Research Article

Distributed Event-Triggered Control of Multiagent Systems with Time-Varying Topology

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This paper studies the consensus of first-order discrete-time multiagent systems, where the interaction topology is time-varying. The event-triggered control is used to update the control input of each agent, and the event-triggering condition is designed based on the combination of the relative states of each agent to its neighbors. By applying the common Lyapunov function method, a sufficient condition for consensus, which is expressed as a group of linear matrix inequalities, is obtained and the feasibility of these linear matrix inequalities is further analyzed. Simulation examples are provided to explain the effectiveness of the theoretical results.

1. Introduction

The distributed control of multiagent systems has received much attention because of its wide applications in scheduling of automated highway systems, cooperation control of multiple vehicles/robots, design of sensor networks, and so forth, and consensus is one of the hot topics in distributed control of multiagent systems [1–3].

Consensus of multiagent systems has been studied by researchers from different viewpoints, such as the dynamics of agents [4–7], the interaction topology [8–10], the convergence rate [11], and the information transmission [12–15]. In [12], consensus of first-order multiagent systems is considered in the cases with sampling delay and without sampling delay, respectively, and some sufficient and necessary conditions are provided in the case of fixed topology. In [13], consensus is analyzed for second-order multiagent systems with time-varying sampling intervals by applying matrix theory and Lyapunov stability theory, respectively. In [14], consensus of second-order multiagent systems is also discussed, where the sampling periods of agents are different and the topology at each time may not have spanning tree. In [15], consensus of second-order multiagent

systems with uniform and nonuniform sampling periods is considered, where only position information is transmitted among agents. It is noted that the work in [12–15] all assumes that the information received by agents at each time will be transmitted to controllers and thus consensus in these cases can be viewed as time-based consensus.

Compared with the time-based consensus, event-based consensus is more realistic, where the information received by agents is transmitted to controllers only when some events occur. Event-triggered control has some advantages such as reducing the number of transmissions and thus it is more suitable for cooperative control over networks with limited bandwidth [16]. There has been some work on event-based consensus of first-order continuous-time multiagent systems [17–22] and second-order continuous-time multiagent systems [20, 23–25]. In [17], a centralized event-triggering condition is provided and a sufficient condition of consensus is established in the cases of fixed topology and switching topology, where the topology at each time is strongly connected and balanced. In [18–20], a distributed event-triggering condition is given and some sufficient conditions for consensus are obtained in the case of fixed topology. Moreover, to avoid continuously monitoring whether the event-triggering

condition is satisfied, the self-triggered control, where the next update time of controller is predetermined, is also discussed. In [21, 22], each agent can only obtain information at some discrete times, and if the information at these discrete times satisfies the distributed event-triggering condition, then they will be used to update control input. Furthermore, sufficient conditions for consensus in the cases of fixed and switching topology, where the topology at each time is connected, are provided. In [20, 23, 24], some centralized or distributed event-triggering conditions are designed for second-order multiagent systems with fixed topology, where the position and velocity information of each agent share a common event-triggering condition. In addition, the leader-follower consensus based on event-triggered control is considered for the case of fixed topology in [25].

However, there is little work [26–28] on the event-based consensus of discrete-time multiagent systems. In [26], a centralized event-triggering condition is provided for first-order multiagent systems and some sufficient conditions are obtained. In [27, 28], the event-based consensus of heterogeneous discrete-time multiagent systems is discussed. Moreover, the above three papers all consider the fixed topology case. Based on the above observation, we study the event-based consensus of first-order discrete-time multiagent systems in the case of time-varying topology. The main contribution of our work is to design a distributed event-triggering condition and obtain a sufficient condition of consensus in the case of time-varying topology.

This paper is organized as follows. In Section 2, we present some concepts in graph theory and formulate the model to be studied. In Section 3, main results are stated. In Section 4, simulations are provided to illustrate the effectiveness of the theoretical results. Conclusion remarks are made in Section 5.

2. Preliminaries

2.1. Graph Theory. Graph plays a key role in modeling the interaction topology among agents. We first introduce some basic definitions in graph theory [29].

A directed graph $\mathcal{G}$ consists of a vertex set $\mathcal{V}(\mathcal{G})$ and an edge set $\mathcal{E}(\mathcal{G})$, where $\mathcal{V}(\mathcal{G}) = \{v_1, \dots, v_n\}$ and $\mathcal{E}(\mathcal{G}) \subset \{(v_j, v_i) : v_j, v_i \in \mathcal{V}(\mathcal{G})\}$. For edge (v_j, v_i) , v_j is called the parent vertex of v_i and v_i is called the child vertex of v_j . If two ends of an edge are the same vertex, then such an edge is called a self-loop. The set of neighbors of vertex v_i is defined by $N(\mathcal{G}, v_i) = \{v_j : (v_j, v_i) \in \mathcal{E}(\mathcal{G}) \text{ and } j \neq i\}$, and the associated index set is denoted by $N(\mathcal{G}, i) = \{j : v_j \in N(\mathcal{G}, v_i)\}$. A (directed) path from v_{i_1} to v_{i_k} is a sequence, $v_{i_1}, \dots, v_{i_k}$, of distinct vertices such that $(v_{i_j}, v_{i_{j+1}}) \in \mathcal{E}(\mathcal{G})$, $j = 1, \dots, k-1$. A directed graph $\mathcal{G}$ is strongly connected if there is a path from every vertex to every other vertex. A directed tree is a directed graph, where every vertex except one special vertex has exactly one parent vertex, and the special vertex, called root vertex, has no parent vertices and can be connected to any other vertices via paths. A subgraph $\mathcal{G}_s$ of $\mathcal{G}$ is a graph such that $\mathcal{V}(\mathcal{G}_s) \subset \mathcal{V}(\mathcal{G})$ and $\mathcal{E}(\mathcal{G}_s) \subset \mathcal{E}(\mathcal{G})$. $\mathcal{G}_s$ is said to be a spanning subgraph if $\mathcal{V}(\mathcal{G}_s) = \mathcal{V}(\mathcal{G})$. For any $v_i, v_j \in \mathcal{V}(\mathcal{G}_s)$, if $(v_i, v_j) \in \mathcal{E}(\mathcal{G}_s) \Leftrightarrow (v_i, v_j) \in \mathcal{E}(\mathcal{G})$, then $\mathcal{G}_s$ is said to be

an induced subgraph of $\mathcal{G}$ and $\mathcal{G}_s$ is also said to be induced by $\mathcal{V}(\mathcal{G}_s)$. A spanning tree of $\mathcal{G}$ is a directed tree which is a spanning subgraph of $\mathcal{G}$. $\mathcal{G}$ is said to have a spanning tree if some edges form a spanning tree of $\mathcal{G}$.

A matrix is called nonnegative if each of its elements is nonnegative. A weighted directed graph $\mathcal{G}(A)$ is a directed graph $\mathcal{G}$ plus a nonnegative matrix $A = [a_{ij}] \in \mathbb{R}^{n \times n}$, where $a_{ij} > 0 \Leftrightarrow (v_j, v_i) \in \mathcal{E}(\mathcal{G})$, and a_{ij} is called the weight of edge (v_j, v_i) . If $\sum_{j=1}^n a_{ij} = \sum_{j=1}^n a_{ji}$, $i = 1, \dots, n$, then $\mathcal{G}(A)$ is called balanced. If $A = A^T$, then $\mathcal{G}(A)$ is also called undirected, and $\mathcal{G}(A)$ is said to be connected if it, as a directed graph, is strongly connected. The Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{n \times n}$ of $\mathcal{G}(A)$ is defined as

$$l_{ij} = \begin{cases} -a_{ij}, & i \neq j \\ \sum_{s=1, s \neq i}^n a_{is}, & i = j. \end{cases} \quad (1)$$

The Laplacian matrix of $\mathcal{G}(A)$ has the following properties.

Lemma 1 (see [9]). *Consider the following:*

- (i) zero is an eigenvalue of L and $\mathbf{1}_n$ is the associated right eigenvector;
- (ii) zero is an algebraically simple eigenvalue of L and all the other eigenvalues are with positive real parts if and only if $\mathcal{G}(A)$ has a spanning tree.

Remark 2. In the time-varying topology case, we use $\mathcal{G}(t)$ to denote the topology graph at time t .

2.2. Model. Consider a group of agents with first-order discrete-time dynamics:

$$x_i(k+1) = x_i(k) + u_i(k), \quad i = 1, \dots, n, \quad k = 0, 1, 2, \dots, \quad (2)$$

where $x_i(k)$ is the state of agent i at time k and $u_i(k)$ is the control input, called the protocol, to be designed based on the information obtained by agent i . Without loss of generality, we assume $x_i \in \mathbb{R}$, $i = 1, \dots, n$.

Given u_i , $i = 1, \dots, n$, u_i or multiagent system (2) solves a consensus problem asymptotically if, for any initial states and any $i, j = 1, \dots, n$, $\lim_{k \rightarrow \infty} (x_i(k) - x_j(k)) = 0$.

Different from the previous work on consensus, not all the information obtained by agent i is used by controller u_i in our work. For agent i , when it receives new information, it will validate whether some condition, which is called event-triggering condition, is satisfied. If the event-triggering condition is satisfied, then the received information will not be used by its controller, or else the received information will be transmitted to its controller. The following event-triggering condition is considered:

$$|e_i(k)| \leq \sigma_i \left| \sum_{j \in N_i(k)} a_{ij}(k) (x_j(k) - x_i(k)) \right|, \quad i = 1, 2, \dots, n, \quad (3)$$

where $e_i(k) = \sum_{j \in N_i(t_l^i)} a_{ij}(t_l^i)(x_j(t_l^i) - x_i(t_l^i)) - \sum_{j \in N_i(k)} a_{ij}(k)(x_j(k) - x_i(k))$ and $t_0^i, t_1^i, \dots$ are event times of agent i ; namely, at these times, (3) is not satisfied and the controller of agent i will be updated. Hence, the controller has the following form:

$$u_i(k) = u_i(t_l^i), k \in \{t_l^i + 1, \dots, t_{l+1}^i - 1\}, i = 1, 2, \dots, n, \quad (4)$$

where $u_i(t_l^i)$ is dependent on the information received at time t_l^i ; namely,

$$u_i(t_l^i) = \varepsilon \sum_{j \in N_i(t_l^i)} a_{ij}(t_l^i)(x_j(t_l^i) - x_i(t_l^i)), \quad (5)$$

where $\varepsilon > 0$.

Remark 3. If (3) is satisfied at all times, then the consensus under consideration becomes the usual consensus of discrete time multiagent systems.

Remark 4. In the following analysis, we use another topology graph $\widehat{\mathcal{G}}(k)$, which is different from the actual topology graph $\mathcal{G}(k)$. $\widehat{\mathcal{G}}(k)$ is defined as follows: $\widehat{\mathcal{G}}(k)$ has the same vertex set as $\mathcal{G}(k)$; if $(v_j, v_i) \in \mathcal{E}(\mathcal{G}(t_l^i))$, then, for any $k \in \{t_l^i, t_l^i + 1, \dots, t_{l+1}^i - 1\}$, $(v_j, v_i) \in \mathcal{E}(\widehat{\mathcal{G}}(k))$, or else, for any $k \in \{t_l^i, t_l^i + 1, \dots, t_{l+1}^i - 1\}$, $(v_j, v_i) \notin \mathcal{E}(\widehat{\mathcal{G}}(k))$. For facilitating the following analysis, we assume that $\{\widehat{\mathcal{G}}(0), \widehat{\mathcal{G}}(1), \dots\}$ is a finite set.

3. Main Results

Obviously, $\sum_{j \in N_i(t_l^i)} a_{ij}(t_l^i)(x_j(t_l^i) - x_i(t_l^i)) = e_i(k) + \sum_{j \in N_i(k)} a_{ij}(k)(x_j(k) - x_i(k))$; then the multiagent system (2) under event-triggering condition (3) and controller (4) can be written as

$$X(k+1) = (I - \varepsilon \widehat{L}_k) X(k) + \varepsilon e(k), \quad (6)$$

where $\sigma = \max\{\sigma_1, \dots, \sigma_n\}$.

Proof. Let $V(k) = \delta(k)^T P \delta(k)$, where $P > 0$; then, by (9), we obtain

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) \\ &= \delta(k)^T \left[(I_{n-1} - \varepsilon \widehat{H}_k)^T P (I_{n-1} - \varepsilon \widehat{H}_k) - P \right] \delta(k) \\ &\quad + \varepsilon^2 e(k)^T U_1 P U_1^T e(k) \\ &\quad + 2\varepsilon e(k)^T U_1 P (I_{n-1} - \varepsilon \widehat{H}_k) \delta(k). \end{aligned} \quad (12)$$

where $X(k) = (x_1(k), x_2(k), \dots, x_n(k))^T$, $e(k) = (e_1(k), e_2(k), \dots, e_n(k))^T$ and $\widehat{L}_k$ denotes the Laplacian matrix of $\widehat{\mathcal{G}}(k)$.

Assume $\widehat{\mathcal{G}}(k)$ has a spanning tree for any $k = 0, 1, \dots$; then there exists an orthogonal matrix $U = [(1/\sqrt{n})\mathbf{1}_n, U_1] \in \mathbb{R}^{n \times n}$ such that

$$U^T \widehat{L}_k U = \begin{bmatrix} 0 & \widehat{h}_k \\ 0 & \widehat{H}_k \end{bmatrix}. \quad (7)$$

Let $\theta(k) = U^T X(k)$, where $\theta(k) = [\delta_1(k), \delta(k)^T]^T$, $\delta(k) \in \mathbb{R}^{n-1}$; then

$$\delta_1(k+1) = \delta_1(k) + \widehat{h}_k \delta(k) + \varepsilon \frac{1}{\sqrt{n}} \mathbf{1}_n^T e(k), \quad (8)$$

$$\delta(k+1) = (I_{n-1} - \varepsilon \widehat{H}_k) \delta(k) + \varepsilon U_1^T e(k). \quad (9)$$

By Remark 4, both $\{\widehat{H}_0, \widehat{H}_1, \dots\}$ and $\{\widehat{L}_0, \widehat{L}_1, \dots\}$ are finite sets; let

$$\begin{aligned} \{\widehat{L}_0, \widehat{L}_1, \dots\} &= \{\overline{L}_1, \overline{L}_2, \dots, \overline{L}_N\}, \\ \{\widehat{H}_0, \widehat{H}_1, \dots\} &= \{\overline{H}_1, \overline{H}_2, \dots, \overline{H}_N\}. \end{aligned} \quad (10)$$

By the result in [30], we obtain the following lemma.

Lemma 5. Multiagent system (2) under event-triggering condition (3) and controller (4) solves a consensus problem asymptotically if system (9) is asymptotically stable.

Hence, we can obtain the following main result by analyzing the stability of system (9). Because the topology graph is time-varying, we apply the common Lyapunov function to analyze the stability of system (9).

Theorem 6. Assume $\widehat{\mathcal{G}}(k)$ has a spanning tree at all times. Multiagent system (2) under event-triggering condition (3) and controller (4) solves a consensus problem asymptotically if there exists $P > 0$ such that the following linear matrix inequalities are satisfied:

$$\begin{pmatrix} (I_{n-1} - \varepsilon \overline{H}_i)^T P (I_{n-1} - \varepsilon \overline{H}_i) - P + \sigma^2 U_1^T \overline{L}_i^T \overline{L}_i U_1 & \varepsilon (I_{n-1} - \varepsilon \overline{H}_i)^T P U_1^T \\ \varepsilon^2 U_1 P U_1^T - I_{n-1} & \end{pmatrix} < 0, \quad i = 1, 2, \dots, N, \quad (11)$$

By event-triggering condition (3),

$$\begin{aligned} e(k)^T e(k) &\leq X(k)^T \widehat{L}_k^T \text{diag}\{\sigma_1^2, \dots, \sigma_n^2\} \widehat{L}_k X(k) \\ &\leq \sigma^2 X(k)^T \widehat{L}_k^T \widehat{L}_k X(k) \\ &= \sigma^2 X(k)^T U (U^T \widehat{L}_k^T \widehat{L}_k U) U^T X(k) \\ &= \sigma^2 \delta(k)^T U_1^T \widehat{L}_k^T \widehat{L}_k U_1 \delta(k). \end{aligned} \quad (13)$$

Thus,

$$\begin{aligned}\Delta V(k) &\leq \delta(k)^T \left[(I_{n-1} - \varepsilon \widehat{H}_k)^T P (I_{n-1} - \varepsilon \widehat{H}_k) - P \right] \delta(k) \\ &\quad + \varepsilon^2 e(k)^T U_1 P U_1^T e(k) \\ &\quad + 2\varepsilon e(k)^T U_1 P (I_{n-1} - \varepsilon \widehat{H}_k) \delta(k)\end{aligned}$$

where

$$\begin{aligned}&+ \sigma^2 \delta(k)^T U_1^T \widehat{L}_k^T \widehat{L}_k U_1 \delta(k) - e(k)^T e(k) \\ &= (\delta(k)^T, e(k)^T) \Phi_k (\delta(k)^T, e(k)^T)^T,\end{aligned}\tag{14}$$

$$\Phi_k = \begin{pmatrix} (I_{n-1} - \varepsilon \widehat{H}_k)^T P (I_{n-1} - \varepsilon \widehat{H}_k) - P + \sigma^2 U_1^T \widehat{L}_k^T \widehat{L}_k U_1 & \varepsilon (I_{n-1} - \varepsilon \widehat{H}_k)^T P U_1^T \\ * & \varepsilon^2 U_1 P U_1^T - I_{n-1} \end{pmatrix}.\tag{15}$$

By (11), $\Phi_k < 0, k = 0, 1, 2, \dots$; that is, system (9) is asymptotically stable. Hence, multiagent system (2) under event-triggering condition (3) and controller (4) solves a consensus problem asymptotically. $\square$

Remark 7. The feasibility of LMI (11) is explained as follows:

$$\begin{pmatrix} (I_{n-1} - \varepsilon H_i)^T P (I_{n-1} - \varepsilon H_i) - P + \sigma^2 U_1^T L_i^T L_i U_1 & \varepsilon (I_{n-1} - \varepsilon H_i)^T P U_1^T \\ * & \varepsilon^2 U_1 P U_1^T - I_{n-1} \end{pmatrix} = \begin{pmatrix} -\varepsilon (H_i^T P + P H_i) + \sigma^2 U_1^T L_i^T L_i U_1 & \varepsilon P U_1^T \\ * & -I_{n-1} \end{pmatrix} + O(\varepsilon^2),\tag{16}$$

where $O(\varepsilon^2)$ denotes a matrix, each element of which is the same order infinitesimal of ε^2 . By Schur complement,

$$\begin{pmatrix} -\varepsilon (H_i^T P + P H_i) + \sigma^2 U_1^T L_i^T L_i U_1 & \varepsilon P U_1^T \\ * & -I_{n-1} \end{pmatrix} < 0\tag{17}$$

if and only if

$$\begin{aligned}-\varepsilon (H_i^T P + P H_i) + \sigma^2 U_1^T L_i^T L_i U_1 &< 0, \\ -\varepsilon (H_i^T P + P H_i) + \sigma^2 U_1^T L_i^T L_i U_1 + \varepsilon^2 P U_1^T U_1 P &< 0.\end{aligned}\tag{18}$$

Hence, if there exists $P > 0$ such that $H_i^T P + P H_i > 0, i = 1, 2, \dots, N$, then (11) must be feasible in the case that ε and σ are small.

4. Simulations

Consider four agents and their interaction topology is time-varying. For convenience, assume that the topology is $\mathcal{G}_1$

and $\mathcal{G}_2$ at odd and even times, respectively. The Laplacian matrices of L_1 and L_2 are

$$\begin{aligned}L_1 &= \begin{pmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ -1 & -1 & 2 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}, \\ L_2 &= \begin{pmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}.\end{aligned}\tag{19}$$

By the LMI Toolbox in Matlab, (11) is feasible for $\varepsilon = 0.1$ and $\sigma = 0.1$. Hence, by Theorem 6, consensus can be reached asymptotically for controller (4) with $\varepsilon = 0.1$ and event-triggering condition (3) with $\max\{\sigma_i\} < 0.1$. The state trajectories of four agents in the case of $\varepsilon = 0.1, \sigma_1 = 0.01, \sigma_2 = 0.02, \sigma_3 = 0.03$, and $\sigma_4 = 0.05$ are shown in Figure 1, which validate the result of Theorem 6.

5. Conclusion

This paper has studied the consensus problem of first-order discrete-time multiagent systems with time-varying topology. Based on the designed event-triggering condition, each agent updates its control input only at event times. By

FIGURE 1: State trajectories of four agents.

applying a state transformation, consensus is transformed into the asymptotical stability of a time-varying system. In virtue of the common Lyapunov function method, we obtain that if the event-triggering conditions and controllers satisfy some linear matrix inequalities, then consensus can be reached asymptotically, and we further analyze the feasibility of these linear matrix inequalities. Our future work will focus on time delays and another time-varying topology, namely, the jointly connected case.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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