Research Article

A Modified D - K Iteration Approach for the Decentralized $\mathcal{H}_\infty$ Control of Civil Structures with Parametric Uncertainties

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This paper presents a robust decentralized $\mathcal{H}_\infty$ controller design method to suppress the vibration of civil structures with the consideration of parametric uncertainties. The decentralized $\mathcal{H}_\infty$ controller design is motivated by the double homotopy approach, which approximates the bilinear matrix inequality (BMI) derived from bounded real lemma to linear matrix inequality (LMI), and gradually deforms a centralized controller to a decentralized controller. The centralized $\mathcal{H}_\infty$ controller can be designed for the civil structures with the parametric uncertainties through D - K iteration method in μ synthesis, which can consider the diagonal block pattern of the uncertain matrix. This paper combines the double homotopy approach and D - K iteration method to design the robust decentralized $\mathcal{H}_\infty$ controller for the civil structures with parametric uncertainties. The proposed method is validated numerically with a four-story building example.

1. Introduction

Civil structures can be vibrated extremely and even damaged by the excessive earthquakes or wind load. Structural control is a new view to suppress the structural vibration. Active or semiactive control attracts more and more researcher attentions [1–7]. In active or semiactive control, a controller should be designed to complete a feedback loop, which contains sensors, actuators, and structures besides the controller. Traditional structural control adopts the centralized control, which needs all the sensor data and delivers the control command to all the actuators. For large-scale civil structures, the feedback loop with the centralized controller has high requirements on data transmission rate and embedded controller processor for computing, which may result in economical and technical difficulties.

Decentralized control is an available method to replace the centralized control, which makes the control decisions only according to the sensor data in the neighborhood of a control device [8, 9]. As a result, the time cost for data transmission and controller computing is reduced a lot. Qiu et al. considered the network-based environment with time delays [10–12]. For the decentralized control in civil structures, it has only been investigated in recent years. For example, Lynch

and Law [13] modified LQR control for decentralized control and used it in the controller design of large-scale civil structures. Loh and Chang [14] made comparisons between some centralized and decentralized control algorithms. Labibi et al. [15] designed the decentralized controller by reducing the multivariable system to an optimization problem with LMI constraints. Wang et al. [16, 17] considered the time delay and presented decentralized dynamic output feedback control schemes through homotopy approach. As an alternative, Mehendale and Grigoriadis [18] proposed a double homotopy approach for decentralized $\mathcal{H}_\infty$ control in continuous time domain. Motivated by Mehendale and Grigoriadis, Qu et al. [19] proposed a time-delayed decentralized $\mathcal{H}_\infty$ controller design scheme through a double homotopy approach in discrete time domain.

It should be noted that the performance of a designed controller is up to how precise the structural model is. For the civil structures, such as tall buildings and long-span bridges, they are more susceptible to the external excitation. Earthquakes or strong wind could vibrate the civil structures and make some damages to the structural components, which would result in the structural parametric uncertainties. On the other hand, civil infrastructures always have large scale and complex structure. The precise numerical model is hard

to build due to the neglect of the model high order or approximation of the structural complex part, which can also cause the uncertainties. These uncertainties can be reflected in the measurement data. The data-driven framework is an available method to obtain the uncertainty information, which is only dependent on the measured process variables [20, 21]. There are also some efforts on the robust controller design preconsidering the uncertainties. Wang et al. [22] studied the robust controller design for the uncertain structure, whose uncertainties located in the system matrices and control input matrices. Wang et al. [23] also developed a state feedback controller for the uncertainties in the disturbance input matrix. In the previous research, the parametric uncertainties are mixed up in the state-space matrices. Young [24] provided an introduction of design techniques for the system with parametric uncertainties through structured singular value approach. Moreno and Thomson [25] used D - K iteration method to design a tuned mass damper (TMD) for one degree-of-freedom system, which can consider the parametric uncertainties separately.

This paper considers the parametric uncertainties and presents a robust decentralized $\mathcal{H}_\infty$ controller design method. The parametric uncertainties exist in the mass, damping, and stiffness matrices and are extracted by linear fractional transformation (LFT). The design method is to modify D - K iteration by combining the double homotopy approach to search the decentralized $\mathcal{H}_\infty$ controller and the D - K iteration approach to consider the uncertainties. The proposed method is validated numerically by a four-story structure with two actuators.

2. Problem Formulation

For an n degree-of-freedom (DOF) shear-frame civil structure instrumented with n_u control devices, the equations of motion can be formulated as

$$M_s \ddot{q}(t) + C_s \dot{q}(t) + K_s q(t) = T_w w(t) + T_u u(t), \quad (1)$$

where $q(t) \in \mathcal{R}^{n \times 1}$ is the displacement vector relative to the ground; the mass, damping, and stiffness constant matrices are denoted by M_s, C_s , and $K_s \in \mathcal{R}^{n \times n}$, respectively; $w(t) \in \mathcal{R}^{n_w \times 1}$ and T_w are the external excitation vector and its location matrix, respectively; $u(t) \in \mathcal{R}^{n_u \times 1}$ and T_u are the control force vector and its location matrix, respectively. If the external excitation w is unidirectional earthquake acceleration $\ddot{q}_g(t)$, the excitation location matrix $T_w = -M\{1\}_{n \times 1}$. However, when there are some uncertainties in these structural parameters, such as mass, damping, and stiffness, the matrices M_s, C_s , and K_s in (1) can be formulated as

$$M_s = M_{\text{nom}} (I + P_M \delta_M), \quad (2a)$$

$$C_s = C_{\text{nom}} (I + P_C \delta_C), \quad (2b)$$

$$K_s = K_{\text{nom}} (I + P_K \delta_K), \quad (2c)$$

where M_s, C_s , and K_s can be recognized as the structural parameters with uncertainties; $M_{\text{nom}}, C_{\text{nom}}$, and K_{nom} are

the nominal value part of M_s, C_s , and K_s , respectively; P_M, P_C , and P_K are the matrices to describe the worst uncertainties; δ_M, δ_C , and δ_K are the norm bounded matrices, whose entry values are perturbed. For example, when the DOF $n = 1$, there are 10%, 20%, and 30% uncertainties in the mass, damping, and stiffness, respectively. It can be derived that $P_M = 0.1$, $P_C = 0.2$, $P_K = 0.3$, and $-1 \leq \delta_M, \delta_C, \delta_K \leq 1$. It is obvious that P_M, P_C , and P_K are constant matrices with given values, which can be recognized as the part of nominal system. And δ_M, δ_C , and δ_K are the uncertainties due to their perturbation. In the state-space representation or the dynamic equation solving process, the mass matrix M_s is always used in its inverse version. Equations (2a), (2b), and (2c) can be deduced as

$$M_s^{-1} = -P_M \delta_M (I + P_M \delta_M)^{-1} M_{\text{nom}}^{-1} + M_{\text{nom}}^{-1}, \quad (3a)$$

$$C_s = C_{\text{nom}} P_C \delta_C (I - 0 \cdot \delta_C)^{-1} \cdot I + C_{\text{nom}}, \quad (3b)$$

$$K_s = K_{\text{nom}} P_K \delta_K (I - 0 \cdot \delta_K)^{-1} \cdot I + K_{\text{nom}}. \quad (3c)$$

It should be noted that the absolute values of the perturbed uncertainties $M_{\text{nom}} P_M \delta_M$, $C_{\text{nom}} P_C \delta_C$, and $K_{\text{nom}} P_K \delta_K$ are smaller than the values of $M_{\text{nom}}, C_{\text{nom}}$, and K_{nom} . So $I + P_M \delta_M$ cannot be zero matrix, which means that the matrix $I + P_M \delta_M$ is invertible. According to the linear fractional transformations (LFTs) in the chapter 9 of [26], (3a), (3b), and (3c) have the similar format to the upper LFT formulation and can be described as in Figures 1(a)–1(c).

From the upper LFTs, the parameter uncertainties can be extracted from the dynamic equation of the civil structure as shown in Figure 1(d). Δ is the diagonal combination of δ_M, δ_C , and δ_K :

$$\Delta = \begin{pmatrix} \delta_M & 0 & 0 \\ 0 & \delta_C & 0 \\ 0 & 0 & \delta_K \end{pmatrix}. \quad (4)$$

P denotes the augmented system and can be represented in state-space:

$$\begin{aligned} \dot{x}_c(t) &= A_P x_c(t) + B_{1P} r(t) + B_{2P} w(t) + B_{3P} u(t), \\ y_r(t) &= C_{1P} x_c(t) + D_{11P} r(t) + D_{12P} w(t) + D_{13P} u(t), \\ z(t) &= C_{2P} x_c(t) + D_{21P} r(t) + D_{22P} w(t) + D_{23P} u(t), \\ m(t) &= C_{3P} x_c(t) + D_{31P} r(t) + D_{32P} w(t) + D_{33P} u(t), \end{aligned} \quad (5)$$

where $x_c(t) = [q \ \dot{q}]^T \in \mathcal{R}^{2n \times 1}$ is the system state vector; $y_r(t) = [y_M \ y_C \ y_K]^T$ denotes the input vector of the uncertainties; $z(t)$ is the system response output vector; $m(t)$ is the system measurement output vector; $r(t) = [u_M \ u_C \ u_K]^T$ is the output vector of the uncertainties. The values for the matrices in the third and fourth expressions of (5) will be determined according to the response output z and measurement m . Other matrices in (5) can be derived as

FIGURE 1: Diagram of upper LFTs for M_s^{-1} , C_s , and K_s .

FIGURE 2: Diagram of the structural control system.

$$\begin{bmatrix} A_P & B_{1P} & B_{2P} & B_{3P} \\ C_{1P} & D_{11P} & D_{12P} & D_{13P} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ -M_{\text{nom}}^{-1}K_{\text{nom}} & -M_{\text{nom}}^{-1}C_{\text{nom}} & -P_M & -M_{\text{nom}}^{-1}C_{\text{nom}}P_C & -M_{\text{nom}}^{-1}K_{\text{nom}}P_K & M_{\text{nom}}^{-1}T_w & M_{\text{nom}}^{-1}T_u \\ -M_{\text{nom}}^{-1}K_{\text{nom}} & -M_{\text{nom}}^{-1}C_{\text{nom}} & -P_M & -M_{\text{nom}}^{-1}C_{\text{nom}}P_C & -M_{\text{nom}}^{-1}K_{\text{nom}}P_K & M_{\text{nom}}^{-1}T_w & M_{\text{nom}}^{-1}T_u \\ 0 & I & 0 & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (6)$$

To facilitate the decentralized feedback formulation, the structural state vector x_c in (5) can be organized as

$$x_T = [q_1 \quad \dot{q}_1 \quad q_2 \quad \dot{q}_2 \quad \cdots \quad q_n \quad \dot{q}_n]^T. \quad (7)$$

The continuous-time system can be discretized to discrete-time system using sampling time ΔT through zero-order hold criteria [27]:

$$\begin{aligned} x_d[k+1] &= A_d x_d[k] + B_{1d} r[k] + B_{2d} w[k] + B_{3d} u[k], \\ y_r[k] &= C_{1P} x_d[k] + D_{11P} r[k] + D_{12P} w[k] + D_{13P} u[k], \\ z[k] &= C_{2P} x_d[k] + D_{21P} r[k] + D_{22P} w[k] + D_{23P} u[k], \\ m[k] &= C_{3P} x_d[k] + D_{31P} r[k] + D_{32P} w[k] + D_{33P} u[k], \end{aligned} \quad (8)$$

where A_d , B_{1d} , B_{2d} , and B_{3d} are given as

$$\begin{aligned} A_d &= e^{A_P \Delta T}, & B_{1d} &= \int_0^{\Delta T} e^{A_P \tau} B_{1P} d\tau, \\ B_{2d} &= \int_0^{\Delta T} e^{A_P \tau} B_{2P} d\tau, & B_{3d} &= \int_0^{\Delta T} e^{A_P \tau} B_{3P} d\tau. \end{aligned} \quad (9)$$

To consider the feedback time delay, an auxiliary LTI (linear time-invariant) system to describe the time delay can be concatenated to the structural system. The structural system in (8) is denoted by P_T and the combined system is shown in Figure 2 [16].

As described in [16], through cascading the structural system P_T and time delay system, the open-loop system in Figure 2 can be formulated as

$$\begin{aligned} x_s[k+1] &= A x_s[k] + B_1 r[k] + B_2 w[k] + B_3 u[k], \\ y_r[k] &= C_1 x_s[k] + D_{11} r[k] + D_{12} w[k] + D_{13} u[k], \\ z[k] &= C_2 x_s[k] + D_{21} r[k] + D_{22} w[k] + D_{23} u[k], \\ y[k] &= C_3 x_s[k] + D_{31} r[k] + D_{32} w[k] + D_{33} u[k], \end{aligned} \quad (10)$$

where $x_s(t) \in \mathcal{R}^{n_{\text{OL}} \times 1}$ is the open-loop system state vector, which contains structural system state vector, $x_d \in \mathcal{R}^{2n \times 1}$, and time-delay system state vector, $x_{\text{TD}} \in \mathcal{R}^{n_{\text{TD}} \times 1}$. $y \in \mathcal{R}^{n_y \times 1}$ is the sensor measurement with time delay. The controller system K in Figure 2, which is described as (11), should

be designed to complete the feedback loop and reduce the response output z :

$$\begin{aligned} x_K[k+1] &= A_K x_K[k] + B_K y[k], \\ u[k] &= C_K x_K[k] + D_K y[k], \end{aligned} \quad (11)$$

where $x_K \in \mathbb{R}^{n_K \times 1}$ is the controller state vector; A_K , B_K , C_K , and D_K are the constant matrices of the controller, whose values should be determined through the controller design.

3. Robust Decentralized $\mathcal{H}_\infty$ Controller Design considering Parametric Uncertainties in Discrete Time

This section first introduces the decentralized controller design scheme for the open-loop system (10) through double homotopy approach. Then D - K method in μ synthesis is invoked to design a robust controller considering the structural parametric uncertainties. Finally, this paper combines the double homotopy approach and the D - K method to design a robust decentralized controller for the uncertain structures.

3.1. Decentralized Controller. An $\mathcal{H}_\infty$ controller is considered to be designed for the open-loop system (10). Take controller system in (11) into the open-loop system in (10) to shape the closed-loop system as shown in Figure 2. The closed-loop

system can be formulated as (12) after assuming $D_{33} = 0$ without the loss of generality [28]:

$$\begin{aligned} \dot{x}_{CL} &= A_{CL} x_{CL} + B_{CL} \begin{bmatrix} r \\ w \end{bmatrix}, \\ \begin{bmatrix} y_r \\ z \end{bmatrix} &= C_{CL} x_{CL} + D_{CL} \begin{bmatrix} r \\ w \end{bmatrix}, \end{aligned} \quad (12)$$

where symbol CL denotes the closed-loop system from $[r \ w]^T$ to $[y_r \ z]^T$; the matrices of the closed-loop system A_{CL} , B_{CL} , C_{CL} , and D_{CL} are denoted as

$$\begin{bmatrix} A_{CL} & B_{CL} \\ C_{CL} & D_{CL} \end{bmatrix} = \begin{bmatrix} \tilde{A} + \tilde{B}_2 K \tilde{C}_2 & \tilde{B}_1 + \tilde{B}_2 K \tilde{D}_{21} \\ \tilde{C}_1 + \tilde{D}_{12} K \tilde{C}_2 & \tilde{D}_{11} + \tilde{D}_{12} K \tilde{D}_{21} \end{bmatrix}. \quad (13)$$

The unknown notations in (13) are defined as

$$\begin{bmatrix} \tilde{A} & \tilde{B}_1 & \tilde{B}_2 \\ \tilde{C}_1 & \tilde{D}_{11} & \tilde{D}_{12} \\ \tilde{C}_2 & \tilde{D}_{21} & K \end{bmatrix} = \begin{bmatrix} A & 0 & B_1 & B_2 & 0 & B_3 \\ 0 & 0_{n_K} & 0 & 0 & I_{n_K} & 0 \\ C_1 & 0 & D_{11} & D_{12} & 0 & D_{13} \\ C_2 & 0 & D_{21} & D_{22} & 0 & D_{23} \\ 0 & I_{n_K} & 0 & 0 & A_K & B_K \\ C_3 & 0 & D_{31} & D_{32} & C_K & D_K \end{bmatrix}, \quad (14)$$

where $A, B_1, \dots, D_{32}$ are the notations of the open-loop system in (10). According to bounded real lemma in discrete time, an $\mathcal{H}_\infty$ controller can be designed to make the closed-loop system (12) stable and its $\mathcal{H}_\infty$ norm smaller than a given scalar γ if and only if there exists a symmetric positive definite matrix $Q > 0$ such that the following matrix inequality holds [29]:

$$\begin{bmatrix} -Q & Q(\tilde{A} + \tilde{B}_2 K \tilde{C}_2) & Q(\tilde{B}_1 + \tilde{B}_2 K \tilde{D}_{21}) & 0 \\ * & -Q & 0 & (\tilde{C}_1 + \tilde{D}_{12} K \tilde{C}_2)^T \\ * & * & -\gamma I & (\tilde{D}_{11} + \tilde{D}_{12} K \tilde{D}_{21})^T \\ * & * & * & -\gamma I \end{bmatrix} < 0, \quad (15)$$

where $(*)$ denotes the terms induced by symmetry. For the centralized controller, the inequality (15) is equivalent to a series of linear matrix inequalities according to the projection lemma [28]. For the decentralized control, the controller system in (11) can be divided into multiple subcontrollers that are independent from each other. Each subcontroller only needs part of sensor measurement data as input and generates only part of control forces as output:

$$\begin{aligned} x_{K_1}[k+1] &= A_{K_1} x_{K_1}[k] + B_{K_1} y_1[k] \\ u_1[k] &= C_{K_1} x_{K_1}[k] + D_{K_1} y_1[k] \\ &\vdots \\ x_{K_i}[k+1] &= A_{K_i} x_{K_i}[k] + B_{K_i} y_i[k] \end{aligned}$$

$$u_i[k] = C_{K_i} x_{K_i}[k] + D_{K_i} y_i[k]$$

$\vdots$

$$x_{K_N}[k+1] = A_{K_N} x_{K_N}[k] + B_{K_N} y_N[k]$$

$$u_N[k] = C_{K_N} x_{K_N}[k] + D_{K_N} y_N[k].$$

(16)

Therefore, the controller matrices in (11) have diagonal block pattern as the following:

$$A_K = \text{diag}(A_{K_1}, A_{K_2}, \dots, A_{K_N}),$$

$$B_K = \text{diag}(B_{K_1}, B_{K_2}, \dots, B_{K_N}),$$

$$\begin{aligned} C_K &= \text{diag}(C_{K_1}, C_{K_2}, \dots, C_{K_N}), \\ D_K &= \text{diag}(D_{K_1}, D_{K_2}, \dots, D_{K_N}). \end{aligned} \quad (17)$$

As discussed in [16], when the controller matrix K in (14) has the sparse pattern as (3.1), (15) is bilinear matrix inequality problem (BMI). It is not a convex problem and cannot be solved by general off-the-shelf numerical package or algorithm. The cone complementarity linearization algorithm is available to solve the BMI problem in the static output feedback controller design process [30–33]. Qu et al. [19] proposed a time-delayed dynamic output feedback decentralized $\mathcal{H}_\infty$ controller design scheme through a double homotopy approach. The BMI is approximately linearized as a linear matrix inequality, which can be solved by some LMI solvers. This approach is employed to solve (15) and introduced briefly as follows.

The double homotopy algorithm approximates BMI to linear matrix inequality (LMI) that can be easily conducted [28]. After initializing a controller without the constraint for the sparse pattern of the controller matrices, the algorithm gradually changes the diagonal-block entries. At the same time, the off-diagonal-block entries are transformed to zero

along the homotopy steps. The initials K and Q are set to the centralized controller matrix K_C and coexisting Q_C :

$$K_0 = K_C = K_{C,\text{diag}} + K_{C,\text{off}}, \quad (18a)$$

$$Q_0 = Q_C, \quad (18b)$$

where $K_{C,\text{diag}}$ is the block diagonal part of K_C that has the same sparse pattern as the desired decentralized controller matrix; $K_{C,\text{off}}$ represents the off-diagonal blocks with the satisfaction of (18a). At the k th homotopy step ($k = 1, 2, \dots, k_{\max}$), K_k and Q_k can be described as follows:

$$\lambda = \frac{1}{k_{\max}}, \quad (19a)$$

$$K_k = K_{k-1} - \lambda K_{C,\text{off}} + \Delta K_k, \quad (19b)$$

$$Q_k = Q_{k-1} + \Delta Q_k. \quad (19c)$$

Equations (19a), (19b), and (19c) with (18a) and (18b) can be taken into (15). The entry parts with the multiplication of ΔK_k and ΔQ_k can be neglected because they are much smaller than other parts. Omitting the derivation, (15) can be approximated to

$$V(\Delta K_k, \Delta Q_k) = \begin{bmatrix} Q_{k-1} + \Delta Q_k & Q_{k-1}\bar{A} + Q_{k-1}\tilde{B}_2\Delta K_k\tilde{C}_2 + \Delta Q_k\bar{A} & Q_{k-1}\bar{B} + Q_{k-1}\tilde{B}_2\Delta K_k\tilde{D}_{21} + \Delta Q_k\bar{B} & 0 \\ * & Q_{k-1} + \Delta Q_k & 0 & (\bar{C} + \tilde{D}_{12}\Delta K_k\tilde{C}_2)^T \\ * & * & -\gamma I & (\bar{D} + \tilde{D}_{12}\Delta K_k\tilde{D}_{21})^T \\ * & * & * & -\gamma I \end{bmatrix} < 0, \quad (20)$$

where

$$\bar{A} = \tilde{A} + \tilde{B}_2(K_{k-1} - \lambda K_{C,\text{off}})\tilde{C}_2, \quad (21a)$$

$$\bar{B} = \tilde{B}_1 + \tilde{B}_2(K_{k-1} - \lambda K_{C,\text{off}})\tilde{D}_{21}, \quad (21b)$$

$$\bar{C} = \tilde{C}_1 + \tilde{D}_{12}(K_{k-1} - \lambda K_{C,\text{off}})\tilde{C}_2, \quad (21c)$$

$$\bar{D} = \tilde{D}_{11} + \tilde{D}_{12}(K_{k-1} - \lambda K_{C,\text{off}})\tilde{D}_{21}. \quad (21d)$$

Equation (20) is an LMI and can be solved easily [28, 34]. After ΔQ_k and ΔK_k are solved from (20), they will be taken back into (19a), (19b), and (19c) to construct K_k for the next homotopy step. When $k = k_{\max}$, the K_k is the desired decentralized controller. It should be noted that the double homotopy approach cannot guarantee convergence. If there is a convergent solution, it may be a local optimization. If not, it does not mean that there is no desired decentralized controller.

It should be pointed out that the decentralized controller stabilizes the closed-loop system and makes the norm of the transfer function from $[r \ w]^T$ to $[y_r \ z]^T$ smaller than γ . It is consistent with small gain theorem described as Figure 3(a).

As shown in Figure 3, Δ_p is the fictitious unstructured uncertainty which is supposed to be existing between the response output vector z and the excitation input vector w . The norm of Δ_p can be determined according to the transfer function $\text{CL}(s)$ and the weighting function of output z that is adjusted during designing the controller K . Δ is the parametric uncertain matrix in (4). Symbol CL denotes the closed-loop transfer function from $[r \ w]^T$ to $[y_r \ z]^T$. $\tilde{\Delta}$ is the diagonal combination of Δ and Δ_p , which is expressed as

$$\tilde{\Delta} = \begin{bmatrix} \Delta & \\ & \Delta_p \end{bmatrix}. \quad (22)$$

According to the small gain theorem, the closed-loop system is internally stable for all $\tilde{\Delta}$ with

$$\|\tilde{\Delta}\|_\infty \leq \frac{1}{\gamma} \text{ iff } \|\text{CL}\|_\infty < \gamma, \quad (23a)$$

$$\|\tilde{\Delta}\|_\infty < \frac{1}{\gamma} \text{ iff } \|\text{CL}\|_\infty \leq \gamma. \quad (23b)$$

Obviously, $\|\text{CL}\|_\infty \leq \gamma$ cannot explain how large the norm of the transfer function $F_u(\text{CL}, \Delta)$ is, which is the combination

FIGURE 3: Diagram of the closed-loop system with parametric uncertainties.

of the uncertainties Δ and the closed-loop system CL as the system from w to z shown in Figure 3(b). Moreover, $\tilde{\Delta}$ in (23a) and (23b) is norm-bounded without consideration of the block diagonal structure of $\tilde{\Delta}$ in (22) and Δ in (4). It will be conservative for the designed controller. So the small gain theorem is not suitable for the controller design.

3.2. Parametric Uncertainties. To consider the block diagonal structure of $\tilde{\Delta}$, the definition of the structured singular value in the μ synthesis theory should be introduced first [35]. The structured singular value for the closed-loop system CL is defined as

$$\mu_{\tilde{\Delta}}(\text{CL}) = \frac{1}{\min \{ \bar{\sigma}(\tilde{\Delta}) \mid \det(I - \text{CL} \cdot \tilde{\Delta}) = 0, \text{ for structured } \tilde{\Delta} \}}, \quad (24)$$

where $\mu_{\tilde{\Delta}}(\text{CL})$ is named as the structured singular value of the closed-loop system CL; $\det$ is the abbreviation of determinant; $\bar{\sigma}(\tilde{\Delta})$ is the largest singular value of $\tilde{\Delta}$. The poles s with $\text{Re } s \geq 0$ of the transfer function $F_u(\text{CL}, \Delta)$ are the zeros of $\det(I - \text{CL}(s) \cdot \tilde{\Delta}(s))$. The system $F_u(\text{CL}, \Delta)$ is stable if and only if there is no zero of $\det(I - \text{CL}(s) \cdot \tilde{\Delta}(s))$. The system CL is stable. So the expression “ $\det(I - \text{CL} \cdot \tilde{\Delta}) = 0$ ” means that the uncertainty $\tilde{\Delta}$ destabilizes the closed-loop system CL. Apparently, the larger the largest singular value of the uncertainty $\tilde{\Delta}$ is, the more hardly the stability of the system $F_u(\text{CL}, \Delta)$ is guaranteed. It is only needed to choose a minimum structured singular value of $\tilde{\Delta}$ such that $F_u(\text{CL}, \Delta)$ is unstable, which is the meaning for the denominator of (24). The structured singular value of the closed-loop system CL is the reciprocal of the minimized $\bar{\sigma}(\tilde{\Delta})$. Obviously, $\tilde{\Delta}$ in (24) is not norm-bounded and its diagonal block structure can be considered. If the system CL is internally stable and $\|\tilde{\Delta}\|_{\infty} < 1/\gamma$, according to Theorem 13.4.2 in [35], the following two inequalities are equivalent:

$$\|F_u(\text{CL}, \Delta)\|_{\infty} \leq \gamma, \quad (25a)$$

$$\mu_{\tilde{\Delta}}(\text{CL}) \leq \gamma. \quad (25b)$$

It means that if there is an $\mathcal{H}_{\infty}$ controller to make the structured singular value $\mu_{\tilde{\Delta}}(\text{CL}) \leq \gamma$ with the uncertainties $\|\tilde{\Delta}\|_{\infty} < 1/\gamma$, then the norm of the uncertain system

$\|F_u(\text{CL}, \Delta)\|_{\infty} \leq \gamma$. An apparent method is to use $\mu_{\tilde{\Delta}}(\text{CL})$ instead of $\|\text{CL}\|_{\infty}$ to design a robust controller. Unfortunately, there is no available method to calculate $\mu_{\tilde{\Delta}}(\text{CL})$. An approximation method of $\mu_{\tilde{\Delta}}(\text{CL})$ is motivated by the following inequality [35]:

$$\mu_{\tilde{\Delta}}(\text{CL}(j\omega)) \leq \inf \bar{\sigma}(D_R(j\omega) \cdot \text{CL}(j\omega) \cdot D_L^{-1}(j\omega)), \quad (26a)$$

$$\text{minimize}_{K,D} \bar{\sigma}(D_R(j\omega) \cdot \text{CL}(\text{OL}, K) \cdot D_L^{-1}(j\omega)), \quad (26b)$$

where $j\omega$ means that the values for two sides of (26a) are changing along the frequency ω ; $\inf$ is the abbreviation of infimum; CL is the closed-loop system as above and contains open loop system named OL and controller K , which can be written as $\text{CL}(\text{OL}, K)$ as described in (26b); D_R and D_L combine a sequence of transfer functions and identity matrix diagonally and are denoted as

$$D_L = \text{diag}(d_1(s)I_{n_1}, \dots, d_{F-1}(s)I_{n_{F-1}}, I_{n_F}), \quad (27a)$$

$$D_R = \text{diag}(d_1(s)I_{m_1}, \dots, d_{F-1}(s)I_{m_{F-1}}, I_{m_F}). \quad (27b)$$

From (26a), the value of the expression in the right side is the upper bound of $\mu_{\tilde{\Delta}}(\text{CL})$. If the triple matrix variables D_L , D_R , and K are chosen appropriately, the difference between $\inf \bar{\sigma}(D_R \cdot \text{CL} \cdot D_L^{-1})$ and $\mu_{\tilde{\Delta}}(\text{CL})$ can be minimized. So $\inf \bar{\sigma}(D_R \cdot \text{CL} \cdot D_L^{-1})$ can be used to be calculated instead of $\mu_{\tilde{\Delta}}(\text{CL})$. The problem for designing a robust controller is transferred to solve the optimal problem as the expression in (26b). In order to approximate $\mu_{\tilde{\Delta}}(\text{CL})$ more precisely, the triple matrix variables D_L , D_R , and K in CL should be decided appropriately. It is not a convex problem for (26b) because the triple matrix variables multiply together. One possible way is to calculate D_L , D_R , and K separately. For the given scaling matrices D_L and D_R , (26b) is turned to a standard $\mathcal{H}_{\infty}$ optimization issue, which can be solved by LMI solvers or DGKF solvers. On the other hand, for a fixed K , (26b) is a convex optimization problem at each frequency point ω_i . A sequence of $D_L(j\omega_i)$ and $D_R(j\omega_i)$ can be optimized by solving (26b) at each frequency point. Through curve fitting for the sequence of $D_L(j\omega_i)$ and $D_R(j\omega_i)$, the rational transfer function matrices $D_L(j\omega)$ and $D_R(j\omega)$ can be approximated, respectively. Then the controller K can be recalculated by fixing the new $D_L(j\omega)$ and $D_R(j\omega)$ in the next iteration. This procedure is called D - K iteration. However, there is no guarantee that D - K iteration can converge to a global minimum due to the nonconvex nature of the D - K iteration.

3.3. Decentralized $\mathcal{H}_{\infty}$ Controller for Structural Parametric Uncertainties. Motivated by the above methods, this paper combines the double homotopy method and the D - K iteration method together to design a robust decentralized controller with the consideration of the structural parametric uncertainties. From the expression in (26b), the open-loop system OL multiplied by $D_L(j\omega)$ and $D_r(j\omega)$ can be written as

$$\text{OL}_{\text{new}}(j\omega) = \begin{bmatrix} D_R(j\omega) & 0 \\ 0 & I \end{bmatrix} \text{OL}(j\omega) \begin{bmatrix} D_L(j\omega)^{-1} & 0 \\ 0 & I \end{bmatrix}, \quad (28)$$

FIGURE 4

where OL_{new} is the new open-loop system, whose system matrices in (14) can be rewritten as

$$\begin{bmatrix} \tilde{A} & \tilde{B}_1 & \tilde{B}_2 \\ \tilde{C}_1 & \tilde{D}_{11} & \tilde{D}_{12} \\ \tilde{C}_2 & \tilde{D}_{21} & K \end{bmatrix} = \begin{bmatrix} A_{\text{new}} & 0 & B_{1\text{new}} & B_{2\text{new}} & 0 & B_{3\text{new}} \\ 0 & 0_{n_K} & 0 & 0 & I_{n_K} & 0 \\ C_{1\text{new}} & 0 & D_{11\text{new}} & D_{12\text{new}} & 0 & D_{13\text{new}} \\ C_{2\text{new}} & 0 & D_{21\text{new}} & D_{22\text{new}} & 0 & D_{23\text{new}} \\ 0 & I_{n_K} & 0 & 0 & A_K & B_K \\ C_{3\text{new}} & 0 & D_{31\text{new}} & D_{32\text{new}} & C_K & D_K \end{bmatrix}. \quad (29)$$

After defining the new open-loop system, a new centralized controller K_C is designed for the new open-loop system in (28) according to the bounded real lemma, which is a convex optimization problem. Then $K_{C,\text{diag}}$ and $K_{C,\text{off}}$ in (18a) and (18b) are then split from the new K_C . The new matrices $\tilde{A}, \tilde{B}_1, \dots, \tilde{D}_{21}$ in (29) can be taken into (20) to solve the variable ΔK_k . K_k would be constructed for the next step to compute ΔK_{k+1} in (20) by taking ΔK_k into (19a), (19b), and (19c). The decentralized controller $K_{k_{\max}}$ for the new open-loop system can be solved gradually. The new pairs of $D_L(j\omega)$ and $D_R(j\omega)$ can be got by fixing the computed new decentralized controller in (26b). So another new open loop system can be formed to replace the previous “new” open loop by (28), and then the iterations for solving decentralized controller for the new open-loop system can be started, that is the procedure to design the robust decentralized controller

for the uncertain system. In other words, the idea is to replace the centralized controller design when $D_L(j\omega)$ and $D_R(j\omega)$ are fixed in D - K iteration by the decentralized controller design schemes. It is a modified D - K iteration method. The proposed process is summarized as follows.

Step 1. Start with initials $D_L(j\omega)$ and $D_R(j\omega)$, usually set $D_L(j\omega) = D_R(j\omega) = I$.

Step 2. Construct a new open-loop system as (28) and replace OL with OL_{new} .

Step 3. Fix $D_L(j\omega)$ and $D_R(j\omega)$, and solve the $\mathcal{H}_\infty$ decentralized controller through double homotopy approach (see Figure 4).

Step 4. Fix the computed decentralized controller K , and solve the convex optimization problem in (26b) for $D_L(j\omega_i)$ and $D_R(j\omega_i)$ at each frequency point ω_i . Then $d_1(j\omega_i), \dots, d_{F-1}(j\omega_i)$ in (27a) and (27b) will be obtained.

Step 5. Fit the values of the sequence of $d_1(j\omega_i), \dots, d_{F-1}(j\omega_i)$ to get the rational transfer function $d_1(j\omega), \dots, d_{F-1}(j\omega)$. Then construct the new matrices $D_{L\text{new}}(j\omega)$ and $D_{R\text{new}}(j\omega)$ as (27a).

Step 6. Compare $D_{L\text{new}}(j\omega)$ and $D_{R\text{new}}(j\omega)$ by curve fitting at each frequency point with $D_L(j\omega)$ and $D_R(j\omega)$ of the previous D - K iteration, respectively. If they are close, the iteration can be stopped, otherwise, replace $D_L(j\omega)$ and $D_R(j\omega)$ with $D_{L\text{new}}(j\omega)$ and $D_{R\text{new}}(j\omega)$ and return to Step 2.

FIGURE 5: A four-story model structure.

It should be noted that the double homotopy method would cost a lot of time to compute the decentralized controller. When this method is integrated into D - K iteration, it would be executed once at each D - K iteration procedure. The computation time may be very large.

4. Numerical Example

This section investigates the influence of the parametric uncertainties on the structural decentralized control performance. Then the performance comparisons are made between the decentralized controller designed by small gain theorem and the proposed modified D - K iteration for the structures with the parametric uncertainties. The numerical model is a four-story structure instrumented with two actuators as shown in Figure 5(a) [23].

The nominal values of the structure parameters are $m_{\text{nom}} = 1.05 \times 10^6$ kg, $c_{\text{nom}} = 1.575 \times 10^6$ N·s/m, and $k_{\text{nom}} = 350 \times 10^6$ N/m. The mass, damping, and stiffness for each floor are shown in Figure 5(a). The nominal values for the structural parametric matrices are denoted as

$$M_{\text{nom}} = \begin{pmatrix} m_{1\text{nom}} & & & \\ & m_{2\text{nom}} & & \\ & & m_{3\text{nom}} & \\ & & & m_{4\text{nom}} \end{pmatrix}, \quad (30a)$$

$$C_{\text{nom}} = \begin{pmatrix} c_{1\text{nom}} + c_{2\text{nom}} & -c_{2\text{nom}} & & & \\ -c_{2\text{nom}} & c_{2\text{nom}} + c_{3\text{nom}} & -c_{3\text{nom}} & & \\ & -c_{3\text{nom}} & c_{3\text{nom}} + c_{4\text{nom}} & -c_{4\text{nom}} & \\ & & -c_{4\text{nom}} & c_{4\text{nom}} & \end{pmatrix}, \quad (30b)$$

$$K_{\text{nom}} = \begin{pmatrix} k_{1\text{nom}} + k_{2\text{nom}} & -k_{2\text{nom}} & & & \\ -k_{2\text{nom}} & k_{2\text{nom}} + k_{3\text{nom}} & -k_{3\text{nom}} & & \\ & -k_{3\text{nom}} & k_{3\text{nom}} + k_{4\text{nom}} & -k_{4\text{nom}} & \\ & & -k_{4\text{nom}} & k_{4\text{nom}} & \end{pmatrix}. \quad (30c)$$

In this numerical example, only the stiffness uncertainties are considered the worst case of which is assumed as

$$K_s = K_{\text{nom}} \pm K_{\text{nom}} \times 0.3. \quad (31)$$

According to (2c), it can be derived that $P_K = 0.3$, $\|\delta_K\| \leq 1$. The 1940 El Centro NS earthquake record is adopted as the ground excitation w . The peak ground acceleration is scaled to be 9.8 m/s^2 . The excitation location matrix T_w and the control force location matrix T_u in (1) are shown as follows:

$$T_w = [-1 \quad -1 \quad -1 \quad -1]^T, \quad (32a)$$

$$T_u = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}^T. \quad (32b)$$

The structural measurement outputs m in (5) are the inter-story drifts as described in (33a). The response outputs z in (5) are the combination of the interstory drifts and control forces with different weightings denoted by

$$y = [q_1 \quad q_2 - q_1 \quad q_3 - q_2 \quad q_4 - q_3]^T, \quad (33a)$$

$$z = [(q_1 \quad q_2 - q_1 \quad q_3 - q_2 \quad q_4 - q_3), 5 \times 10^{-8} (u_1 \quad u_2)]^T. \quad (33b)$$

To design the decentralized controllers, three different cases represented by different degrees of centralization (DC) are set and shown in Figure 5(b). DC1 means that each sub-controller system only needs two neighbouring floor measurements and generates one control force. Each sub-controller system in DC2 covers three floor measurements and generates one control force. For case DC3, the controller system covers all four floor measurements and generates two control forces, which is obviously the centralized controller. In practical implementation, the data transmitting and the embedded controller computing can cost time and result in time delay in the feedback loop. In this study, the continuous structural system is discretized by the sampling period that is

FIGURE 6: Simulation results for decentralized control without consideration of structural parametric uncertainties.

TABLE 1: Feedback time delay and sampling period for four three DCs (unit: ms).

DC1	DC2	DC3
15	20	40

determined by the cost time. The time delay is assumed to be equal to one sampling period. Obviously, a larger number of DC indicates that more time is required for the transmitting and computing. The sampling time or the delayed time from DC1 to DC3 is listed in Table 1.

To describe the following comparison simply, the comparing cases are first listed in Table 2 and explained in the following figure comparisons.

In this numerical example, the uncertainties are the stiffness uncertainties that are the worst case in (31) and reduce the stiffness of the nominal structural system. The decentralized controllers can be designed for the uncertain structural system through the double homotopy approach by the small gain theorem, which use Step 3 in the modified D - K iteration with the setting $D_L(j\omega) = D_R(j\omega) = I$. When the designed decentralized controllers are used in the nominal structural system, the comparison is shown in Figure 6(a). The solid line named Bare is the nominal structural response without control, and DC1nom, DC2nom, and DC3nom represent that the decentralized and centralized controllers are designed by small gain theorem and used in nominal system as denoted in Table 2. From Figure 6(a), the three controllers can reduce the bare structural response comparing with the solid line. DC1nom has the better control performance than DC2 nom, and DC3nom has the worst. Obviously, the decentralized controllers have a better work than the centralized controller.

Furthermore, it can be concluded that the longer the delayed time is, the worse the performance of the decentralized controller has.

The decentralized controllers designed by small gain theorem but used in the uncertain structure are denoted as DC1un, DC2un, and DC3un in Table 2. The performance of these controllers would be influenced extremely by the uncertainties as shown in Figure 6(b). The solid line named Bareun is the uncertain structural response without control. DC1un, DC2un, and DC3un in Figure 6(b) have worse performance than that in Figure 6(a). Furthermore, DC2un in Figure 6(b) enlarges the response at the third story. It can be concluded that the decentralized controllers designed by the small gain theorem are conservative, because the controller design scheme through the small gain theorem cannot consider the diagonal structure of the uncertainties, even though, there are only stiffness uncertainties in this numerical example. It should be noted that the uncertainties from the output to input of the closed-loop can be supposed as the notation Δ_p in (22), which can compose the block diagonal structured uncertainties $\tilde{\Delta}$ as described in (22).

Through the modified D - K iteration method with the combination of D - K iteration and double homotopy approach, which is proposed in the previous section, the decentralized controllers are designed for the uncertain structures and denoted as DCDK1, DCDK2, and DCDK3 in Table 2. The computation time to solve the robust decentralized controllers are listed in Table 3.

From Table 3, it demonstrates that the higher level of centralization results in the less time cost to search the robust controller. The comparison of the interstory drift peak values is shown in Figure 7(a). The relationship between DCDK1, DCDK2, and DCDK3 is similar with that in Figure 6(a).

TABLE 2: Notations for different comparing cases.

	Without control	Decentralized control		Centralized control
Nominal structure	Bare	DC1nom	DC2nom	DC3nom
Uncertain structure				
Controller designed by small gain theorem	Bareun	DC1un	DC2un	DC3un
Controller designed by the proposed modified D - K iteration	Bareun	DCDK1	DCDK2	DCDK3

FIGURE 7: Simulation results for decentralized control with and without consideration of structured parametric uncertainties.

TABLE 3: Computation time to solve robust controllers for three different DCs (unit: ms).

DC1	DC2	DC3
1 d 23 hr 8 min 59 s	17 hr 30 min 10 s	36 s

DCDK1 has the best performance, while DCDK3 has the worst due to the longest delayed time. To make the comparison clearly between the decentralized controllers designed by the small gain theorem and the proposed combined method, each DC is compared separately in Figures 7(b), 7(c), and 7(d). The three figures have the similar trend. Figure 7(c) can be taken for an example. When there exist stiffness uncertainties in the structure, the performance for DC2un is worse than that for DC2nom. And sometimes the designed controller by small gain theorem may enlarge the response of the uncertain structure, which can be seen from the comparison from DC2un and Bareun. DCDK2 is the decentralized controller designed by the proposed method, which has better robust performance than DC2un. For the comparisons in Figures 7(b) and 7(d), the performance of the controllers DC1un and DC3un which are designed by small gain theorem, is worse than the performance of DC1nom and DC3nom, but not so much. It illustrates that a robust controller can be designed based on the small gain theorem. However, the comparisons with the DCDK1 and DCDK3 demonstrates that the decentralized controllers designed by the proposed method can contain more robustness than these designed by small gain theorem.

Beside the comparisons in time domain, the controller performance can be checked in frequency domain as shown in Figure 8. Figure 8(a) illustrates the $\mathcal{H}_\infty$ norm comparison for the closed-loop system with the decentralized controller DC1 in (14). The closed-loop systems with DC1un and DCDK1 have the similar $\mathcal{H}_\infty$ norm valued about 1.25. When considering the structure of uncertainties, the structured singular values μ in (24) have been compared as shown in Figure 8(b). Their values are smaller than their $\mathcal{H}_\infty$ norm, respectively; that is, $\mu = 0.8475$ for DC1un and $\mu = 0.6793$ for DCDK1. The reason is that μ synthesis as describe in the second part of the last section can consider the uncertain structure but $\mathcal{H}_\infty$ norm cannot. It should be noted that $\mathcal{H}_\infty$ norm is used in small gain theorem and more conservative than μ value. Another important view can be concluded from the comparison of DCDK1 between Figures 8(a) and 8(b). The $\mathcal{H}_\infty$ norm of the closed-loop with DCDK1 is 1.2751, which means that if using the $\mathcal{H}_\infty$ optimization to analyses the system stability, the $\mathcal{H}_\infty$ norm of the uncertainties that can be tolerated is smaller than $1/1.2751$. On the other hand, the μ value for the system with DCDK1 is 0.6793, and the system stability is preserved for $1/\|\Delta\|_\infty < 1/0.6793$. It illustrates that the same closed-loop systems with the same controllers have different stabilities due to using different criteria to analyses the system performance. Figures 8(c) and 8(d) have the same trend with Figures 8(a) and 8(b). The $\mathcal{H}_\infty$ norm of the

system with DC2un in Figure 8(c) is 1.4760 and the μ value of the system with DCDK2 is 0.5925 in Figure 8(d), whose value distance between 1.4760 (DC2un) and 0.5925 (DCDK2) is larger than that in Figures 8(a) and 8(b), the value distance between 1.25 (DC1un) and 0.6793 (DCDK1). So that is the reason why the response differences are more obvious in time domain between DC2un and DCDK2 in Figure 7(c) than that between DC1un and DCDK1 in Figure 7(b).

From the comparisons in frequency domain, the controllers designed by the proposed method can contain more robustness than these designed by the small gain theorem. Because the former uses the singular value concept, and the latter takes the $\mathcal{H}_\infty$ criterion that cannot consider the diagonal block structure of the uncertainties.

5. Conclusions

This paper presents a robust decentralized $\mathcal{H}_\infty$ controller design method for the civil structures with the parametric uncertainties. The parametric uncertainties are first extracted from the civil structural model by LFT. The controller design architecture with parametric uncertainties and time delay is then modeled in discrete time. The double homotopy method to design a decentralized $\mathcal{H}_\infty$ controller for the structure without uncertainties and the D - K iteration method to design a centralized $\mathcal{H}_\infty$ controller for the structure with parametric uncertainties are introduced, respectively. Finally, the schemes to design the robust decentralized $\mathcal{H}_\infty$ controller considering parametric uncertainties are proposed, which modified the D - K iteration by combining the double homotopy approach and D - K iteration method.

The controller performance is validated through a four-story numerical example. The centralized and decentralized $\mathcal{H}_\infty$ controllers designed by small gain theorem and used in the nominal system have good performance to reduce the structural vibration. The larger the delayed time is, the worse the controller performance is (e.g., the comparison between DC1nom, DC2nom, and DC3nom). If the designed controllers by small gain theorem are used in uncertain system, the decentralized $\mathcal{H}_\infty$ controllers will have worse control performance and even enlarge the structural response as described as DC1un, DC2un, and DC3un. Through the proposed controller design schemes, they have more robust control performance. DCDK1, DCDK2, and DCDK3 have better performance than DC1un, DC2un, and DC3un, respectively.

It is noted that the computation time of the proposed method deserves to be reduced. The method to solve the BMI problem should be investigated further more. The effectiveness should be validated by some experiments. All these problems are the future works.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

FIGURE 8: Comparisons of $\mathcal{H}_\infty$ norm and μ values between decentralized controllers designed for nominal and uncertain systems.

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