phi_f(t) := \left[ \varphi_{f_1}(t) \otimes I_{m}, \varphi_{f_2}(t) \right] \in \mathbb{R}^{mrs}, \] (24)

\[ \varphi_{f_1}(t) := \left[ \tilde{u}_f(t-1), \tilde{u}_f(t-2), \ldots, \tilde{u}_f(t-n_b) \right]^T \in \mathbb{R}^{m}, \] (25)

\[ \varphi_{f_2}(t) := \left[ \zeta_1(t), \zeta_2(t), \ldots, \zeta_{n_b}(t) \right] \in \mathbb{R}^{m}, \] (26)

Since the polynomial \( N(z) \) is unknown and to be estimated, it is impossible to use \( \tilde{u}_f(t) \) to construct \( \varphi_{f_1}(t) \) in (25). Here, we adopt the principle of the MRLS algorithm in Section 2 and replace the unmeasurable variables and vectors with their estimates to derive the following algorithm.

By using the parameter estimates \( \hat{\Theta}_1(t) \) and \( \hat{\Theta}_n(t) \), the estimates of polynomials \( B(z) \) and \( N(z) \) at time \( t \) can be constructed as

\[ \tilde{B}(t, z) := I_{mrx} + \sum_{i=1}^{n_b} B_i(t) z^{-i}, \]

\[ \tilde{N}(t, z) := 1 + \tilde{d}_1(t) z^{-1} + \tilde{d}_2(t) z^{-2} + \cdots + \tilde{d}_{n_b}(t) z^{-n_b}, \] (27)

\( w(t) \) in model (1) can be rewritten as

\[ w(t) = y(t) - x(t) = y(t) - \left[ I_{mrx} + \sum_{i=1}^{n_b} B_i z^{-i} \right] \tilde{u}(t) \] (28)

\[ = y(t) - \varphi_f(t) \theta_f. \]

Let \( \tilde{w}(t) \) be the estimate of \( w(t) \). Replacing \( w(t), y(t), \varphi_f(t), \varphi_{f_1}(t), \varphi_{f_2}(t), \theta_f, \) with their estimates \( \tilde{w}(t), \tilde{y}(t), \tilde{\varphi}_f(t), \tilde{\varphi}_{f_1}(t), \tilde{\varphi}_{f_2}(t), \tilde{\theta}_f(t), \) and \( \tilde{\theta}_f(t-1) \) leads to

\[ \tilde{w}(t) = y(t) - \tilde{\varphi}_f(t) \tilde{\theta}_f(t-1). \] (29)

Defining and minimizing the cost function

\[ J(\theta_f) := \sum_{j=1}^{l} \left[ w(j) - \varphi_f(j) \theta_f \right]^2 \] (30)
and using the least squares search principle, we list the recursive least squares algorithm to compute $\tilde{\theta}_n(t)$:

$$\tilde{\theta}_n(t) = \tilde{\theta}_n(t-1) + L_n(t) \left[ \tilde{w}(t) - \tilde{\phi}_n(t) \tilde{\theta}_n(t-1) \right],$$

$$L_n(t) = P_n(t-1) \tilde{\phi}_n^T(t) \left[ I_{\text{m}} + \tilde{\phi}_n(t) P_n(t-1) \tilde{\phi}_n^T(t) \right]^{-1},$$

$$P_n(t) = \left[ I_{\text{m}} - L_n(t) \tilde{\phi}_n(t) \right] P_n(t-1), \quad P_n(0) = p_0 I.$$  

(31)

Let $\tilde{c}(t) := \tilde{c}_1(t), \tilde{c}_2(t), \ldots, \tilde{c}_n(t) \in \mathbb{R}^n$ be the estimate of $c$ at time $t$, filtering $y(t)$ with $1/N(t,z)$ to obtain the estimate $\tilde{y}_f(t)$:

$$\tilde{y}_f(t) = \frac{1}{N(t,z)} y(t)$$

$$= \left[ 1 - \tilde{N}(t,z) \right] \tilde{y}_f(t) + y(t)$$

$$= -\sum_{i=1}^{n_z} \tilde{d}_i(t) \tilde{y}_f(t-i) + y(t).$$  

(32)

The estimate of $U_j(t)$ can be computed by

$$\tilde{U}_j(t) = \frac{1}{N(t,z)} y_j(u(t))$$

$$= \left[ 1 - \tilde{N}(t,z) \right] \tilde{U}_j(t) + y_j(u(t))$$

$$= -\sum_{i=1}^{n_z} \tilde{d}_i(t) \tilde{U}_j(t-i) + y_j(u(t)).$$  

(33)

Define the estimate of $\tilde{\zeta}_j(t)$ by

$$\tilde{\zeta}_j(t) := \begin{bmatrix} \tilde{U}_j(t) \\ 0 \end{bmatrix} \in \mathbb{R}^m, \quad j = 1, 2, \ldots, n_z,$$  

(34)

and construct the estimate of $\Psi_f(t)$ with $\tilde{\phi}_{f1}(t)$ and $\tilde{\phi}_{f2}(t)$ as follows:

$$\tilde{\Psi}_f(t) = \left[ \tilde{\phi}_{f1}(t) \otimes I_m, \tilde{\phi}_{f2}(t) \right] \tilde{\phi}_{f1}^T(t) \tilde{\phi}_{f2}(t),$$

$$\tilde{\phi}_{f1}(t) = \left[ \tilde{u}_f(t-1), \tilde{u}_f(t-2), \ldots, \tilde{u}_f(t-n_b) \right]^T,$$

$$\tilde{\phi}_{f2}(t) = \left[ \tilde{\zeta}_1(t), \tilde{\zeta}_2(t), \ldots, \tilde{\zeta}_{n_z}(t) \right].$$  

(35)

The filtered model in (21) can be rewritten in a matrix form:

$$y_f(t) = \Psi_f(t) \theta_x + v(t),$$  

or

$$v(t) = y_f(t) - \Psi_f(t) \theta_x.$$  

(36)

(37)

Based on the MRLS search principle, we can obtain the estimate of $\theta_x$ by the following algorithm:

$$\tilde{\theta}_x(t) = \tilde{\theta}_x(t-1) + L_f(t) \left[ \tilde{y}_f(t) - \tilde{\Psi}_f(t) \tilde{\theta}_x(t-1) \right],$$

$$L_f(t) = P_f(t-1) \tilde{\Psi}_f^T(t) \left[ I_m + \tilde{\Psi}_f(t) P_f(t-1) \tilde{\Psi}_f^T(t) \right]^{-1},$$

$$P_f(t) = \left[ I_{n_z+n_b} - L_f(t) \tilde{\Psi}_f(t) \right] P_f(t-1), \quad P_f(0) = p_0 I.$$  

(38)

The estimate $\tilde{u}(t)$ can be computed by

$$\tilde{u}(t) = \tilde{c}_1(t) y_1(u(t)) + \tilde{c}_2(t) y_2(u(t))$$

$$+ \cdots + \tilde{c}_{n_z}(t) y_{n_z}(u(t)).$$  

(39)

Filter $\tilde{u}(t)$ by $1/N(t,z)$ to obtain the estimate $\tilde{u}_f(t)$:

$$\tilde{u}_f(t) = \frac{1}{N(t,z)} \tilde{u}(t)$$

$$= \frac{1}{N(t,z)} \sum_{i=1}^{n_z} \tilde{c}_i(t) y_i(u(t))$$

$$= \tilde{c}_1(t) \tilde{U}_1(t) + \tilde{c}_2(t) \tilde{U}_2(t) + \cdots + \tilde{c}_{n_z}(t) \tilde{U}_{n_z}(t).$$  

(40)

Replacing $v(t)$, $y_f(t)$, $\Psi_f(t)$, and $\theta_x$ in (37) with their estimates $\tilde{v}(t)$, $\tilde{y}_f(t)$, $\tilde{\Psi}_f(t)$, and $\tilde{\theta}_x(t)$ at time $t$, the noise vector can be computed by

$$\tilde{v}(t) = \tilde{y}_f(t) - \tilde{\Psi}_f(t) \tilde{\theta}_x(t).$$  

(41)

In conclusion, we can summarize the following filtering based recursive least squares algorithm for multivariable Hammerstein models (the F-MRLS algorithm for short):

$$\tilde{\theta}_x(t) = \tilde{\theta}_x(t-1) + L_f(t) \left[ \tilde{y}_f(t) - \tilde{\Psi}_f(t) \tilde{\theta}_x(t-1) \right],$$

$$L_f(t) = \frac{P_f(t-1) \tilde{\Psi}_f^T(t)}{I_m + \tilde{\Psi}_f(t) P_f(t-1) \tilde{\Psi}_f^T(t)},$$

$$P_f(t) = \left[ I_{n_z+n_b} - L_f(t) \tilde{\Psi}_f(t) \right] P_f(t-1),$$

$$\tilde{\Psi}_f(t) = \left[ \tilde{\phi}_{f1}(t) \otimes I_m, \tilde{\phi}_{f2}(t) \right],$$

$$\tilde{\phi}_{f1}(t) = \left[ \tilde{u}_f(t-1), \tilde{u}_f(t-2), \ldots, \tilde{u}_f(t-n_b) \right]^T,$$

$$\tilde{\phi}_{f2}(t) = \left[ \tilde{\zeta}_1(t), \tilde{\zeta}_2(t), \ldots, \tilde{\zeta}_{n_z}(t) \right],$$

$$\tilde{\zeta}_j(t) := \begin{bmatrix} \tilde{U}_j(t) \\ 0 \end{bmatrix}.$$  

(42)

(43)

(44)

(45)

(46)

(47)

(48)
\[ \mathbf{U}_f(t) = -\sum_{i=1}^{nd} \mathbf{d}_i(t) \mathbf{U}_f(t-i) + \mathbf{y}_f(t), \quad (49) \]

and \( j = 1, 2, \ldots, n_r \), \( \mathbf{P}_f(0) = p_0 \mathbf{I}_{nr \times nr} \), \( \mathbf{P}_n(0) = p_0 \mathbf{I}_{n_x} \), \( p_0 = 10^6 \), and give the basis functions \( \mathbf{y}_f(t) \).

(2) Collect the input-output data \( \mathbf{u}(t) \) and \( \mathbf{y}(t) \), and construct \( \mathbf{\eta}_1(t) \) by (60), and \( \mathbf{\bar{\phi}}_1(t), \mathbf{\bar{\phi}}_2(t) \) by (58), (59). Form the information vectors \( \mathbf{\bar{\phi}}_1(t) \) by (57), \( \mathbf{\bar{\phi}}_n(t) \) by (61), and \( \mathbf{\Psi}(t) \) by (56), respectively.

(3) Compute \( \mathbf{\bar{w}}(t) \) by (55), the gain vector \( \mathbf{L}_n(t) \) by (53), and the covariance matrix \( \mathbf{P}_n(t) \) by (54), respectively. Update the parameter estimate \( \mathbf{\bar{\theta}}_n(t) \) by (52).

(4) Compute \( \mathbf{\bar{y}}_f(t) \) by (51), \( \mathbf{\bar{U}}_f(t) \) by (49). Construct \( \mathbf{\bar{\xi}}_1(t), \mathbf{\bar{\xi}}_2(t), \mathbf{\bar{\phi}}_{f1}(t) \) by (48), (47), and (46), respectively. Compute \( \mathbf{\bar{\Psi}}_f(t) \) by (45).

(5) Compute the gain vector \( \mathbf{L}_1(t) \) by (43) and the covariance matrix \( \mathbf{P}_1(t) \) by (44). Update the parameter estimate \( \mathbf{\bar{\theta}}_1(t) \) by (42).

(6) Construct the parameter vectors \( \mathbf{\bar{\theta}}_1(t), \mathbf{\bar{\theta}}_2(t), \mathbf{\bar{\theta}}_n(t) \) by (66), (67), and (68). Form \( \mathbf{\bar{\theta}}(t) \) and \( \mathbf{\bar{\theta}}(t) \) by (65) and (64). Compute \( \mathbf{\bar{u}}_f(t) \) by (50), \( \mathbf{\bar{v}}(t) \) by (62), and \( \mathbf{\bar{u}}_j(t) \) by (63).

(7) Increase \( t \) by 1 and go to step 2.

### 4. Examples

**Example 1.** Consider the following 2-input 2-output Hammerstein FIR-MA system:

\[
\begin{bmatrix}
\mathbf{y}_1(t) \\
\mathbf{y}_2(t)
\end{bmatrix} =
\begin{bmatrix}
\mathbf{U}(t) \\
\mathbf{U}(t)
\end{bmatrix} + \mathbf{B}
\begin{bmatrix}
\mathbf{I}_1(t-1) \\
\mathbf{I}_2(t-1)
\end{bmatrix}
+ \begin{bmatrix}
\mathbf{v}_1(t) \\
\mathbf{v}_2(t)
\end{bmatrix} + d_1 \begin{bmatrix}
\mathbf{v}_1(t-1) \\
\mathbf{v}_2(t-1)
\end{bmatrix},
\]

where

\[
\mathbf{B}(z) = \begin{bmatrix}
0.13 & 0.25 \\
-1.21 & 0.13
\end{bmatrix} z^{-1},
\]

\[
D(z) = 1 + d_1 z^{-1} = 1 + 0.68 z^{-1},
\]

\[
\mathbf{U}(t) = c_1 \mathbf{u}(t) + c_2 \mathbf{u}^2(t),
\]

\[
\mathbf{\theta} = [c_1, c_2]^T = [1.00, -0.14]^T,
\]

\[
\mathbf{c} = [0.13, -1.21, 0.25, 0.13, -0.14, 0.68]^T.
\]

In simulation, the inputs \{\( \mathbf{u}_1(t) \)\} and \{\( \mathbf{u}_2(t) \)\} are taken as two uncorrelated persistent excitation signal sequences with zero mean and unit variances and \{\( \mathbf{v}_1(t) \)\} and \{\( \mathbf{v}_2(t) \)\} as two white noise sequences with zero mean and variances \( \sigma_1^2 = \sigma_2^2 = 0.50^2 \) and \( \sigma_1^2 = 1.00^2 \). Applying the MRLS algorithm in (19) and the F-MRLS algorithm in (42)–(68) to estimate the parameters of this multivariable Hammerstein system, the F-MRLS parameter estimates and their estimation errors are shown in Table 1, the comparison between the F-MRLS algorithm and the MRLS algorithm in the estimation error \( \mathbf{\delta} = \||\mathbf{\bar{\theta}}(t) - \mathbf{\theta}||\) versus \( t \) is shown in Table 2, and the
Table 1: The F-MRLS estimates and errors in Example 1 ($\sigma^2 = 0.50^2$ and $\sigma^2 = 1.00^2$).

<table>
<thead>
<tr>
<th>$\sigma^2$</th>
<th>$t$</th>
<th>$B(1, 1)$</th>
<th>$B(2, 1)$</th>
<th>$B(1, 2)$</th>
<th>$B(2, 2)$</th>
<th>$c_2$</th>
<th>$d_1$</th>
<th>$\delta$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50^2</td>
<td>100</td>
<td>0.10999</td>
<td>-1.60279</td>
<td>0.39771</td>
<td>0.12515</td>
<td>-0.16795</td>
<td>0.46319</td>
<td>33.14292</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.08155</td>
<td>-1.44298</td>
<td>0.31145</td>
<td>0.09590</td>
<td>-0.13875</td>
<td>0.51078</td>
<td>21.01533</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.10144</td>
<td>-1.33934</td>
<td>0.25923</td>
<td>0.08705</td>
<td>-0.15442</td>
<td>0.60335</td>
<td>11.18663</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.12452</td>
<td>-1.30385</td>
<td>0.26703</td>
<td>0.11599</td>
<td>-0.14796</td>
<td>0.65282</td>
<td>7.04142</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>0.13930</td>
<td>-1.27241</td>
<td>0.26711</td>
<td>0.12703</td>
<td>-0.14550</td>
<td>0.69136</td>
<td>4.66378</td>
</tr>
<tr>
<td></td>
<td>3000</td>
<td>0.14008</td>
<td>-1.25781</td>
<td>0.26567</td>
<td>0.11928</td>
<td>-0.14361</td>
<td>0.70371</td>
<td>4.03373</td>
</tr>
<tr>
<td>1.00^2</td>
<td>100</td>
<td>0.09530</td>
<td>-1.72159</td>
<td>0.52726</td>
<td>0.13387</td>
<td>-0.15869</td>
<td>0.52551</td>
<td>42.21841</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.03631</td>
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<td>0.36553</td>
<td>0.07113</td>
<td>-0.11792</td>
<td>0.58022</td>
<td>25.56872</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.07490</td>
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<td>0.25492</td>
<td>0.04715</td>
<td>-0.16037</td>
<td>0.64763</td>
<td>15.50868</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.12015</td>
<td>-1.36311</td>
<td>0.27803</td>
<td>0.01606</td>
<td>-0.15335</td>
<td>0.68001</td>
<td>11.08080</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>0.14849</td>
<td>-1.31457</td>
<td>0.28056</td>
<td>0.12704</td>
<td>-0.15048</td>
<td>0.70624</td>
<td>7.98334</td>
</tr>
<tr>
<td></td>
<td>3000</td>
<td>0.15005</td>
<td>-1.29138</td>
<td>0.27909</td>
<td>0.10998</td>
<td>-0.14680</td>
<td>0.71335</td>
<td>6.79480</td>
</tr>
</tbody>
</table>

True values: $\theta = [1.00, 0.14, -0.19, 1.19, 0.50]^T$.

Table 2: The MRLS and F-MRLS estimates and errors in Example 1 ($\sigma^2 = 0.50^2$).

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>$t$</th>
<th>$B(1, 1)$</th>
<th>$B(2, 1)$</th>
<th>$B(1, 2)$</th>
<th>$B(2, 2)$</th>
<th>$c_2$</th>
<th>$d_1$</th>
<th>$\delta$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRLS</td>
<td>100</td>
<td>-0.01946</td>
<td>-1.84785</td>
<td>0.44644</td>
<td>0.04712</td>
<td>-0.16528</td>
<td>0.39802</td>
<td>52.11950</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>-0.03340</td>
<td>-1.60645</td>
<td>0.31299</td>
<td>0.05091</td>
<td>-0.14456</td>
<td>0.46017</td>
<td>34.45327</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.03788</td>
<td>-1.39775</td>
<td>0.28600</td>
<td>0.03749</td>
<td>-0.14940</td>
<td>0.56178</td>
<td>18.20046</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.08074</td>
<td>-1.34458</td>
<td>0.28178</td>
<td>0.07949</td>
<td>-0.14078</td>
<td>0.61722</td>
<td>11.71737</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>0.11167</td>
<td>-1.28751</td>
<td>0.27285</td>
<td>0.10106</td>
<td>-0.13785</td>
<td>0.66411</td>
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<tr>
<td></td>
<td>3000</td>
<td>0.12600</td>
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<td>0.26866</td>
<td>0.10656</td>
<td>-0.13729</td>
<td>0.68367</td>
<td>4.99772</td>
</tr>
<tr>
<td>F-MRLS</td>
<td>100</td>
<td>0.10919</td>
<td>-1.60279</td>
<td>0.39771</td>
<td>0.12515</td>
<td>-0.16795</td>
<td>0.46319</td>
<td>33.14292</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.08155</td>
<td>-1.44298</td>
<td>0.31145</td>
<td>0.09590</td>
<td>-0.13875</td>
<td>0.51078</td>
<td>21.01533</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.01444</td>
<td>-1.33934</td>
<td>0.25923</td>
<td>0.08705</td>
<td>-0.15442</td>
<td>0.60335</td>
<td>11.18663</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.02452</td>
<td>-1.30385</td>
<td>0.26703</td>
<td>0.11599</td>
<td>-0.14796</td>
<td>0.65282</td>
<td>7.04142</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>0.13930</td>
<td>-1.27241</td>
<td>0.26711</td>
<td>0.12703</td>
<td>-0.14550</td>
<td>0.69136</td>
<td>4.66378</td>
</tr>
<tr>
<td></td>
<td>3000</td>
<td>0.14008</td>
<td>-1.25781</td>
<td>0.26567</td>
<td>0.11928</td>
<td>-0.14361</td>
<td>0.70371</td>
<td>4.03373</td>
</tr>
</tbody>
</table>

True values: $\theta = [1.00, 0.14, -0.19, 1.19, 0.50]^T$.

Estimation errors $\delta = \| \tilde{\theta}(t) - \theta \| / \| \theta \|$ versus $t$ are shown in Figures 1 and 2.

Example 2. Consider the following 2-input 2-output Hammerstein FIR-MA system:

$$
\begin{align*}
\begin{bmatrix}
    y_1(t) \\
    y_2(t)
\end{bmatrix} &= \begin{bmatrix}
    \overline{u}_1(t) \\
    \overline{u}_2(t)
\end{bmatrix} + B \begin{bmatrix}
    \overline{u}_1(t-1) \\
    \overline{u}_2(t-1)
\end{bmatrix} \\
&+ \begin{bmatrix}
    v_1(t) \\
    v_2(t)
\end{bmatrix} + d_1 \begin{bmatrix}
    v_1(t-1) \\
    v_2(t-1)
\end{bmatrix},
\end{align*}
$$

where

$$
B(z) = \begin{bmatrix}
    0.14 & 0.25 \\
   -0.15 & 0.125
\end{bmatrix} z^{-1},
$$

$$
D(z) = 1 + d_2 z^{-1} = 1 + 0.50 z^{-1},
$$

$$
\overline{u}(t) = c_1 u(t) + c_2 u^2(t) + c_3 u^3(t),
$$

with

$$
c = [c_1, c_2, c_3]^T = [1.00, -0.19, 1.19]^T,
$$

$$
\theta = [0.14, -0.15, 0.25, 0.125, -0.19, 1.19, 0.50]^T.
$$

FIGURE 1: The F-RLS estimation errors $\delta$ versus $t$ in Example 1 ($\sigma^2 = 0.50^2$ and $\sigma^2 = 1.00^2$).
Table 3: The F-RLS estimates and errors in Example 2 ($\sigma^2 = 0.50^2$ and $\sigma^2 = 1.00^2$).

<table>
<thead>
<tr>
<th>$\sigma^2$</th>
<th>$t$</th>
<th>$B(1,1)$</th>
<th>$B_{2,1}$</th>
<th>$B_{2,2}$</th>
<th>$c^2_1$</th>
<th>$c^2_2$</th>
<th>$d_1$</th>
<th>$\delta$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50^2</td>
<td>100</td>
<td>0.12373</td>
<td>-1.02307</td>
<td>0.27283</td>
<td>0.10358</td>
<td>-0.39300</td>
<td>1.25093</td>
<td>0.32512</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.12306</td>
<td>-1.03981</td>
<td>0.24977</td>
<td>0.11748</td>
<td>-0.29306</td>
<td>1.19925</td>
<td>0.35407</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.13109</td>
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<td>0.24655</td>
<td>0.11563</td>
<td>-0.25013</td>
<td>1.19098</td>
<td>0.40948</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.13728</td>
<td>-1.06400</td>
<td>0.25207</td>
<td>0.12027</td>
<td>-0.22484</td>
<td>1.21519</td>
<td>0.45612</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>0.14372</td>
<td>-1.06203</td>
<td>0.25315</td>
<td>0.12346</td>
<td>-0.20822</td>
<td>1.20110</td>
<td>0.49970</td>
</tr>
<tr>
<td></td>
<td>3000</td>
<td>0.14389</td>
<td>-1.05999</td>
<td>0.25321</td>
<td>0.12156</td>
<td>-0.20281</td>
<td>1.19897</td>
<td>0.51458</td>
</tr>
<tr>
<td>1.00^2</td>
<td>100</td>
<td>0.10227</td>
<td>-0.88895</td>
<td>0.29620</td>
<td>0.02428</td>
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<td>0.08815</td>
<td>-0.27093</td>
<td>1.24343</td>
<td>0.43981</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.13195</td>
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<td>0.25483</td>
<td>0.10541</td>
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<td>0.47397</td>
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<tr>
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<td>-0.21751</td>
<td>1.22939</td>
<td>0.51038</td>
</tr>
<tr>
<td></td>
<td>3000</td>
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<td>-1.06041</td>
<td>0.25650</td>
<td>0.11459</td>
<td>-0.20918</td>
<td>1.22061</td>
<td>0.52227</td>
</tr>
<tr>
<td>True values</td>
<td>0.14000</td>
<td>-1.05000</td>
<td>0.25000</td>
<td>0.12500</td>
<td>-0.19000</td>
<td>1.19000</td>
<td>0.50000</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: The comparison of parameter estimates and errors in Example 2 ($\sigma^2 = 0.50^2$).

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>$t$</th>
<th>$B_{2,1}$</th>
<th>$B_{2,2}$</th>
<th>$c^2_1$</th>
<th>$c^2_2$</th>
<th>$d_1$</th>
<th>$\delta$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRLS</td>
<td>100</td>
<td>0.10379</td>
<td>-1.49022</td>
<td>0.30877</td>
<td>0.11454</td>
<td>-0.37596</td>
<td>0.96745</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.08551</td>
<td>-1.31358</td>
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<td>3000</td>
<td>0.13715</td>
<td>-1.08439</td>
<td>0.25990</td>
<td>0.11297</td>
<td>-0.20411</td>
<td>1.18517</td>
</tr>
<tr>
<td>F-MRLS</td>
<td>100</td>
<td>0.12373</td>
<td>-1.02307</td>
<td>0.27283</td>
<td>0.10358</td>
<td>-0.39300</td>
<td>1.25093</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.12306</td>
<td>-1.03981</td>
<td>0.24977</td>
<td>0.11748</td>
<td>-0.29306</td>
<td>1.19925</td>
</tr>
<tr>
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<td>500</td>
<td>0.13109</td>
<td>-1.05853</td>
<td>0.24655</td>
<td>0.11563</td>
<td>-0.25013</td>
<td>1.19098</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.13728</td>
<td>-1.06400</td>
<td>0.25207</td>
<td>0.12027</td>
<td>-0.22484</td>
<td>1.21159</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>0.14372</td>
<td>-1.06203</td>
<td>0.25315</td>
<td>0.12346</td>
<td>-0.20822</td>
<td>1.20110</td>
</tr>
<tr>
<td></td>
<td>3000</td>
<td>0.14389</td>
<td>-1.05999</td>
<td>0.25321</td>
<td>0.12156</td>
<td>-0.20281</td>
<td>1.19897</td>
</tr>
<tr>
<td>True values</td>
<td>0.14000</td>
<td>-1.05000</td>
<td>0.25000</td>
<td>0.12500</td>
<td>-0.19000</td>
<td>1.19000</td>
<td>0.50000</td>
</tr>
</tbody>
</table>

Table 5: Comparison of the computational efficiency of the F-MRLS and MRLS algorithms.

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Number of multiplications</th>
<th>Number of additions</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRLS</td>
<td>$2n^2 + 5n$</td>
<td>$2n^2 + 2n$</td>
</tr>
<tr>
<td>F-MRLS</td>
<td>$2n^3 + 2n^2 + 3n^2 + 5n$</td>
<td>$2n^3 + 2n^2 + 3n^2 + 2n + 1$</td>
</tr>
</tbody>
</table>

In simulation, the inputs $\{u_1(t)\}$ and $\{u_2(t)\}$ and noise data $\{v_1(t)\}$ and $\{v_2(t)\}$ are settled in the same way as in Example 1, where variances $\sigma_1^2 = \sigma_2^2 = \sigma^2$ ($\sigma^2 = 0.50^2$). Applying the MRLS algorithm and the F-MRLS algorithm in (42)–(68), the F-MRLS parameter estimates and their estimation errors are shown in Table 3, the comparison between the F-MRLS algorithm and the MRLS algorithm in the estimation error $\delta := \|\hat{\theta}(t) - \theta\|/\|\theta\|$ versus $t$ is shown in Table 4, and the estimation errors $\hat{\theta} := \|\hat{\theta}(t) - \theta\|/\|\theta\|$ versus $t$ are shown in Figures 3 and 4.

To illustrate the advantages of the proposed algorithm, the numbers of multiplications and additions for each step of the F-MRLS algorithm and the MRLS algorithm are listed in Table 5.

From Tables 1–5, Figures 1–4, we can draw the following conclusions.

(i) The parameter estimation errors are getting smaller with $t$ increasing, which proves that the proposed algorithms are effective.
The F-MRLS algorithm is more accurate than the MRLS algorithm, which means the proposed F-MRLS algorithm has a better performance compared with the MRLS algorithm.

(iii) The parameter estimates given by the F-MRLS algorithm have faster convergence than those given by the MRLS algorithm.

5. Conclusions

This paper presents a data filtering based recursive least squares algorithm for MIMO nonlinear FIR-MA systems. The simulation results show that the proposed data filtering based recursive least squares algorithm is more accurate and reduces computational burden compared with the recursive least squares algorithm.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China (nos. 61174032 and 61202473), the Doctoral Foundation of Higher Education Priority Fields of Scientific Research (no. 20110093130001), and the Scientific Research Foundation of Jiangnan University (no. 1252050205135110) and by the PAPD of Jiangsu Higher Education Institutions.

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