Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2014, Article ID 421510, 6 pages
http://dx.doi.org/10.1155/2014/421510

Research Article

The Application of Time-Delay-Dependent H_∞ Control in the Transmission Effect of Monetary Policy on Real Estate Market

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Received 6 June 2014; Accepted 7 July 2014; Published 22 July 2014

Academic Editor: Ligang Wu

This paper uses a time-delay-dependent H_∞ control model to analyze the transmission effect of monetary policy on the real estate market. We establish a theoretical framework between monetary policy and real estate market based on the three channels, that is, interest rate channel, monetary supply channel, and credit channel. Then we construct a basic model using H_∞ control method. By analyzing the effect of the time-delay characteristics of monetary policy on the real estate market, we introduce time variables and propose a time-delay-dependent H_∞ control model. We test the robustness of the model using the Chinese data of the monetary policy and the real estate market and prove that this model can be well employed in the reality.

1. Introduction

The real estate market is increasingly becoming a mature monetary policy transmission channel. Many empirical studies find that the potential influence of monetary policies on assets prices, especially on real estate market. Ben and Bernanke find that real estate investments react greatly to the impact of short-term interest rate, while they react mildly to the impact of long-term interest rate [1]. Aoki et al. introduce BGG model [2] to study how the real estate market reacts in the transmission of monetary policy. The result is that, in some circumstances, monetary policy is influenced by financial accelerator effect that its impact on real estate price, real estate investment, and consumption is magnified.

The empirical studies in this field can be sorted into two types. The first type of studies employs econometrics models and the other type of studies uses DSGE model. For the first type of studies, many scholars would like to use VAR models. For example, MatteoIa and Raoul construct multiple VAR models and analyze the different reactions of real estate markets to the impact of the monetary policies of Finland, UK, Norway, and Germany [3]. Liu and He, Gao and Wang use Chinese data and make empirical studies using VAR model or SVAR model in the related field [4, 5]. For the other type of studies using DSGE model, the first study can be traced to Kyland and Prescott, who use DSGE to analyze economic cycles [6]. After the Lucas critique, DSGE model has been developed from RBC theory framework into the theory framework of new Keynes. For example, Liang introduces the DSGE model into the real estate market and finds that the monetary policy has a strong effect on the real estate market [7].

However, the research methods in the transmission effect of are supposed to develop. Take DSGE model as an example, Cogley and Nason, Su propose that DSGE model is not well implemented in China because the assumptions are not designed and related to Chinese economic situations [8, 9]. Meanwhile, the above models are most based on discrete system, which is lack of the continuous data.

Economic system can be viewed as a complex system which is full of uncertain factors. H_∞ control method can minimize the effect of disturbance on the state, which indicates that H_∞ control can be used to solve economic questions. H-infinite control is widely used in the field of engineering. Saifia et al., Krokavec and Filasová, Wu et al., and Wu and Zheng have carried some detailed research [10–13]. Meanwhile there are several articles using H-infinite control in economy, such as Li et al. [14]. And others like that of

Huang and Zhong employ H_∞ control in Nash equilibrium and find that this method has a good explanation ability in economic problems [15]. H_∞ control can manage risks and thus minimize the disturbing effect in an uncertain environment. Also, through the state feedback controller, the monetary policy can keep the house price stable as optimal as possible.

However, there are few articles introducing robust control into monetary policy. Always, the articles on monetary policy and real estate market focus on positive transmission mechanism, but few focus on reverse. Therefore, this paper introduces H_∞ control method and constructs a time-delay-dependent model to adjust the monetary policy on the real estate market, which is a new attempt. The advantage of this model is that it can deal with the continuous system in economic data; that is, the previous studies usually simplify economic problem as a discrete system; it is a new attempt to study the feedback of real state marker to design optimal monetary policy. Also being different from the usual measuring method and empirical method, this paper mainly adopts the method of simulation. Section 2 proposes the theoretical framework. Section 3 delineates the basic H_∞ control model of monetary policy on real estate market and Section 4 explains the time-delay-dependent feedback model. Section 5 puts forward a design example. Section 6 is conclusion.

2. Theoretical Framework

Monetary policy is supposed to manage real estate market, so the transmission effect of monetary policy is a crucial problem in economy. There have been some theories in this field. Many key issues of the theories agree that the following channels are well accepted ways for transmission of monetary policy, such as interest rate channel, credit channel, and monetary supply channel.

As for interest rate channel, the most famous theory is Keynes's interest rate channel theory. When the monetary policy maker sets a new interest rate, the demand, capital supply, and capital cost will change, and thus the supply and demand of the real estate will change. Take the loose monetary policy as an example, interest rates decrease and thus the cost of capital will decrease. So the cost of real estate will decrease and thus the real estate market will boom.

In the aspect of credit channel, under loose monetary policy, the credit amount grows with the increase of money supply, which leads to high currency liquidity in society. During boom years, land agents will buy lands at a high price that the house price in future will grow due to the increasing cost, which causes the rise of house price anticipation. Besides, when credit amount grows with the increase of money supply, it is easier for residents to get housing loans. This will expand house demands and raise house prices and, at the same time, encourage people's desire to earn more money. In a word, under loose monetary policy, real estate investment and house prices promote each other and grow together.

About the monetary supply channel, Friedman and Schwartz propose that monetary supply will change the economic cycles and thus change the real estate market [16]. With the increase of the monetary supply, the gross revenue will increase; that is, this study proves the existence of monetary supply channel. Zhao et al. and Sun use VAR model to test the monetary supply channel and find that this channel has a very important role in the monetary policy transmission effect [17, 18].

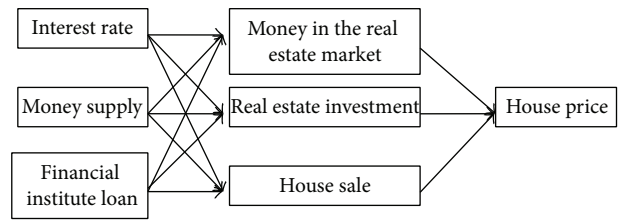

Figure 1: The framework model for the transmission of monetary policy on real estate market.

Meanwhile, the real estate market can be explained using demand and supply theory, which means that the house price is the trade balance result of house demand and supply and can be affected by factors related to supply and demand.

For house supply, it means the quantity of house which can be provided in a certain time and price. This indicates two aspects: the willingness and ability that real estate companies can supply. The willingness can be seen from the investment of real estate developers. Besides the general real estate companies' supply, the government can assist and affect the supply based on its purpose of manage people's living. In order to reflect this supply, this paper uses the monetary in the real estate market to reflect how many houses can be supplied. For the demand, it means the quantity of houses which can be bought in a certain price and time. In China, the house demand is quite rigid because Chinese people prefer buying not renting houses. Especially in current situation, the high profit attracts more and more demand in real estate market.

Based on the above theories, we construct a framework model for the transmission of monetary policy on real estate market. In Figure 1, the monetary policy affects the real estate market via interest rate channel, monetary supply channel, and credit channel. And then monetary policy can manage the real estate investment, the money in the real estate market, and the sales in the real estate market. Finally, the house price can be set.

What is more, the relevant indicators are as follows: we select interest rate (R), money supply (M), and financial institute loan (L) to represent the monetary policy; the monetary in the real estate market (HM), the real estate investment (HI), and house sale (HS) to reflect the supply and demand of real estate market; the house price (HP) is the target item.

Then, assume that we have known the details of this framework model above. While monetary policy changes someday, we can infer how the house price goes. Also, if there is disturbance out of system affecting the house price and deviating from what is excepted, we can design new monetary policies to keep the price stable by the feedback of real estate market. The latter is the main idea of this paper. The following will be introduced in detail.

3. Basic Model

In this section, the theoretical framework above will be converted into mathematical model, and the method of state feedback controller will be introduced.

The key idea is to build a continuous H_∞ control, as

$$\begin{aligned} \dot{X}(t) &= AX(t) + Bu(t) + B_1\omega(t), \\ Y(t) &= CX(t) + D\omega(t), \end{aligned} \tag{1}$$

where $u(t)$ is the control input and $X(t)$ is the measured output or the so called state variable. $Y(t)$ is the control output. So we can get

$$\begin{aligned} \begin{bmatrix} \dot{\mathrm{HM}}(t) \\ \dot{\mathrm{HI}}(t) \\ \dot{\mathrm{HS}}(t) \end{bmatrix} &= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \mathrm{HM}(t) \\ \mathrm{HI}(t) \\ \mathrm{HS}(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} R(t) \\ M(t) \\ L(t) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \omega(t), \\ \mathrm{HP}(t) &= \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} \mathrm{HM}(t) \\ \mathrm{HI}(t) \\ \mathrm{HS}(t) \end{bmatrix} + D\omega(t), \end{aligned} \tag{2}$$

where HP means the house price, HM means the money supply in the real estate market, HI means the investment in the real estate market, HS means the house sell, R means the interest rate, M means the money supply, L means the financial institute loans, ω means the disturbance term, and the others are the undetermined coefficients.

Then, C. Romer and D. Romer find that, in the long term, the monetary policy is "neutral" [19, 20], which means that the monetary policy does not have a real impact on output or other actual economic indicators; though in the short term, it can affect the economy. Obviously, in the long term, the output depends on the input of resources, the level of technology, and the work efficiency. The monetary policy mainly causes short-term impact, which would finally decay to the equilibrium state. What is more, Wu and Hu do an impulse response research on the transmission of monetary policy in China's real estate market [21]. They find that, in the short term, the monetary policy's conduction in the real estate market has a time lag. Therefore, in this section, we will add a lag period (h) in the monetary policy. So, we can get a delay-dependent H_∞ control as

$$\begin{aligned} \begin{bmatrix} \dot{\mathrm{HM}}(t) \\ \dot{\mathrm{HI}}(t) \\ \dot{\mathrm{HS}}(t) \end{bmatrix} &= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \mathrm{HM}(t) \\ \mathrm{HI}(t) \\ \mathrm{HS}(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} R(t-h) \\ M(t-h) \\ L(t-h) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \omega(t), \\ \mathrm{HP}(t) &= \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} \mathrm{HM}(t) \\ \mathrm{HI}(t) \\ \mathrm{HS}(t) \end{bmatrix} + D\omega(t). \end{aligned} \tag{3}$$

According to the economic theory, we can build such assumptions.

Assumption 1. Supply and demand are independent of each other. It means that a_{13}, a_{23}, a_{31}, and a_{32} are zero, while HM, HI are not affected by HS, and HS is not affected by HM, HI.

Assumption 2. HM means money supply in real estate market. So the key influencing factor is money supply. Then we can set b_{11} and b_{13} to be zero.

Assumption 3. b_{22} is zero, while HI is not affected by M. It is because money supply is determined by central bank but the investment in real estate is determined by estate agent. The estate agent invests in the real estate market with the money of his own or borrowed from the bank.

With the assumptions above, we can get

$$\begin{aligned} \begin{bmatrix} \dot{\mathrm{HM}}(t) \\ \dot{\mathrm{HI}}(t) \\ \dot{\mathrm{HS}}(t) \end{bmatrix} &= \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} \mathrm{HM}(t) \\ \mathrm{HI}(t) \\ \mathrm{HS}(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 & b_{12} & 0 \\ b_{21} & 0 & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} R(t-h) \\ M(t-h) \\ L(t-h) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \omega(t), \\ \mathrm{HP}(t) &= \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} \mathrm{HM}(t) \\ \mathrm{HI}(t) \\ \mathrm{HS}(t) \end{bmatrix} + D\omega(t). \end{aligned} \tag{4}$$

In the uncertain environment, the economic system will be affected by error, asymmetric information, credit risk, psychological factors, and so on [22]. To avoid these disturbances, we use the continuous delay-dependent H_∞ control. So that we can make an optimal control of monetary policy's conduction in the real estate market at the "worst case," which means $\|Y(t)\|_2 < \gamma^2\|\omega\|_2$. Then the differential equation can be further simplified into

$$\begin{aligned} \dot{X}(t) &= AX(t) + Bu(t-h) + B_1\omega(t), \\ Y(t) &= CX(t) + D\omega(t), \end{aligned} \tag{5}$$

where

$$\begin{gathered} \dot{X}(t) = \begin{bmatrix} \dot{\mathrm{HM}}(t) \\ \dot{\mathrm{HI}}(t) \\ \dot{\mathrm{HS}}(t) \end{bmatrix}, \qquad X(t) = \begin{bmatrix} \mathrm{HM}(t) \\ \mathrm{HI}(t) \\ \mathrm{HS}(t) \end{bmatrix}, \\ u(t-h) = \begin{bmatrix} R(t-h) \\ M(t-h) \\ L(t-h) \end{bmatrix}, \qquad Y(t) = \mathrm{HP}(t), \\ A = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & b_{12} & 0 \\ b_{21} & 0 & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}, \\ C = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}. \end{gathered} \tag{6}$$

For the design of an H_∞ control, we are interested in designing a memoryless state feedback controller

$$u(t) = KX(t), \tag{7}$$

where K is the state feedback gain matrix to be designed, so that

(1) the closed-loop system is asymptotically stable;

(2) under zero initial condition, the closed-loop system guarantees that $\|Y(t)\|_2 < \gamma^2\|\omega\|_2$ for all nonzero $\omega \in L_2[0,\infty)$, where $\gamma > 0$ is a prescribed constant. Thus, the closed-loop system is given by

$$\begin{aligned} \dot{X}(t) &= AX(t) + BKX(t-h) + B_1\omega(t), \\ Y(t) &= CX(t) + D\omega(t). \end{aligned} \tag{8}$$

4. Delay-Dependent H_∞ Control Model

The following theorem presents the sufficient conditions for the existence of a delay-dependent H_∞ state feedback control.

Theorem 1. *Given $\gamma > 0$ and $h > 0$, if there exit matrices $P = P^T > 0$, $Q = Q^T > 0$, and S_i, satisfying matrix inequalities* (5). *Moreover, a desired delay-dependent H_∞ state feedback control law is given by $u(t) = KX(t)$:*

$$\begin{bmatrix} A^TP + PA + S_1 + S_1^T & PBK - S_1 + S_2^T & PB_1 + S_3^T & \sqrt{h}S_1 & \sqrt{h}A^T & C^T \\ * & -S_2 - S_2^T & -S_3^T & \sqrt{h}S_2 & \sqrt{h}K^TB^T & 0 \\ * & * & -\gamma^2 I & \sqrt{h}S_3 & \sqrt{h}B_1^T & D^T \\ * & * & * & -Q & 0 & 0 \\ * & * & * & * & -Q^{-1} & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0. \tag{9}$$

Proof. Under the condition of the theorem, firstly, we choose a Lyapunov functional candidate for system (5) as

$$V(t) = X^T(t)PX(t) + \int_{-h}^{0}\int_{t+\beta}^{t} \dot{X}^T(\alpha)Q\dot{X}(\alpha)\,d\alpha\,d\beta, \tag{10}$$

where $P > 0$ and $Q > 0$ are matrices to be determined.

The derivative of $V(t)$ satisfies

$$\begin{aligned} \dot{V}(t) = {} & \dot{X}^T(t)PX(t) + X^T(t)P\dot{X}(t) + h\dot{X}^T(t)Q\dot{X}(t) \\ & - \int_{t-h}^{t} \dot{X}^T(\alpha)Q\dot{X}(\alpha)\,d\alpha. \end{aligned} \tag{11}$$

So we design that

$$Z(t) = \dot{V}(t) + Y^T(t)Y(t) - \gamma^2\omega^T(t)\omega(t); \tag{12}$$

then,

$$\begin{aligned} Z(t) &= \dot{V}(t) + Y^T(t)Y(t) - \gamma^2\omega^t(t)\omega(t) \\ &= (AX + BKX_1 + B_1\omega)^T PX \\ &\quad + X^TP(AX + BKX_1 + B_1\omega) \\ &\quad + (CX + D\omega)^T(CX + D\omega) \\ &\quad + h(AX + BKX_1 + B_1\omega)^T Q(AX + BKX_1 + B_1\omega) \\ &\quad - \int_{t-h}^{t} \dot{X}^T(\alpha)Q\dot{X}(\alpha)\,d\alpha - \gamma^2\omega^t\omega \\ &= X^T\left(A^TP + PA + hA^TQA + C^TC\right)X \\ &\quad + X^T\left(PBK + hA^TQBK\right)X_1 \\ &\quad + X^T\left(PB_1 + hA^TQB_1 + C^TD\right)\omega \\ &\quad + X_1^T\left(K^TB^TP + hK^TB^TQA\right)X \\ &\quad + X_1^T\left(hK^TB^TQBK\right)X_1 + X_1^T\left(hK^TB^TQB_1\right)\omega \\ &\quad - \int_{t-h}^{t} \dot{X}^T(\alpha)Q\dot{X}(\alpha)\,d\alpha \\ &\quad + \omega^T\left(B_1^TP + hB_1^TQA + D^TC\right)X \\ &\quad + \omega^T\left(hB_1^TQBK\right)X_1 \\ &\quad + \omega^T\left(hB_1QB_1 + D^TD - \gamma^2 I\right)\omega, \end{aligned} \tag{13}$$

where $X = X(t)$, $X_1 = X(t-h)$, and $\omega = \omega(t)$.

For simplification, we make

$$\psi_1 = \begin{bmatrix} A^TP + PA + C^TC & PBK & PB_1 + C^TD \\ K^TB^TP & 0 & 0 \\ B_1^TP + D^TC & 0 & D^TD - \gamma^2 I \end{bmatrix}, \tag{14}$$

$$\psi_2 = h[A \;\; BK \;\; B_1]^T Q [A \;\; BK \;\; B_1].$$

Thus, we have

$$Z = \zeta^T(t)(\psi_1 + \psi_2)\zeta(t) - \int_{t-h}^{t} \dot{X}^T(\alpha)Q\dot{X}(\alpha)\,d\alpha. \tag{15}$$

In addition, by the Leibniz-Newton formula, for any appropriate matrices $S = \sum_{i=1}^{t} k_i S_i = \sum_{i=1}^{t} \left[S_{1i}^T \;\; S_{2i}^T \;\; S_{3i}^T\right]^T$, we have

$$\zeta^T(t) S \left[X(t) - X(t-h) - \int_{t-h}^{t} \dot{X}(\alpha)\, d\alpha\right] = 0, \tag{16}$$

where $\zeta(t) = \left[X^T(t) \;\; X^T(t-h) \;\; \omega^T(t)\right]^T$. So we can get

$$\begin{aligned} Z &= \zeta^T(t)(\psi_1 + \psi_2)\zeta(t) - \int_{t-h}^{t} \dot{X}^T(\alpha) Q \dot{X}(\alpha)\, d\alpha \\ &\quad + \zeta^T(t) S \left[X(t) - X(t-h) - \int_{t-h}^{t} \dot{X}(\alpha)\, d\alpha\right] \\ &\quad + \left[X(t) - X(t-h) - \int_{t-h}^{t} \dot{X}(\alpha) d\alpha\right]^T S^T \zeta(t) \\ &= \zeta^T(t)(\psi_1 + \psi_2)\zeta(t) + \zeta^T(t) S (X - X_1) \\ &\quad + \left(X^T - X_1^T\right) S^T \zeta(t) \\ &\quad - \int_{t-h}^{t} \left[\dot{X}^T(\alpha) Q \dot{X}(\alpha) + \zeta^T(t) S \dot{X}(\alpha) \right. \\ &\qquad \left. + \dot{X}^T(\alpha) S^T \zeta(t)\right] d\alpha. \end{aligned} \tag{17}$$

With the addition of $h\zeta^T(t) S Q^{-1} S^T \zeta(t) - \int_{t-h}^{t} \zeta^T(t) S Q^{-1} S^T \zeta(t) d\alpha = 0$, then we have

$$\begin{aligned} Z &= \zeta^T(t)\left(\psi_1 + \psi_2 + \psi_3 + \psi_3^T\right)\zeta(t) + h\zeta^T(t) S Q^{-1} S^T \zeta(t) \\ &\quad - \int_{t-h}^{t} \left[\zeta^T(t) S + \dot{X}(\alpha) Q\right] Q^{-1} \left[Q \dot{X}^T(\alpha) + S^T \zeta(t)\right] d\alpha, \end{aligned} \tag{18}$$

where $\psi_3 = [S \;\; -S \;\; 0]$, $S = \left[S_1^T \;\; S_2^T \;\; S_3^T\right]^T$.

Note that $Q > 0$; thus $-\int_{t-h}^{t} [\zeta^T(t) S + \dot{X}(\alpha) Q] Q^{-1} [Q \dot{X}^T(\alpha) + S^T \zeta(t)] d\alpha$ is positive. By Schur complement, inequalities (9) guarantee

$$\psi_1 + \psi_2 + \psi_3 + \psi_3^T + h S Q^{-1} S^T < 0. \tag{19}$$

Therefore, we have

$$Z(t) = \dot{V}(t) + Y^T(t) Y(t) - \gamma^2 \omega^T(t) \omega(t) < 0, \tag{20}$$

for all nonzero $\omega \in L_2[0, \infty)$, and the asymptotic stability of system (8) is established. Under zero initial condition, we have $V(0) = 0$ and $V(\infty) \geq 0$. Integrating both sides of inequality (20), we obtain

$$0 < V(\infty) - V(0) < \gamma^2 \|\omega\|_2 - \|Y(t)\|_2. \tag{21}$$

So, the inequality above means $\|Y(t)\|_2 < \gamma^2 \|\omega\|_2$ for all nonzero $\omega \in L_2[0, \infty)$, and the H_∞ performance is established. This completes the proof. □

5. A Design Example

The time-delay-dependent H_∞ control has been proved to be useful in many aspects, [23–26]. In this section, we will provide an example to illustrate the effectiveness of the proposed delay-dependent state feedback H_∞ control based on the theories above. Through the demonstration of the quarterly data from 2005 to 2013, we have the parameters as the following values:

$$\begin{aligned} A &= \begin{bmatrix} 0.401404 & 2.504153 & 0 \\ -0.368434 & 2.013139 & 0 \\ 0 & 0 & 0.914773 \end{bmatrix}; \\ B &= \begin{bmatrix} 0 & -3.821928 & 0 \\ -0.283832 & 0 & -2.102299 \\ 0.331962 & -11.82723 & 10.80898 \end{bmatrix}; \\ C &= \left[-0.230125 \;\; 0.310886 \;\; 0.256653\right]; \\ B_1 \omega(t) &= \begin{bmatrix} 28.51301 \\ 14.20208 \\ 10.01136 \end{bmatrix}; \qquad D\omega(t) = 5.219788. \end{aligned} \tag{22}$$

Now, we design a continuous delay-dependent H_∞ control, based on the theory of system (5). To further research, we set the lag phase (h) to be 0.2 and the D to be 1. So we can choose $B_1 = \left[5.46248 \;\; 2.72082 \;\; 1.91796\right]^T$. By solving the matrices inequalities (9) with LMI tools of MATLAB, while there exit matrices $P > 0$, $Q > 0$, and S with a given scalar $\gamma > 0$, the state feedback controller is established. Then we can find that, when the best $\gamma = 1.0923$, the linear matrix inequality system is feasible. And we can get the following controller gain matrix based on system (7):

$$K = \begin{bmatrix} 13.4161 & -17.0838 & -21.1605 \\ -4.7350 & 7.6431 & 5.9833 \\ -4.2677 & 7.1089 & 5.4628 \end{bmatrix}. \tag{23}$$

Assume that the original system is stable and the fluctuations of house price have been within the expected range. But suddenly, some disturbance out of system happens, such as system (5), with the parameters of (22). It causes the price gravely deviating from the forecast. Then the robust control militates. We have to design some new monetary policy to keep its stability, where the design is mentioned in Section 3 and the strict proof in Section 4. So we can build a state feedback controller as system (7) with the coefficient of (23). Then we can find that $\|Y(t)\|_2 < \gamma^2 \|\omega\|_2$ with the smallest possible $\gamma = 1.0923$, and the house price is asymptotically stable.

It means that this example further proves the possibility of delay-dependent H-infinity control's application in monetary policy. And the economy system has robustness. In other words, through the state feedback controller, we can adjust the monetary policy in time to keep the house price stable as optimal as possible, in uncertain case. Also, if the lag phase changes or the external disturbance is different, we can find a new state feedback controller to make the best decision.

6. Conclusion

This paper uses a time-delay-dependent H_∞ control model to analyze the transmission effect of monetary policy on the real estate market and finds the result which meets the optimal feedback requirement. We test the robustness of the model using the Chinese data of the monetary policy and the real estate market and prove that this model has a good ability to be employed in the reality. Based on the simulation test, we find that under an uncertain circumstance, the model can give an optimal result to fulfill the goal of controlling the risk and improve the policy's effectiveness. In the real estate market, the monetary policy can have a good effect on the market when the policies are well combined based on the simulation test.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This study was sponsored by the NSFC (nos. 71031003, 71103049, and 71303069), the Program for New Century Excellent Talents in University (NCET-13-0167), the Chinese Distinguished Postdoctoral Fund (2012T50369), Humanities and Social Science Foundation of Ministry of Education (no. 10YJC790070), The Natural Science Foundation of Heilongjiang Province of China (Grant no. F201209). The Research Fund for the Doctoral Program of Higher Education (20112302120038), and the Fundamental Research Funds for the Central Universities (HIT.BRETIII.201408).

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