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Research Article

A Cross-Efficiency Based Ranking Method for Finding the Most Efficient DMU

S. Sadeghi Gavgani and M. Zohrehbandian

Department of Mathematics, Islamic Azad University, Karaj Branch, Karaj, Iran

Correspondence should be addressed to M. Zohrehbandian; zohrebandian@yahoo.com

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In many applications of DEA, ranking of DMUs and finding the most efficient DMU are desirable, as reported by Toloo (2013). In this paper, after introducing an improvement to the measure of cross-efficiency by Jahanshahloo et al. (2011), we develop a new ranking method under the condition of variable returns to scale (VRS). Numerical example illustrates the effectiveness of the proposed cross-efficiency based ranking method and demonstrates the advantages of our proposal, against the other ranking approaches.

1. Introduction

Data envelopment analysis (DEA) provides a relative efficiency measure to evaluate decision making units (DMUs) with multiple inputs and multiple outputs. While it is an effective approach in identifying the best practice frontier, its flexibility in selecting the input/output weights and its nature of self-evaluation may result in a relatively high number of efficient DMUs. The lack of discrimination between efficient DMUs has been considered as an important problem in DEA models and subgroups of papers have been developed in this field in which many researchers have sought to improve the differential capabilities of DEA and to fully rank both efficient and inefficient DMUs.

Since these ranking methods have been developed based on some of the aspects of production possibility set (PPS), in certain cases, different calculations are reached in applying the alternative ranking methods. Furthermore, for each method, there are problematic areas, for example, infeasibility and instability of the proposed model. Hence, whilst each ranking technique is useful in a specialist area, no methodology can be prescribed as the complete solution to the question of ranking.

To increase the discrimination power of DEA models and make its weights more realistic, cross-efficiency evaluation has been suggested by Sexton et al. [1] and was later investigated by Doyle and Green [2, 3]. The basic idea of the cross-efficiency evaluation is to evaluate the overall efficiencies of the DMUs through both self- and peer-evaluations and can usually provide a full ranking for the DMUs to be evaluated. Therefore, it has found a significant number of applications in various fields; see Green et al. [4], Sun and Lu [5], Bao et al. [6], Wu et al. [7, 8], and Yang et al. [9].

However, the nonuniqueness of the DEA optimal weights possibly reduces the usefulness of cross-efficiency evaluation. It is due to this reason that the cross-efficiency evaluation has also been extensively investigated theoretically. Sexton et al. [1] were the first who recommended the use of aggressive and benevolent formulations of secondary objectives to deal with the nonuniqueness issue. Recently, Jahanshahloo et al. [10] proposed the symmetric weight assignment technique (SWAT) that does not affect feasibility and rewards DMUs that make a symmetric selection of weights. Similar thoughts also appeared in the articles of Anderson et al. [11], Liang et al. [12, 13], Wu et al. [14–16], Lam [17], Ramón et al. [18, 19], Wang et al. [20–24], Örkcü and Bal [25], Contreras [26], Lim [27], Ruiz and Sirvent [28], Jeong and Ok [29], Zerafat Angiz et al. [30], and Washio et al. [31].

This paper develops a new ranking method under the condition of VRS, which is based on introducing an improvement to Jahanshahloo et al.'s measure of cross-efficiency [10]. That is, the proposed cross-evaluation method is developed as a BCC extension tool that can be utilized to rank the DMUs using cross-efficiency scores and to identify the most BCC efficient DMU.

The rest of the paper is organized as follows. Section 2 gives a brief introduction to the DEA models. The mathematical foundation of our method to propose a secondary objective function in VRS cross-evaluation is discussed in Section 3. In Section 4, numerical example illustrates the effectiveness of the proposed cross-efficiency based ranking method. Finally, Section 5 is devoted to concluding remarks.

2. DEA Background

Consider n DMUs that are evaluated in terms of m inputs and s outputs. Let x_{ij} and y_{rj}, $i = 1, 2, \ldots, m$, $r = 1, 2, \ldots, s$, be their input and output values for DMU_j $j = 1, 2, \ldots, n$. The CCR efficiency of n DMUs is measured by

$$\begin{aligned} \max \quad & \frac{u^t y_0}{v^t x_0} \\ \text{s.t} \quad & \frac{u^t y_j}{v^t x_j} \leq 1, \quad j = 1, 2, \ldots, n, \\ & u \geq 0, \quad v \geq 0, \end{aligned} \tag{1}$$

where u and v are input and output weight vectors and x_0 and y_0 are the input and output vectors for the DMU under evaluation, respectively. Model (1) can be transformed to the following LP format, called input oriented CCR model:

$$\begin{aligned} \max \quad & u^t y_0 \\ \text{s.t} \quad & v^t x_0 = 1, \\ & u^t Y - v^t X \leq 0, \\ & u \geq 0, \quad v \geq 0. \end{aligned} \tag{2}$$

Note that model CCR in oriented output is as follows:

$$\begin{aligned} \min \quad & v^t x_0 \\ \text{s.t} \quad & u^t y_0 = 1, \\ & u^t Y - v^t X \leq 0, \\ & u \geq 0, \quad v \geq 0. \end{aligned} \tag{3}$$

Previous models have constant returns to scale (CRS) characteristic. Banker et al. [32] proposed BCC model which has variable returns to scale (VRS). The LP form of this model in oriented output is as follows:

$$\begin{aligned} \min \quad & v^t x_0 - v_0 \\ \text{s.t} \quad & u^t y_0 = 1, \\ & u^t Y - v^t X + v_0 \leq 0, \\ & u \geq 0, \quad v \geq 0, \quad v_0 \text{ free.} \end{aligned} \tag{4}$$

The optimal objective value of this model is BCC efficiency score of DMU_0 in oriented output.

However, in classical DEA models, no preference information is needed and the weights are allowed total flexibility to obtain maximum efficiency. Thompson et al. [33] were the first who studied the role of weight restrictions in DEA models. Note that the above DEA models have alternative optimal solutions and free selection of weights may lead one DMU to have two types of weights. In one type, all of positive weights are on one group of variables and another with its weights symmetrically allocated to all variables. Dimitrov and Sutton [34] proposed symmetry measure which was the relative weight of each output dimension to all other output dimensions as follows:

$$\left| u_i y_{i0} - u_j y_{j0} \right| = Z_{ij}, \quad \forall i, j, \tag{5}$$

where Z_{ij} is the difference in symmetry between output dimension i and dimension j for all DMUs under evaluation. They minimized the sum of all the Z_{ij} values ($\sum_{i,j} Z_{ij} = e^T Z e$, where $e = [1, 1, \ldots, 1]^T$ is an s-dimensional vector) and then effectively rewarded symmetry with asymmetry scaling factor $\beta \geq 0$. By adding this constraint to model (3), we have

$$\begin{aligned} \min_{u,v} \quad & v^t x_0 + \min_{Z} \beta e^t Z e \\ \text{s.t} \quad & u^t y_0 = 1, \\ & u^t Y - v^t X \leq 0, \\ & \left| u_i y_{i0} - u_j y_{j0} \right| = Z_{ij}, \quad \forall i, j \\ & u \geq 0, \quad v \geq 0. \end{aligned} \tag{6}$$

But model (6) is not an LP model. Fortunately, as we are minimizing $e^t Z e$, we may change the equality to $\leq$ as any optimal solution will have the equality constraint satisfied. With this observation, Dimitrov and Sutton [34] rewrite (6) as follows, where its objective function is $\in [1, \infty)$ and has the same feasibility region as LP model (3):

$$\begin{aligned} \min_{u,v,z} \quad & v^t x_0 + \beta e^t Z e \\ \text{s.t} \quad & u^t y_0 = 1, \\ & u^t Y - v^t X \leq 0, \\ & u_i y_{i0} - u_j y_{j0} \leq Z_{ij}, \quad \forall i, j \\ & - u_i y_{i0} + u_j y_{j0} \leq Z_{ij}, \quad \forall i, j \\ & u \geq 0, \quad v \geq 0. \end{aligned} \tag{7}$$

The resulting objective function value of the LP (7) is referred to as the SWAT score with smaller scores being more desirable. Here, we want to extend the above model to BCC model and variable returns to scale environment. It is straightforward that the SWAT model can easily adapt to situation with variable returns to scale in BCC model as follows:

$$\begin{aligned}\min_{u,v,v_0,z} \quad & v^t x_0 - v_0 + \beta e^t Z e \\ \text{s.t} \quad & u^t y_0 = 1, \\ & u^t Y - v^t X + v_0 \le 0, \\ & u_i y_{i0} - u_j y_{j0} \le Z_{ij}, \quad \forall i, j \\ & -u_i y_{i0} + u_j y_{j0} \le Z_{ij}, \quad \forall i, j \\ & u \ge 0, \quad v \ge 0, \quad v_0 \text{ free},\end{aligned} \tag{8}$$

where the objective function is $\in [1, \infty)$ and the β factor is a nonnegative factor, which determines how much a particular DMU will be penalized for an asymmetric selection of virtual weights. Note that $\beta = 0$ is equivalent to the original oriented output BCC model and $\beta \to \infty$ is equivalent to the situation that all DMUs select equal virtual weights for all outputs. However, ideal β value can be determined by the decision maker (DM).

3. BCC Cross-Efficiency Evaluation

Jahanshahloo et al. [10], based on model (7), suggested a secondary goal for cross-efficiency evaluation in CRS environment. In this section and based on model (8), we propose an improvement to their method to propose a secondary objective function in VRS cross-evaluation.

3.1. Cross-Efficiency for CCR Model. Let u^*_{r0} $(r = 1, 2, \ldots, s)$ and v^*_{i0} $(i = 1, 2, \ldots, m)$ be the optimal solution to model (7). Then, $\theta^*_{00} = \sum_{r=1}^{s} u^*_{r0} y_{r0}$ is referred to as the CCR efficiency of DMU_0 by self-evaluation. If $\theta^*_{00} = 1$, DMU_0 is CCR efficient and otherwise it is CCR inefficient. Moreover,

$$\theta_{j0} = \frac{\sum_{r=1}^{s} u^*_{r0} y_{rj}}{\sum_{i=1}^{m} v^*_{i0} x_{ij}}, \quad j = 1, 2, \ldots, n, \tag{9}$$

is referred to as DMU_j efficiency peer-evaluated by DMU_0 $(j = 1, 2, \ldots, n,\ j \ne 0)$. As a result, each DMU has one CCR efficiency and $(n - 1)$ peer-efficiencies and CCR cross-efficiency score of DMU_0 is calculated as follows:

$$\overline{\theta}_0 = \frac{\sum_{j=1}^{n} \theta_{j0}}{n}. \tag{10}$$

It is noticed that DEA models may have multiple optimal solutions. This nonuniqueness of input/output optimal weights would damage the use of cross-efficiency concept due to the ambiguity in using weights for execution of final results. To resolve this ambiguity, alternative secondary goals in cross-efficiency evaluation have been introduced.

3.2. Cross-Efficiency for BCC Model. Let u^*_{r0}, $r = 1, 2, \ldots, s$, v^*_{i0}, $i = 1, 2, \ldots, m$, v^*_0, be the optimal solution of model (4). Then, $E^*_{00} = \sum_{i=1}^{m} v_{i0} x_{i0} - v^*_0$ is BCC efficiency score of DMU_0 by self-evaluation. If $E^*_{00} = 1$, the DMU_0 is called BCC efficient and otherwise it is called BCC inefficient. Moreover, the BCC peer-evaluated efficiency of DMU_j $(j = 1, 2, \ldots, n)$ by DMU_0 is calculated by

Table 1: Data set for six nursing homes.

DMUs	O_1	O_2	I_1	I_2
DMU_1	1.4	0.35	1.5	0.2
DMU_2	1.4	2.1	4	0.7
DMU_3	4.2	1.05	3.2	1.2
DMU_4	2.8	4.2	5.2	2
DMU_5	1.9	2.5	3.5	1.2
DMU_6	1.4	1.5	3.2	0.7

$$E^*_{j0} = \frac{\sum_{i=1}^{m} v^*_{i0} x_{ij} - v_0}{\sum_{r=1}^{s} u^*_{i0} y_{rj}}. \tag{11}$$

The average of all E^*_{j0} is called BCC cross-efficiency score which is calculated as follows:

$$E_0 = \frac{\sum_{j=1}^{n} E^*_{j0}}{n} \quad j = 1, 2, \ldots, n. \tag{12}$$

Note that, because of using oriented output of BCC model, the DMU with lower cross-efficiency score has better rank.

However, the weights obtained from model (4) are usually not unique. As a result, the cross-efficiency is determined depending on the optimal solution arising from the particular LP software in use. As discussed before, Jahanshahloo et al. [10] suggested the use of symmetric weights as a secondary goal in CCR cross-efficiency evaluation. Here, to propose a secondary objective function in VRS cross-evaluation, we propose the following algorithm.

Step 1. Solve model (4) to determine BCC efficiency scores.

Step 2. To select the best weight from alternative optimal weights of model (4), based on the concept of symmetric weights introduced by model (8), solve LP model (13) to select suitable weight between alternative solutions. Consider

$$\begin{aligned}\min \quad & e^t Z_0 e \\ \text{s.t} \quad & \sum_{r=1}^{s} u_{r0} y_{r0} = 1, \\ & \sum_{i=1}^{m} v_{i0} x_{i0} - v_0 = E^*_{00} \\ & u_0^t Y - v_0^t X + v_0 \le 0, \\ & u_{i0} y_{i0} - u_{j0} y_{j0} \le Z_{ij}, \quad \forall i, j \ (I) \\ & -u_{i0} y_{i0} + u_{j0} y_{j0} \le Z_{ij}, \quad \forall i, j \ (II) \\ & u_0 \ge 0, \quad v_0 \ge 0, \quad v_0 \text{ free}.\end{aligned} \tag{13}$$

We add constraints $(I), (II)$ for selecting symmetric weights in BCC model by minimizing the sum of difference in

Table 2: Alternative optimal solutions for model proposed by Toloo and Nalchigar [35].

DMUs	M^*	v^*	u^*	v_0^*	d^*	β^*
DMU1	1	0.0001 0.4997	0.0001 0.0001	−0.0999	$d_7 = 0$ $d_j = 1, \quad j \neq 1$	(0.000, 0.75004, 0.5004,0.1006, 0.5003, 0.75)
DMU_2	1	0.00003 0.499	0.0003 0.143	−0.0498	$d_2 = 0$ $d_j = 1, \quad j \neq 2$	(0.9999, 0.000, 0.6015,0.6517, 0.808, 0.9142)
DMU_3	1	0.192 0.001	0.303 0.001	0.658	$d_3 = 0$ $d_j = 1, \quad j \neq 3$	(0.479, 0.000, 0.000,0.194, 0.247, 0.153)
DMU_4	1	0.192 0.001	0.078 0.157	−0.124	$d_4 = 0$ $d_j = 1, \quad j \neq 4$	(0.9999, 0.7942, 0.9994, 0.000, 0.9915, 0.8536)
DMU_5	1	0.192 0.00067	0.0451 0.1684	−0.1664	$d_5 = 0$ $d_j = 1, \quad j \neq 5$	(0.999, 0.8147, 0.9175,0.999, 0.000, 0.8673)

symmetry between output dimension i and dimension j for DMU_0 in objective function. Selecting symmetric weights is desirable, because, in application, centralization of weights in one group of variables may not be acceptable.

Step 3. Compute the BCC cross-efficiency score of DMU_j based on the formulations (11) and (12), where $(u_0^*, v_0^*, v_0'^*)$ is optimal solution of model (13). Finally, DMU_0 with the lowest cross-efficiency score is the most BCC efficient DMU.

Note that we can use model (8) instead of Steps 1 and 2 of the above algorithm. As discussed before, β is nonnegative factor where $\beta = 0$ results in the classical BCC model in oriented output and, for $\beta = 1$, model (8) is equivalent to model (13).

However, the ideal value for parameter β can be determined by DM. In other words, as it is discussed in Dimitrov and Sutton [34], instead of (8) and for greater flexibility, we can use the following model which puts greater burden on the DM to define the appropriate β_{ij} values:

$$\begin{aligned} \min_{u,v,v_0,Z} \quad & v^t x_0 - v_0 + \sum_{i,j} \beta_{ij} Z_{ij} \\ \text{s.t} \quad & u^t y_0 = 1, \\ & u^t Y - v^t X + v_0 \leq 0, \\ & u_i y_{i0} - u_j y_{j0} \leq Z_{ij}, \quad \forall i, j \\ & -u_i y_{i0} + u_j y_{j0} \leq Z_{ij}, \quad \forall i, j \\ & u \geq 0, \quad v \geq 0, \quad v_0 \text{ free.} \end{aligned} \tag{14}$$

In the next section, we use the proposed BCC cross-evaluation approach for ranking of DMUs in a numerical example and finding the most BCC efficient DMU. To demonstrate the advantages of our proposal, against the other approaches, we compare the results to the results of using the approach proposed by Toloo and Nalchigar [35].

4. Numerical Example

Six nursing homes are evaluated in terms of two inputs and two outputs. The data set is reported in Table 1. Input variables are

> I_1: staff hours per day including nurses and physicians,
>
> I_2: supplier per day, measured in thousands of dollars.

Output variables are

> O_1: total medicative plus medicated reimbursed patient days,
>
> O_2: total privately paid patient days.

By using mixed integer linear problem proposed by Toloo and Nalchigar [35], the most BCC efficient DMU is not unique and, based on the results depicted in Table 2, all of BCC efficient DMUs can choose as most BCC efficient DMU. This is due to the fact that the model proposed by Toloo and Nalchigar [35] has alternative optimal solutions.

Results of the proposed model (13) in this paper are presented in Table 3. The second column of this table shows BCC efficiency, and BCC cross-efficiency scores produced by our proposed method are depicted in the third column. Based on our procedure, DMU_2 is the most BCC efficient unit, alone. Moreover, to compare the obtained results with BCC model, in BCC model, five of six units are most efficient and choosing single most efficient unit is impossible.

Moreover, Table 4 shows cross-efficiency score based on model (14) with different amounts for β. Rank of DMUs is depicted in parentheses. It represents that DMU_2 is the most efficient DMU with three types of β, where, as mentioned before, factor β shows how a particular DMU will be penalized for an asymmetric choice of virtual weights.

5. Conclusion

In this paper, we proposed BCC cross-efficiency with model based on symmetric weight selection and by using it we discussed a procedure for finding the most BCC efficient DMU. Considerably, advantages of our cross-efficiency model are as

Table 3: Efficiency scores based on BCC model and BCC cross-efficiency model.

DMUs	BCC EFF	Cross-EFF (model (13))	Rank
DMU_1	1	1.18	3
DMU_2	1	1.091	1
DMU_3	1	1.152	2
DMU_4	1	1.445	6
DMU_5	1	1.181	4
DMU_6	1.1525	1.182	5

Table 4: Results for model (14) with different amounts of β.

DMUs	$\beta = 0.18$	$\beta = 1$	$\beta \gg 1$
DMU_1	1.382(4)	1.18(3)	1.155(3)
DMU_2	1.115(1)	1.091(1)	1.086(1)
DMU_3	1.38(3)	1.152(2)	1.087(2)
DMU_4	1.42(6)	1.445(6)	1.46(6)
DMU_5	1.2(2)	1.181(4)	1.18(5)
DMU_6	1.391(5)	1.182(5)	1.16(4)

follows: it is a linear program which is always feasible, it gives full ranking for DMUs, and it can find single most efficient unit.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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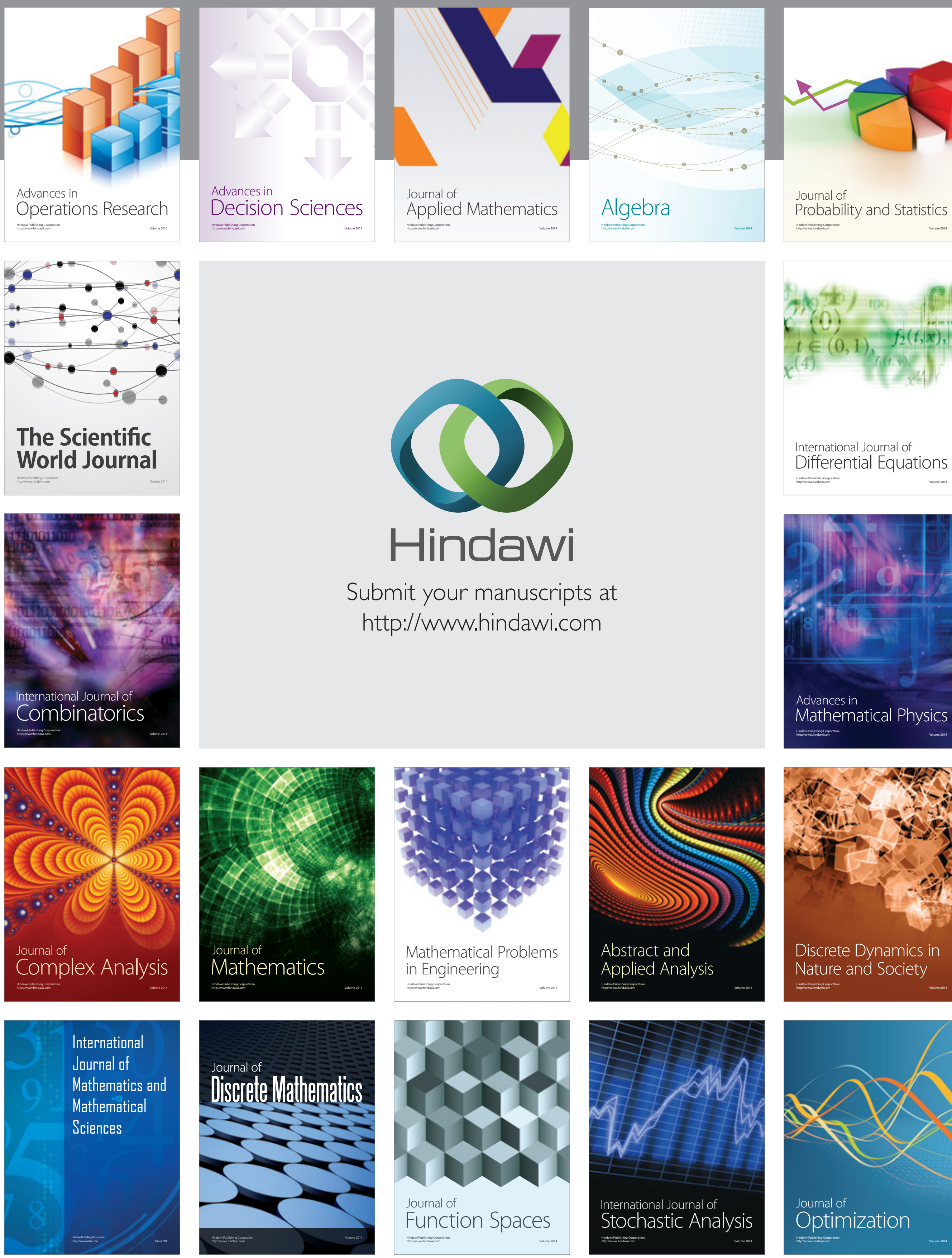
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