Research Article

Productivity for Horizontal Wells in Low-Permeability Reservoir with Oil/Water Two-Phase Flow

Yu-Long Zhao,¹ Lie-Hui Zhang,¹ Zhi-Xiong He,² and Bo-Ning Zhang¹

¹ State Key Laboratory of Oil and Gas Reservoir Geology and Exploitation, Southwest Petroleum University, Chengdu, Sichuan 610500, China

² School of Sciences, Southwest Petroleum University, Chengdu, Sichuan 610500, China

Correspondence should be addressed to Yu-Long Zhao; 373104686@qq.com

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This paper presents a semianalytical steady-state productivity of oil/water two-phase flow in low-permeability reservoirs with both top and bottom boundaries closed which takes the permeability stress-sensitive and threshold pressure gradient into account. Using the similar approach as Joshi's (1988), the three-dimensional (3D) horizontal well problem is divided into two two-dimensional problems (2D), and then the corresponding nonlinear steady seepage mathematical models in vertical and horizontal planes are established. Through the separation of variables method and equivalent flow resistance principle, the productivity equation of horizontal well is obtained. The liquid and oil productivity with different influential factors are plotted, and the related effects are also analyzed. This paper expanded the conventional productivity equations of single phase into multiphase flow which have both theoretical and practical significance in predicting production behaviors in such reservoirs.

1. Introduction

With the development of drilling technology and the reduction of its cost, more and more horizontal wells have been used in low-permeability reservoirs, fractured reservoirs, multilayered reservoirs, and bottom water drive reservoirs. And the steady-state productivity of horizontal well is always a hot topic for the petroleum engineers. After several decades of development and research, many methods, including analytical method, conformal transformation method, potential superposition method, equivalent flow resistance method, and the point source function method, were proposed to calculate it [1–9].

Merkulov [1] and Borisov [2] derived analytical productivity equation of the horizontal well with single oil phase flow. Giger et al. [3, 4] and Karcher and Giger [5] developed a concept of replacement ratio, FR , which indicates the required number of vertical wells to produce at the same rate as that of a single phase from formation well to horizontal well. Reiss [6] proposed an equation to calculate the productivity index for horizontal well. Thereafter, through subdividing the 3D flow of horizontal well into two 2D problems (flow on

horizontal plane and vertical plane, resp.), Joshi [7] derived an equation to calculate the productivity of steady-state horizontal well, which is the most popular method nowadays.

Babu and Odeh [8] proposed an equation to calculate the productivity of horizontal well under the assumption that the shape of the drainage volume is box and all the boundaries are closed. Renard and Dupuy [9] derived the flow efficiency equation for horizontal well in anisotropic reservoir with the consideration of skin factor. Then, Helmy and Wattenbarger [10] and Billiter et al. [11] also proposed their corresponding equations to calculate the productivity. Anklam and Wiggins [12] obtained the steady-state productivity with the consideration of the mechanical properties of fluid flow into the wellbore. Using the steady-state point source function theory, Lu [13] achieved the productivity equations for horizontal wells under different boundary conditions.

Most of the productivity equations of horizontal well mentioned above are mainly concentrated on single oil phase flow, but the ones related to multiphase are rare. In this paper, we employ the same method described by Joshi [7] and derived a steady-state productivity formula for a horizontal well with oil/water two-phase flow problem

FIGURE 1: Schematic of a horizontal well in a top and bottom boundaries closed reservoir.

in low-permeability reservoir, which takes the permeability stress-sensitive and threshold pressure gradient into account.

2. Theoretical Analysis

2.1. Physical Model. Figure 1 is a schematic of a horizontal well drilled in low-permeability oil reservoirs. To make the problem more tractable, the following assumptions are made: (1) the reservoir is horizontal, homogeneous with uniform thickness of h , and both the top and the bottom boundaries are closed; (2) the well length and radius are L and r_w , respectively, the well is located at the center of the formation, and the pressure at the drainage boundary is p_i with the radius of r_e ; (3) two-phase fluid flows from the reservoir to the well at a constant bottomhole pressure p_{wf} and ignore the gravity and capillary pressure effects.

To simplify the mathematical solution, the 3D problem is subdivided into two 2D problems [7]. Figure 2 shows the following subdivision of the ellipsoidal drainage problem: (1) fluid flow into the horizontal well in the horizontal plane; (2) fluid flow into the horizontal well in the vertical plane.

2.2. Permeability Model in Low-Permeability Reservoir. In this paper, we use the exponential permeability model to describe the relationship between permeability and pore pressure [14–20]. This model is based on a permeability modulus, α_k , which is defined as the following:

$$\alpha_k = \frac{1}{k} \frac{\partial k}{\partial p}. \quad (1)$$

Integrating of (1) with p from p_i to p yields

$$\frac{k}{k_i} = \exp[-\alpha_k (p_i - p)], \quad (2)$$

where α_k is the permeability modulus, MPa^{-1} ; p is the formation pressure, MPa; p_i is the initial pressure, MPa; k_i is the permeability at initial condition, D; k is the permeability at current condition, D.

2.3. Threshold Pressure Gradient in Oil/Water Two-Phase Flow. According to the experimental investigations (the schematic

of core sample experiments is shown in Figure 3), an appropriate form of Darcy's law with a threshold gradient should be used as [21–28]

$$v = \begin{cases} -\frac{k}{\mu} \left(\frac{\partial p}{\partial r} - G \right), & \frac{\partial p}{\partial r} > G, \\ 0, & \frac{\partial p}{\partial r} \leq G, \end{cases} \quad (3)$$

where v is velocity of the fluid flow, m/s; μ is the fluid viscosity, cp; r is the radius, m; G is the threshold pressure gradient, MPa/m.

2.4. Flow in a Horizontal Plane. Figure 2(a) shows a schematic of fluid flow to a horizontal well in a horizontal plane. The drainage area is an ellipse and introduces

$$\frac{z}{L/2} = \frac{1}{2} \left(\xi + \frac{1}{\xi} \right), \quad (4)$$

by defining $z = x + i \cdot y$ and $\xi = u + i \cdot v = r_{eH} \cdot e^{i\theta}$. Substituting them into (4) and then equating the real and imaginary parts yield

$$\begin{aligned} \frac{x}{L/2} &= \frac{1}{2} \left[r_{eH} + \frac{1}{r_{eH}} \right] \cos(\theta), \\ \frac{y}{L/2} &= \frac{1}{2} \left[r_{eH} - \frac{1}{r_{eH}} \right] \sin(\theta). \end{aligned} \quad (5)$$

The real equation of the elliptical drainage area can be described as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad (6)$$

where a and b are the major and minor axes of the ellipse, m.

Combing (5) with (6), we have

$$r_{eH} = \frac{a + b}{0.5L}. \quad (7)$$

Moreover, $+L/2$ and $-L/2$ represent foci of the ellipse, which have the following relationship with a and b :

$$a^2 - b^2 = \left(\frac{L}{2} \right)^2. \quad (8)$$

The drainage radius of horizontal well, r_e , can be obtained by equaling the areas of a circle and ellipse, which is

$$r_e = \sqrt{ab}. \quad (9)$$

Combing (9) with (8), we have

$$a = 0.5L \left[0.5 + \sqrt{0.25 + \left(\frac{2r_e}{L} \right)^4} \right]^{0.5}, \quad (10)$$

$$b = \sqrt{a^2 - \left(\frac{L}{2} \right)^2}.$$

FIGURE 2: Schematic of the fluid flow to a horizontal well: (a) horizontal plane and (b) vertical plane.

FIGURE 3: A schematic of core sample laboratory experiments in low-permeability reservoir.

So, as shown in Figure 4, the fluid flow in the u - v plane can be viewed as a unit radius vertical well produced at a circle drainage area with the radius of $(a + b)/(0.5L)$. Combining the modified Darcy flow equation with the steady-state governing equation, the fluid flow in the horizontal plane can be described as

$$\frac{1}{r} \frac{d}{dr} \left[r e^{-\alpha_k(p_i - p)} \lambda_t \left(\frac{dp}{dr} - G' \right) \right] = 0, \quad (11)$$

$$p|_{r=1} = p_c, \quad (12)$$

$$p|_{r=r_{eH}} = p_i, \quad (13)$$

where λ_t is the total mobility of the oil/water phase, $\lambda_t = k_{ro}/\mu_o + k_{rw}/\mu_w$, which is a function of the radius, and G' is the equivalent threshold pressure gradient in the horizontal plane, which is

$$G' = \frac{G(r_e - L/2)}{((a + b)/0.5L) - 1}. \quad (14)$$

The solution of mathematical models of (11)–(13) is (the detailed derivations are showed in the Appendix)

$$\begin{aligned} p(r) = & p_i \\ & + \frac{1}{\alpha_k} \ln \left\{ e^{\alpha_k G' r} \left[e^{-\alpha_k (G' + p_i - p_c)} \right. \right. \\ & + \frac{e^{-\alpha_k G' r_{eH}} - e^{-\alpha_k (G' + p_i - p_c)}}{\int_1^{r_{eH}} e^{-\alpha_k G' r} \cdot (1/r \lambda_t(r)) dr} \\ & \left. \left. \cdot \int_1^r e^{-\alpha_k G' r} \frac{1}{r \lambda_t(r)} dr \right] \right\}. \end{aligned} \quad (15)$$

The liquid flow rate in horizontal plane can be calculated by

$$q_{LH} = Chk\lambda_t(1) \cdot \left(r \frac{dp}{dr} \right)_{r=1}, \quad (16)$$

where C is the unit conversion factor, $C = 542$; q_{LH} is the liquid flow rate at reservoir condition, m^3/d .

Combining (15) with (16), the liquid flow rate in the horizontal plane is

$$\begin{aligned} q_{LH} = & Ck_i e^{-\alpha_k(p_i - p_c)} h \\ & \cdot \left\{ \lambda_t(1) \cdot G' \right. \\ & \left. + \frac{\left[e^{-\alpha_k[G' r_{eH} - (p_i - p_c)]} - e^{-\alpha_k G'} \right]}{\left(\alpha_k \int_1^{r_{eH}} e^{-\alpha_k G' r} \cdot (1/r \lambda_t(r)) dr \right)} \right\}. \end{aligned} \quad (17)$$

The corresponding oil production rate in the horizontal plane is

$$\begin{aligned} q_{oH} = & Ck_i e^{-\alpha_k(p_i - p_c)} h \lambda_o(1) \\ & \cdot \left[G' + \frac{1}{\lambda_t(1)} \cdot \frac{e^{-\alpha_k[G' r_{eH} - (p_i - p_c)]} - e^{-\alpha_k G'}}{\alpha_k \int_1^{r_{eH}} e^{-\alpha_k G' r} \cdot (1/r \lambda_t(r)) dr} \right], \end{aligned} \quad (18)$$

FIGURE 4: The schematic of fluid flow in horizontal plane after conformal transformation.

FIGURE 5: The schematic of fluid flow in vertical plane after conformal transformation.

where q_{oH} is the oil flow rate at reservoir condition, m^3/d ; $\lambda_t(r_w)$ is the total mobility of the oil/water phase in the well bottomhole, D/cp .

If we do not take into account the effects of permeability stress-sensitive and threshold pressure gradient on the productivity equation in horizontal plane ($\alpha_k = G = 0$) and assume only oil flow in the formation ($\lambda_t(r) = 1/\mu_o$), then (17) becomes

$$q_{\text{SOH}} = \frac{0.542 \times 10^{-3} k_i h (p_i - p_c)}{B_o \mu_o \ln(2(a+b)/L)}, \quad (19)$$

where q_{SOH} is the oil flow rate at surface condition, m^3/d ; B_o is the oil volume factor, sm^3/m^3 .

Equation (19) is the same as the productivity equation in horizontal plane of (1) derived by Joshi [7].

2.5. Calculation of Flow in a Vertical Plane. The fluid flow in the vertical plane with top and bottom boundaries closed reservoir can be viewed as a vertical well with the radius of $\xi_w = 2\pi r_w/h$ produced in a unit circle area after the conformal transformation (as shown in Figure 5).

The mathematical models to describe the steady-state fluid flow of oil/water flow in the vertical plane are

$$\begin{aligned} \frac{1}{r} \frac{d}{dr} \left[r e^{-\alpha_k(p_i-p)} \lambda_t \left(\frac{dp}{dr} - G'' \right) \right] &= 0, \\ p|_{r=\xi_w} &= p_{wf}, \\ p|_{r=1} &= p_c, \end{aligned} \quad (20)$$

where G'' is the equivalent threshold pressure gradient in the vertical plane, which is

$$G'' = \frac{(h/2) - r_w}{1 - (2\pi r_w/h)} G. \quad (21)$$

The solution of the pressure distribution along the radius can be solved with the same method showed in the Appendix. Then, the productivity equation in the vertical plane can be calculated by the following equation:

$$q_{LV} = CLk\lambda_t(\xi_w) \cdot \left(r \frac{dp}{dr} \right)_{r=\xi_w}. \quad (22)$$

The liquid and oil productivity in the vertical plane are

$$\begin{aligned} q_{LV} &= Ck_i e^{-\alpha_k(p_c-p_{wf})} L \\ &\cdot \left\{ \lambda_t(\xi_w) \cdot G'' \cdot \xi_w \right. \\ &\quad \left. + \frac{\left[e^{-\alpha_k[G''-(p_c-p_{wf})]} - e^{-\alpha_k G'' \xi_w} \right]}{\left(\alpha_k \int_{\xi_w}^1 e^{-\alpha_k G'' r} \cdot (1/r \lambda_t(r)) dr \right)} \right\}, \quad (*) \\ q_{oV} &= Ck_i e^{-\alpha_k(p_c-p_{wf})} L \lambda_o(\xi_w) \\ &\times \left\{ G'' \cdot \xi_w \right. \\ &\quad \left. + \frac{\left[e^{-\alpha_k[G''-(p_c-p_{wf})]} - e^{-\alpha_k G'' \xi_w} \right]}{\left(\lambda_t(\xi_w) \alpha_k \int_{\xi_w}^1 e^{-\alpha_k G'' r} \cdot (1/r \lambda_t(r)) dr \right)} \right\}. \quad (**) \end{aligned}$$

Assuming the reservoir with single phase flow and neglecting the permeability stress-sensitive and threshold pressure gradient ($\alpha_k = G = 0$, $\lambda_t(r) = 1/\mu_o$) yield

$$q_{oV} = \frac{Ck_i L (p_c - p_{wf})}{B_o \mu_o \ln(h/2r_w)}. \quad (23)$$

Equation (23) is similar to the productivity equation in vertical plane of (D-3) derived by Joshi [7], which proves the correctness of the productivity equation in this paper.

2.6. Horizontal Well Eccentricity. Figure 5 and (*)-(**) are obtained under the assumption that the horizontal well is located at the center of the reservoir in the vertical plane. According to Muskat's [29] formulation for off-centered wells, the liquid production rate of a well placed at a distance δ from the mid-height of the reservoir in a vertical plane is

$$q_{LV} = Ck_i e^{-\alpha_k(p_c - p_{wf})} h \cdot \left\{ \lambda_t(\xi_w) \cdot G' \cdot \xi_w + \frac{[e^{-\alpha_k[G'\beta - (p_c - p_{wf})]} - e^{-\alpha_k G' \xi_w}]}{(\alpha_k \int_{\xi_w}^{\beta} e^{-\alpha_k G' r} \cdot (1/r \lambda_t(r)) dr)} \right\}. \quad (*')$$

The oil production rate is

$$q_{oV} = Ck_i e^{-\alpha_k(p_c - p_{wf})} h \lambda_o(\xi_w) \times \left\{ G'' \cdot \xi_w + \frac{[e^{-\alpha_k[G''\beta - (p_c - p_{wf})]} - e^{-\alpha_k G'' \xi_w}]}{(\lambda_t(\xi_w) \alpha_k \int_{\xi_w}^{\beta} e^{-\alpha_k G'' r} \cdot (1/r \lambda_t(r)) dr)} \right\}, \quad (**')$$

where δ is the vertical distance between the reservoir center and horizontal well location, m; $\beta = 1 - (2\delta/h)^2$.

3. The Solving Method of $\lambda_t(r)$

In order to correctly calculate the productivity of horizontal well with oil/water phase flow, we must determinate the expression of total mobility, $\lambda_t(r)$, along the radius r . The following steps are the procedure to calculate it.

Step 1. According to the relative permeability curves and the viscosity of oil and water, the relationship between $\lambda_t(r)$ and S_w can be obtained.

Step 2. When the water displacement front breaks through the oil well, the distribution of water saturation along the well radius satisfies the following Buckley-Leverett equation:

$$\pi h \phi (r_e^2 - r^2) = \frac{df_w}{ds_w} \cdot \sum Q_l, \quad (24)$$

where h is the formation thickness, m; ϕ is the porosity, fraction; $\sum Q_l$ is the cumulative fluid production, m³/d; f_w is the water ratio, fraction, $f_w = (k_{rw}/\mu_w)/(k_{ro}/\mu_o) + (k_{rw}/\mu_w)$.

FIGURE 6: The productivity calculation diagram.

In (24), the expressions of the df_w/dS_w versus s_w can be calculated by the relative permeability curves.

Step 3. Combining the relationship of $\lambda_t(r)$ versus S_w and S_w versus r derived in Steps 1 and 2, the relationship of $\lambda_t(r)$ with r can be obtained.

Statistical results show that the relationships of S_w versus r , $\lambda_t(r)$ versus S_w , and $\lambda_t(r)$ versus r can be approximated by the following analytical functions:

$$\begin{aligned} S_w(r) &= a_1 + b_1 r^2, \\ \lambda_t(S_w) &= a_2 + b_2 S_w + c_2 S_w^2, \\ \lambda_t(r) &= a_3 + b_3 r^2 + c_3 r^4. \end{aligned} \quad (25)$$

4. The Productivity Calculation Process

Using the electrical analog concept, the well production rate in the horizontal plane must equal the production rate in the vertical plane. Because of the nonlinearity of (17)-(18) and (*)-(**), we cannot obtain productivity expressions similar to Joshi [7]. With the aid of computer programs, the results can be obtained and the calculation diagram is showed in Figure 6.

5. Results and Their Sensitive Analysis

In this section, the liquid and oil production rate are calculated, and the essential parameters of well, reservoir, and fluid properties are listed in Table 1, and the relative permeability curves are showed in Figure 7.

According to the relative permeability curves and the parameters in Table 1, the relationship of $\lambda_t(S_w)$ versus S_w

FIGURE 7: The relative permeability curves.

FIGURE 8: The relationship between $\lambda_t(S_w)$ and S_w .

TABLE 1: The parameters of reservoir and fluid properties.

Variables	value
Porosity, ϕ (fraction)	0.05
Drainage radius, r_e (m)	600
Horizontal well length, L (m)	300
Initial permeability of the reservoir, k_i (D)	2×10^{-3}
Cumulative fluid production, $\sum Q_i$ (m ³)	1.14×10^6
Oil viscosity in the reservoir, μ_o (mPa·s)	3
Formation thickness, h (m)	20
Initial reservoir pressure, p_e (MPa)	30
Permeability modulus, α_k (MPa ⁻¹)	0.1
Water saturation in the bottomhole, S_{wB} (fraction)	0.6
Threshold pressure gradient, G (MPa/m)	0.001
Water viscosity in the reservoir, μ_w (mPa·s)	1

and df_w/dS_w versus S_w can be plotted when water cut ratio reaches 0.6 (as shown in Figures 8 and 9). The corresponding regression curve equations can be obtained as follows:

$$\lambda_t = 0.4917 - 2.3096S_w + 3.2547S_w^2, \quad (26)$$

$$\frac{df_w}{dS_w} = -6.0226S_w + 4.3193, \quad S_w > 0.6.$$

FIGURE 9: The relationship between df_w/dS_w and S_w . FIGURE 10: The effect of permeability modulus (α_k) on liquid production rate.

Taking (26) as well as the cumulative fluid production into (24), the expressions between the $\lambda_t(r)$ and r can be obtained, which is

$$\lambda_t = 0.2 + 1.53 \times 10^{-6} r^2 + 6.8 \times 10^{-13} r^4. \quad (27)$$

Combining (27) and other parameters in Table 1 with (17)-(18) and (*)-(**), the steady-state fluid productivity can be calculated.

Figure 10 shows the effect of permeability modulus α_k on liquid and oil productivity of horizontal well in low permeability reservoir. It can be seen from the figure that the permeability stress-sensitive has a significant effect on the productivity; the bigger the α_k is, the smaller the liquid and oil productivity are, which is mainly because, with the same pressure drop of the reservoir, big α_k will lead to a serious permeability decreasing. When we do not take into account the permeability stress sensitive ($\alpha_k = 0$), the liquid and oil productivity can be calculated with the limit of α_k tending to zero for (17)-(18) and (*)-(**).

Figures 11 and 12 show the effect of threshold pressure gradient (G) and well length (L) on liquid and oil productivity. It can be seen from the chart that the threshold pressure gradient has small effect on the productivity of horizontal well for big drainage volume. In general, the bigger

TABLE 2: The productivity of liquid and oil in different bottomhole pressure.

p_{wf} (MPa)	$\alpha_k = 0.1, G = 0.001$		$\alpha_k = 0, G = 0$	
	Liquid production rate (m ³ /d)	Oil production rate (m ³ /d)	Liquid production rate (m ³ /d)	Oil production rate (m ³ /d)
30.00	0.00	0.00	0.00	0.00
29.80	0.00	0.00	0.39	0.19
29.60	0.00	0.00	0.79	0.37
29.50	0.33	0.16	0.98	0.47
27.00	4.58	2.18	5.89	2.80
24.00	8.61	4.09	11.77	5.60
21.00	11.71	5.56	17.66	8.40
18	14.07	6.69	23.5	11.19
15	15.85	7.54	29.44	13.99
2	17.20	8.18	35.32	16.79

FIGURE 11: The effect of threshold pressure gradient (G) on liquid production rate. FIGURE 12: The effect of well length (L) on liquid production rate.

the G is, the smaller the liquid and oil productivity are. When the reservoir has both threshold pressure gradient and permeability stress sensitive, the longer the well length is, the bigger the productivity is.

FIGURE 13: The productivity of horizontal well in different bottom-hole pressure.

Figure 13 shows the liquid productivity with different bottomhole pressure when $\alpha_k = 0.1, G = 0.001$ and $\alpha_k = 0, G = 0$, the corresponding values are listed in Table 2. It can be clearly seen that the permeability stress-sensitive and threshold pressure gradient have significant effects on the well productivity, and the bigger α_k and G are, the more obvious the effect is. And when the pressure drop is small, the fluid cannot flow for the existing of threshold pressure gradient, which is mainly because only the fluid can flow when the pressure drop overcomes the threshold pressure for multiphase flow.

6. Conclusions

In this paper, a semianalytical productivity equation of horizontal well in low-permeability oil reservoir with oil/water two-phase flow is established with the consideration of permeability stress-sensitive and threshold pressure gradient. Based on the above study, the following conclusions can be summarized.

- (1) The steady-state percolation mathematical models of horizontal well with oil/water two-phase flow are established and the corresponding solutions are solved by the method of separation of variables.
- (2) For low-permeability reservoir, there always exists the phenomenon of permeability stress-sensitive (α_k), which has a significant influence on the well productivity; the bigger the α_k is, the smaller the productivity is.
- (3) Due to the existence of capillary pressure of two-phase flow, there always is threshold pressure gradient (G) in the fluid seepage process. Although the G has a smaller effect on the productivity of the horizontal well for a big drainage volume, we cannot neglect its effect on the productivity.

Appendix

From the expressions of (11), we have

$$r\lambda_t e^{-\alpha_k(p_i-p)} \left(\frac{dp}{dr} - G' \right) = c, \quad (\text{A.1})$$

where c is a constant.

We define the following expression:

$$y = e^{-\alpha_k(p_i-p)}. \quad (\text{A.2})$$

Substituting (A.2) into (A.1) and (12)-(13) yields

$$\frac{dy}{dr} = \alpha_k G' y + \frac{c\alpha_k}{r\lambda_t}, \quad (\text{A.3})$$

$$y|_{r=1} = e^{-\alpha_k(p_i-p_c)}, \quad (\text{A.4})$$

$$y|_{r=r_{eH}} = 1. \quad (\text{A.5})$$

According to the general solution of the Bernoulli differential equation [30], the solution of (A.3) can be obtained as

$$y(r) = e^{\alpha_k G' r} \cdot \left[e^{-\alpha_k G' r} e^{-\alpha_k(p_i-p_c)} + \int_{r_w}^r e^{-\alpha_k G' r} \frac{c\alpha_k}{r\lambda_t(r)} dr \right]. \quad (\text{A.6})$$

Substituting (A.6) into (A.5), the value of constant c can be solved, and then combining it with (A.6), we have

$$\begin{aligned} p(r) &= p_i + \frac{1}{\alpha_k} \ln \left\{ e^{\alpha_k G' r} \cdot \left[e^{-\alpha_k(G' + p_i - p_c)} + \frac{e^{-\alpha_k G' r_{eH}} - e^{-\alpha_k(G' + p_i - p_c)}}{\int_{r_w}^{r_{eH}} e^{-\alpha_k G' r} \cdot (1/r\lambda_t(r)) dr} \right. \right. \\ &\quad \left. \left. \cdot \int_{r_w}^r e^{-\alpha_k G' r} \frac{1}{r\lambda_t(r)} dr \right] \right\}. \end{aligned} \quad (\text{A.7})$$

Equation (A.7) is the pressure distribution relation along the radius r with the oil/water two-phase flows in the horizontal plane of the horizontal well.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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