Vehicle Delay Estimation for an Isolated Intersection under Actuated Signal Control

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Vehicle delay at an actuated signal control (ASC) intersection is analyzed and computed with the aim of reflecting the effects of ASC and optimizing its parameters. The operation characteristics of traffic flows at the intersections serve as the foundation to present the mechanism of vehicle delay under the ASC; given that the arrival of vehicles obeys the Poisson distribution, probability algorithms of vehicle delay are put forward under the semi-ASC and fully ASC depending on discretization of green time. Computation process is illustrated by case study and CORSIM simulation experiments. Finally, the primary results indicate that the appropriate control modes can decrease vehicle delay, which is significantly affected by unit extension. Hopefully, the study will provide useful information for selecting different ASC modes and optimizing signal control parameters.

1. Introduction

Delay, as one of the main measures of effectiveness (MOE) to evaluate effect of traffic management, reflects the travelling quality of traffic participation fundamentally. Although certain research efforts have been achieved on delay at uncontrolled intersection [1] and timing signal control intersection [2–4], the delay at ASC intersections can only be studied using simulation model. The results are greatly influenced by the simulation software due to the different control logic and parameter settings, hardly reflecting the actual traffic situation [5].

Although it is hard to set up an analytical model to compute vehicle delays at an ASC intersection, several researchers tried to build relevant models to get vehicle delays by calculation or simulation. Here are the main theoretical and simulation methods on delay at an ASC intersection published before.

Belker [6] put forward several vehicular delay models for traffic-actuated control signals. Roushail et al. [7] used TRAF-NETSIM and field data to evaluate a generalized delay model developed for vehicle-actuated traffic intersections, and the delay values obtained from NETSIM were compared with those estimated by the generalized delay model. Ding et al. [8] proposed a delay model of an actuated controlled intersection based on the maximum queue length of each phase according to the Gamma statistics probabilities of traffic volume and states of signal phase, which was proved by a two-phase fully actuated controlled intersection. Hanabusa et al. [9] developed a real time delay estimation method using probe data with adaptive signal control algorithm and validated it using traffic simulation. The Highway Capacity Manual (HCM) [10] reported an average control delay, which included the deceleration, the queue moving time, the stopped time, and the acceleration. However, the length of an approach and the speed limitation that may contribute to the control delay were not specifically considered. In general, different methods were usually used to compute vehicle delays in analytical and simulation models with some certain limitations. However, there is a little gap between existing achievements and application demand.

This paper starts from the operation mechanism and control logic of ASC and relies on the thought of green time discretization using the methods of probability algorithm.
(PA). The vehicle arrival is assumed to follow the Poisson distribution (PD) to analyze the vehicle delay at the ASC intersections.

2. Delay Generation Mechanism under ASC

The green light to an approach turns on, taking this as a starting point, while the vehicles waiting at the stop line are discharged in sequence. In Figure 1, \( t_{sc} \) is the discharge time of the vehicles arrival during red interval and initial green time. Subsequently, arriving vehicles can pass the intersection directly without waiting after discharging the queued vehicles and vehicles that arrived during late green time or yellow time determine to pass or not based on the actual situation. Here, \( t_{yc} \) represents the loss time of the green time and the yellow time. Vehicles that arrived during the red interval need to queue and wait for the next phase of green light. And \( C \) is the cycle length.

Here, the vehicle delay is divided into the stopped delay \( (d_{tc}) \) and the discharged delay \( (d_{sc}) \). \( d_{tc} \) is the average stopped time of the vehicles, and \( d_{sc} \) is the time difference between the discharge time of queued vehicles and time of passing the intersections without signal control.

3. Vehicle Delay on Semi-ASC

3.1. Detectors on the Main Road. If vehicles cannot be detected on the main road for a certain period of time, the controller then switch phase and the green light of the side road is turned on. When vehicles arrive on the main road continuously during unit extension, then the maximum green time is judged to reach or not; the green light of the side road is turned on if it reaches, otherwise the green time of the main road is extended. Figures 2 and 3 illustrate the signal control and operation process with detectors on the main road, where control parameters are the minimum green time of the main road \( (G_{min,m}) \), unit extension of the main road \( (G_{0,m}) \), the maximum green time of the main road \( (G_{max,m}) \), and the minimum green time of the side road \( (G_{min,s}) \).

Vehicle delays are significantly affected by the green time and the cycle length of an intersection. Unit extension, one of the factors influencing the green time and the cycle length, is the parameter used to decide when to switch phase. In this part, unit extension is discrete and vehicle arrival is computed using the probability algorithms (PA). Procedure of ASC is combined to further determine the vehicle delay.

The test starts from the green light of the main road and during unit extension after the end of initial green light; and a green time interval of unit extension is extended since vehicle is detected. The arrival of the following vehicles is set as \( x \), and then the green time is extended to \( x \); if no following vehicle arrives during any unit extension, the phase, showed in Figure 4, is switched.

Ordinarily, vehicle arrival of an intersection follows Poisson distribution, negative binomial distribution, and/or gamma distribution [8, 11, 12]. Here, the arrival of vehicles is supposed to obey Poisson distribution and the probability distribution is as follows:

\[
P_k(t) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}, \quad k = 0, 1, 2 \ldots
\]  

where \( P_k(t) \) is the probability of \( k \)th vehicle arrival during the interval \( t \); \( \lambda \) is the average arrival probability per unit; \( t \) is the time interval; and \( e \) is a mathematical constant.

\[
\]
Traffic volume and vehicle arrival rate of each approach are $d_{ck}$ and $\lambda_{ck}$, respectively (with $k = 1, 2, 3, 4$ representing western, northern, eastern, and southern approaches).

So, the probability of no vehicle arrival on the main road during $G_{0,m}$ is as follows:

$$P_0(G_{0,m}) = e^{-(\lambda_{c1}+\lambda_{c3})G_{0,m}},$$

(2)

When $h_m \leq G_{0,m}$, $x = h_m$, while $h_m \geq G_{0,m}$, $x$ is assigned the value of the probability of one arriving vehicle during $G_{0,m}$. That is,

$$x = P_1(G_{0,m}) = G_{0,m}(\lambda_{c1} + \lambda_{c3}) e^{-(\lambda_{c1}+\lambda_{c3})G_{0,m}},$$

(3)

where $h_m$ is the average headway time of vehicles arrival on the main road.

Discretization of $G_{0,m}$ is conducted in seconds as shown in Figure 5.

Therefore, the probability of one vehicle arrival per second is therefore as follows:

$$P_1(1) = (\lambda_{c1} + \lambda_{c3}) e^{-(\lambda_{c1}+\lambda_{c3})}.$$  

(4)

The probability of one vehicle arrival in the first second is $P_1(1)/P_1(G_{0,m})$, it is $P_1(1)(1 - P_1(1))/P_1(G_{0,m})$ for the second second, and it is $P_1(1)(1 - P_1(1))^{(G_{0,m}-1)}/P_1(G_{0,m})$ for the last one interval. Thus, the expectation of vehicle arrival time intervals on the main road during $G_{0,m}$ is given as follows:

$$E(x) = \left(1 P_1(1) + 2 (1 - P_1(1)) P_1(1) \right)$$

$$\hspace{1cm} + \cdots + G_{0,m}(1 - P_1(1))^{(G_{0,m}-1)} P_1(1) \right) \left( P_1(1) \right)^{-1}$$

$$\times \left(1 - (1 - P_1(1))^{G_{0,m}} / P_1(1) - G_{0,m}(1 - P_1(1))^{G_{0,m}} / P_1(1) \right)^{-1}$$

$$= \left[1 - (1 - P_1(1))^{G_{0,m}} / P_1(1) - G_{0,m}(1 - P_1(1))^{G_{0,m}} / P_1(1) \right].$$

(5)

Accordingly, the operation of traffic lights is showed in Figure 6, where $n$ is the number of times for continuous vehicle arrival during unit extension on the main road in maximum green time and its value is actually unfixed. Figure 6 indicates that the probability of extending one $x$ is $1 - P_1(G_{0,m})$, $2x$ is $[1 - P_1(G_{0,m})]^2$, and $nx$ is $[1 - P_1(G_{0,m})]^n$. So, the expectation of continuous vehicle arrival number on the main road, $E(n)$, should be

$$E(n) = 1 [1 - P_0(G_{0,m})] + 2 [1 - P_0(G_{0,m})]^2$$

$$\hspace{1cm} + \cdots + n [1 - P_0(G_{0,m})]^n = \frac{[1 - P_0(G_{0,m})]}{P_0(G_{0,m})}.$$  

(6)

For traffic flow on the side road, delays mainly depend on the green time extension of the main road. The expectation of vehicle delay on the main road, $E(n)$, is given as follows:

$$d_{cj} = \frac{1}{2} \left[G_{min,s} + E(n) E(x) \right] \sum_{i=2}^{4} \frac{\lambda_{cj} S_{ci}}{(S_{ci} - \lambda_{cj})}.$$  

(7)

The fixed value of green time of the side road is $G_{min,s}$ and the vehicle delay on the main road is therefore concluded as

$$d_{cj} = \frac{1}{2} \left[G_{min,s} \right] \sum_{j=1}^{3} \frac{\lambda_{cj} S_{cj}}{(S_{cj} - \lambda_{cj})},$$  

(8)

where $S_{cj}$ and $S_{cj}$ are saturation flow rates of vehicle on the side road and the main road (vps).

Therefore, the total number of vehicle arrivals in one signal circle is given as

$$Q = \sum_{k} d_{ck} = \left[G_{min,m} + E(n) E(x) + G_{min,s} \right] \sum_{k} \lambda_{ck}.$$  

(9)

Meanwhile, the average vehicle delay at the intersection is computed as

$$\overline{d_c} = \frac{d_{cj} + d_{cj}}{Q}.$$  

(10)
Maximum
Green time of
main road

Any vehicle on
side road? No

Yes

Minimum green
time of
main road?

No

Yes

Green time of
side road

Any vehicle on
side road? No

Yes

Maximum green
time of
side road?

No

Yes

Figure 7: Flow chart of semi-ASC with detectors on the side road.

3.2. Detectors on the Side Road. Under this control mode, the main road is usually green light. If vehicle arrival is detected on the side road, the phase is switched and green light on the side road is turned on. No vehicle arrives during unit extension of the side road and green light on the side road ends up and phase is switched to the main road; or the phase is forced to be switched after reaching the maximum green time. This is shown in Figure 7.

The probability of no vehicle arrival on the side road during the minimum green time of the main road is as follows:

$$P_0(G_{min,m}) = e^{-(\lambda_{ci}+\lambda_{ci})G_{min,m}}.$$  \hspace{1cm} (11)

If no vehicle arrives during the minimum green time of the main road, the green light of the main road is continuously extended. The extending number of minimum green lights on the main road is supposed as \(m\); the operation procedure of signal on the main road is shown in Figure 8.

As demonstrated in Figure 8, the expectation of extending number of minimum green lights on the main road, \(E(m)\), for the minimum green light on the main road is as follows with no vehicle arrival of the side road:

$$E(m) = 1 + P_0(G_{min,m}) + 2P_0(G_{min,m})^2 + \cdots + mP_0(G_{min,m})^m = 1 + \frac{P_0(G_{min,m})}{(1 - P_0(G_{min,m}))^2}.$$  \hspace{1cm} (12)

The green light on the main road ends and light on the side road is turned on when vehicle arrival is detected on the side road. The time interval of the subsequent vehicle arrival is set as \(y\) after the initial green time ending; the green time on the side road is extended by \(y\). The value of \(y\) is determined by the same method as \(x\). The expectation of time interval for the subsequent vehicle arrival is shown as follows if vehicle arrival on the side road is available in \(G_{0,c}\):

$$E(y) = \left[\frac{1 - (1 - P_1(1))^{G_{0,c}}}{P_1(G_{0,c})}\right] \left[\frac{1 - (1 - P_1(1))^{G_{0,m}}}{P_1(G_{0,m})}\right].$$  \hspace{1cm} (13)

where \(l\) is supposed to be the number of subsequent vehicle arrivals after the minimum green light ending on the side road, and the expectation for continuous vehicle arrival number on the side road is concluded from (6). Consider

$$E(l) = \frac{[1 - P_0(G_{0,c})]}{P_0(G_{0,c})}.$$  \hspace{1cm} (14)

The vehicle delays on the side road and the main road are computed as follows, respectively:

$$d'_{cj} = \frac{1}{2} G_{min,m}^2 \sum_i \frac{\lambda_{ci} S_{ci}}{(S_{ci} - \lambda_{ci})^2}.$$  \hspace{1cm} (15)

$$d'_c = \frac{1}{2} \left[G_{min,s} + E(l) E(y)\right]^2 \sum_j \frac{\lambda_{cj} S_{cj}}{(S_{cj} - \lambda_{cj})^2}.$$  \hspace{1cm} (16)

Hence, the total amount of vehicle arrival is shown as follows in one signal cycle length:

$$Q' = \sum_k q_{ck} = [E(m) G_{min,m} + G_{min,s} + E(l) E(y)] \sum_k \lambda_{ck}.$$  \hspace{1cm} (17)

And the average vehicle delay at the intersection is

$$\overline{d'_c} = \frac{d'_{cj} + d'_c}{Q'}.$$  \hspace{1cm} (18)

4. Vehicle Delay on Fully ASC

Fully ASC is a way to install detectors on all approaches, which is applicable to considerably various intersections with large traffic amount between the same leveled roads, where other two control parameters are unit extension of the side road \((G_{0,s})\) and the maximum green time of the side road \((G_{max,s})\).

The vehicle delay of the main road is the same as the side road in process generation at this time and the vehicle delay on the main road is taken as the example to conduct derivation (Figure 9).

If vehicle arrival is detected on the main road during \(G_{0,m}\) of the minimum green time, the time interval of arrival is set as \(x_1\) and the expectation of time interval for vehicle arrival on the main road is \(E(x_1)\). Consider

$$E(x_1) = \left[\frac{1 - (1 - P_1(1))^{G_{0,m}}}{P_1(G_{0,m})}\right] \left[\frac{1 - (1 - P_1(1))^{G_{0,m}}}{P_1(G_{0,m})}\right].$$  \hspace{1cm} (19)
where \( n_1 \) is the times of continuous vehicle arrival during the maximum green time of the main road. The expectation of times for continuous vehicle arrival on the main road is concluded from (6). Consider

\[
E(n_1) = \frac{1 - P_1(G_{0,m})}{P_0(G_{0,m})^2}. \tag{20}
\]

Vehicle delays on the main road and the side road are computed as

\[
d_{ci}' = \frac{1}{2}[G_{\text{min},m} + E(n_1)E(x_1)]^2 \sum_j \frac{\lambda_{ci}S_{ci}}{S_{ci} - \lambda_{ci}}, \tag{21}
\]

\[
d_{cj}' = \frac{1}{2}[G_{\text{min},s} + E(n_2)E(x_2)]^2 \sum_j \frac{\lambda_{cj}S_{cj}}{S_{cj} - \lambda_{cj}}. \tag{22}
\]

So, the total amount of vehicle arrival in one cycle length is shown as follows:

\[
Q'' = [G_{\text{min},m} + G_{\text{min},s} + E(n_1)E(x_1) + E(n_2)E(x_2)] \cdot \sum_k \lambda_{ck}. \tag{23}
\]

And the average vehicle delay at the intersection is

\[
\overline{d_c''} = \frac{d_{ci}'' + d_{cj}''}{Q''}. \tag{24}
\]

5. Case Study and Simulation Analysis

The relevant parameters are illustrated for ASC and traffic flow on the main road and the side road in Table 1.

5.1. Semi-ASC with Detectors on the Main Road

Each probability is computed based on (1) to (4). Consider the following:

\[
P_1(G_{0,m}) = \lambda_mG_0e^{-\lambda_m G_{0,m}} = 0.42 \times 3 \times e^{(-0.42 \times 3)} = 0.36,
\]

\[
P_1(1) = \lambda_m e^{-\lambda_m} = 0.42 \times e^{(-0.42)} = 0.27,
\]

\[
P_0(G_{0,m}) = e^{-\lambda_m G_{0,m}} = e^{(-0.42 \times 3)} = 0.29. \tag{25}
\]

Time interval and the expectation of arrival times are computed based on (5) and (6). Consider the following:

\[
E(x) = \frac{[1 - (1 - P_1(1)G_{0,m})]/P_1(1) - G_0(1 - P_1(1))G_{0,m}}{P_1(G_{0,m})} = 3.09 \text{ s},
\]

\[
E(n) = \frac{[1 - P_0(G_{0,m})]}{P_0(G_{0,m})^2} = 8.69. \tag{26}
\]
Table 1: Parameters of ASC and traffic flow of the intersection.

<table>
<thead>
<tr>
<th>Intersection approach</th>
<th>Minimum green time (sec)</th>
<th>Unit extension (sec)</th>
<th>Maximum green time (sec)</th>
<th>Arrival probability (vph)</th>
<th>Saturation flow (vph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>The main road</td>
<td>20</td>
<td>3</td>
<td>40</td>
<td>1500</td>
<td>2000</td>
</tr>
<tr>
<td>The side road</td>
<td>10</td>
<td>3</td>
<td>20</td>
<td>800</td>
<td>1000</td>
</tr>
</tbody>
</table>

The vehicle delays of the side road and the main road and the average delay are computed based on (7) to (10). Consider the following:

\[ d_{ci} = \frac{1}{2} (G_{\text{min,m}} + E(n)E(x))^2 \sum \frac{\lambda_{ci}S_{ci}}{(S_{ci} - \lambda_{ci})} = 2439.8 \text{ s}, \]

\[ d_{cj} = \frac{1}{2} G_{\text{min,s}}^2 \sum \frac{\lambda_{cj}S_{cj}}{(S_{cj} - \lambda_{cj})} = 166.7 \text{ s}, \]

\[ \overline{d_c} = \frac{d_{ci} + d_{cj}}{\sum k q_{ck}} = 17.9 \text{ s}. \]  

(27)

5.2. Semi-ASC with Detectors on the Side Road. Each probability is computed based on (1), (4), and (11). Consider the following:

\[ P_0 (G_{\text{min,m}}) = e^{-\lambda_s G_{\text{min,m}}} = e^{(-0.22 \times 20)} = 0.01, \]

\[ P_1 (1) = \lambda_s e^{-\lambda_s} = 0.22 \times e^{(-0.22)} = 0.18, \]

\[ P_1 (G_{0,c}) = \lambda_s G_{0,c} e^{-\lambda_s G_{0,c}} = 0.22 \times 3 \times e^{(-0.22 \times 3)} = 0.34, \]

\[ P_0 (G_{0,s}) = e^{-\lambda_s G_{0,s}} = e^{(-0.22 \times 3)} = 0.51. \]  

(28)

\[ E(m), E(y), \text{ and } E(l) \] are computed based on (12) to (14). Consider the following:

\[ E(m) = 1 + \frac{P_0 (G_{\text{min,m}})}{(1 - P_0 (G_{\text{min,m}}))^2} = 1.01, \]

\[ E(y) = \left[ (1 - (1 - P_1 (1))^G_{0,s}^0) / P_1 (1) - G_{0,c} (1 - P_1 (1))^G_{0,c} \right] / P_1 (G_{0,c}) = 11.45 \text{ s}, \]

\[ E(l) = \left[ 1 - P_0 (G_{0,s}) \right] / P_0 (G_{0,s}) = 1.85. \]  

(29)

The vehicle delays on the side road and the main road and the average delay are computed based on (15) to (18) to conclude that \( \overline{d_{ci}} = 444.4 \text{ s}, \overline{d_{cj}} = 1617.1 \text{ s}, \) and \( \overline{d_c} = 15.7 \text{ s}. \)

5.3. Fully ASC. The above results are also applicable to the parameters under the fully ASC. The vehicle delays on the side road and the main road and the average delay are computed based on (21) to (24) to conclude that \( \overline{d_{ci}} = 2439.8 \text{ s}, \overline{d_{cj}} = 1617.1 \text{ s}, \) and \( \overline{d_c} = 20.4 \text{ s}. \)

5.4. CORSIM Simulation. Here, we suppose that there are 2 through lanes, 1 right-turn and through lane and 1 left-turn pocket lane at the main road approaches, and 1 through lane, 1 right-turn and through lane, and 1 left-turn pocket lane at the side road approaches separately. Also, the presence detection system is at the stop line; and the detectors are 12.2 to 18.3 meters in length.

The proportions of left-turn, through, and right-turn vehicles are set as 15%, 65%, and 20%. And there is no truck in the system. Further, 3-second yellow time and 1-second all red time are chosen for every phase, respectively. Finally, the speed limitation is set as 35 mph for the intersection. Figure 10 shows the CORSIM simulation interface of an isolated intersection under fully ASC.

From simulation, the average vehicle delays of the intersection are 16.5 s, 17.7 s, and 18.8 s for the semi-ASC with detectors on the main road and with detectors on the side road and for the fully ASC. Meanwhile, the relative errors are 7.8%, −12.7%, and 8.0%, respectively. The results show that the relative errors are not so distinct. So, it is approved that the calculating results should be acceptable.

6. Conclusions

Vehicle delay, a crucial index to evaluate the traffic operation situation at intersections, also serves as an important parameter to determine the level of service and develop the channelization and the signal control scheme. The vehicle delay of an ASC intersection is analyzed to offer the information for the evaluation of ASC effect and optimization of signal control parameters. Here are some primary findings.

(1) The calculations of vehicle delay are presented in three situations, which are tested with case study and simulation analysis.
(2) Vehicle delays on the main road vary with the different locations of detectors and logics of ASC. The appropriate control ways can decrease the vehicle delay.

(3) Among the parameters of ASC, the vehicle delay is significantly affected by the unit extension. The maximum green time is hard to get and it is acceptable to ignore it. The value of the minimum green time is relatively fixed with less impact.

(4) The calculating results of these methods are approved by the CORSIM simulations.

**Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

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**References**


