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Research Article

An Uncertain Programming for the Integrated Planning of Production and Transportation

Deyi Mou and Xiaoding Chang

Institute of Mathematics for Applications, Civil Aviation University of China, Tianjin 300300, China

Correspondence should be addressed to Deyi Mou; deyimou@hotmail.com

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The goal of this paper is to tackle joint decisions in assigning production and organizing transportation for single product in a production-transportation network system with multiple manufacturers and multiple demands. In order to meet practical situation, assume that the variant costs and the amounts of the consumption of raw materials that every manufacturer produces per unit product are all uncertain variables in manufacturing processes; meanwhile, the demands that each destination needs are random variables in the transportation problem. Then, a joint optimization model of production and transportation is developed, in which the uncertain chance constraint and the stochastic chance constraint are applied in the manufacturing processes and the transporting processes, respectively, and transformed into a deterministic form by taking expected value on objective function and confidence level on the constraint functions. Finally, a practical example points out the effectiveness of our model.

1. Introduction

Production problem and transportation problem, which were solved separately in the past, are two important problems for the manufacturing firms. But in order to pursue the maximization of benefits, the firms have to consider the whole process of production and transportation which contains not only how to assign the production but also how to design the transportation plan. Hence, many researches have been made to coordinate production and transportation network, and more effort is now being made to do this work. Glover et al. [1] presented the production, distribution, and inventory planning system (PDI) in the form of supply chain management (SCM). In the network, by introducing the supply policy node and supply policy arcs to each of the supply sites and demand policy arcs from each demand site to a demand policy node, the Agrico PDI network model was established to minimize the costs. After that, the integrated models in production and transportation ([2–5]) and production and inventory [6–8], as well as transportation and inventory were investigated one after another. All of these models are directly or indirectly linked with inventory and inventory cost is a significant portion of the total cost. Additionally, these models were also formulated into different versions of single or multiple periods, single or multiple products [9], and so on.

On the other hand, in almost all the works mentioned above, the conventional mathematical programming methods are used to solve problems in SCM and the goals and relevant inputs are generally assumed to be deterministic/crisp. However, in real-world problems in supply chains, environmental coefficients and/or parameters, involving available supply, resources, capacities, demand, and related operating costs, are often uncertain owing to information being incomplete and unavailable over the planning horizon. Therefore, conventional deterministic mathematical programming methods cannot solve all programming problems in uncertain environments. Holmberg and Tuy [10] developed a stochastic programming to deal with the integrated decisions of production and transportation modeled by the convex nonlinear production costs and stochastic demand. They reported computational tests that indicated that quite large problems can be solved efficiently. Liang [11] presented a possibilistic linear programming (PLP) method for solving the integrated manufacturing/distribution planning decision problems with multiple imprecise goals in supply chains

under an uncertain environment. An industrial case was used to demonstrate the feasibility of applying the proposed method to a real problem.

Because of the lack of the information about the raw materials, changes in sales, weather and road conditions, and so forth, there are unpredicted things which will make the amounts of the supply of the raw materials, the costs of production and transportation, and the demands be uncertain values. There are some limitations when the traditional stochastic models and possibilistic methods above deal with such problems at these situations. In such cases, we have to invite some experts to evaluate their degree of belief that each event will occur. However, humans tend to overweigh unlikely events [12]; thus, the degree of belief may have a much larger range than the real frequency. In this situation, if we insist on dealing with the degree of belief using the probability theory, some counterintuitive results will be obtained. In order to deal with the uncertain situations, Liu [13] presented the uncertainty theory. After that, Liu [14, 15] made some researches about uncertainty theory and proposed the uncertain programming and then applied the uncertainty theory to model human uncertainty [16] and refined the uncertainty theory in 2013 [17]. More and more researchers have contributed to this area. Liu and Ha [18] researched the expected value of function of uncertain variables. Rong [19] proposed two new uncertain programming models of inventory with uncertain cost. So the uncertain programming began to be regarded as a new technique of mathematic programming and be applied widely. Based on the uncertain programming, the transportation problem [20, 21], the facility location problem [22], and the vehicle routing problem [23] were all discussed in the simplest basic form. Until now, no studies are found on the integrated decisions by exploiting uncertainty theory about production, transportation, or inventory. In other ways of application of the uncertainty theory, uncertain hypothesis testing [24] and uncertain optimal control [25] were also studied, respectively.

Motivated by the above-described applications in the production problem and transportation problem, in this paper we study an integrated optimization model of production and distribution operations which applies the uncertain programming from uncertainty theory into the joint model. Therefore, optimizing the trade-off between the production cost and the transportation cost is the goal of our systems. The problem is to find an optimal operation of production and transportation at the level of detailed scheduling such that the raw material gets used adequately and the demands of transportation are satisfied completely. As we know, when the production and transportation happen in reality, the different production plans need to arrange different transportation solutions; inversely, the different transportation demands also need the production plans to be adjusted accordingly. The joint decision problem addressed here attempts to make the total cost of production and transportation be the least.

The rest of this paper is organized as follows. In Section 2, some preliminaries from the uncertainty theory are provided for use in the next few sections, and, in Section 3, the problem formulation and some basic assumptions about this paper are given. Then, Section 4 will develop the uncertain production model and the stochastic transportation model, respectively. Subsequently, the joint optimization model of production and transportation for a single product is obtained. In Section 5, one numerical example with multiple manufacturers and multiple destinations of demand is presented. At the end of this paper, the conclusions are proposed in Section 6.

2. Preliminaries

In this section, we will introduce some basic knowledge on uncertainty theory which will be used in the next few sections.

Definition 1 (see [17]). An uncertain variable ξ is a measurable function from an uncertainty space $(\Gamma, \mathscr{L}, \mathscr{M})$ to the set of real numbers; that is, for any Borel set B of real numbers, the set

$$\{\xi \in B\} = \{\gamma \in \Gamma \xi(\gamma) \in B\} \tag{1}$$

is an event.

For a sequence of uncertain variables $\xi_1, \xi_2, \ldots, \xi_n$ and a measurable function f, Liu [17] proved that

$$\xi = f(\xi_1, \xi_2, \ldots, \xi_n), \tag{2}$$

defined as $\xi(\gamma) = f(\xi_1(\gamma), \xi_2(\gamma), \ldots, \xi_n(\gamma))$, $\forall \gamma \in \Gamma$, is also an uncertain variable.

In order to research an uncertain variable more deeply, a concept of uncertainty distribution is given as follows.

Definition 2 (see [17]). The uncertainty distribution Φ of an uncertain variable ξ is defined by

$$\Phi(x) = \mathscr{M}\{\xi \le x\}, \tag{3}$$

for any real number x.

To calculate the uncertain measure from an uncertainty distribution, Liu presented the measure inversion theorem.

Theorem 3 (see [17]). *Let ξ be an uncertain variable with continuous uncertainty distribution Φ. Then, for any real number x, one has*

$$\mathscr{M}\{\xi \le x\} = \Phi(x), \qquad \mathscr{M}\{\xi \ge x\} = 1 - \Phi(x). \tag{4}$$

Theorem 4 (see [17]). *Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. If $f(x_1, x_2, \ldots, x_n)$ is strictly increasing with respect to $x_1, x_2, \ldots, x_m$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \ldots, x_n$, then*

$$\xi = f(\xi_1, \xi_2, \ldots, \xi_n) \tag{5}$$

is an uncertain variable with inverse uncertainty distribution

$$\begin{aligned}\Phi^{-1}(\alpha) = f\Big(&\Phi_1^{-1}(\alpha), \ldots, \Phi_m^{-1}(\alpha), \\ &\Phi_{m+1}^{-1}(1-\alpha), \ldots, \Phi_n^{-1}(1-\alpha)\Big).\end{aligned} \tag{6}$$

Definition 5 (see [17]). Let ξ be an uncertain variable. Then the expected value of ξ is defined by

$$E[\xi] = \int_0^{\infty} \mathcal{M}\{\xi \geq r\}\, dr - \int_{-\infty}^{0} \mathcal{M}\{\xi \leq r\}\, dr, \tag{7}$$

provided that at least one of the two integrals is finite.

Theorem 6 (see [17]). *Let ξ be an uncertain variable with regular uncertainty distribution Φ; if the expected value exists, then*

$$E[\xi] = \int_0^1 \Phi^{-1}(\alpha)\, d\alpha. \tag{8}$$

For $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$, we have

$$E[\xi] = \int_0^1 f\left(\Phi_1^{-1}(\alpha)\right), \ldots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \ldots, \Phi_n^{-1}(1-\alpha)\, d\alpha. \tag{9}$$

At the same time, Liu proved the linearity of expected value operator, that is,

$$E[a\xi + b\eta] = aE[\xi] + bE[\eta], \tag{10}$$

for any two independent uncertain variables ξ, η with finite expected values and any two real numbers a, b.

3. Problem Formulation

For a manufacturing enterprise, there are plants each producing multiple parts and multiple assemblies that serve different assembly plants in a year; meanwhile, each assembly plant demands multiple parts from many different manufacturers. Hence, the two processes of production and transportation are integrated into a combined production-transportation network consisting of multiple manufacturers and multiple demands. The basic objective for an effective supply-demand system is the integration of production and transportation functions, which needs the simultaneous consideration of cost of consumption in production and cost-savings opportunity in transportation. In production, in order to minimize production cost which contains the setup cost and processing cost of each manufacturer, they must finish the task that they are asked according to their own producing capacities. After that, the transportation will come as follows. The produced final goods are packed in a cargo, from which unit delivery cost is incurred, and are then delivered to different destinations via transportation fleets such as trucks, railroads, or aircrafts. Therefore, a reasonable transportation program based on demands must be drawn up so as to reduce unnecessary deliveries and lower delivery cost.

In this study, a single-product problem is considered for uncertain demands under the integrated production and transportation operations. The joint decision problem addressed here is, based on the delivery amounts from each manufacturer to individual destinations to meet their respective total demands at a minimum total cost in the system, to determine production quantity to be assigned to each manufacturer. Then, an uncertain and stochastic programming model about production and transportation is developed under the assumptions that production costs and the amounts of the consumption of raw materials are uncertain variables and the delivery demands are random variables.

4. Joint Decision Model Development

4.1. Uncertain Production Model. Assume that there are sets of m manufacturers and n destinations, where each of the manufacturers and destinations is indicated by the subscripts i and j, respectively. The manufacturers produce a certain product P for the destinations, where the P is processed by kinds of l ($l = 1, 2, \ldots, k$) raw materials $h_1, h_2, \ldots, h_k$. In order to describe the production model, the following notations are applied to serve a single product:

c_{i0}: the setup cost at manufacturer i;

η_i: uncertain variable, the unit variant cost for producing the product P at manufacturer i;

ξ_{ih_l}: uncertain variable, the amounts of the consumption of raw materials h_l ($l = 1, 2, \ldots, k$) for producing per unit product P at manufacturer i;

φ_i: the uncertain distribution of the uncertain variable η_i;

Φ_{ih_l}: the uncertain distribution of the uncertain variable ξ_{ih_l};

S_{h_l}: the amounts of the supply of raw materials h_l;

y_i: decision variable, the yield for producing the product P at manufacturer i;

V_i: production capacity for producing the product P at manufacturer i;

α_{h_l}: the predetermined confidence level.

Based on the above assumptions, not only do we minimize the production cost, but also we should make best use of the raw materials in the whole producing process. So we have proposed the uncertain production-producing model as follows:

$$\begin{aligned} &\text{Min} \quad Z_1 = \sum_{i=1}^{m} c_{i0} + E\left[\sum_{i=1}^{m} \eta_i y_i\right] \\ &\text{subject to} \\ &\mathcal{M}\left\{\sum_{i=1}^{m} \xi_{ih_l} y_i \geq S_{h_l}\right\} \geq \alpha_{h_l}, \quad (l = 1, 2, \ldots, k) \\ &0 \leq y_i \leq V_i, \quad (i = 1, 2, \ldots, m). \end{aligned} \tag{11}$$

In this model, the objective is to minimize the expected production cost, and the major constraint is an uncertain chance constraint which will make raw materials be fully used. In order to solve this model, we will have the following corollary according to the uncertainty theory that was introduced in the preliminaries.

Corollary 7. *Assume the constraint function $g(x, \xi_1, \xi_2, \ldots, \xi_n)$ is strictly increasing with respect to $\xi_1, \xi_2, \ldots, \xi_k$ and strictly decreasing with respect to $\xi_{k+1}, \xi_{k+2}, \ldots, \xi_n$. If $\xi_1, \xi_2, \ldots, \xi_n$ are independent uncertain variables with uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively, then the chance constrain*

$$\mathcal{M}\{g(x, \xi_1, \xi_2, \ldots, \xi_n) \geq 0\} \geq \alpha \tag{12}$$

holds if and only if

$$g\left(x, \Phi_1^{-1}(\alpha), \ldots, \Phi_k^{-1}(\alpha), \Phi_{k+1}^{-1}(1-\alpha), \ldots, \Phi_n^{-1}(1-\alpha)\right) \geq 0. \tag{13}$$

Proof. It follows from Theorem 4 that the inverse uncertainty distribution of $g(x, \xi_1, \xi_2, \ldots, \xi_n)$ is

$$\Psi^{-1}(x, \alpha) = g\left(x, \Phi_1^{-1}(\alpha), \ldots, \Phi_k^{-1}(\alpha), \Phi_{k+1}^{-1}(1-\alpha), \ldots, \Phi_n^{-1}(1-\alpha)\right). \tag{14}$$

Thus $\mathcal{M}\{g(x, \xi_1, \xi_2, \ldots, \xi_n) \geq 0\} \geq \alpha$ holds if and only if $\Psi^{-1}(x, \alpha) \geq 0$. The theorem is thus verified. □

Theorem 8. *The uncertain chance constraint*

$$\mathcal{M}\left\{\sum_{i=1}^{m} \xi_{ih_l} y_i \geq S_{h_l}\right\} \geq \alpha_{h_l}, \tag{15}$$

in model (11), *is equivalent to*

$$\sum_{i=1}^{m} \Phi_{ih_l}^{-1}\left(\alpha_{h_l}\right) y_i \geq S_{h_l}. \tag{16}$$

Proof. It is obvious that the function

$$g(y, \xi) = \sum_{i=1}^{m} \xi_{ih_l} y_i \tag{17}$$

is strictly increasing with respect to $\xi_{1h_l}, \xi_{2h_l}, \ldots, \xi_{mh_l}$. And ξ_{ih_l} are independent uncertain variables with uncertainty distributions Φ_{ih_l}, respectively. Therefore

$$\mathcal{M}\left\{\sum_{i=1}^{m} \xi_{ih_l} y_i \geq S_{h_l}\right\} = \mathcal{M}\left\{g(y, \xi) - S_{h_l} \geq 0\right\}. \tag{18}$$

It follows from Corollary 7 that we have immediately that

$$\sum_{i=1}^{m} \Phi_{ih_l}^{-1}\left(\alpha_{h_l}\right) y_i - S_{h_l} \geq 0. \tag{19}$$

The theorem is proved. □

Then, the uncertain production-producing model is expressed in the following deterministic model with the goal of minimizing the production cost:

$$\begin{aligned} &\text{Min} \quad Z_1 = \sum_{i=1}^{m} c_{i0} + \sum_{i=1}^{m} y_i \int_0^1 \varphi_i^{-1}(\alpha)\, d\alpha \\ &\text{subject to} \\ &\sum_{i=1}^{m} \Phi_{ih_l}^{-1}\left(\alpha_{h_l}\right) y_i \geq S_{h_l}, \quad (l = 1, 2, \ldots, k) \\ &0 \leq y_i \leq V_i, \quad (i = 1, 2, \ldots, m). \end{aligned} \tag{20}$$

4.2. Transportation Model with Stochastic Demands. Assume that the demands at the n destinations are stochastic and are supplied by the above-described sets of m manufacturers. Additionally, there are some parameters which will be used in the transportation problem as follows:

c_{ij}: the transportation cost of delivering unit value of product from manufacturer i to destination j;

x_{ij}: decision variable, the number of units shipped from manufacturer i to destination j;

b_j: independent random variable, the demand at destination j which follows the normal distribution $N(\mu_j, \sigma_j^2)$;

β_j: the predetermined confidence level ($j = 1, 2, \ldots, n$);

$\Phi(\theta)$: the standard normal distribution function.

Thus, on the basis of the typical transportation problem, the transportation model with stochastic demands is formulated into the following form:

$$\begin{aligned} &\text{Min} \quad Z_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \\ &\text{subject to} \\ &\Pr\left\{\sum_{i=1}^{m} x_{ij} \geq b_j\right\} \geq \beta_j, \quad (j = 1, 2, \ldots, n) \\ &\sum_{j=1}^{n} x_{ij} \leq V_i, \quad (i = 1, 2, \ldots, m) \\ &x_{ij} \geq 0, \quad (i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n). \end{aligned} \tag{21}$$

Based on the possibility theory, we also get the theorem below.

Theorem 9. *Assume that b_j ($j = 1, 2, \ldots, n$) are random variables which follow the normal distribution of $N(\mu_j, \sigma_j^2)$. Then, the chance constraint*

$$\Pr\left\{\sum_{i=1}^{m} x_{ij} \geq b_j\right\} \geq \beta_j \tag{22}$$

in model (21) *is equivalent to*

$$\sum_{i=1}^{m} x_{ij} \geq \mu_j + \sigma_j \Phi^{-1}\left(\beta_j\right). \tag{23}$$

Proof. Since b_j are random variables, $y(x) = \sum_{i=1}^{m} x_{ij} - b_j$ are also random variables. Then we have

$$E\left(y\left(x\right)\right) = \sum_{i=1}^{m} x_{ij} - \mu_j, \qquad V\left(y\left(x\right)\right) = \sigma_j^2, \tag{24}$$

where $E(\cdot)$ and $V(\cdot)$ denote expected value and variance, respectively.

On the other hand, since the inequalities $\sum_{i=1}^{m} x_{ij} \geq b_j$ are equivalent to

$$-\frac{\sum_{i=1}^{m} x_{ij} - b_j - \left[\sum_{i=1}^{m} x_{ij} - E\left(b_j\right)\right]}{\sigma\left(b_j\right)} \leq \frac{\sum_{i=1}^{m} x_{ij} - E\left(b_j\right)}{\sigma\left(b_j\right)},$$

$$\eta = -\frac{\sum_{i=1}^{m} x_{ij} - b_j - \left[\sum_{i=1}^{m} x_{ij} - E\left(b_j\right)\right]}{\sigma\left(b_j\right)}, \tag{25}$$

are random variables which follow standard normal distribution of $N(0,1)$ with possibility distribution function $\Phi(\eta) = (1/\sqrt{2\pi}) \int_{-\infty}^{\eta} e^{-t^2/2} dt$. Thus, we have

$$\Pr\left\{\eta \leq \frac{\sum_{i=1}^{m} x_{ij} - E\left(b_j\right)}{\sigma\left(b_j\right)}\right\} \geq \beta_j. \tag{26}$$

According to the definition of possibility distribution function, we also have

$$\Phi\left\{\frac{\sum_{i=1}^{m} x_{ij} - E\left(b_j\right)}{\sigma\left(b_j\right)}\right\} \geq \beta_j. \tag{27}$$

So, we have

$$\sum_{i=1}^{m} x_{ij} \geq \mu_j + \sigma_j \Phi^{-1}\left(\beta_j\right). \tag{28}$$

The theorem is proved. □

According to Theorem 9, the improved transportation model with stochastic demands will be transferred into the following model:

$$\begin{aligned}
&\text{Min} \quad Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} \\
&\text{subject to} \\
&\sum_{i=1}^{m} x_{ij} \geq \mu_j + \sigma_j \Phi^{-1}\left(\beta_j\right), \quad (j = 1, 2, \ldots, n) \\
&\sum_{j=1}^{n} x_{ij} \leq V_i, \quad (i = 1, 2, \ldots, m) \\
&x_{ij} \geq 0, \quad (i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n).
\end{aligned} \tag{29}$$

4.3. The Joint Optimization Model of Production and Transportation. In actual life, the production and transportation are usually combining closely. So in order to reduce the total cost of the production and transportation, we have to consider the two great problems comprehensively rather than one by one. Firstly, we must organize the production effectively and arrange the transportation reasonably for reaching the aim of saving the cost. Thus, we could get the number of units that every manufacturer produces to be transported completely. That is,

$$y_i = \sum_{j=1}^{n} x_{ij}. \tag{30}$$

Upon this, we can effectively adjust the production of the product according to the changes of the demands and then reach the goal of reducing the cost of each of the counterparts and optimizing the production-transportation network.

Then, combining models (11) and (21), we can get the joint optimization model of production and transportation under uncertain environment:

$$\begin{aligned}
&\text{Min} \quad Z = Z_1 + Z_2 = \sum_{i=1}^{m} c_{i0} + E\left[\sum_{i=1}^{m} \eta_i y_i\right] + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} \\
&\text{subject to} \\
&\mathcal{M}\left\{\sum_{i=1}^{m} \xi_{ih_l} y_i \geq S_{h_l}\right\} \geq \alpha_{h_l}, \quad (l = 1, 2, \ldots, k) \\
&\Pr\left\{\sum_{i=1}^{m} x_{ij} \geq b_j\right\} \geq \beta_j, \quad (j = 1, 2, \ldots, n) \\
&0 \leq y_i = \sum_{j=1}^{n} x_{ij} \leq V_i, \quad (i = 1, 2, \ldots, m) \\
&x_{ij} \geq 0, \quad (i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n).
\end{aligned} \tag{31}$$

Combining models (20) and (29), we can get the equivalent model of model (31) immediately:

$$\begin{aligned}
&\text{Min} \quad Z = \sum_{i=1}^{m} c_{i0} + \sum_{i=1}^{m} y_i \int_0^1 \varphi_i^{-1}(\alpha)\, d\alpha + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} \\
&\text{subject to} \\
&\sum_{i=1}^{m} \Phi_{ih_l}^{-1}\left(\alpha_{h_l}\right) y_i \geq S_{h_l}, \quad (l = 1, 2, \ldots, k) \\
&\sum_{i=1}^{m} x_{ij} \geq \mu_j + \sigma_j \Phi^{-1}\left(\beta_j\right), \quad (j = 1, 2, \ldots, n) \\
&0 \leq y_i = \sum_{j=1}^{n} x_{ij} \leq V_i, \quad (i = 1, 2, \ldots, m) \\
&x_{ij} \geq 0, \quad (i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n).
\end{aligned} \tag{32}$$

This model is a typical linear programming which can be solved easily by the simplex method.

5. Numerical Experiment

To evaluate and test the performance of the above uncertain and stochastic model, this section presents an example with three manufacturers and four destinations to illustrate the application of the model.

Assume that the production of P, which is made up of three raw materials h_1,h_2,h_3, is assigned to be produced by three manufacturers A_1,A_2,A_3, and then the production of P will be delivered to four destinations B_1,B_2,B_3,B_4. According to the expertise experience, the amounts of the consumption of raw materials h_1,h_2,h_3 for producing per unit product P follow linear uncertain distribution $L(a_{ih_i}, b_{jh_j})$, $i = 1, 2, 3$, $j = 1, 2, 3$, respectively. The unit variant cost for producing the product P and the demand at destination j follow the zigzag uncertain distribution $Z(a_i, b_i, c_i)$, $i = 1, 2, 3$, and the normal random distribution $N(\mu_j, \sigma_j^2)$, $j = 1, 2, 3, 4$, respectively. Tables 1, 2, and 3 give the value of the parameters and others.

It follows from model (31) that we have the following model about this problem:

$$\begin{aligned}
&\text{Min} \quad Z = \sum_{i=1}^{3} c_{i0} + E\left[\sum_{i=1}^{3} \eta_i y_i\right] + \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij} x_{ij} \\
&\text{subject to} \\
&\mathcal{M}\left\{\sum_{i=1}^{3} \xi_{ih_l} y_i \geq S_{h_l}\right\} \geq \alpha_{h_l}, \quad (l = 1, 2, 3) \\
&\Pr\left\{\sum_{i=1}^{3} x_{ij} \geq b_j\right\} \geq \beta_j, \quad (j = 1, 2, 3, 4) \\
&0 \leq y_i = \sum_{j=1}^{4} x_{ij} \leq V_i, \quad (i = 1, 2, 3) \\
&x_{ij} \geq 0, \quad (i = 1, 2, 3,\ j = 1, 2, 3, 4).
\end{aligned} \tag{33}$$

Note that the confidence levels are $\alpha_{h_l} = 0.9$, $l = 1, 2, 3$, $\beta_j = 0.95$, and $j = 1, 2, 3, 4$. Since the inverse uncertainty distribution of linear uncertain variable $L(a, b)$ is

$$\Phi^{-1}(\alpha) = (1 - \alpha) a + \alpha b, \tag{34}$$

the inverse uncertainty distribution of zigzag uncertain variable $Z(a, b, c)$ is

$$\Phi^{-1}(\alpha) = (2 - 2\alpha) b + (2\alpha - 1) c, \quad \text{if } \alpha \geq 0.5. \tag{35}$$

Table 1: Parameters of linear distribution of the consumption and the supply of the raw materials.

	h_1	h_2	h_3
A_1	$L(2, 4)$	$L(3, 5)$	$L(4, 6)$
A_2	$L(0, 2)$	$L(1, 3)$	$L(2, 4)$
A_3	$L(1, 3)$	$L(2, 4)$	$L(3, 5)$
S_{h_l}	60	80	100

Table 2: Parameters of the setup cost, the capacity, and the unit variant cost at the manufacturers.

	c_{i0}	V_i	η_i
A_1	35	15	$Z(90, 100, 110)$
A_2	30	25	$Z(50, 60, 70)$
A_3	35	10	$Z(70, 80, 90)$

Table 3: Parameters of the unit delivery cost and the normal distribution of demands at the destinations.

	B_1	B_2	B_3	B_4
A_1	7	5	12	9
A_2	6	3	6	5
A_3	9	5	8	5
(μ_j, σ_j^2)	$(5, 2^2)$	$(10, 2^2)$	$(8, 2^2)$	$(6, 2^2)$

For the standard normal distribution, we have that $\Phi^{-1}(0.95) = 1.645$. Thus, model (33) has the form as follows:

$$\begin{aligned}
&\text{Min} \quad Z = \sum_{i=1}^{3} c_{i0} + \sum_{i=1}^{3} y_i \int_0^1 \varphi_i^{-1}(\alpha)\, d\alpha + \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij} x_{ij} \\
&\text{subject to} \\
&\sum_{i=1}^{3} \left(0.1 a_{ih_l} + 0.9 b_{ih_l}\right) y_i \geq S_{h_l}, \quad (l = 1, 2, 3) \\
&\sum_{i=1}^{3} x_{ij} \geq \mu_j + 1.645\sigma_j, \quad (j = 1, 2, 3) \\
&0 \leq y_i = \sum_{j=1}^{4} x_{ij} \leq V_i, \quad (i = 1, 2, 3) \\
&x_{ij} \geq 0, \quad (i = 1, 2, 3,\ j = 1, 2, 3, 4).
\end{aligned} \tag{36}$$

By means of the simplex method, the optimal production plan, transportation scheme, and the demands are listed in Table 4.

The production cost is 302.0000, the transportation cost is 21.2600, and the total cost is 333.2600.

6. Conclusions

This paper considers the whole process of production and transportation and establishes the joint optimal model for a single product with multiple manufacturers and multiple destinations. By introducing uncertainty theory, we presented

Table 4: The results of the transportation scheme, the yields of the manufacturers, and the demands of the destinations.

x_{ij}	B_1	B_2	B_3	B_4	y_i
A_1	7.2000	0.0000	0.0000	0.0000	7.2000
A_2	1.1000	12.6000	11.3000	0.0000	25.0000
A_3	0.0000	0.7000	0.0000	9.3000	10.0000
Demands	8.3000	13.3000	11.3000	9.3000	

uncertain production model, and in the transportation process, we assumed that the demands are random variables and proposed the stochastic transportation model. Based on these above, the uncertain and stochastic model was developed by taking expected value on objective function and confidence level on the constraint functions. In the end of this paper, a numerical example was given and showed the effectiveness of the method by which we establish the model. Then, the production plan and transportation scheme were obtained by the simplex method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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