Research Article
Effects of Car Accidents on Three-Lane Traffic Flow

Jianzhong Chen, Zhiyuan Peng, and Yuan Fang

College of Automation, Northwestern Polytechnical University Xi’an, Shaanxi 710072, China

Correspondence should be addressed to Jianzhong Chen; jzhchen@nwpu.edu.cn

Received 26 March 2014; Revised 6 July 2014; Accepted 6 July 2014; Published 21 July 2014

Academic Editor: Miguel A. F. Sanjuan

Copyright © 2014 Jianzhong Chen et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

A three-lane traffic flow model is proposed to investigate the effect of car accidents on the traffic flow. The model is an extension of the full velocity difference (FVD) model by taking into account the lane changing. The extended lane-changing rules are presented to model the lane-changing behaviour. The cases that the car accidents occupy the exterior or interior lane, the medium lane, and two lanes are studied by numerical simulations. The time-space diagrams and the current diagrams are presented, and the traffic jams are investigated. The results show that the car accident has a different effect on the traffic flow when it occupies different lanes. The car accidents have a more serious effect on the whole road when they occupy two lanes. The larger the density is, the greater the influence on the traffic flow becomes.

1. Introduction

Traffic jam is a complicated traffic phenomenon and has been investigated by various traffic models [1–18]. Traffic accidents are one of the important sources of traffic jams. The accident usually leads to vehicle delay and reduces the capacity of the roads.

Recently, the studies of the influence of the car accident on traffic flow have attracted considerable attention. Nagatani [19] applied a cellular automaton (CA) model to study the effect of a traffic accident on the dynamical jamming. Zhu et al. [20] investigated the physics of congested traffic pattern induced by a car accident using a two-lane CA model. Kerner et al. [21] proposed a simple two-lane CA model to explain the physics of traffic breakdown, highway capacity, and synchronized flow. Qian et al. [22] presented an improved CA model to evaluate the effect of an accident. Sheng et al. [23] proposed a new CA model to investigate the temporary bottleneck by a special accident.

Kurata and Nagatani [24] extended the optimal velocity (OV) model [25] to two-lane traffic by introducing the lane-changing rules and used this model to study traffic jams induced by a car accident. Sugiyama and Nagatani [26] presented an extended OV model to simulate the multiple-vehicle collision when a vehicle decelerates suddenly in single-lane traffic flow. Based on the full velocity difference (FVD) model [27], Tang et al. [28] suggested a new car-following model with consideration of the effect of the traffic interruption probability.

As for the study of the macroscopic model, Tang et al. [29] proposed a new macromodel taking into account the traffic interruption probability. This model was derived from the microscopic model [28] by using the relationship between the microscopic and macroscopic variables. Zhao [30] adopted this macromodel to explore the effect of the accident position on shock and rarefaction. Tian and Sun [31] extended this model to two-lane traffic flow. To explore the effects of static bottleneck on traffic flow, Tang et al. [32] proposed a new macromodel. Tang et al. [33] further developed an extended macromodel considering effects of multistatic bottlenecks on traffic flow.

The abovementioned models concentrate on modeling the effects of car accidents on traffic flow at a single-lane or two-lane roadway. However, there exist a large number of three-lane roads in real traffic. In comparison with two-lane roads, the greater traffic flow and more frequent lane changing in three-lane roads make the accident easier to happen. It is thus necessary to further investigate the effect of the car accidents on traffic flow in three-lane roads.

In this paper, we extend the FVD model to the three-lane traffic by taking into account the lane changing. We investigate the traffic jams caused by the car accidents which
occupy the exterior or interior lane, the medium lane, and two lanes. We derive the time-space diagrams and the current diagrams to study the effect of the car accidents on traffic flow (Figure 1).

2. Model

We consider traffic flow on a highway which consists of three lanes: the exterior lane (lane 1), the medium lane (lane 2), and the interior lane (lane 3). We discuss the cases where a car accident is located at lane 1 or lane 2, two car accidents occupy lane 1 and lane 2, and two car accidents occupy lane 1 and lane 3, as shown in Figure 2, where the car accidents are marked as black. Cars can overtake the car accident by changing lanes if the lane-changing criteria are fulfilled.

2.1. Basic Model. The car motion is divided into two parts: the forward motion and the sideways motion. We employ the FVD model [27] to describe the forward motion. This model has the form

\[
\frac{d^2 x_i(t)}{dt^2} = a \left\{ V(\Delta x_i(t)) - \frac{dx_i(t)}{dt} \right\} + \lambda \Delta v_i, \tag{1}
\]
Figure 3: The time-space diagrams of three lanes for the case that a car accident is in lane 1 at density $\rho = 0.072$. (a) lane 1; (b) lane 2; (c) lane 3.

Figure 4: The time-space diagrams of three lanes for the case that a car accident is in lane 1 at density $\rho = 0.1$. (a) Lane 1; (b) lane 2; (c) lane 3.
where \( x_i(t) \) is the position of car \( i \) at time \( t \), \( \Delta x_i(t) = x_{i+1}(t) - x_i(t) \) is the headway of car \( i \), \( V \) denotes the optimal velocity, \( a \) is the sensitivity, \( \Delta v_i(t) = v_{i+1}(t) - v_i(t) \) represents the velocity difference between car \( i \) and its adjacent front car \( i + 1 \), and \( \lambda \) is the sensitivity coefficient of a driver to the velocity difference.

The optimal velocity function \( V(\Delta x_i) \) can take many forms. We adopt the form suggested by Bando et al. [25]:

\[
V(\Delta x_i) = \frac{v_{\text{max}}}{2} \left[ \tanh (\Delta x_i - x_c) - \tanh (x_c) \right],
\]

where \( v_{\text{max}} \) is the maximal velocity and \( x_c \) is the safety distance.

2.2. Lane-Changing Rules. In general, a driver has an incentive to change lane if the adjacent lane can provide better driving conditions. Lane-changing decision of a driver needs to satisfy two criteria: (1) incentive criterion (i.e., the adjacent lane has a better driving condition) and (2) the security criterion (i.e., a driver must ensure the safety in the process of lane changing). As an example, Figure 2 shows a schematic illustration of lane changing for car \( i \) on lane 2. In Figure 2, \( \Delta x_f_i^{(1)} \), \( \Delta x_f_i^{(2)} \), and \( \Delta x_f_i^{(3)} \) denote, respectively, the distances between car \( i \) and its adjacent front car in lane 1, lane 2, and lane 3, while \( \Delta x_b_i^{(1)} \), \( \Delta x_b_i^{(2)} \), and \( \Delta x_b_i^{(3)} \) represent, respectively, the distances between car \( i \) and its following car in lane 1, lane 2, and lane 3. In this paper, we adopt the symmetric lane-changing rules. That is, cars on lane 2 can change to lane 1 or lane 3 while cars on lane 1 and lane 3 can also change to lane 2.

Based on the lane-changing rules [24, 34], we propose the following extended lane-changing rules at time \( t \).

(a) cars on lane 1 change to lane 2.

We adopt the rules of Kurata and Nagatani [24]:

\[
\begin{align*}
\Delta x_f_i^{(1)} > 1.02 v_{i+1}^{(1)}, \quad \Delta x_f_i^{(1)} &< 4x_c & \text{for the incentive criterion,} \\
\Delta x_f_i^{(2)} > 2x_c, \quad \Delta x_b_i^{(2)} &> x_c & \text{for the security criterion,} \\
\end{align*}
\]

or

\[
\begin{align*}
\Delta x_f_i^{(1)} > 1.02 v_{i+1}^{(1)}, \quad \Delta x_f_i^{(1)} &< 4x_c & \text{for the incentive criterion,} \\
\Delta x_f_i^{(2)} > v_i^{(1)}, \quad \Delta x_f_i^{(2)} &> v_{i+1}^{(1)} & \text{for the security criterion,}
\end{align*}
\]

or

\[
\begin{align*}
\Delta x_f_i^{(1)} &< 2x_c & \text{for the incentive criterion,} \\
\Delta x_f_i^{(2)} > \Delta x_f_i^{(1)}, \quad \Delta x_b_i^{(2)} &> x_c & \text{for the security criterion.}
\end{align*}
\]

(b) Change from lane 3 to lane 2, if

\[
\begin{align*}
\Delta x_f_i^{(3)} &> 1.02 v_{i+1}^{(3)}, \quad \Delta x_f_i^{(3)} &< 4x_c, \\
\Delta x_f_i^{(2)} &> 2x_c, \quad \Delta x_b_i^{(2)} &> x_c,
\end{align*}
\]

or

\[
\begin{align*}
\Delta x_f_i^{(3)} &> 1.02 v_{i+1}^{(3)}, \quad \Delta x_f_i^{(3)} &< 4x_c, \quad v_i^{(2)} &> v_{i+1}^{(3)}, \quad \Delta x_f_i^{(2)} > \Delta x_f_i^{(3)}, \\
\Delta x_b_i^{(2)} &> x_c,
\end{align*}
\]

or

\[
\begin{align*}
\Delta x_f_i^{(3)} &< 2x_c, \quad \Delta x_f_i^{(2)} > \Delta x_f_i^{(3)}, \\
\Delta x_b_i^{(2)} &> x_c.
\end{align*}
\]

(c) Change from lane 2 to lane 1, if

\[
\begin{align*}
\Delta x_f_i^{(2)} &> 1.02 v_{i+1}^{(2)}, \quad \Delta x_f_i^{(2)} &< 4x_c, \quad \Delta x_f_i^{(1)} > 2x_c, \\
\Delta x_b_i^{(1)} &> x_c, \quad \Delta x_b_i^{(3)} &\leq x_c,
\end{align*}
\]

or

\[
\begin{align*}
\Delta x_f_i^{(2)} &> 1.02 v_{i+1}^{(2)}, \quad \Delta x_f_i^{(2)} &< 4x_c, \quad \Delta x_f_i^{(1)} > 2x_c, \\
\Delta x_b_i^{(1)} &> x_c, \quad \Delta x_b_i^{(3)} > x_c, \quad \Delta x_f_i^{(3)} &\leq 2x_c,
\end{align*}
\]

or

\[
\begin{align*}
\Delta x_f_i^{(2)} &> 1.02 v_{i+1}^{(2)}, \quad \Delta x_f_i^{(2)} &< 4x_c, \quad v_i^{(1)} &> v_{i+1}^{(2)}, \quad \Delta x_f_i^{(1)} > 2x_c, \quad v_i^{(2)} &> v_{i+1}^{(3)}, \quad \Delta x_b_i^{(1)} > x_c, \quad \Delta x_b_i^{(3)} > x_c, \quad v_i^{(2)} &> v_{i+1}^{(3)}, \quad \Delta x_f_i^{(3)} &\leq 2x_c,
\end{align*}
\]
or

\[ v_i^{(2)} > 1.02 v_{i+1}^{(2)}, \quad \Delta x f_i^{(2)} < 4 x_c, \quad v_{i+1}^{(1)} > v_i^{(2)}, \quad \Delta x f_i^{(1)} > x_c, \quad \Delta x b_i^{(1)} > x_c, \quad \Delta x f_i^{(3)} > x_c, \quad \Delta x b_i^{(3)} \leq x_c, \]  

(13)

\[ \Delta x f_i^{(2)} < 2 x_c, \quad \Delta x f_i^{(1)} > \Delta x f_i^{(2)}, \quad \Delta x b_i^{(1)} > x_c, \]  

(14)

\[ \Delta x f_i^{(2)} < 2 x_c, \quad \Delta x f_i^{(3)} > \Delta x f_i^{(2)}, \quad \Delta x b_i^{(3)} > x_c. \]  

(15)

(d) Cars on lane 2 change to lane 3.

The rules are similar to those of cars on lane 2 changing to lane 1 (consult (9)–(15)).

(e) Change from lane 2 to lane 1 and lane 3 with equal probability, if

\[ v_i^{(2)} > 1.02 v_{i+1}^{(2)}, \quad \Delta x f_i^{(2)} < 4 x_c, \quad \Delta x f_i^{(1)} > 2 x_c, \quad \Delta x f_i^{(3)} > 2 x_c, \quad \Delta x b_i^{(1)} > x_c, \quad \Delta x f_i^{(3)} > x_c, \quad \Delta x b_i^{(3)} \geq x_c. \]  

(16)

or

\[ v_i^{(2)} > 1.02 v_{i+1}^{(2)}, \quad \Delta x f_i^{(2)} < 4 x_c, \quad \Delta x f_i^{(1)} > 2 x_c, \quad \Delta x f_i^{(3)} > x_c, \quad \Delta x b_i^{(1)} > x_c, \]  

(17)

or

\[ \Delta x f_i^{(2)} < 2 x_c, \quad \Delta x f_i^{(3)} > \Delta x f_i^{(2)}, \quad \Delta x b_i^{(3)} > x_c. \]  

(18)

In (3)–(18), \( v_i^{(1)}, v_i^{(2)}, \) and \( v_i^{(3)} \) denote the velocity of car \( i \) on lane 1, lane 2, and lane 3, respectively.

The explanation of lane-changing rules is as follows. The rule described by (3) means that a driver has a motivation to change lane when the velocity of his car is 1.02 times larger than that of the front car and the distance between them is less than \( 4 x_c \), and his car can change lane successfully if the distance between his car and its adjacent front car in lane 2 (target lane) is larger than \( 2 x_c \) and the distance between his car and its following car in lane 2 is larger than \( x_c \). In comparison with (3), the rule given by (4) the incentive criterion remains, and the security criterion is that the velocity of adjacent front car in lane 2 (target lane) is larger than that of car \( i \), and the distance between car \( i \) and its adjacent front car and the distance between car \( i \) and its following car in lane 2 are both larger than \( x_c \). The rule given by (5) means that a driver has an incentive to change lane
Figure 7: The time-space diagrams of three lanes for the case that a car accident is in lane 2 at density $\rho = 0.1$. (a) Lane 1; (b) lane 2; (c) lane 3.

Figure 8: Plots of traffic currents against density for the case that a car accident is in lane 2.

when the headway is less than $2x_c$, and his car can change lane successfully if the distance between his car and its adjacent front car in lane 2 (target lane) is larger than his headway and the distance between his car and its following car in lane 2 is larger than $x_c$. The rules given by (6)–(8) are similar to those described by (3)–(5). The rules given by (9)–(10) are an extension of two-lane rule described by (3) for three-lane. A car in lane 2 can change to lane 1 because two criteria of lane changing are fulfilled, but it is not able to change to lane 3 because the security criterion $\Delta x f_{ij}^{(3)} > x_c$ or $\Delta x f_{ij}^{(3)} > 2x_c$ is not satisfied. Similarly, the rules given by (11)–(13) are an extension of the rule described by (4), and the rules given by (14)–(15) are an extension of the rule described by (5). The rules given by (16)–(18) mean that when a car $i$ in lane 2 can change to either lane 1 or lane 3, then it changes two lanes with equal probability.

3. Simulation and Results

In this section, we use the above model to explore the effects of car accidents on traffic flow. There are a certain number of cars in each lane at the beginning of simulation. The cars’ motion is split into two parts: one is the forward motion and the other is the lane changing. The forward motion is described by (1). The judging criteria of the lane changing are determined by (3)–(18). In the simulation, model (1) is solved by using the Euler method, where the time interval is $\Delta t = 1/128$, the sensitivity $a = 2.0$, the maximal velocity $v_{\text{max}} = 2.0$, the safety distance $x_c = 4.0$, and the sensitivity coefficient $\lambda = 0.4$.

The simulation is performed by varying the initial density $\rho$ (or initial headway $\Delta x_{\text{ini}}$) for road length $L = 400$. All cars in each lane are numbered in order from 1 to $N$, where $N$ is the total number of cars on the lane. We adopt the open boundary condition for each lane. At each time of interval $\Delta t$, we check whether the position of the leading car is longer than the length of the road $L$, and if so we will remove the leading car and set the car behind as a new leading car. If the distance between the last car and the entrance is greater than the initial headway $\Delta x_{\text{ini}}$, a new car is added as the last car.
Figure 9: The time-space diagrams of three lanes for the case that two car accidents occupy lane 1 and lane 2 at density $\rho = 0.072$. (a) Lane 1; (b) lane 2; (c) lane 3.

Figure 10: The time-space diagrams of three lanes for the case that two car accidents occupy lane 1 and lane 2 at density $\rho = 0.1$. (a) Lane 1; (b) lane 2; (c) lane 3.
3.1. The Car Accident in the Exterior or Interior Lane. We first investigate the case that the accident happens on the exterior lane (i.e., lane 1). The situation that the car accident is in the interior lane (i.e., lane 3) can be discussed similarly. Assume that a car accident is located at \( x = 300 \) in lane 1. Figures 3(a)–3(c), respectively, show the time-space diagrams of lane 1, lane 2, and lane 3 at a low density, \( \rho = 0.072 \). The car accident has a little effect on the traffic flow. This is because lane 2 and lane 3 can ease the effect of the car accident at this initial low density. Not all cars in lane 1 which pass through the car accident change lane back to lane 1 since the traffic conditions of lane 2 and lane 3 are also good. Figures 4(a)–4(c), respectively, show the time-space diagrams of lane 1, lane 2, and lane 3 at a high density, \( \rho = 0.1 \), using the same parameters as those in Figure 3. The jam appears just behind the car accident in lane 1. This is caused by the cars that are hindered on lane 1 and cannot change lane to the right in time. Corresponding to the local jam of lane 1, a region with an increase in density appears in lane 2. The result is due to the lane changing. The car behind the car accident in lane 1 has to change to lane 2. It will prevent the following car in lane 2 from getting ahead. Then the following car in lane 2 slows down and a localized high density region eventually is formed. When the traffic circumstance of lane 2 becomes worse, more cars will change to lane 3. The density of lane 3 increases.

Figure 5 shows the plots of traffic currents against density in lane 1 and lane 2.

Figure 11: Plots of traffic currents against density for the case that two car accidents occupy lane 1 and lane 2.

3.2. The Car Accident in the Middle Lane. For a comparison with the case that the car accident is in lane 1, we next study the case that a car accident occupies the middle lane. The car accident is located at \( x = 300 \) in lane 2. Figures 6(a)–6(c) show the time-space diagrams of lane 1, lane 2, and lane 3 at a low density, \( \rho = 0.072 \), respectively. It can be seen that the effect of the car accident is eased by lane 2 and lane 3 at this initial low density. Figures 7(a)–7(c) show the time-space diagrams of lane 1, lane 2, and lane 3 at a high density, \( \rho = 0.1 \), respectively. A localized jam appears behind the car accident in lane 2. Comparing with the results of the car accident located at lane 1, we can find that the length of jam behind the car accident is shorter when the car accident is located at lane 2. This is because we adopt the symmetric lane-changing rules and the cars behind the car accident in lane 2 can directly change to lane 1 or lane 3.

3.3. The Car Accidents in Two Lanes. In real traffic, many traffic accidents happen when cars are changing lanes. Under this situation, the car accidents usually occupy two lanes at the same time. Thus, we finally explore the cases that there are two car accidents.

The first situation is that two car accidents are, respectively, located at \( x = 288 \) in lane 1 and \( x = 300 \) in lane 2. Figures 9(a)–9(c) illustrate the time-space diagrams of lane 1, lane 2, and lane 3 at a low density, \( \rho = 0.072 \), respectively. From Figure 9, we can see that the jam is formed behind the car accident in lane 1, and a region with a slight increase in density appears in lane 3 because all cars behind the accident car in lane 1 and lane 2 need to change to lane 3 and go through the accident car. Figures 10(a)–10(c) show the time-space diagrams of lane 1, lane 2, and lane 3 at a high density, \( \rho = 0.1 \), respectively. The high density region appears and extends behind the car accident in lane 1 and lane 2. Correspondingly, a region with a very large increase in density appears in lane 3. At this initial high density, all cars behind the car accident in lane 1 and lane 2 need to change to lane 3 while the capacity of lane 3 is limited. Thus more cars in lane 1 and lane 2 are not able to change lane with time and the extended jams are formed behind the car accidents. On the other hand, the cars changing from lane 1 to lane 2 gradually lead to the extended high density region in lane 3. In addition, from Figures 9 and 10 we can see that the density in the downstream of the car accident in lane 2 and lane 3 is lower than that in lane 1. The reason is as follows. After the cars go through the car accident, many of them attempt to change from lane 3 to lane 1 where the driving condition is better. Thus the accident not only has
Figure 12: The time-space diagrams of three lanes for the case that two car accidents occupy lane 1 and lane 3 at density $\rho = 0.072$. (a) Lane 1; (b) lane 2; (c) lane 3.

Figure 13: The time-space diagrams of three lanes for the case that two car accidents occupy lane 1 and lane 3 at density $\rho = 0.1$. (a) Lane 1; (b) lane 2; (c) lane 3.
a significant effect on the upstream of the car accident, but also considerably affects the downstream of the car accident.

Figure 11 shows the currents against the density when two car accidents occupy lane 1 and lane 2. When the density is low, vehicles move freely. With the increase of density, the current of lane 3 increases and then keeps a high value, while the current of lane 1 decreases and then keeps a low value, and the current of lane 2 almost saturates and keeps a constant value. The current of lane 2 is higher than of lane 1. When the density is higher, the currents decrease with increasing density.

The second situation is that two car accidents are located at \( x = 288 \) in lane 1 and \( x = 300 \) in lane 3, respectively. Figures 12(a)−12(c) illustrate the time-space diagrams of lane 1, lane 2, and lane 3 at a low density, \( \rho = 0.072 \), respectively. It can be seen that the localized jam is formed behind the car accident in lane 1 and lane 3, and a region with an increase in density arises in lane 2. Figures 13(a)−13(c) show the time-space diagrams of lane 1, lane 2, and lane 3 at a high density, \( \rho = 0.1 \), respectively. The high density region appears and extends behind the car accidents in lane 1 and lane 3, because there are few cars changing from lane 1 and lane 3 to lane 2 at this initial high density. Correspondingly, a region with a very large increase in density appears in lane 2. Comparing Figure 10 with Figure 13, we can see that the effect of the car accidents located at lane 1 and lane 3 on the traffic flow is greater than the case of the car accidents located at lane 1 and lane 2 because the cars behind the accident cars in lane 1 and lane 3 all try to change lane to lane 2, but this causes more serious traffic confusion.

Figure 14 shows the currents against the density when two car accidents occupy lane 1 and lane 3. The current increases with density at a low density. With the increase of density, the currents of lane 1 and lane 3 decrease and then keep at a very low value because of the existence of the car accident, while the current of lane 2 almost saturates and keeps a high value. When the density is higher, the currents of each lane decrease. The current on lane 3 is consistent with that on lane 1 since the symmetric lane-changing rules are adopted.

\[ \text{Figure 14: Plots of traffic currents against density for the case that two car accidents occupy lane 1 and lane 3.} \]

Toledo et al. [35], Varas et al. [36], Pastén et al. [37], and Toledo et al. [38].

**Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

**Acknowledgments**

This paper is supported by the National Natural Science Foundation of China (Grant no. 11102165), the Natural Science Basis Research Plan in Shaanxi Province of China (Grant nos. 2012JM1001 and 2013JQ7014), and NPU Foundation for Fundamental Research (Grant no. NPU-FFR-JC201254). The authors would like to thank two anonymous referees for their valuable suggestions on improving the paper.

**References**
