Research Article

Peristaltic Flow of a Nanofluid under the Effect of Hall Current and Porous Medium

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The problem of peristaltic flow of an incompressible viscous electrically conducting nanofluid in a vertical asymmetric channel through a porous medium is investigated by taking the Hall effects into account. The governing equations are formulated and simplified under the assumptions of long wavelength and low Reynolds number. The solutions for temperature and nanoparticle profiles are obtained by using the homotopy perturbation method (HPM) and closed form solutions for stream function and pressure gradient are developed. Finally, the effects of various emerging parameters on the physical quantities of interest are plotted and discussed.

1. Introduction

The peristaltic flow problems have attracted the attention of many researchers because of its increasing importance, specially in engineering, physiology, and biological systems. This is due to its many applications in real life such as urine transport from kidney to bladder through the ureter; transport of lymph in the lymphatic vessels; swallowing food through the esophagus; the movement of chyme in the gastrointestinal tract; ovum movement in the fallopian tube; vasomotion of small blood vessels such as venules, capillaries, and arterioles; and also in sanitary fluid transportation of corrosive fluids. Many modern mechanical devices have been designed on the principal of the peristaltic pumping for transporting fluids without internal moving parts, for example, the blood pump in the heart-lung machine and the peristaltic transport of noxious fluid in nuclear industry. For more details in this direction see, for example, [1–22].

The study of magnetohydrodynamics flows with Hall currents has important engineering applications in problems of magnetohydrodynamic generators and of Hall accelerators as well as in flight magnetohydrodynamics. Siddiqui et al. [23], studied effects of Hall current and heat transfer on MHD flow of a Burgers fluid due to a pull of eccentric rotating disks. Hall effects on peristaltic flow of a Maxwell fluid in

a porous medium have been studied by Hayat et al. [24]. Hayat et al. [25], studied effects of Hall current and heat transfer on rotating flow of a second grade fluid through a porous medium. The effect of Hall currents on interaction of pulsatile and peristaltic transport induced flows of a particle fluid suspension that had been examined by Gad [26]. Recently the analytical solution for Hall and ion-slip effects on mixed convection flow of couple stress fluid between parallel disks is considered by Srinivasacharya and Kaladhar [27]. Narayana et al. [28] studied the effects of Hall current and radiation absorption on MHD micropolar fluid in a rotating system.

Nanofluid is a fluid containing nanometer-sized particles, called nanoparticles. These fluids are engineered colloidal suspensions of nanoparticles in a base fluid. The nanoparticles used in nanofluids are typically made of metals (Al, Cu), oxides (Al_2O_3), carbides (SiC), nitrides (AlN, SiN), or nonmetals (Graphite, carbon nanotubes) and the base fluid is usually a conductive fluid, such as water or ethylene glycol. Other base fluids are oil and other lubricants, biofluids, and polymer solutions. Nanoparticles range in diameter between 1 and 100 nm. Nanofluids commonly contain up to a 5% volume fraction of nanoparticles to ensure effective heat transfer enhancements. Nanofluids have novel properties that make them potentially useful in many applications in heat transfer, including microelectronics, fuel cells, pharmaceutical

processes, and hybrid-powered engines. In engineering devices it has been widely used for engine cooling/vehicle thermal management, domestic refrigerator, chiller, heat exchanger, and nuclear reactor; in grinding, in machining, in space, defense, and ships; and in boiler flue gas temperature reduction.

Nowadays, there is a continuous focus of the researchers in the flow analysis of nanofluids because of, in addition to, the above mentioned applications these fluids enhance thermal conductivity of the base fluid enormously, which is beyond the explanation of any existing theory. They are also very stable and have no additional problems, such as sedimentation, erosion, and additional pressure drop. Choi [29] was the first who initiated this nanofluid technology. A detailed analysis of nanofluids was discussed by Buongiorno [30]. Khan and Pop [31] considered the boundary layer flow of a nanofluid passing a stretching sheet. Recently, Mustafa et al. [32] examined the influence of wall properties on the peristaltic flow of a nanofluid. The endoscopic effects on peristaltic flow of a nanofluid are introduced by Akbar and Nadeem [33]. The same authors Akbar and Nadeem studied the peristaltic flow of a Phan-Thien-Tanner nanofluid in a diverging tube [34]. Akbar et al. [35, 36] studied peristaltic flow of a nanofluid with slip effects and peristaltic flow of a nanofluid in a nonuniform tube.

To the best of the author's knowledge, no attempt is available in the literature which deals with the peristalsis of nanofluid including the Hall effect through a porous medium. Even the study of peristaltic flow of an electrically conducting nanofluid in a porous medium is not available. Also, the study of peristaltic flow of a nanofluid through a porous medium with Hall effects has not been made yet. The present study fills the gap in these directions.

The aim of this work is to investigate the peristaltic flow of an electrically conducting nanofluid in a vertical asymmetric channel through a porous medium with constant permeability. Modified Darcy's law for a nanofluid including the Hall current has been used for the modeling. In fact, the Hall effect is important when the Hall parameter is high. This happens when the magnetic field is strong. In most cases, the Hall term has been ignored in applying Ohm's law as it has no marked effects for small and moderate values of the magnetic field. However, the current trend in the application of magnetohydrodynamics is towards a strong magnetic field, so that the influence of electromagnetic force is noticeable. Under these conditions, the Hall current is important and it has marked effects on the magnitude and direction of the current density and consequently on the magnetic-force term. Therefore, it is important to study the effect of the Hall current on the flow. The governing equations of motion, energy, and nanoparticles for nanofluids have been reduced under the assumption of long wavelength and low Reynolds number. The reduced equations are then solved analytically via homotopy perturbation method [37–39]. The physical behaviors of emerging parameters are discussed through graphs. At the end the trapping phenomenon is discussed.

FIGURE 1: Geometry of the problem.

2. Problem Formulation

Consider the peristaltic flow of an incompressible viscous electrically conducting nanofluid in a two-dimensional vertical asymmetric channel of width $d_1 + d_2$ through a porous medium. Asymmetry in the channel is produced by assuming the peristaltic wave trains propagating with constant speed c along the walls (H_1 is the right hand side wall and H_2 is the left hand side wall) to have different amplitudes and phases as shown in Figure 1. The shapes of the channel walls are represented as

$$Y = H_1(X, t) = d_1 + a_1 \cos\left(\frac{2\pi}{\lambda} [X - ct]\right), \text{ right wall,}$$

$$Y = H_2(X, t) = -d_2 - b_1 \cos\left(\frac{2\pi}{\lambda} [X - ct] + \phi\right), \text{ left wall,} \quad (1)$$

where a_1, b_1 are the amplitudes of the right and left waves, λ is the wave length, and ϕ is the phase difference. Further, d_1, d_2, a_1, b_1 , and ϕ satisfy the following inequality:

$$a_1^2 + b_1^2 + 2a_1b_1 \cos \phi \leq (d_1 + d_2)^2, \quad (2)$$

so that the walls will not intersect with each other. A uniform magnetic field with magnetic flux density vector $\mathbf{B} = (0, 0, B_0)$ is applied; the induced magnetic field is neglected by taking a very small magnetic Reynolds number. The expression for the current density $\mathbf{J}$ including the Hall effect and neglecting ion-slip and thermoelectric effects is given by [40]

$$\mathbf{J} = \sigma \left[\mathbf{E} + \mathbf{V} \wedge \mathbf{B} - \frac{1}{en_e} (\mathbf{J} \wedge \mathbf{B}) \right], \quad (3)$$

where $\mathbf{J}$ is the current density, σ is the electric conductivity of the fluid, $\mathbf{V}$ is the velocity vector, and $m = \sigma B_0 / en_e$ is the Hall parameter. It is also assumed that $\mathbf{E} = 0$ (since there

FIGURE 2: Variations of velocity profile U with y for various values of Hall parameter m (panel a), porosity parameter s (panel b), thermophoresis parameter N_t (panel c), and local temperature Grashof number G_r (panel d). The other parameters chosen are $F = 2$, $x = 0.1$, $d = 1$, $M = 1.2$, $a = 0.1$, $b = 0.5$, $N_b = 0.8$, $N_t = 0.5$, $s = 0.5$, $\phi = 0.2$, $G_r = 0.5$, and $B_r = 0.5$ (panel a); $F = 2$, $x = 0.1$, $d = 1$, $M = 1.2$, $a = 0.1$, $b = 0.5$, $N_b = 0.8$, $N_t = 0.5$, $m = 1$, $\phi = 0.2$, $G_r = 0.5$, and $B_r = 0.5$ (panel b); $F = 2$, $x = 0.1$, $d = 1$, $M = 1.2$, $a = 0.1$, $b = 0.5$, $N_b = 0.8$, $m = 1$, $s = 0.5$, $\phi = 0.2$, $G_r = 0.5$ and $B_r = 0.5$ (panel c); and $F = 2$, $x = 0.1$, $d = 1$, $M = 1.2$, $a = 0.1$, $b = 0.5$, $N_b = 0.8$, $N_t = 0.5$, $m = 1$, $s = 0.5$, $\phi = 0.2$ and $B_r = 0.5$ (panel d).

is no applied polarization voltage), e is the electric charge and n_e is the number density of electrons. The heat transfer and nanoparticle processes are maintained by considering temperatures T_0 , T_1 and nanoparticle phenomena C_0 , C_1 to the right and left sides walls of the channel, respectively. The fundamental equations governing this model [41] together with the generalized Ohm's law taking the effects of Hall currents and Maxwell's equations into account are

$$\nabla \cdot \mathbf{V} = 0,$$

$$\begin{aligned} \rho_f \frac{d\mathbf{V}}{dt} &= -\nabla P + \mu \nabla^2 \mathbf{V} - \frac{\mu}{k_1} \mathbf{V} + \mathbf{J} \times \mathbf{B} + \mathbf{f}, \\ (\rho c)_f \frac{dT}{dt} &= k \nabla^2 T + (\rho c)_p \left[D_B \nabla C \nabla T + \left(\frac{D_T}{T_0} \right) \nabla T \nabla T \right], \\ \frac{dC}{dt} &= D_B \nabla^2 C + \left(\frac{D_T}{T_0} \right) \nabla^2 T, \end{aligned} \quad (4)$$

FIGURE 3: Variations of the axial pressure gradient dp/dx with x for various values of Hall parameter m (panel a), porosity parameter s (panel b), amplitude of right wave a (panel c), and local nanoparticle Grashof number B_r (panel d). The other parameters chosen are $F = -2$, $y = 0.1$, $d = 1$, $M = 1.2$, $a = 0.1$, $b = 0.5$, $N_b = 0.8$, $N_t = 0.5$, $s = 0.5$, $\phi = 0.2$, $G_r = 0.5$, and $B_r = 0.5$ (panel a); $F = -2$, $y = 0.1$, $d = 1$, $M = 1.2$, $a = 0.1$, $b = 0.5$, $N_b = 0.8$, $N_t = 0.5$, $m = 1$, $\phi = 0.2$, $G_r = 0.5$, and $B_r = 0.5$ (panel b); $F = -2$, $y = 0.1$, $d = 1$, $M = 1.2$, $N_t = 0.5$, $b = 0.5$, $N_b = 0.8$, $m = 1$, $s = 0.5$, $\phi = 0.2$, $G_r = 0.5$, and $B_r = 0.5$ (panel c); and $F = -2$, $y = 0.1$, $d = 1$, $M = 1.2$, $a = 0.1$, $b = 0.5$, $N_b = 0.8$, $N_t = 0.5$, $m = 1$, $s = 0.5$, $\phi = 0.2$, and $G_r = 0.5$ (panel d).

where P is the pressure, k_1 is the permeability parameter, μ is the dynamic viscosity coefficient, $\mathbf{f}$ is the body forces, ∇^2 is the Laplacian operator, d/dt is the material derivative, t is the time, ρ_f is the density of the fluid, ρ_p is the density of the particle, T is the temperature of the fluid, c is the volumetric volume expansion, C is the nanoparticle phenomena, the ambient values of T and C as y tends to h_1 are denoted by T_0 and C_0 , D_B is the Brownian diffusion coefficient, and D_T is the thermophoretic diffusion coefficient.

The equations governing the two-dimensional motion of this model are

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0,$$

$$\rho_f \left(\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right)$$

FIGURE 4: Variations of pressure rise ΔP versus flow rate Q for different values of Hall parameter m (panel a), local temperature Grashof number G_r (panel b), porosity parameter s (panel c), and the Brownian motion parameter N_b (panel d). The other parameters chosen are $d = 1$, $y = 1$, $M = 1.2$, $a = 0.3$, $b = 0.5$, $N_b = 0.8$, $N_t = 0.5$, $s = 0.5$, $\phi = 0.2$, $G_r = 0.5$, and $B_r = 0.5$ (panel a); $d = 1$, $y = 1$, $M = 1.2$, $a = 0.3$, $b = 0.5$, $N_b = 0.8$, $N_t = 0.5$, $s = 0.5$, $\phi = 0.2$, $m = 1$, and $B_r = 0.5$ (panel b); $d = 1$, $y = 1$, $M = 1.2$, $a = 0.3$, $b = 0.5$, $N_b = 0.8$, $N_t = 0.5$, $m = 1$, $\phi = 0.2$, $G_r = 0.5$, and $B_r = 0.5$ (panel c); and $d = 1$, $y = 1$, $M = 1.2$, $a = 0.3$, $b = 0.5$, $m = 1$, $N_t = 0.5$, $s = 0.5$, $\phi = 0.2$, $G_r = 0.5$, and $B_r = 0.5$ (panel d).

$$\begin{aligned}
 &= -\frac{\partial P}{\partial X} + \mu \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \\
 &\quad - \frac{\mu}{k_1} U + \rho_f g \alpha (T - T_0) \\
 &\quad + \rho_f g \alpha (C - C_0) + \frac{\sigma B_0^2}{1 + m^2} (mV - U), \\
 &\quad \rho_f \left(\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right) \\
 &= -\frac{\partial P}{\partial Y} + \mu \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) \\
 &\quad - \frac{\mu}{k_1} V - \frac{\sigma B_0^2}{1 + m^2} (mU + V),
 \end{aligned}$$

FIGURE 5: Variations of temperature profile $\Theta(x, y)$ with y for different values of thermophoresis parameter N_t (panel a) and the Brownian motion parameter N_b (panel b). The other parameters chosen are $d = 1$, $x = 0.05$, $a = 0.1$, $b = 0.5$, $N_b = 0.8$, and $\phi = 0.2$ (panel a); $d = 1$, $x = 0.05$, $a = 0.3$, $b = 0.5$, $N_t = 1.5$, and $\phi = 0.2$ (panel b).

FIGURE 6: Variations of nanoparticle phenomena $\Omega(x, y)$ with y for different values of thermophoresis parameter N_t (panel a) and the Brownian motion parameter N_b (panel b). The other parameters chosen are $d = 1$, $x = 0.25$, $a = 0.3$, $b = 0.5$, $N_b = 1.5$, and $\phi = 0.2$ (panel a); $d = 1$, $x = 0.25$, $a = 0.3$, $b = 0.5$, $N_t = 1.5$, and $\phi = 0.2$ (panel b).

$$\begin{aligned}
 & \frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} \\
 & = \left(\frac{k}{\rho c} \right)_f \left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right) + \frac{(\rho c)_p}{(\rho c)_f} \\
 & \quad \times \left[D_B \left(\frac{\partial C}{\partial X} \frac{\partial T}{\partial X} + \frac{\partial C}{\partial Y} \frac{\partial T}{\partial Y} \right) \right. \\
 & \quad \left. + \frac{D_T}{T_0} \left(\left(\frac{\partial T}{\partial X} \right)^2 + \left(\frac{\partial T}{\partial Y} \right)^2 \right) \right],
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} \\
 & = D_B \left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2} \right) + \frac{D_T}{T_0} \left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right),
 \end{aligned} \tag{5}$$

where U and V are the velocity components in the laboratory frame (X, Y) . Following Shapiro et al. [3] we introduce a wave frame of reference (x, y) moving with velocity c_1 in which the motion becomes independent of time when the channel

FIGURE 7: Streamlines for two different values of Hall parameter m (panels a, b) and two different values of the Brownian motion parameter N_b (panels c, d). The other parameters chosen are $Q = 2$, $d = 1$, $M = 1.2$, $a = 0.3$, $b = 0.5$, $N_b = 0.8$, $N_t = 0.5$, $s = 0.5$, $\phi = 0.02$, $G_r = 0.5$, and $B_r = 0.5$ (panels a, b); $Q = 2$, $d = 1$, $M = 1.2$, $a = 0.3$, $b = 0.5$, $m = 1$, $N_t = 0.5$, $s = 0.5$, $\phi = 0.02$, $G_r = 0.5$, and $B_r = 0.5$ (panels c, d).

is an integral multiple of the wavelength and the pressure difference at the ends of the channel is a constant. The transformation from the laboratory frame of reference (X, Y) to the wave frame of reference (x, y) is given by

$$\begin{aligned} x &= X - c_1 t, & y &= Y, & u &= U - c_1, \\ v &= V, & p(x) &= P(x, t), \end{aligned} \quad (6)$$

where (u, v) , p and (U, V) , P are the velocity components and pressure in the wave and laboratory frames of reference, respectively. The pressure $p(x)$ remains a constant across any axial station of the channel under the assumption that the wavelength is large and the curvature effects are negligible.

The following are nondimensional variables:

$$\begin{aligned} x^* &= \frac{x}{\lambda}, & y^* &= \frac{y}{d_1}, & u^* &= \frac{u}{c_1}, & v^* &= \frac{v}{c_1}, \\ \delta &= \frac{d_1}{\lambda}, & p^* &= \frac{p d_1^2}{c_1 \lambda \mu}, & t^* &= \frac{t}{\lambda}, \\ h_1 &= \frac{h_1}{d_1}, & h_2 &= \frac{h_2}{d_1}, & d &= \frac{d_2}{d_1}, \end{aligned}$$

FIGURE 8: Streamlines for two different values of local temperature Grashof number G_r (panels a, b) and two different values of porosity parameter s (panels c, d). The other parameters chosen are $Q = 2$, $d = 1$, $M = 1.2$, $a = 0.3$, $b = 0.5$, $N_b = 0.8$, $N_t = 0.5$, $s = 0.5$, $\phi = 0.02$, $m = 1$, and $B_r = 0.5$ (panels a, b); $Q = 2$, $d = 1$, $M = 1.2$, $a = 0.3$, $b = 0.5$, $N_b = 0.8$, $N_t = 0.5$, $m = 1$, $\phi = 0.02$, $G_r = 0.5$, and $B_r = 0.5$ (panels c, d).

$$a = \frac{a_1}{d_1}, \quad b = \frac{b_1}{d_1}, \quad \text{Re} = \frac{\rho_f d_1 c_1}{\mu}, \quad s = \frac{k_1}{d_1^2},$$

$$\theta = \frac{T - T_0}{T_1 - T_0}, \quad \Omega = \frac{C - C_0}{C_1 - C_0}, \quad \alpha = \frac{k}{(\rho c)_f},$$

$$N_b = \frac{\rho c_p D_B (c_1 - c_0)}{(\rho c)_f \alpha}, \quad \text{Pr} = \frac{\nu}{\alpha},$$

$$N_t = \frac{\rho c_p D_T (T_1 - T_0)}{(\rho c)_f \alpha T_0}, \quad G_r = \frac{\alpha g (T_1 - T_0) d_1^2}{\nu c_1},$$

$$B_r = \frac{\alpha g (C_1 - C_0) d_1^2}{\nu c_1}, \quad M = \frac{\sigma B_0^2 d_1^2}{\mu}, \quad \nu = \frac{\mu}{\rho_f},$$

(7)

where Re is the Reynolds number, δ is the dimensionless wave number, s is the porosity parameter, Pr is the Prandtl number, N_b is the Brownian motion parameter, N_t is

the thermophoresis parameter, G_r is the local temperature Grashof number, B_r is the local nanoparticle Grashof number, and M is the magnetic parameter. Using (6) and (7) together with the stream function Ψ defined by

$$u = \frac{\partial \Psi}{\partial y}, \quad v = -\delta \frac{\partial \Psi}{\partial x} \quad (8)$$

in (5), adopting the long wave length and low Reynolds number assumptions, we get the following:

$$\frac{\partial p}{\partial x} = \frac{\partial^3 \Psi}{\partial y^3} - \Gamma^2 \frac{\partial \Psi}{\partial y} + G_r \theta + B_r \Omega - \Gamma^2, \quad (9)$$

$$\frac{\partial p}{\partial y} = 0, \quad (10)$$

$$0 = \frac{\partial^2 \theta}{\partial y^2} + N_b \frac{\partial \theta}{\partial y} \frac{\partial \Omega}{\partial y} + N_t \left(\frac{\partial \theta}{\partial y} \right)^2, \quad (11)$$

$$0 = \frac{\partial^2 \Omega}{\partial y^2} + \frac{N_t}{N_b} \frac{\partial^2 \theta}{\partial y^2}. \quad (12)$$

Eliminating the pressure gradient between (9) and (10) we get

$$\frac{\partial^4 \Psi}{\partial y^4} - \Gamma^2 \frac{\partial^2 \Psi}{\partial y^2} + G_r \frac{\partial \theta}{\partial y} + B_r \frac{\partial \Omega}{\partial y} = 0, \quad (13)$$

where

$$\Gamma^2 = \left(\frac{1}{s} + \frac{M}{1+m^2} \right). \quad (14)$$

The corresponding boundary conditions are

$$\begin{aligned} \Psi &= \frac{F}{2}, & \frac{\partial \Psi}{\partial y} &= -1, & \theta &= 0, & \Omega &= 0 & \text{at } y = h_1, \\ \Psi &= -\frac{F}{2}, & \frac{\partial \Psi}{\partial y} &= -1, & \theta &= 1, & \Omega &= 1 & \text{at } y = h_2, \end{aligned} \quad (15)$$

where

$$h_1 = 1 + a \cos 2\pi x, \quad h_2 = -d - b \cos (2\pi x + \phi), \quad (16)$$

and a, b, d , and ϕ satisfy the relation

$$a^2 + b^2 + 2ab \cos \phi \leq (1+d)^2. \quad (17)$$

And, F is the dimensionless average flux in the wave frame defined by

$$F = \int_{h_2(x)}^{h_1(x)} \frac{\partial \Psi}{\partial y} dy = \Psi(h_1(x)) - \Psi(h_2(x)). \quad (18)$$

3. Method of Solution

3.1. Homotopy Perturbation Method. The homotopy perturbation method is a combination of the perturbation method

and the homotopy method which eliminates the drawbacks of the traditional perturbation methods while keeping all their advantages. On the basis of the homotopy perturbation method [37–39] we can write (11) and (12) as follows:

$$\begin{aligned} H(q, \theta) &= (1-q) [L(\theta) - L(\theta_{10})] \\ &+ q \left[L(\theta) + N_b \frac{\partial \theta}{\partial y} \frac{\partial \Omega}{\partial y} + N_t \left(\frac{\partial \theta}{\partial y} \right)^2 \right], \end{aligned} \quad (19)$$

$$H(q, \Omega) = (1-q) [L(\Omega) - L(\Omega_{10})]$$

$$+ q \left[L(\Omega) + \frac{N_t}{N_b} \frac{\partial^2 \theta}{\partial y^2} \right].$$

Or

$$H(q, \theta) = L(\theta) - L(\theta_{10}) + qL(\theta_{10})$$

$$+ q \left[N_b \frac{\partial \theta}{\partial y} \frac{\partial \Omega}{\partial y} + N_t \left(\frac{\partial \theta}{\partial y} \right)^2 \right], \quad (20)$$

$$H(q, \Omega) = L(\Omega) - L(\Omega_{10})$$

$$+ qL(\Omega_{10}) + q \left[\frac{N_t}{N_b} \frac{\partial^2 \theta}{\partial y^2} \right],$$

with $L = \partial^2 / \partial y^2$ as the linear operator, the initial approximations θ_{10} and Ω_{10} can be defined as

$$\theta_{10}(x, y) = \frac{y - h_1}{h_1 - h_2} = \Omega_{10}(x, y). \quad (21)$$

And assume that

$$\theta(y, q) = \theta_0 + q\theta_1 + q^2\theta_2 + \dots, \quad (22)$$

$$\Omega(y, q) = \Omega_0 + q\Omega_1 + q^2\Omega_2 + \dots.$$

The solution of temperature and nanoparticle phenomenon (for $q = 1$) are constructed by adopting the same procedure as done by [37–39]

$$\begin{aligned} \theta(x, y) &= \frac{A_1 (N_b + 2N_t)}{6(h_2 - h_1)} y^3 \\ &- \frac{A_1 [(N_b + 2N_t)(h_1 + h_2) + 2(h_2 - h_1)]}{4(h_2 - h_1)} y^2 \end{aligned}$$

$$\begin{aligned}
& + \left((A_1 [(N_b + 2N_t)(h_1^2 + 4h_1h_2 + h_2^2) \right. \\
& \quad \left. + 6(h_2^2 - h_1^2)] + 12) \right. \\
& \quad \left. \times (12(h_2 - h_1))^{-1} \right) y \\
& - (A_1 h_1 h_2 [(N_b + 2N_t)(h_1 + h_2) + 6(h_2 - h_1)] \\
& \quad + 12h_1)(12(h_2 - h_1))^{-1}, \\
\Omega(x, y) &= \frac{A_1 N_t}{2N_b} y^2 + \frac{2N_b - A_1 N_t (h_2^2 - h_1^2)}{2N_b (h_2 - h_1)} y \\
& + \frac{A_1 N_t h_1 h_2 (h_2 - h_1) - 2h_1 N_b}{2N_b (h_2 - h_1)},
\end{aligned} \tag{23}$$

where

$$A_1 = \frac{N_b + N_t}{(h_2 - h_1)^2}. \tag{24}$$

Substituting (23) in (13), the exact solution for velocity is

$$\begin{aligned}
\Psi(x, y) &= C_{11} + C_{12}y + C_{13} \cosh(\Gamma y) \\
& + C_{14} \sinh(\Gamma y) - \frac{A_9}{12\Gamma^2} y^4 - \frac{A_{10}}{6\Gamma^2} y^3 - \frac{A_9}{\Gamma^4} y^2 \\
& - \left(\frac{A_{11}}{2\Gamma^2} + \frac{A_{10}}{\Gamma^4} \right) y - \left(\frac{A_{11}}{\Gamma^4} + \frac{2A_9}{\Gamma^6} \right).
\end{aligned} \tag{25}$$

And the pressure gradient is

$$\begin{aligned}
\frac{\partial P}{\partial x} &= \left(A_2 G_r + \frac{A_9}{3} \right) y^3 + \left(A_3 G_r + A_6 B_r + \frac{A_{10}}{2} \right) y^2 \\
& + (A_4 G_r + A_7 B_r) y \\
& + \left(A_5 G_r + A_8 B_r + \frac{A_{11}}{2} - \Gamma^2 (C_{12} + 1) \right).
\end{aligned} \tag{26}$$

The dimensionless pressure rise ΔP is given by

$$\Delta P = \int_0^1 \left(\frac{\partial P}{\partial x} \right) dx, \tag{27}$$

where

$$A_2 = \frac{A_1 (N_b + 2N_t)}{6(h_2 - h_1)}, \tag{28}$$

$$A_3 = -\frac{A_1 [(N_b + 2N_t)(h_1 + h_2) + 2(h_2 - h_1)]}{4(h_2 - h_1)}, \tag{29}$$

$$\begin{aligned}
A_4 &= \frac{A_1 [(N_b + 2N_t)(h_1^2 + 4h_1h_2 + h_2^2) + 6(h_2^2 - h_1^2)] + 12}{12(h_2 - h_1)}, \\
\end{aligned} \tag{30}$$

$$\begin{aligned}
A_5 &= -\frac{A_1 h_1 h_2 [(N_b + 2N_t)(h_1 + h_2) + 6(h_2 - h_1)] + 12h_1}{12(h_2 - h_1)}, \\
\end{aligned} \tag{31}$$

$$A_6 = \frac{N_t (N_b + N_t)}{2N_b (h_2 - h_1)^2}, \tag{32}$$

$$A_7 = \frac{2N_b - N_t A_1 (h_2^2 - h_1^2)}{2N_b (h_2 - h_1)}, \tag{33}$$

$$A_8 = \frac{2N_t A_1 h_1 h_2 (h_2 - h_1) - 2h_1 N_b}{2N_b (h_2 - h_1)}, \tag{34}$$

$$\begin{aligned}
A_9 &= -G_r (A_1 [(N_b + 2N_t)(h_1^2 + 4h_1h_2 + h_2^2) \\
& + 6(h_2^2 - h_1^2)] + 12)
\end{aligned} \tag{35}$$

$$\times (4(h_2 - h_1))^{-1},$$

$$\begin{aligned}
A_{10} &= \frac{G_r A_1 [(N_b + 2N_t)(h_1 + h_2) + 2(h_2 - h_1)]}{2(h_2 - h_1)} \\
& - \frac{B_r A_1 N_t}{N_b},
\end{aligned} \tag{36}$$

$$\begin{aligned}
A_{11} &= -G_r (A_1 [(N_b + 2N_t)(h_1^2 + 4h_1h_2 + h_2^2) \\
& + 6(h_2^2 - h_1^2)] + 12) \\
& \times (12(h_2 - h_1))^{-1}
\end{aligned} \tag{37}$$

$$- \frac{B_r (2N_b - N_t A_1 (h_2^2 - h_1^2))}{2N_b (h_2 - h_1)},$$

$$\begin{aligned}
C_{11} &= - \left(\operatorname{csch} \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) \right. \\
&\quad \times (2 (6 (A_{11} (2 \cosh (\Gamma (h_1 - h_2)) \\
&\quad - \Gamma \sinh (\Gamma (h_1 - h_2)) h_1 \\
&\quad + \Gamma \sinh (\Gamma (h_1 - h_2)) h_2 - 2) \\
&\quad + \Gamma (h_1 + h_2) \\
&\quad \times (3 (2 \cosh (\Gamma (h_1 - h_2)) \\
&\quad + F \Gamma \sinh (\Gamma (h_1 - h_2)) - 2) \Gamma^3 \\
&\quad + A_{10} (\Gamma (\cosh (\Gamma (h_1 - h_2)) \\
&\quad + \Gamma \sinh (\Gamma (h_1 - h_2)) \\
&\quad \times h_2 + 2) h_1^2 \\
&\quad - (\Gamma^2 \sinh (\Gamma (h_1 - h_2)) h_2^2 \\
&\quad + 2 \Gamma (2 \cosh (\Gamma (h_1 - h_2)) \\
&\quad + 1) h_2 \\
&\quad + 3 \sinh (\Gamma (h_1 - h_2))) \\
&\quad \times h_1 + h_2 \\
&\quad \times (3 \sinh (\Gamma (h_1 - h_2)) + \Gamma \\
&\quad \times (\cosh (\Gamma (h_1 - h_2)) + 2) \\
&\quad \times h_2))) \Gamma^2 + A_9 \\
&\quad \times (\Gamma^4 \\
&\quad \times (\cosh (\Gamma (h_1 - h_2)) \\
&\quad + \Gamma \sinh (\Gamma (h_1 - h_2)) h_2 + 3) \\
&\quad \times h_1^4 - 4 \Gamma^3 (\sinh (\Gamma (h_1 - h_2)) \\
&\quad + \Gamma (\cosh (\Gamma (h_1 - h_2)) h_2) \\
&\quad \times h_1^3 + 12 \Gamma^2 \\
&\quad \times (\cosh (\Gamma (h_1 - h_2)) \\
&\quad + \Gamma \sinh (\Gamma (h_1 - h_2)) \\
&\quad \times h_2 + 1) \\
&\quad \times h_1^2 - \Gamma \\
&\quad \times (\Gamma^4 \sinh (\Gamma (h_1 - h_2)) \\
&\quad \times h_2^4 + 4 \Gamma^3 \\
&\quad \times \cosh (\Gamma (h_1 - h_2)) h_2^3 \\
&\quad + 12 \Gamma^2 \\
&\quad \times \sinh (\Gamma (h_1 - h_2)) \\
&\quad \times h_2^2 + 48 \Gamma \\
&\quad \times \cosh (\Gamma (h_1 - h_2)) h_2 \\
&\quad + 48 \sinh (\Gamma (h_1 - h_2)) \\
&\quad \times h_1 + \Gamma^4 \\
&\quad \times (\cosh (\Gamma (h_1 - h_2)) \\
&\quad + 3) \\
&\quad \times h_2^4 + 4 \Gamma^3 \\
&\quad \times \sinh (\Gamma (h_1 - h_2)) \\
&\quad \times h_2^3 + 24 \Gamma^2 \\
&\quad \times \cosh^2 \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) \\
&\quad \times h_2^2 + 48 \\
&\quad \times (\cosh (\Gamma (h_1 - h_2)) \\
&\quad - 1) \\
&\quad + 48 \Gamma \sinh (\Gamma (h_1 - h_2)) \\
&\quad \times h_2))) \\
&\quad \times \left(24 \Gamma^6 \left(-2 \sinh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) \right. \right. \\
&\quad \left. \left. + \Gamma \cosh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) h_1 \right. \right. \\
&\quad \left. \left. - \Gamma \cosh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) h_2 \right) \right)^{-1}, \\
C_{12} &= - \left(-12 F \cosh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) \Gamma^5 \right. \\
&\quad - 24 \sinh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) \Gamma^4 \\
&\quad - \cosh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) A_9 h_1^4 \Gamma^3 \\
&\quad + \cosh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) A_9 h_2^4 \Gamma^3 \\
&\quad + 4 \sinh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) \\
&\quad \times A_9 h_1^3 \Gamma^2 + 4 \sinh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) A_9 h_2^3 \Gamma^2 \\
&\quad - 6 A_{11} \left(-2 \sinh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) \right. \\
&\quad \left. + \Gamma \cosh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) h_1 \right. \\
&\quad \left. - \Gamma \cosh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) h_2 \right) \Gamma^2
\end{aligned} \tag{38}$$

$$\begin{aligned}
& -12 \cosh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) \\
& \times A_9 h_1^2 \Gamma + 12 \cosh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) \\
& \times A_9 h_2^2 \Gamma + 24 \sinh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) \\
& \times A_9 h_1 + 24 \sinh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) A_9 h_2 \\
& + 2A_{10} \left(-\Gamma^3 \cosh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) \right. \\
& \quad \times h_1^3 + 3\Gamma^2 \sinh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) h_1^2 \\
& \quad - 6\Gamma \cosh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) h_1 \\
& \quad + \Gamma^3 \cosh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) h_2^3 \\
& \quad + 3\Gamma^2 \sinh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) h_2^2 \\
& \quad + 12 \sinh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) \\
& \quad \left. + 6\Gamma \cosh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) h_2 \right) \\
& \times \left(12\Gamma^4 \left(-2 \sinh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) \right. \right. \\
& \quad + \Gamma \cosh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) h_1 \\
& \quad \left. \left. - \Gamma \cosh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) h_2 \right) \right)^{-1}, \\
& C_{13} = \left(\operatorname{csch} \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) \right. \\
& \quad \times \left(12F (\cosh (\Gamma h_1) - \cosh (\Gamma h_2)) \Gamma^5 \right. \\
& \quad + (\cosh (\Gamma h_1) + 3 \cosh (\Gamma h_2)) A_9 h_1^4 \Gamma^3 \\
& \quad + (3 \cosh (\Gamma h_1) + \cosh (\Gamma h_2)) A_9 h_2^4 \Gamma^3 \\
& \quad + 2 (2 (\sinh (\Gamma h_1) - \sinh (\Gamma h_2)) A_9 + \Gamma \\
& \quad \quad \times (2 \cosh (\Gamma h_1) + \cosh (\Gamma h_2)) A_{10}) \\
& \quad \times h_2^3 \Gamma^2 + 2h_1^3 \\
& \quad \times (\Gamma (\cosh (\Gamma h_1) + 2 \cosh (\Gamma h_2)) A_{10} \\
& \quad \quad - 2A_9 (\sinh (\Gamma h_1) - \sinh (\Gamma h_2)) \\
& \quad \quad \left. + \Gamma \cosh (\Gamma h_2) h_2) \right) \Gamma^2 \\
& \quad + 6 (2 (\cosh (\Gamma h_1) + \cosh (\Gamma h_2)) A_9 \\
& \quad \quad + \Gamma \sinh (\Gamma h_1) - \sinh (\Gamma h_2)) A_{10} \\
& \quad \times h_2^2 \Gamma - 6h_1^2 \\
& \quad \times (\Gamma A_{10} (\sinh (\Gamma h_1) - \sinh (\Gamma h_2)) \\
& \quad \quad + \Gamma \cosh (\Gamma h_2) h_2) \\
& \quad \quad - 2 (\cosh (\Gamma h_1) + \cosh (\Gamma h_2)) A_9) \\
& \quad \left. \times \Gamma - 12 \right)
\end{aligned} \tag{39}$$

$$\begin{aligned}
& \times \left(\Gamma^5 (\cosh (\Gamma h_1) - \cosh (\Gamma h_2)) \right. \\
& \quad - 2 (\sinh (\Gamma h_1) - \sinh (\Gamma h_2)) \\
& \quad \times A_9 h_2 + 2h_1 \\
& \quad \times (6 (\cosh (\Gamma h_1) - \cosh (\Gamma h_2)) \Gamma^5 \\
& \quad \quad - 3 \cosh (\Gamma h_1) A_{10} h_2^2 \Gamma^3 - 2A_9 \\
& \quad \quad \times (\Gamma^3 \cosh (\Gamma h_1) h_2^3 + 6\Gamma \\
& \quad \quad \times (\cosh (\Gamma h_1) + \cosh (\Gamma h_2)) \\
& \quad \quad \times h_2 \\
& \quad \quad \left. + 6 (\sinh (\Gamma h_1) \right. \\
& \quad \quad \left. \left. - \sinh (\Gamma h_2)) \right) \right) \Gamma^5 \\
& \times \left(24\Gamma^5 \left(-2 \sinh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) \right. \right. \\
& \quad + \Gamma \cosh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) h_1 \\
& \quad \left. \left. - \Gamma \cosh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) h_2 \right) \right)^{-1}, \\
& C_{14} = - \left(\operatorname{csch} \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) \right.
\end{aligned} \tag{40}$$

$$\begin{aligned}
& \times \left(2 (-\cosh (\Gamma h_1) + \cosh (\Gamma h_2)) \right. \\
& \quad + \Gamma \sinh (\Gamma h_2) h_1 - \Gamma \sinh (\Gamma h_2) h_2) \\
& \quad \times \left(3A_{10} (h_1^2 - h_2^2) \Gamma^2 + 2A_9 \right. \\
& \quad \quad \times (\Gamma^2 h_1^3 + 6h_1 - h_2 (\Gamma^2 h_2^2 + 6)) \\
& \quad \quad + \Gamma (\sinh (\Gamma h_1) - \sinh (\Gamma h_2)) \\
& \quad \times (12F\Gamma^4 - 12h_2\Gamma^4 + A_9 h_1^4 \Gamma^2 \\
& \quad \quad + 3A_9 h_2^4 \Gamma^2 + 2A_{10} h_1^3 \Gamma^2 \\
& \quad \quad + 4A_{10} h_2^3 \Gamma^2 \\
& \quad \quad + 12A_9 h_1^2 + 12A_9 h_2^2 + 2h_1 \\
& \quad \quad \times (6\Gamma^4 - 3A_{10} h_2^2 \Gamma^2 \\
& \quad \quad \quad \left. - 2A_9 h_2 (\Gamma^2 h_2^2 + 6)) \right) \\
& \times \left(24\Gamma^5 \left(-2 \sinh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) \right. \right. \\
& \quad + \Gamma \cosh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) h_1 \\
& \quad \left. \left. - \Gamma \cosh \left(\frac{1}{2} \Gamma (h_1 - h_2) \right) h_2 \right) \right)^{-1}.
\end{aligned} \tag{41}$$

When calculating the constants C_{11} , C_{12} , C_{13} , and C_{14} we used the mathematica program.

4. Results and Discussion

In this section we have studied the velocity, pressure gradient, pressure rise, temperature, and nanoparticle phenomena graphically. The effects of Hall parameter m , porosity parameter s , thermophoresis parameter N_t , and local temperature Grashof number G_r on the velocity profile have been displayed in Figure 2. Figures 2(a) and 2(b) declare that velocity profile increases with an increase in the value of each of m and s , Figure 2(c) announces that the velocity near the channel walls are not similar in view of N_t , and the velocity decreases by increasing N_t . In Figure 2(d) it is found that the velocity field increases in the region $y \in (-0.8, -0.16)$ and it decreases in the region $y \in (-0.16, 0.45)$ with an increase in G_r . Figures 3(a) to 3(d) examine the pressure gradient for different values of Hall parameter m , porosity parameter s , amplitude of right wave a , and local nanoparticle Grashof number B_r . It is observed that the magnitude of pressure gradient decreases in view of an increase in m and s Figures 3(a) and 3(b). The situation is reversed in Figures 3(c) and 3(d), the magnitude of pressure gradient increases with an increase in a and B_r . The pressure rise versus volume flow rate is shown in Figures 4(a) to 4(d). It is noticed from Figures 4(a) and 4(c) that in the peristaltic pumping region ($\Delta P > 0$), the pressure rise increases with an increase in Hall parameter m and porosity parameter s , while decreases in the augmented pumping region ($\Delta P < 0$). Free pumping region holds when ($\Delta P = 0$). It is also noticed that the pressure rise is an increasing function of the increase in local temperature Grashof number G_r (Figure 4(b)); moreover, by increasing the value of Brownian motion parameter N_b , the pressure rise is a decreasing function (Figure 4(d)). Figures 5 and 6 demonstrate the variation of temperature profile and nanoparticle phenomena for different values of thermophoresis parameter N_t and Brownian motion parameter N_b . Here the temperature profile decreases when N_t and N_b are increased and nanoparticle phenomenon decreases by increasing N_t , while it increases when N_b is increased.

5. Trapping Phenomenon

The formation of an internally circulating bolus of fluid is called trapping. The trapped bolus moves along with the wave. The effects of Hall parameter m and Brownian motion parameter N_b on trapping can be observed through Figure 7, where the number of trapping bolus increases with an increase in m and N_b in the left half of the channel. Figure 8 depicts that increasing the local temperature Grashof number G_r leads to a decrease in the number of trapping bolus in the left half of the channel but it increases in both the left and right halves of the channel with an increase in the porosity parameter s .

6. Conclusions

The effects of Hall current and porous media on peristaltic transport of a nanofluid are investigated. The main results are summarized as follows.

- (1) Effects of Hall m and porosity s parameters are similar on velocity profile but thermophoresis parameter N_t has opposite effect.
- (2) The velocity near the channel walls is not similar in view of the local temperature Grashof number G_r .
- (3) The magnitude of pressure gradient decreases by increasing Hall m and porosity s parameters. However it increases with an increase in amplitude of right wave a .
- (4) The behaviors of Hall parameter m and porosity parameter s on the pressure rise are similar.
- (5) The behaviors of local temperature Grashof number G_r and Brownian motion parameter N_b on the pressure rise are opposite.
- (6) Temperature profile decreases when thermophoresis parameter N_t and Brownian motion parameter N_b are increased.
- (7) Nanoparticle phenomenon decreases by increasing thermophoresis parameter N_t while it increases by increasing Brownian motion parameter N_b .
- (8) The number of trapping bolus increases in the left half of the channel due to an increase in Hall parameter m and Brownian motion parameter N_b . On the other hand it decreases in the left half of the channel with an increase in local temperature Grashof number G_r .
- (9) Increasing the magnitude of porosity parameter s led to an increase in the number of trapping bolus in both halves of the channel.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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