Research Article

Positive Solutions of a Third-Order Three-Point BVP with Sign-Changing Green's Function

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We are concerned with the following third-order three-point boundary value problem: $u'''(t) = f(t, u(t))$, $t \in [0, 1]$, $u'(0) = u(1) = 0$ and $u''(\eta) - \alpha u'(1) = 0$, where $\alpha \in [0, 1)$ and $\eta \in [(14 + \alpha)/(24 - 3\alpha), 1)$. Although the corresponding Green's function is sign-changing, we still obtain the existence of at least two positive and decreasing solutions under some suitable conditions on f by using the two-fixed-point theorem due to Avery and Henderson.

1. Introduction

Third-order differential equations arise from a variety of different areas of applied mathematics and physics, for example, in the deflection of a curved beam having a constant or varying cross-section, a three-layer beam, electromagnetic waves or gravity driven flows, and so on [1].

Recently, the existence of single or multiple positive solutions to some third-order three-point boundary value problems (BVPs for short) has received much attention from many authors; see [2–11] and the references therein. However, all the above-mentioned papers are achieved when the corresponding Green's functions are nonnegative, which is a very important condition.

In 2008, Palamides and Smyrlis [12] studied the existence of at least one positive solution to the singular third-order three-point BVP with an indefinitely signed Green's function:

$$\begin{aligned} u'''(t) &= a(t)f(t, u(t)), \quad t \in (0, 1), \\ u(0) &= u(1) = u''(\eta) = 0, \end{aligned} \quad (1)$$

where $\eta \in (17/24, 1)$. Their technique was a combination of the Guo-Krasnoselskii fixed-point theorem and properties of the corresponding vector field.

Very recently, for the third-order three-point BVP with sign-changing Green's function

$$\begin{aligned} u'''(t) &= f(t, u(t)), \quad t \in [0, 1], \\ u'(0) &= u(1) = u''(\eta) = 0, \end{aligned} \quad (2)$$

Sun and Zhao proved the existence of at least one or three positive solutions when $\eta \in (1/2, 1)$ by using the Guo-Krasnoselskii and Leggett-Williams fixed-point theorems in [13, 14] and obtained the existence of a positive solution when $\eta \in [2 - \sqrt{2}, 1)$ by using iterative technique in [15].

In 2013, Li et al. [16] discussed the existence of a positive solution to the third-order three-point BVP with sign-changing Green's function:

$$\begin{aligned} u'''(t) &= f(t, u(t)), \quad t \in [0, 1], \\ u'(0) &= u(1) = 0, \quad u''(\eta) + \alpha u(0) = 0, \end{aligned} \quad (3)$$

where $\alpha \in [0, 2)$ and $\eta \in [(\sqrt{121 + 24\alpha} - 5)/3(4 + \alpha), 1)$. The main tool used was the Guo-Krasnoselskii fixed-point theorem.

It is worth mentioning that there are other types of works on sign-changing Green's functions which prove the existence of sign-changing solutions, positive in some cases; see Infante and Webb's papers [17–19].

Motivated greatly by the above-mentioned works, in this paper, we consider the following third-order three-point BVP:

$$\begin{aligned} u'''(t) &= f(t, u(t)), \quad t \in [0, 1], \\ u'(0) &= u(1) = 0, \quad u''(\eta) - \alpha u'(1) = 0. \end{aligned} \quad (4)$$

Throughout this paper, we always assume that $\alpha \in [0, 1]$ and $\eta \in [(14 + \alpha)/(24 - 3\alpha), 1]$. Although the corresponding Green's function is sign-changing, we still obtain the existence of at least two positive and decreasing solutions under some suitable conditions on f . Our main tool is the well-known Avery-Henderson two-fixed-point theorem.

To end this section, we state some fundamental definitions and the two-fixed-point theorem due to Avery and Henderson [20].

Let K be a cone in a real Banach space E .

Definition 1. A functional $\psi : K \rightarrow \mathbb{R}$ is said to be increasing on K provided $\psi(x) \leq \psi(y)$ for all $x, y \in K$ with $x \leq y$, where $x \leq y$ if and only if $y - x \in K$.

Definition 2. Let $\gamma : K \rightarrow [0, +\infty)$ be continuous. For each $d > 0$, we define the set

$$K(\gamma, d) = \{u \in K : \gamma(u) < d\}. \quad (5)$$

Theorem 3. Let ψ and γ be increasing, nonnegative, and continuous functionals on K , and let ω be a nonnegative continuous functional on K with $\omega(0) = 0$ such that, for some $c > 0$ and $M > 0$,

$$\gamma(u) \leq \omega(u) \leq \psi(u), \quad \|u\| \leq M\gamma(u) \quad (6)$$

for all $u \in \overline{K(\gamma, c)}$. Suppose that there exist a completely continuous operator $T : \overline{K(\gamma, c)} \rightarrow K$ and $0 < a < b < c$ such that

$$\omega(\lambda u) \leq \lambda \omega(u) \quad \text{for } 0 \leq \lambda \leq 1, \quad u \in \partial K(\omega, b), \quad (7)$$

and

- (1) $\gamma(Tu) > c$ for all $u \in \partial K(\gamma, c)$;
- (2) $\omega(Tu) < b$ for all $u \in \partial K(\omega, b)$;
- (3) $K(\psi, a) \neq \emptyset$, and $\psi(Tu) > a$ for all $u \in \partial K(\psi, a)$.

Then T has at least two fixed points u_1 and u_2 in $\overline{K(\gamma, c)}$ such that

$$\begin{aligned} a &< \psi(u_1) \quad \text{with } \omega(u_1) < b, \\ b &< \omega(u_2) \quad \text{with } \gamma(u_2) < c. \end{aligned} \quad (8)$$

2. Preliminaries

For the BVP

$$\begin{aligned} u'''(t) &= 0, \quad t \in [0, 1], \\ u'(0) &= u(1) = 0, \quad u''(\eta) - \alpha u'(1) = 0, \end{aligned} \quad (9)$$

we have the following lemma.

Lemma 4. The BVP (9) has only trivial solution.

Proof. It is simple to check. $\square$

Now, for any $y \in C[0, 1]$, we consider the BVP

$$\begin{aligned} u'''(t) &= y(t), \quad t \in [0, 1], \\ u'(0) &= u(1) = 0, \quad u''(\eta) - \alpha u'(1) = 0. \end{aligned} \quad (10)$$

After a direct computation, one may obtain the expression of Green's function $G(t, s)$ of the BVP (10) as follows:

$$G(t, s) = g(t, s) + k(t, s) + \begin{cases} 0, & s \geq \eta, \\ \frac{1-t^2}{2(1-\alpha)}, & s < \eta, \end{cases} \quad (11)$$

where

$$\begin{aligned} g(t, s) &= -\frac{\alpha(1-t^2)(1-s)}{2(1-\alpha)} - \frac{(1-s)^2}{2}, \\ (t, s) &\in [0, 1] \times [0, 1], \end{aligned} \quad (12)$$

$$k(t, s) = \begin{cases} 0, & 0 \leq t \leq s \leq 1, \\ \frac{(t-s)^2}{2}, & 0 \leq s \leq t \leq 1. \end{cases}$$

It is not difficult to verify that the $G(t, s)$ has the following properties:

$$\begin{aligned} G(t, s) &\geq 0 \quad \text{for } 0 \leq s < \eta, \\ G(t, s) &\leq 0 \quad \text{for } \eta \leq s \leq 1. \end{aligned} \quad (13)$$

Moreover, for $s \geq \eta$,

$$\max \{G(t, s) : t \in [0, 1]\} = G(1, s) = 0,$$

$$\begin{aligned} \min \{G(t, s) : t \in [0, 1]\} &= G(0, s) \\ &= -\frac{\alpha(1-s)}{2(1-\alpha)} - \frac{(1-s)^2}{2} \end{aligned} \quad (14)$$

and for $s < \eta$,

$$\begin{aligned} \max \{G(t, s) : t \in [0, 1]\} &= G(0, s) \\ &= \frac{1-\alpha(1-s)}{2(1-\alpha)} - \frac{(1-s)^2}{2}, \end{aligned} \quad (15)$$

$$\min \{G(t, s) : t \in [0, 1]\} = G(1, s) = 0.$$

Let

$$\begin{aligned} K_0 &= \{y \in C[0, 1] : y(t) \text{ is nonnegative} \\ &\quad \text{and decreasing on } [0, 1]\}. \end{aligned} \quad (16)$$

Then K_0 is a cone in $C[0, 1]$.

Lemma 5. Let $y \in K_0$ and $u(t) = \int_0^1 G(t, s)y(s)ds$, $t \in [0, 1]$. Then u is the unique solution of the BVP (10) and $u \in K_0$. Moreover, $u(t)$ is concave on $[0, \eta]$.

Proof. For $t \in [0, \eta]$, we have

$$\begin{aligned} u(t) &= \int_0^t \left[g(t, s) + \frac{(t-s)^2}{2} + \frac{1-t^2}{2(1-\alpha)} \right] y(s) ds \\ &\quad + \int_t^\eta \left[g(t, s) + \frac{1-t^2}{2(1-\alpha)} \right] y(s) ds \\ &\quad + \int_\eta^1 g(t, s) y(s) ds. \end{aligned} \quad (17)$$

Since $y \in K_0$ and $\eta \geq (14 + \alpha)/(24 - 3\alpha)$ implies that $\eta > \alpha/(1 + \alpha)$, we get

$$\begin{aligned} u'(t) &= -\frac{\alpha t}{1-\alpha} \int_0^\eta sy(s) ds - \int_0^t sy(s) ds \\ &\quad - t \int_t^\eta y(s) ds + \frac{\alpha t}{1-\alpha} \int_\eta^1 (1-s) y(s) ds \\ &\leq y(\eta) \left[-\frac{\alpha t}{1-\alpha} \int_0^\eta s ds - \int_0^t s ds \right. \\ &\quad \left. - t \int_t^\eta ds + \frac{\alpha t}{1-\alpha} \int_\eta^1 (1-s) ds \right] \\ &= ty(\eta) \left[\frac{\alpha(1-2\eta)}{2(1-\alpha)} - \eta + \frac{t}{2} \right] \\ &\leq ty(\eta) \left[\frac{\alpha(1-2\eta)}{2(1-\alpha)} - \frac{\eta}{2} \right] \\ &\leq 0, \quad t \in [0, \eta]. \end{aligned} \quad (18)$$

At the same time, $y \in K_0$ and $\eta \geq (14 + \alpha)/(24 - 3\alpha) > 1/2$ shows that

$$\begin{aligned} u''(t) &= -\frac{\alpha}{1-\alpha} \int_0^\eta sy(s) ds - \int_t^\eta y(s) ds \\ &\quad + \frac{\alpha}{1-\alpha} \int_\eta^1 (1-s) y(s) ds \\ &\leq y(\eta) \left[-\frac{\alpha}{1-\alpha} \int_0^\eta s ds - \int_t^\eta ds + \frac{\alpha}{1-\alpha} \int_\eta^1 (1-s) ds \right] \\ &= y(\eta) \left[\frac{\alpha(1-2\eta)}{2(1-\alpha)} - \eta + t \right] \\ &\leq \frac{y(\eta)\alpha(1-2\eta)}{2(1-\alpha)} \\ &\leq 0, \quad t \in [0, \eta]. \end{aligned} \quad (19)$$

For $t \in (\eta, 1]$, we have

$$\begin{aligned} u(t) &= \int_0^\eta \left[g(t, s) + \frac{(t-s)^2}{2} + \frac{1-t^2}{2(1-\alpha)} \right] y(s) ds \\ &\quad + \int_\eta^t \left[g(t, s) + \frac{(t-s)^2}{2} \right] y(s) ds \\ &\quad + \int_t^1 g(t, s) y(s) ds. \end{aligned} \quad (20)$$

In view of $y \in K_0$ and $\eta > 1/2$, we get

$$\begin{aligned} u'(t) &= -\frac{\alpha t}{1-\alpha} \int_0^\eta sy(s) ds + \int_\eta^t (t-s) y(s) ds \\ &\quad - \int_0^\eta sy(s) ds + \frac{\alpha t}{1-\alpha} \int_\eta^1 (1-s) y(s) ds \\ &\leq y(\eta) \left[-\frac{\alpha t}{1-\alpha} \int_0^\eta s ds + \int_\eta^t (t-s) ds \right. \\ &\quad \left. - \int_0^\eta s ds + \frac{\alpha t}{1-\alpha} \int_\eta^1 (1-s) ds \right] \\ &= ty(\eta) \left[\frac{\alpha(1-2\eta)}{2(1-\alpha)} + \frac{t-2\eta}{2} \right] \\ &\leq ty(\eta) \left[\frac{1-2\eta}{2(1-\alpha)} \right] \\ &\leq 0, \quad t \in (\eta, 1]. \end{aligned} \quad (21)$$

Obviously, $u'''(t) = y(t)$ for $t \in [0, 1]$, $u'(0) = u(1) = 0$, and $u''(\eta) - \alpha u'(1) = 0$. This shows that u is a solution of the BVP (10). The uniqueness follows immediately from Lemma 4. Since $u'(t) \leq 0$ for $t \in [0, 1]$ and $u(1) = 0$, we have $u(t) \geq 0$ for $t \in [0, 1]$. So, $u \in K_0$. In view of $u''(t) \leq 0$ for $t \in [0, \eta]$, we know that $u(t)$ is concave on $[0, \eta]$. $\square$

Throughout this paper, for any $u \in C[0, 1]$, we define $\|u\| = \max_{t \in [0, 1]} |u(t)|$.

Lemma 6. Let $y \in K_0$. Then the unique solution u of the BVP (10) satisfies

$$\min_{t \in [0, \theta]} u(t) \geq \theta^* \|u\|, \quad (22)$$

where $\theta \in (0, 1/2]$ and $\theta^* = (\eta - \theta)/\eta$.

Proof. By Lemma 5, we know that $u(t)$ is concave on $[0, \eta]$; thus, for $t \in [0, \eta]$,

$$u(t) \geq \frac{\eta-t}{\eta} u(0) + \frac{t}{\eta} u(\eta). \quad (23)$$

At the same time, it follows from $u \in K_0$ that $\|u\| = u(0)$, which together with (23) implies that

$$u(t) \geq \frac{\eta-t}{\eta} \|u\|, \quad t \in [0, \eta]. \quad (24)$$

Consequently,

$$\min_{t \in [0, \theta]} u(t) = u(\theta) \geq \frac{\eta - \theta}{\eta} \|u\| = \theta^* \|u\|. \quad (25)$$

□

3. Main Results

In the remainder of this paper, we always assume that $f : [0, 1] \times [0, +\infty) \rightarrow [0, +\infty)$ is continuous and satisfies the following two conditions:

(H1) for each $u \in [0, +\infty)$, the mapping $t \mapsto f(t, u)$ is decreasing;

(H2) for each $t \in [0, 1]$, the mapping $u \mapsto f(t, u)$ is increasing.

Let

$$K = \left\{ u \in K_0 : \min_{t \in [0, \theta]} u(t) \geq \theta^* \|u\| \right\}. \quad (26)$$

Then it is easy to see that K is a cone in $C[0, 1]$.

Now, we define an operator T as follows:

$$(Tu)(t) = \int_0^1 G(t, s) f(s, u(s)) ds, \quad u \in K, \quad t \in [0, 1]. \quad (27)$$

First, it is obvious that if u is a fixed point of T in K , then u is a nonnegative and decreasing solution of the BVP (4). Next, by Lemmas 5 and 6, we know that $T : K \rightarrow K$. Furthermore, although $G(t, s)$ is not continuous, it follows from known textbook results, for example, see [21], that $T : K \rightarrow K$ is completely continuous.

For convenience, we denote

$$A = \int_0^\theta G(\eta, s) ds, \quad B = \int_0^\eta G(\theta, s) ds. \quad (28)$$

Theorem 7. Suppose that there exist numbers a, b , and c with $0 < a < b < \theta^* c$ such that

$$f(\theta, c) > \frac{c}{A}, \quad (29)$$

$$f\left(0, \frac{b}{\theta^*}\right) < \frac{b}{B}, \quad (30)$$

$$f(\theta, \theta^* a) > \frac{a}{A}. \quad (31)$$

Then the BVP (4) has at least two positive and decreasing solutions.

Proof. First, we define the increasing, nonnegative, and continuous functionals γ, ω , and ψ on K as follows:

$$\gamma(u) = \min_{t \in [0, \theta]} u(t) = u(\theta),$$

$$\omega(u) = \max_{t \in [0, 1]} u(t) = u(\theta), \quad (32)$$

$$\psi(u) = \max_{t \in [0, 1]} u(t) = u(0).$$

Obviously, for any $u \in K$, $\gamma(u) = \omega(u) \leq \psi(u)$. At the same time, for each $u \in K$, in view of $\gamma(u) = \min_{t \in [0, \theta]} u(t) \geq \theta^* \|u\|$, we have

$$\|u\| \leq \frac{1}{\theta^*} \gamma(u) \quad \text{for } u \in K. \quad (33)$$

In addition, we also note that

$$\omega(\lambda u) = \lambda \omega(u) \quad \text{for } 0 \leq \lambda \leq 1, \quad u \in K. \quad (34)$$

Next, for any $u \in K$, we claim that

$$\int_\theta^1 G(\eta, s) f(s, u(s)) ds \geq 0. \quad (35)$$

In fact, it follows from (H1), (H2), and $\eta \geq (14 + \alpha)/(24 - 3\alpha)$ that

$$\begin{aligned} & \int_\theta^1 G(\eta, s) f(s, u(s)) ds \\ &= \int_\theta^\eta G(\eta, s) f(s, u(s)) ds + \int_\eta^1 G(\eta, s) f(s, u(s)) ds \\ &\geq f(\eta, u(\eta)) \left[\int_\theta^\eta G(\eta, s) ds + \int_\eta^1 G(\eta, s) ds \right] \\ &= f(\eta, u(\eta)) \\ &\quad \times \left[\int_\theta^\eta \left(g(\eta, s) + \frac{(\eta - s)^2}{2} + \frac{1 - \eta^2}{2(1 - \alpha)} \right) ds \right. \\ &\quad \left. + \int_\eta^1 g(\eta, s) ds \right] \\ &= \frac{(1 - \eta) f(\eta, u(\eta))}{12(1 - \alpha)} \\ &\quad \times \left[(4 + 2\alpha)\eta^2 + (4 - \alpha)\eta - \alpha - 2 + (3\alpha - 3\alpha\eta - 6)\theta^2 \right] \\ &\geq \frac{(1 - \eta) f(\eta, u(\eta))}{12(1 - \alpha)} \\ &\quad \times \left[(4 + 2\alpha)\eta^2 + \left(4 - \frac{7}{4}\alpha \right)\eta - \frac{1}{4}\alpha - \frac{7}{2} \right] \\ &\geq \frac{(1 - \eta) f(\eta, u(\eta))}{12(1 - \alpha)} \left[\left(6 - \frac{3}{4}\alpha \right)\eta - \frac{1}{4}\alpha - \frac{7}{2} \right] \\ &\geq 0. \end{aligned} \quad (36)$$

Now, we assert that $\gamma(Tu) > c$ for all $u \in \partial K(\gamma, c)$.

To prove this, let $u \in \partial K(\gamma, c)$; that is, $u \in K$ and $\gamma(u) = u(\theta) = c$. Then

$$u(t) \geq u(\theta) = c, \quad t \in [0, \theta]. \quad (37)$$

Since $(Tu)(t)$ is decreasing on $[0, 1]$, it follows from (29), (35), (37), (H1), and (H2) that

$$\begin{aligned}
 \gamma(Tu) &= (Tu)(\theta) \\
 &\geq (Tu)(\eta) \\
 &= \int_0^1 G(\eta, s) f(s, u(s)) ds \\
 &\geq \int_0^\theta G(\eta, s) f(s, u(s)) ds \\
 &\geq \int_0^\theta G(\eta, s) f(\theta, c) ds \\
 &> \frac{c}{A} \int_0^\theta G(\eta, s) ds \\
 &= c.
 \end{aligned} \tag{38}$$

Then, we assert that $\omega(Tu) < b$ for all $u \in \partial K(\omega, b)$.

To see this, suppose that $u \in \partial K(\omega, b)$; that is, $u \in K$ and $\omega(u) = b$. Since $\|u\| \leq (1/\theta^*)\gamma(u) = (1/\theta^*)\omega(u)$, we have

$$0 \leq u(t) \leq \|u\| \leq \frac{b}{\theta^*}, \quad t \in [0, \eta]. \tag{39}$$

In view of the properties of $G(t, s)$, (30), (39), (H1), and (H2), we get

$$\begin{aligned}
 \omega(Tu) &= (Tu)(\theta) \\
 &= \int_0^1 G(\theta, s) f(s, u(s)) ds \\
 &\leq \int_0^\eta G(\theta, s) f(s, u(s)) ds \\
 &\leq \int_0^\eta G(\theta, s) f\left(0, \frac{b}{\theta^*}\right) ds \\
 &< \frac{b}{B} \int_0^\eta G(\theta, s) ds \\
 &= b.
 \end{aligned} \tag{40}$$

Finally, we assert that $K(\psi, a) \neq \emptyset$ and $\psi(Tu) > a$ for all $u \in \partial K(\psi, a)$.

In fact, the constant function $a/2 \in K(\psi, a)$. Moreover, for $u \in \partial K(\psi, a)$, that is, $u \in K$ and $\psi(u) = u(0) = a$, we know that

$$u(t) \geq \theta^* \|u\| = \theta^* u(0) = \theta^* a, \quad t \in [0, \theta]. \tag{41}$$

Since $(Tu)(t)$ is decreasing on $[0, 1]$, it follows from (31), (35), (41), (H1), and (H2) that

$$\begin{aligned}
 \psi(Tu) &= (Tu)(0) \\
 &\geq (Tu)(\eta) \\
 &= \int_0^1 G(\eta, s) f(s, u(s)) ds \\
 &\geq \int_0^\theta G(\eta, s) f(s, u(s)) ds \\
 &\geq \int_0^\theta G(\eta, s) f(\theta, \theta^* a) ds \\
 &> \frac{a}{A} \int_0^\theta G(\eta, s) ds \\
 &= a.
 \end{aligned} \tag{42}$$

To sum up, all the hypotheses of Theorem 3 are satisfied. Hence, T has at least two fixed points u_1 and u_2 ; that is, the BVP (4) has at least two positive and decreasing solutions u_1 and u_2 satisfying

$$\begin{aligned}
 a &< \max_{t \in [0, 1]} u_1(t) \quad \text{with} \quad \max_{t \in [0, 1]} u_1(t) < b, \\
 b &< \max_{t \in [\theta, 1]} u_2(t) \quad \text{with} \quad \min_{t \in [0, \theta]} u_2(t) < c.
 \end{aligned} \tag{43}$$

□

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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