Research Article

On H_∞ Fault Estimator Design for Linear Discrete Time-Varying Systems under Unreliable Communication Link

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This paper investigates the H_∞ fixed-lag fault estimator design for linear discrete time-varying (LDTV) systems with intermittent measurements, which is described by a Bernoulli distributed random variable. Through constructing a novel partially equivalent dynamic system, the fault estimator design is converted into a deterministic quadratic minimization problem. By applying the innovation reorganization technique and the projection formula in Krein space, a necessary and sufficient condition is obtained for the existence of the estimator. The parameter matrices of the estimator are derived by recursively solving two standard Riccati equations. An illustrative example is provided to show the effectiveness and applicability of the proposed algorithm.

1. Introduction

To satisfy the growing demands for reliability and safety in control systems, more and more research efforts are made for model-based fault detection (FD) during the past decades; see [1–6] and references therein. Basically, the FD issue concerns designing a fault detection filter (FDF) for generating a residual signal such that the sensitivity of residual to fault is intensified by enhancing the robustness to the disturbance. In reviewing of the development of FD, with the aid of linear matrix inequality (LMI) techniques, much attention has been paid to linear time-invariant (LTI) systems with various characteristics such as time-delay, model inaccuracy, time-dependent switching mode, and uncertain observations; see [7–10] and related works. Recently, some contributions are devoted to linear time-varying (LTV) systems since most practical industrial processes can be represented or well approximated by time-varying dynamics [11]. For example, in [11, 12], unified optimal solutions are derived in the framework of maximizing H_-/H_∞ and H_∞/H_∞ FD performance indices for linear continuous time-varying (LCTV) and linear discrete time-varying (LDTV) systems, respectively. In [13–15], the H_∞ filtering based fault estimation methods are proposed for LDTV systems in virtue of the Krein space based

reorganized innovation analysis and projection theory in the background of [16–21].

On another front line, with the rapid progress of networked control systems and distributed sensor/actuator systems, the packet dropout caused by sensor gain reductions may happen when transmitting information under unreliable links. The random uncertainty introduced by packet dropouts evidently deteriorates the performance of the FDF. Many contributions are dedicated to FD issue for systems with incomplete measurements by employing the LMI formulated H_∞ fault estimation approach over infinite horizon; we refer to [22–26] and references therein. For finite-horizon case, an H_∞ fault estimator for LDTV systems with multiple packet dropouts is designed in [27] based on the stochastic bounded real lemma (BRL), while a two-objective optimization FD method for LDTV systems with intermittent observations is addressed in [28]. Unfortunately, if there is no sensor fault in the measurement channel or the data packet is not time-stamped, the algorithms proposed in [27, 28] will fail. This indicates that research on FD problem for LDTV systems subject to intermittent measurements has not been fully investigated yet, which is the main motivation of the present study.

To overcome the drawbacks in the existing results, a novel fault estimator design method for LDTV systems with intermittent observations is proposed. The contribution of this paper consists in three aspects as follows:

- (1) an H_∞ fixed-lag fault estimator design problem is formulated by establishing an equivalent system and its corresponding deterministic performance index;
- (2) by employing the reorganized innovation analysis approach and the projection theory in Krein space, a necessary and sufficient condition of the existence of the estimator is derived;
- (3) a recursive fault estimation algorithm is proposed, which is apt to be online applied for finite-horizon.

The rest of the content is organized as follows. Section 2 provides the formulation of the concerned problem. Section 3 presents our main results of designing the fault estimator. The proposed approach is applied to a time-varying model to illustrate its applicability in Section 4. Finally, the paper is ending with some conclusions.

Notations. Throughout this paper, vectors in the Krein space are represented by **boldface** letters, and vectors in the Euclidean space are denoted by normal letters. For a matrix X , X^T and X^{-1} stand for the transpose and inverse of X , respectively. $X > 0$ ($X < 0$) denotes X is positive (negative) definite. R^n means the set of n -dimensional real vectors. I and 0 denote identity matrix and zero matrix with appropriate dimensions, respectively. $E\{\vartheta(k)\}$ means the mathematical expectation of $\vartheta(k)$. $\vartheta(k) \in l_2[0, N]$ means $\sum_{k=0}^N \vartheta^T(k)\vartheta(k) < \infty$, where N is a positive integer. The symbol $\mathcal{L}\{\{\vartheta(i)\}_{i=j}^k\}$ represents the linear space spanned by the sequence $\vartheta(k)$ taking values in the time interval $[j, k]$. $\text{Prob}\{Y\}$ denotes the occurrence probability of the event “ Y ”. δ_{ij} represents the Kronecker delta function, which is equal to unity for $i = j$ and zero for $i \neq j$. $\text{diag}\{S_1, S_2, \dots, S_n\}$ means a block diagonal matrix with diagonal blocks $S_1, S_2, \dots, S_n$.

2. Problem Formulation and Preliminaries

Consider the following LDTV system:

$$\begin{aligned} x(k+1) &= A(k)x(k) + B_f(k)f(k) + D(k)d(k), \\ y(k) &= \theta(k)C(k)x(k) + v(k), \end{aligned} \quad (1)$$

$$x(0) = x_0,$$

where $x(k) \in R^n$, $y(k) \in R^q$, $d(k) \in R^{n_d}$, $v(k) \in R^{n_v}$, and $f(k) \in R^{n_f}$ denote the state, sensor measurement, process noise, observation noise, and fault, respectively. $f(k)$, $d(k)$, and $v(k)$ belong to $l_2[0, N]$. $A(k)$, $B_f(k)$, $C(k)$, and $D(k)$ are known time-varying matrices with appropriate dimensions. $\theta(k)$ is a Bernoulli distributed binary stochastic variable to describe the measurement packet dropouts, which satisfies

$$\begin{aligned} \text{Prob}\{\theta(k) = 1\} &= E\{\theta(k)\} = \rho, \\ \text{Prob}\{\theta(k) = 0\} &= 1 - E\{\theta(k)\} = 1 - \rho, \end{aligned} \quad (2)$$

with ρ being a known constant. The value of ρ can be obtained by empirical observations, experimentations, and statistical analysis [29].

The main purpose of this paper is as follows: given a prescribed disturbance attenuation level γ , by collecting the observations $y(0), \dots, y(k)$, find $\check{f}(k-l|k)$ as a suitable estimation of the fault signal $f(k)$ such that the following l -step delayed H_∞ performance index is fulfilled with l being a positive integer:

$$\begin{aligned} \sup_{(x_0, f_k, d_k, v_k) \neq 0} E \left\{ \sum_{k=l}^N \left(\check{f}(k-l|k) - f(k-l) \right)^T \right. \\ \left. \times \left(\check{f}(k-l|k) - f(k-l) \right) \right\} \\ \times \left(x_0^T P_0^{-1} x_0 + \sum_{k=0}^N f^T(k) f(k) \right. \\ \left. + \sum_{k=0}^{N-1} d^T(k) d(k) + \sum_{k=0}^N v^T(k) v(k) \right)^{-1} < \gamma^2, \end{aligned} \quad (3)$$

where $f_k = [f^T(0) \cdots f^T(k)]^T$, $d_k = [d^T(0) \cdots d^T(k)]^T$, $v_k = [v^T(0) \cdots v^T(k)]^T$.

Due to the fact that the denominator of the left side of (3) is positive, (3) can be rewritten as

$$\begin{aligned} J_0 &= x_0^T P_0^{-1} x_0 + \sum_{k=0}^N f^T(k) f(k) \\ &+ \sum_{k=0}^{N-1} d^T(k) d(k) + \sum_{k=0}^N v^T(k) v(k) \\ &- E \left\{ \gamma^{-2} \sum_{k=l}^N v_s^T(k) v_s(k) \right\} > 0, \end{aligned} \quad (4)$$

where $v_s(k) = \check{f}(k-l|k) - f(k-l)$. Consequently, according to [30], the H_∞ fixed-lag fault estimation problem can be restated as follows: given a constant $\gamma > 0$, design an estimator in the following way:

$$\check{f} = \Psi(y) = \bar{\Psi}(f, d, v), \quad (5)$$

where Ψ denotes a stable operator which generates a bounded operator $\bar{\Psi}$ mapping from f, d, v to $\check{f}$, such that the indefinite cost function (4) has a positive minimum with respect to f, d , and v .

Remark 1. In the existing results, for example, [22–28], the Bernoulli distributed random variables are introduced to describe the packet dropping or finite step measurement time-delay phenomenon. It is noteworthy that the designed estimators only depend on the probability, that is, ρ , rather than $\theta(k)$. This indicates that the desired fault estimator does not require the time stamp of the data packet.

Remark 2. Notice that when $y(k)$ is affected by the so-called sensor fault with the following form:

$$y(k) = \theta(k) C(k) x(k) + D_f(k) f(k) + v(k), \quad (6)$$

the existing BRL based H_∞ fault estimation algorithm in [27] is applicable in a “filter” manner. In the case that $D_f(k) = 0$, the estimator is supposed to be designed as a “smoother” with the proposed performance index (4). In this scenario, the methodology in [27] may induce computational burden via state augmentation approach and the gain matrices of the estimator are arduous to be derived due to some coupled product terms. In what follows, a Krein space based fault estimator design scheme will be addressed to overcome the aforementioned defects.

3. Main Results

In this section, inspired by [31, 32], an equivalent Krein space stochastic system and a corresponding H_∞ performance index are first introduced. Then, by exploiting the reorganized innovation analysis and the projection theory in Krein space, the H_∞ fault estimator is derived.

3.1. Krein Space Model Design. Before we proceed, we would like to propose the following lemma to construct an auxiliary stochastic system in Krein space.

Lemma 3. Given a scalar $\gamma > 0$ and an integer $l > 0$, then the H_∞ performance (4) is fulfilled if and only if there exists a fault estimator $\check{f}(k-l|k)$ such that the following inequality holds:

$$\begin{aligned} J &= x_0^T P_0^{-1} x_0 + \sum_{k=0}^N f^T(k) f(k) + \sum_{k=0}^N v_0^T(k) v_0(k) \\ &+ \sum_{k=0}^{N-1} d^T(k) d(k) + \sum_{k=0}^N v_z^T(k) v_z(k) \\ &- \gamma^{-2} \sum_{k=l}^N v_s^T(k) v_s(k) > 0, \end{aligned} \quad (7)$$

subject to the following dynamic constraints:

$$\begin{aligned} x(k+1) &= A(k) x(k) + B_f(k) f(k) + D(k) d(k), \\ y_0(k) &= \rho C(k) x(k) + v_0(k), \\ y_z(k) &= \sqrt{\rho(1-\rho)} C(k) x(k) + v_z(k), \\ \check{f}(k-l|k) &= f(k-l) + v_s(k), \\ x(0) &= x_0, \end{aligned} \quad (8)$$

where $y_0(k)$ and $y_z(k)$ are the fictitious observations with their corresponding observation noises $v_0(k)$ and $v_z(k)$, respectively. The instantaneous value of y_0 at each time instant k is equal to $y(k)$ along with $y_z(k) \equiv 0$.

Proof. Consider the following.

Necessity. From (1), the state transition matrix Φ is defined as

$$\Phi(k, j) = \begin{cases} A(k-1) \cdots A(j), & 0 < k < j, \\ I, & k = j; \end{cases} \quad (9)$$

hence, we have

$$\begin{aligned} x(k) &= \Phi(k, 0) x_0 + \sum_{i=0}^{k-1} \Phi(k, i+1) B_f(i) f(i) \\ &+ \sum_{i=0}^{k-1} \Phi(k, i+1) D(i) d(i). \end{aligned} \quad (10)$$

Define

$$\begin{aligned} y_k &= [y^T(0) \cdots y^T(k)]^T, \\ v_{s,k} &= [v_s^T(0) \cdots v_s^T(k)]^T, \\ \check{f}_k &= [\check{f}^T(0|l) \cdots \check{f}^T(k-l|k)]^T. \end{aligned} \quad (11)$$

Then, in view of (10), we have

$$\begin{aligned} y_N &= \Xi(k) G_x x_0 + \Xi(k) G_f f_N + \Xi(k) G_d d_N + v_N, \\ \check{f}_N &= f_{N-l} + v_{s,N}, \end{aligned} \quad (12)$$

where

$$\begin{aligned} \Xi(k) &= \text{diag}\{\theta(1), \dots, \theta(k)\}, \\ G_f(k, i) &= C(k) \Phi(k, i+1) B_f(i), \\ G_d(k, i) &= C(k) \Phi(k, i+1) D(i), \end{aligned}$$

$$\begin{aligned} G_x &= \begin{bmatrix} C(0) \Phi(0, 0) \\ C(1) \Phi(1, 0) \\ \vdots \\ C(N) \Phi(N, 0) \end{bmatrix}, \\ G_f &= \begin{bmatrix} 0 & \cdots & \cdots & 0 \\ G_f(1, 0) & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ G_f(N, 0) & G_f(N, 1) & \cdots & 0 \end{bmatrix}, \\ G_d &= \begin{bmatrix} 0 & \cdots & \cdots & 0 \\ G_d(1, 0) & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ G_d(N, 0) & G_d(N, 1) & \cdots & 0 \end{bmatrix}. \end{aligned} \quad (13)$$

Thus, by substituting (12) into (4) and taking (2) into consideration, we have

$$\begin{aligned}
J_0 &= E \left\{ x_0^T P_0^{-1} x_0 + \sum_{k=0}^N f^T(k) f(k) + \sum_{k=0}^{N-1} d^T(k) d(k) \right. \\
&\quad - (y_N - \Xi(k) G_x x_0 - \Xi(k) G_f f_N - \Xi(k) G_d d_N)^T \\
&\quad \times (y_N - \Xi(k) G_x x_0 - \Xi(k) G_f f_N - \Xi(k) G_d d_N) \\
&\quad \left. - \gamma^{-2} \sum_{k=l}^N (\check{f}(k-l|k) - f(k-l))^T \right. \\
&\quad \left. \times (\check{f}(k-l|k) - f(k-l)) \right\} \\
&= x_0^T P_0^{-1} x_0 + \sum_{k=0}^N f^T(k) f(k) + \sum_{k=0}^{N-1} d^T(k) d(k) \\
&\quad + (y_{0,N} - \bar{\Xi} G_x x_0 - \bar{\Xi} G_f f_N - \bar{\Xi} G_d d_N)^T \\
&\quad \times (y_{0,N} - \bar{\Xi} G_x x_0 - \bar{\Xi} G_f f_N - \bar{\Xi} G_d d_N) \\
&\quad + (y_{z,N} - \bar{\Xi} G_x x_0 - \bar{\Xi} G_f f_N - \bar{\Xi} G_d d_N)^T \\
&\quad \times (y_{z,N} - \bar{\Xi} G_x x_0 - \bar{\Xi} G_f f_N - \bar{\Xi} G_d d_N) \\
&\quad - \gamma^{-2} \sum_{k=l}^N (\check{f}(k-l|k) - f(k-l))^T \\
&\quad \times (\check{f}(k-l|k) - f(k-l)), \tag{14}
\end{aligned}$$

where

$$\begin{aligned}
y_{0,k} &= [y_0^T(0) \cdots y_0^T(k)]^T, \quad y_{z,k} = [y_z^T(0) \cdots y_z^T(k)]^T, \\
y_0(i) &= y(i), \quad y_z(i) = 0, \quad (i = 0, \dots, k), \\
\bar{\Xi} &= \rho I, \quad \tilde{\Xi} = \sqrt{\rho(1-\rho)} I. \tag{15}
\end{aligned}$$

Therefore, if the H_∞ performance index (4) is satisfied, then, following the same line with the correlation between (1) and (4), we have $J > 0$ subject to the dynamics (8) over x_0 , f_k , and d_k .

Sufficiency. For (8), since the value of $y_0(k)$ is equivalent to $y(k)$ and $y_z(k) \equiv 0$, in light of (14), it is easy to find out that for a given constant $\gamma > 0$ and an integer $l > 0$, $J_0 = J$, which indicates that if $J > 0$ holds, then the H_∞ performance (4) is satisfied. Combining the sufficiency and necessity part, the proof is complete. $\square$

In virtue of Lemma 3, the auxiliary performance index J in (7) can be converted into the following compact form:

$$J = \begin{bmatrix} x_0 \\ d_N \\ f_N \\ v_{a,N} \end{bmatrix}^T \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & Q_{a,N} \end{bmatrix}^{-1} \begin{bmatrix} x_0 \\ d_N \\ f_N \\ v_{a,N} \end{bmatrix}, \tag{16}$$

where

$$v_a(k) = \begin{cases} v_1(k) = \begin{bmatrix} v_0(k) \\ v_z(k) \end{bmatrix}, & 0 \leq k < l, \\ v_2(k) = \begin{bmatrix} v_0(k) \\ v_z(k) \\ v_s(k) \end{bmatrix}, & k \geq l, \end{cases} \tag{17}$$

$$v_{a,N} = [v_a^T(0) \cdots v_a^T(N)]^T,$$

$$Q_a(k) = \begin{cases} Q_{v_1}(k) = \text{diag}\{I, I\}, & 0 \leq k < l, \\ Q_{v_2}(k) = \text{diag}\{I, I, -\gamma^2 I\}, & k \geq l, \end{cases} \tag{18}$$

$$Q_{a,N} = \text{diag}\{Q_a(0), \dots, Q_a(N)\}.$$

From (8) and (17), we have

$$\begin{aligned}
y_f(k) &= \begin{bmatrix} y(k) \\ y_z(k) \end{bmatrix} = C_1(k) x(k) + v_1(k), \\
y_a(k) &= \begin{cases} y_f(k), & 0 \leq k < l, \\ \begin{bmatrix} y_f(k) \\ \check{f}(k-l|k) \end{bmatrix} \\ = C_2(k) x(k) \\ + Hf(k-l) + v_2(k), & k \geq l, \end{cases} \tag{19}
\end{aligned}$$

where

$$\begin{aligned}
C_1(k) &= \begin{bmatrix} \rho C(k) \\ \sqrt{\rho(1-\rho)} C(k) \end{bmatrix}, \quad C_2(k) = \begin{bmatrix} C_1(k) \\ 0 \end{bmatrix}, \\
H &= [0 \ 0 \ I]^T. \tag{20}
\end{aligned}$$

Thus, according to [20, 21], we introduce the following Krein space system associated with (8), (16), (18), and (19):

$$\begin{aligned}
x(k+1) &= A(k) x(k) + B_f(k) f(k) + D(k) d(k), \\
y_a(k) &= \begin{cases} y_f(k) = C_1(k) x(k) + v_1(k), & 0 \leq k < l, \\ \begin{bmatrix} y_f(k) \\ \check{f}(k-l|k) \end{bmatrix} \\ = C_2(k) x(k) + Hf(k-l) + v_2(k), & k \geq l, \end{cases} \\
x(0) &= x_0, \tag{21}
\end{aligned}$$

where $\mathbf{x}_0(i)$, $\mathbf{d}(i)$, $\mathbf{f}(i)$, $\mathbf{v}_1(i)$, and $\mathbf{v}_2(i)$ are uncorrelated white noises in Krein space satisfying

$$\left\langle \begin{bmatrix} \mathbf{x}_0 \\ \mathbf{d}(i) \\ \mathbf{f}(i) \\ \mathbf{v}_a(i) \end{bmatrix}, \begin{bmatrix} \mathbf{x}_0 \\ \mathbf{d}(i) \\ \mathbf{f}(i) \\ \mathbf{v}_a(i) \end{bmatrix} \right\rangle = \begin{bmatrix} I\delta_{ij} & 0 & 0 & 0 \\ 0 & I\delta_{ij} & 0 & 0 \\ 0 & 0 & I\delta_{ij} & 0 \\ 0 & 0 & 0 & Q_a(i)\delta_{ij} \end{bmatrix}, \quad (22)$$

$$\mathbf{v}_a(k) = \begin{cases} \mathbf{v}_1(k) = \begin{bmatrix} \mathbf{v}_0(k) \\ \mathbf{v}_z(k) \end{bmatrix}, & 0 \leq k < l, \\ \mathbf{v}_2(k) = \begin{bmatrix} \mathbf{v}_0(k) \\ \mathbf{v}_z(k) \\ \mathbf{v}_s(k) \end{bmatrix}, & k \geq l, \end{cases} \quad (23)$$

with $\mathbf{v}_0(k)$, $\mathbf{v}_z(k)$, and $\mathbf{v}_s(k)$ being fictitious noise in Krein space corresponding to (17).

Consequently, on the basis of Lemma 4.2.1 in [20], we have the following lemma.

Lemma 4. For (8), given a scalar $\gamma > 0$ and an integer $l > 0$, then the H_∞ performance (7) has a minimum over $\mathbf{x}_0$, $\mathbf{f}$, $\mathbf{d}$ if and only if $Q_a(k)$ and $Q_w(k)$ have the same inertia, where $Q_w(k) = \langle \mathbf{w}(k), \mathbf{w}(k) \rangle$ is the covariance matrix of innovation sequence $\mathbf{w}(k)$ given by

$$\mathbf{w}(k) = \mathbf{y}_a(k) - \hat{\mathbf{y}}_a(k), \quad (24)$$

where $\hat{\mathbf{y}}_a(k)$ is the projection of $\mathbf{y}_a(k)$ onto $\mathcal{L}\{\{\mathbf{y}_a(j)\}_{j=0}^{k-1}\}$. Furthermore, the minimum value of J is

$$\begin{aligned} J_{\min} &= \sum_{k=0}^{l-1} \left[\mathbf{y}_f(k) - C_1(k) \hat{\mathbf{x}}(k) \right]^T \\ &\quad \times Q_w^{-1}(k) \left[\mathbf{y}_f(k) - C_1(k) \hat{\mathbf{x}}(k) \right] \\ &\quad + \sum_{k=l}^N \left[\begin{array}{c} \mathbf{y}_f(k) - C_1(k) \hat{\mathbf{x}}(k) \\ \tilde{\mathbf{f}}(k-l|k) - \tilde{\mathbf{f}}(k-l|k-1) \end{array} \right]^T \\ &\quad \times Q_w^{-1}(k) \left[\begin{array}{c} \mathbf{y}_f(k) - C_1(k) \hat{\mathbf{x}}(k) \\ \tilde{\mathbf{f}}(k-l|k) - \tilde{\mathbf{f}}(k-l|k-1) \end{array} \right], \end{aligned} \quad (25)$$

where $\hat{\mathbf{x}}(k)$ and $\tilde{\mathbf{f}}(k-l|k-1)$ are, respectively, calculated from the Krein space projections of $\mathbf{x}(k)$ and $\mathbf{f}(k-l)$ onto $\mathcal{L}\{\{\mathbf{y}_a(j)\}_{j=0}^{k-1}\}$.

Remark 5. According to Lemmas 3 and 4, the purpose of establishing the dynamic model (8) associated with (7) is to derive a positive minimum of the cost function (4) by applying the projection theory in Krein space. Notice that although the measurement $\{y(k)\}_{k=0}^N$ is a substantially stochastic sequence, the instantaneous values of $y(k)$ and $\tilde{\mathbf{f}}(k-l|k)$ at each instant are available for the estimator. Thus, the equivalent cost function (7) and its corresponding dynamic constraint are constructed in a “conditional expectation” sense by gathering up $\{y(k)\}_{k=0}^N$ (cf. (14) in the proof of Lemma 3).

3.2. Kalman Filtering in Krein Space. From the analysis above, the key step to achieve our goal is to find a suitable $\hat{\mathbf{x}}(k)$ and $\tilde{\mathbf{f}}(k-l|k-1)$. To this end, let

$$\mathbf{y}_1(k) = \mathbf{y}_f(k), \quad \mathbf{y}_2(k) = \begin{bmatrix} \mathbf{y}_f(k) \\ \tilde{\mathbf{f}}(k-l|k-1) \end{bmatrix}; \quad (26)$$

then

$$\begin{aligned} \mathbf{y}_1(k-l+i) &= C_1(k-l+i) \mathbf{x}(k-l+i) + \tilde{\mathbf{v}}_1(k-l+i), \\ &\quad i = 1, \dots, l, \\ \mathbf{y}_2(i) &= C_2(i) \mathbf{x}(i) + H\mathbf{f}(i) + \tilde{\mathbf{v}}_2(i), \quad i = 0, \dots, k-l, \end{aligned} \quad (27)$$

where $\tilde{\mathbf{v}}_1(k) = \mathbf{v}_1(k)$ and $\tilde{\mathbf{v}}_2 = [\mathbf{v}_1^T(k) \quad \mathbf{v}_s^T(k+l)]^T$ are zero-mean white noises with the following covariance matrices, respectively:

$$Q_{\tilde{\mathbf{v}}_1}(k) = \text{diag}\{I, I\}, \quad Q_{\tilde{\mathbf{v}}_2}(k) = \text{diag}\{I, I, -\gamma^2 I\}. \quad (28)$$

It is easy to check out that $\{\mathbf{y}_2(0), \dots, \mathbf{y}_2(k-l); \mathbf{y}_1(k-l+1), \dots, \mathbf{y}_1(k)\}$ span the same linear space as $\mathcal{L}\{\{\mathbf{y}_a(j)\}_{j=0}^k\}$.

To proceed, the following definition is introduced.

Definition 6 (see [32]). For $t > k-l$, the estimator $\hat{\eta}(t, 1)$ is the optimal estimation of $\eta(t)$ on the observation $\mathcal{L}\{\{\mathbf{y}_2(t)\}_{t=0}^{k-l-1}; \{\mathbf{y}_1(t)\}_{t=k-l}^{t=k-1}\}$. For $0 < t \leq k-l$, the estimator $\hat{\eta}(t, 2)$ is the optimal estimation of $\eta(t)$ on the observation $\mathcal{L}\{\{\mathbf{y}_2(t)\}_{t=0}^{t=k-1}\}$.

In accordance with (24), the innovation sequence is defined as follows:

$$\begin{aligned} \mathbf{w}_1(k-l+i) &= C_1(k-l+i) \mathbf{e}_1(k-l+i) \\ &\quad + \tilde{\mathbf{v}}_1(k-l+i), \quad i = 0, \dots, l, \\ \mathbf{w}_2(i) &= C_2(i) \mathbf{e}_2(i) + H\mathbf{f}(i) + \tilde{\mathbf{v}}_2(i), \quad i = 0, \dots, k-l, \end{aligned} \quad (29)$$

where

$$\begin{aligned} \mathbf{e}_1(k-l+i) &= \mathbf{x}(k-l+i) - \hat{\mathbf{x}}(k-l+i, 1), \quad i = 0, \dots, l, \\ \mathbf{e}_2(i) &= \mathbf{x}(i) - \hat{\mathbf{x}}(i, 2), \quad i = 0, \dots, k-l, \end{aligned} \quad (30)$$

with the corresponding covariance matrices given as

$$\begin{aligned} P_1(k-l+i) &= \langle \mathbf{e}_1(k-l+i), \mathbf{e}_1(k-l+i) \rangle, \quad i = 0, \dots, l, \\ P_2(i) &= \langle \mathbf{e}_2(i), \mathbf{e}_2(i) \rangle, \quad i = 0, \dots, k-l. \end{aligned} \quad (31)$$

In light of Lemma 2.2.1 in [20], the innovation sequences $\mathcal{L}\{\{\mathbf{w}_2(t)\}_{t=0}^{k-l-1}; \{\mathbf{w}_1(t)\}_{t=k-l}^{k-1}\}$ are uncorrelated white noises and span the same linear space as $\mathcal{L}\{\{\mathbf{y}_a(j)\}_{j=0}^k\}$.

For deriving $\hat{\mathbf{x}}(k-l, 2)$ ($k = l+1, l+2, \dots$), applying the Krein space based projection formula in [21] by taking (21) and (22) into account, we have that

$$\begin{aligned}\hat{\mathbf{x}}(k-l, 2) &= A(k-l-1) \hat{\mathbf{x}}(k-l-1, 2) \\ &\quad + \langle \mathbf{x}(k-l), \mathbf{w}_2(k-l-1) \rangle \\ &\quad \times \langle \mathbf{w}_2(k-l-1), \mathbf{w}_2(k-l-1) \rangle^{-1} \\ &\quad \times \mathbf{w}_2(k-l-1) \\ &= A(k-l-1) \hat{\mathbf{x}}(k-l-1, 2) \\ &\quad + K_2(k-l-1) \mathbf{w}_2(k-l-1), \\ \hat{\mathbf{x}}(0) &= 0,\end{aligned}\tag{32}$$

where

$$\begin{aligned}K_2(k-l-1) &= (A(k-l-1) P_2(k-l-1) C_2^T(k-l-1) \\ &\quad + B_f(k-l-1) H) Q_2^{-1}(k-l-1),\end{aligned}\tag{33}$$

with $Q_2(k-l-1) = C_2(k-l-1) P_2(k-l-1) C_2^T(k-l-1) + HH^T + Q_{\tilde{\mathbf{y}}_2}(k-l-1)$.

In addition, following the definition of $P_2(i)$ and (32), $P_2(i)$ ($i = 0, 1, \dots, k-l-1$) is the solution to the following standard Riccati equation:

$$\begin{aligned}P_2(i+1) &= A(i) P_2(i) A^T(i) + B_f(i) B_f^T(i) \\ &\quad + D(i) D^T(i) - K_2(i) Q_2^{-1}(i) K_2^T(i), \\ P_2(0) &= P_0.\end{aligned}\tag{34}$$

For calculating $\hat{\mathbf{x}}(k-l+i, 1)$ ($i = 1, \dots, l$) with the initial condition $\hat{\mathbf{x}}(k-l, 1) = \hat{\mathbf{x}}(k-l, 2)$, we apply the projection formula once again such that

$$\begin{aligned}\hat{\mathbf{x}}(k-l+i+1, 1) &= A(k-l+i) \hat{\mathbf{x}}(k-l+i, 1) \\ &\quad + A(k-l+i) \langle \mathbf{x}(k-l+i), \mathbf{w}_1(k-l+i) \rangle \\ &\quad \times \langle \mathbf{w}_1(k-l+i), \mathbf{w}_1(k-l+i) \rangle^{-1} \mathbf{w}_1(k-l+i) \\ &= A(k-l+i) \hat{\mathbf{x}}(k-l+i, 1) + K_1(k-l+i) \mathbf{w}_1(k-l+i),\end{aligned}\tag{35}$$

where

$$\begin{aligned}K_1(k-l-1) &= A(k-l+i) P_1(k-l+i) \\ &\quad \times C_1^T(k-l+i) Q_1^{-1}(k-l+i),\end{aligned}\tag{36}$$

with $Q_1(k-l+i) = C_1(k-l+i) P_1(k-l+i) C_1^T(k-l+i) + Q_{\tilde{\mathbf{y}}_1}(k-l+i)$, and $P_1(k-l+i)$ is computed recursively in the following form:

$$\begin{aligned}P_1(k-l+i+1) &= A(k-l+i) P_1(k-l+i) A^T(k-l+i) \\ &\quad + B_f(k-l+i) B_f^T(k-l+i) \\ &\quad + D(k-l+i) D^T(k-l+i) \\ &\quad - K_1(k-l+i) Q_2^{-1}(k-l+i) K_1^T(k-l+i), \\ P_1(k-l) &= P_2(k-l).\end{aligned}\tag{37}$$

Similarly, the projection formula is reutilized to compute $\hat{\mathbf{f}}(k-l | k-1)$; that is,

$$\begin{aligned}\hat{\mathbf{f}}(k-l | k-1) &= \sum_{i=0}^{l-1} \langle \mathbf{f}(k-l), \mathbf{w}_1(k-l+i) \rangle Q_1^{-1}(k-l+i) \mathbf{w}_1(k-l+i) \\ &= \sum_{i=0}^{l-1} \Omega_{k-l+i}^{k-l} C_1^T(k-l+i) Q_1^{-1}(k-l+i) \mathbf{w}_1(k-l+i), \\ i &= 1, \dots, l-1,\end{aligned}\tag{38}$$

where Ω_{k-l+i}^{k-l} , $i = 1, \dots, l-1$ is obtained recursively in terms of

$$\begin{aligned}\Omega_{k-l+i}^{k-l} &= \Omega_{k-l+i-1}^{k-l} [A(k-l+i-1) - K_1(k-l+i-1) \\ &\quad \times C_1(k-l+i-1)]^T, \\ \Omega_{k-l+1}^{k-l} &= B_f^T(k-l).\end{aligned}\tag{39}$$

Finally, in order to calculate $Q_w(k)$ which is associated with $J_{\min}$ and $\hat{\mathbf{f}}(k-l | k)$, define $\tilde{\mathbf{f}}(k-l) = \mathbf{f}(k-l) - \hat{\mathbf{f}}(k-l | k-1)$, and then, from (38), we know that

$$\begin{aligned}\langle \tilde{\mathbf{f}}(k-l), \tilde{\mathbf{f}}(k-l) \rangle &= I - \sum_{i=0}^{l-1} \Omega_{k-l+i}^{k-l} C_1^T(k-l+i) Q_1^{-1}(k-l+i) \\ &\quad \times (\Omega_{k-l+i}^{k-l} C_1^T(k-l+i))^T.\end{aligned}\tag{40}$$

Combining (29) and (40), we have

$$Q_w(k) = \begin{cases} C_1(k) P_1(k) C_1^T(k) + I, & 0 < k < l, \\ \begin{bmatrix} C_1(k) P_1(k) C_1^T(k) + I & C_1(k) (\Omega_k^{k-l})^T \\ \Omega_k^{k-l} C_1^T(k) & -\gamma^2 I + I - \langle \tilde{f}(k-l), \tilde{f}(k-l) \rangle \end{bmatrix}, & k \geq l, \end{cases} \quad (41)$$

where $P_1(k)$ and Ω_{k-l+i}^{k-l} are the same as in (37) and (39).

3.3. H_∞ Fault Estimator Design. From analysis and lemmas above, we are now in the position to give our main results for designing the fault estimator, which is summarized in the following theorem.

Theorem 7. For (8), given a scalar $\gamma > 0$ and an integer $l > 0$, then the H_∞ fixed-lag fault estimator that satisfies (7) exists if and only if

$$\begin{aligned} \Lambda_1(k) &= C_1(k) P_1(k) C_1^T(k) + I > 0, \\ \Lambda_3(k) &= -\gamma^2 I + I - \sum_{i=0}^{l-1} \Omega_{k-l+i}^{k-l} C_1^T(k-l+i) Q_1^{-1}(k-l+i) \\ &\quad \times (\Omega_{k-l+i}^{k-l} C_1^T(k-l+i))^T \\ &\quad - \Omega_k^{k-l} C_1^T(k) \Lambda_1^{-1}(k) (\Omega_k^{k-l} C_1^T(k))^T < 0. \end{aligned} \quad (42)$$

In this case, a feasible fault estimator is given by

$$\begin{aligned} \tilde{f}(k-l|k) &= \sum_{i=0}^l \Omega_{k-l+i}^{k-l} C_1^T(k-l+i) Q_1^{-1}(k-l+i) \\ &\quad \times [y_f(k-l+i) \\ &\quad - C_1(k-l+i) \hat{x}(k-l+i, 1)], \end{aligned} \quad (43)$$

where $\hat{x}(k-l+i, 1)$, $Q_1(k-l+i)$, and Ω_{k-l+i}^{k-l} are calculated by (32), (34), (35), (37), and (39).

Proof. For $k \geq l$, applying the block triangular factorization technique to $Q_w(k)$ in (41), we have

$$\begin{aligned} Q_w(k) &= \begin{bmatrix} I & 0 \\ \Lambda_2(k)^T \Lambda_1(k)^{-1} & I \end{bmatrix} \begin{bmatrix} \Lambda_1(k) & 0 \\ 0 & \Lambda_3(k) \end{bmatrix} \\ &\quad \times \begin{bmatrix} I & 0 \\ \Lambda_2(k)^T \Lambda_1(k)^{-1} & I \end{bmatrix}^T, \end{aligned} \quad (44)$$

where $\Lambda_2(k) = \Omega_k^{k-l} C_1^T(k)$. Thus, from Lemma 4, we know that $Q_a(k)$ and $Q_w(k)$ have the same inertia if and only if

$\Lambda_1(k) > 0$ as well as $\Lambda_3(k) < 0$. Furthermore, based on (25) and (44), J has a minimum $J_{\min}$ if (42) are satisfied, where

$$\begin{aligned} J_{\min} &= \sum_{k=0}^{l-1} [y_f(k) - C_1(k) \hat{x}(k)]^T \\ &\quad \times \Lambda_1^{-1}(k) [y_f(k) - C_1(k) \hat{x}(k)] \\ &\quad + \sum_{k=l}^N \left(\begin{bmatrix} I & 0 \\ -\Lambda_2(k) \Lambda_1^{-1}(k) & I \end{bmatrix} \right. \\ &\quad \times \begin{bmatrix} y_f(k) - C_1(k) \hat{x}(k) \\ \tilde{f}(k-l|k) - \hat{f}(k-l|k-1) \end{bmatrix} \Big)^T \\ &\quad \times \begin{bmatrix} \Lambda_1^{-1}(k) & 0 \\ 0 & \Lambda_3^{-1}(k) \end{bmatrix} \begin{bmatrix} I & 0 \\ -\Lambda_2(k) \Lambda_1^{-1}(k) & I \end{bmatrix} \\ &\quad \times \begin{bmatrix} y_f(k) - C_1(k) \hat{x}(k) \\ \tilde{f}(k-l|k) - \hat{f}(k-l|k-1) \end{bmatrix} \Big). \end{aligned} \quad (45)$$

Since $\Lambda_3(k) < 0$, to guarantee $J_{\min} > 0$, combining (38) with (45), we know that a possible choice of $\tilde{f}(k-l|k)$ is

$$\begin{aligned} \tilde{f}(k-l|k) &= \hat{f}(k-l|k-1) \\ &\quad + \Lambda_2(k) \Lambda^{-1} (y_f(k) - C_1 \hat{x}(k)) \\ &= \sum_{i=0}^l \Omega_{k-l+i}^{k-l} C_1^T(k-l+i) Q_1^{-1}(k-l+i) \\ &\quad \times [y_f(k-l+i) - C_1(k-l+i) \\ &\quad \times \hat{x}(k-l+i, 1)], \end{aligned} \quad (46)$$

which indicates (43). This completes the proof. $\square$

Remark 8. It can be seen from Theorem 7 that the superiority of the proposed algorithm lies in three aspects:

- (i) in contrast with the results in [22–26], the proposed algorithm can be applied to systems with time-varying $\rho(k)$;
- (ii) comparing to the result in [27], the parameter matrices of the addressed estimator are given in terms of standard Riccati equations with the same dimension “ n ” of system (8), where no coupled Lyapunov equation with higher dimension is needed;
- (iii) the fault can be estimated in an arbitrary fixed-lag “ l ”

FIGURE 1: The change mode of $\theta(k)$.

4. An Illustrative Example

To illustrate the effectiveness and the applicability of the proposed method, we will implement our algorithm on a time-varying model. The following system matrices are adopted which are borrowed from [33, 34]:

$$\begin{aligned} A(k) &= (1 + 0.2 \sin(0.02k\pi)) \times \begin{bmatrix} 0.8 & 0 \\ 0.9 & 0.2 \end{bmatrix}, \\ B_f(k) &= [0.5 \ 0.5]^T, \quad C(k) = [1 \ 1], \\ D(k) &= [0.3 \ 0.25]^T. \end{aligned} \quad (47)$$

The process noise $d(k)$ is uniformly randomly chosen from the interval $[-0.5, 0.5]$ and the measurement noise $v(k)$ is assumed as $v(k) = 0.5 \sin(0.2k)$. The fault signal $f(k)$ is assumed to be time-varying in the following sinusoidal form:

$$f(k) = \begin{cases} \sin(0.5k), & k \in [30, 80], \\ 0, & \text{otherwise,} \end{cases} \quad (48)$$

and the expectation of $\theta(k)$ is assumed as $\rho = 0.8$, where Figure 1 displays the switching mode of $\theta(k)$.

Set $l = 10$, $\gamma = 1.52$, $x_0 = [0.2 \ 0]^T$, and $P_0 = 0.1I$; we design the fault estimator by applying Theorem 7. Figure 2 displays the fault signal and its estimation simultaneously. Figure 3 shows the value of $f(k-l) - \hat{f}(k-l|k)$ which is the error between the fault and its estimation. It can be seen from the results that our algorithm can track the fault signal no matter whether the random packet dropouts occur.

5. Conclusions

The problem of H_∞ fixed-lag fault estimator design for LDTV systems subject to intermittent observations has been dealt with. Special efforts have been made to handle the multiplicative uncertainty introduced by the random measurement packet dropouts. Through defining a couple of

FIGURE 2: Fault and its estimation.

FIGURE 3: Fault estimation error.

equivalent dynamic system and H_∞ performance index, the fault estimator has been derived by using the projection formula in Krein space based on the reorganized innovation approach. The parameter matrices of the estimator have been calculated by solving two standard Riccati equations. The proposed algorithm has been applied to an LDTV model to illustrate its effectiveness and applicability.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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