Research Article

Consensus of Multiagent Systems with Directed Topology and Communication Time Delay Bases on the Laplace Transform

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This paper investigates the consensus problem of multiagent systems with directed topologies. Different from the literatures, a new method, the Laplace transform, to study the consensus of multiagent systems with directed topology and communication time delay is proposed. The accurate state of the consensus center and the upper bound of the communication delay to make the agents reach consensus are given. It is proved that all the agents could aggregate and eventually form a cohesive cluster in finite time under certain conditions, and the consensus center is only determined by the initial states and the communication configuration among the agents. Finally, simulations are given to illustrate the theoretical results.

1. Introduction

In recent years, there has been an increasing interest in the study of consensus and rendezvous problems in the multiagent systems [1–16]. This is partly due to the wide applications in cooperative control of unmanned air vehicles, formation control of mobile robots, design of sensor networks, flocking of ants and birds, distributed decision making, and so on [5–10]. Vicsek et al. [1] proposed a discrete model of autonomous agents. Each agent of such model was moving at a constant identical velocity, and the direction was updated via a local rule based on the average of the directions of its neighbors. At the same time, they gave some numerical simulations to describe the dynamic behavior of the model. Jadbabaie et al. [3] gave a theoretical explanation for the numerical results of Vicsek's model. In [5], Olfati-Saber and Murray analyzed the consensus problem of multiagent systems with time delay and obtained the accurate bound of the time delay with undirected topology. In [7, 9], the authors discussed the consensus of second-order multiagent systems with time delay by Lyapunov approach. However, due to certain limitations, the bound of delay in those papers is not specific.

The analysis of the coupling topology plays an important part in discussing the consensus problems. Jadbabaie et al. [3]

discussed the communication information by applying an undirected graph to model the coupling topology among the agents. Olfati-Saber and Murray [5] investigated the average consensus problem with directed topology, where the coupling topology is undirected and needs to satisfy the balance condition, which is a strong condition for the consensus. Ren and Beard [11] introduced the definition of spanning tree to depict the coupling topology. It was shown that consensus can be achieved asymptotically if the directed interaction graph contains a spanning tree as the system evolves. A useful Lemma about the Laplacian matrix was given in [11], and Laplacian matrix has a simple zero eigenvalue if and only if the directed graph has a spanning tree. Lin et al. [12] introduced the definition of globally reachable node to describe the coupling topology and gave a similar lemma about the Laplacian matrix; that is, the digraph has a globally reachable node if and only if 0 is a simple eigenvalue of Laplacian matrix.

In this paper, we discuss the consensus problem of the multiagent system mentioned in [5] with directed topology and time delay. Compared with the previous references, the main contribution of this paper is to study the coupling topology in a more general case. According to the Laplace transform, we can not only have the specific consensus center of the model but also obtain the accurate upper bound of

the communication delay value to achieve consensus. By the Laplace transform, we can eliminate some assumptions used in the Lyapunov approach and give further simplification of the assumptions.

This paper is organized as follows. Section 2 presents some preliminaries of graph theory. Section 3 proposes the model and gives the analysis for the consensus of the model. In Section 3, some extensions of the studies are discussed. Section 4 gives simulations to verify the theoretical results. Finally, we summarize our main contribution in Section 5.

2. Some Preliminaries

To discuss the coupling topology of the communication configuration of the agents, graph theory is a very effective tool. If each agent is regarded as a node, then the coupling topology is conveniently described by a directed graph. In particular, in the definition of the directed graph, self-loops are excluded. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ be a weighted digraph of order n with the set of nodes $\mathcal{V} = \{1, 2, 3, \dots, n\}$ and set of arcs $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and $A = [a_{ij}]$ is the adjacency matrix of graph $\mathcal{G}$. An arc of $\mathcal{G}$ is denoted by (i, j) , which is from i to j . The set of neighbors of node i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$. A digraph $\mathcal{G}$ is strongly connected if there exists a path between any two distinct nodes. For a node j , if there exists at least a path from every other node i in $\mathcal{G}$ to node j , we say that node j is globally reachable.

A diagonal matrix $D = \text{diag}\{d_1, d_2, \dots, d_n\} \in \mathbb{R}^{n \times n}$ is a degree matrix of $\mathcal{G}$, and its diagonal elements $d_i = \sum_{j \in \mathcal{N}_i} a_{ij}$ for $i = 1, 2, \dots, n$. The Laplacian of the weighted digraph $\mathcal{G}$ (or matrix A) is denoted as

$$L = D - A \in \mathbb{R}^{n \times n}. \quad (1)$$

It is obvious that the digraph $\mathcal{G}$, the adjacency matrix A , and the Laplacian matrix L are peer-to-peer. Some basic properties of the Laplacian matrix need to be introduced in the following.

Lemma 1 (see [12]). *The digraph $\mathcal{G}$ has a globally reachable node if and only if the Laplacian matrix L has a simple zero eigenvalue (with eigenvector $\mathbf{1} = (1, 1, \dots, 1)^T \in \mathbb{R}^n$).*

Lemma 2 (see [5]). *The nonzero eigenvalues of L are of positive real part.*

Lemma 3. *For arbitrary row of the Laplacian matrix L , the cofactor of any elements is equal.*

Proof. For Laplacian L

$$\begin{pmatrix} l_{11} & l_{12} & \cdots & l_{1n} \\ l_{21} & l_{22} & \cdots & l_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ l_{n1} & l_{n2} & \cdots & l_{nn} \end{pmatrix}, \quad (2)$$

without loss of generality, the first row of L is chosen to discuss. The cofactors of element l_{1j} and $l_{1(j+1)}$, $j \in \{1, 2, \dots, n-1\}$, are, respectively, denoted by

$$M_{1j} = (-1)^{1+j} \begin{vmatrix} l_{21} & l_{22} & \cdots & l_{2(j-1)} & l_{2(j+1)} & \cdots & l_{2n} \\ l_{31} & l_{32} & \cdots & l_{3(j-1)} & l_{3(j+1)} & \cdots & l_{3n} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ l_{n1} & l_{n2} & \cdots & l_{n(j-1)} & l_{n(j+1)} & \cdots & l_{nn} \end{vmatrix}, \quad (3)$$

$$M_{1(j+1)} = (-1)^{1+j+1} \begin{vmatrix} l_{21} & l_{22} & \cdots & l_{2j} & l_{2(j+2)} & \cdots & l_{2n} \\ l_{31} & l_{32} & \cdots & l_{3j} & l_{3(j+2)} & \cdots & l_{3n} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ l_{n1} & l_{n2} & \cdots & l_{nj} & l_{n(j+2)} & \cdots & l_{nn} \end{vmatrix}.$$

For $M_{1(j+1)}$, adding all the other columns to the j th column, we can have

$$M_{1(j+1)} = (-1)^{1+j+1} \begin{vmatrix} l_{21} & l_{22} & \cdots & -l_{2(j+1)} & l_{2(j+2)} & \cdots & l_{2n} \\ l_{31} & l_{32} & \cdots & -l_{3(j+1)} & l_{3(j+2)} & \cdots & l_{3n} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ l_{n1} & l_{n2} & \cdots & -l_{n(j+1)} & l_{n(j+2)} & \cdots & l_{nn} \end{vmatrix} \quad (4)$$

$$= (-1)^{1+j+1+1} \begin{vmatrix} l_{21} & l_{22} & \cdots & l_{2(j+1)} & l_{2(j+2)} & \cdots & l_{2n} \\ l_{31} & l_{32} & \cdots & l_{3(j+1)} & l_{3(j+2)} & \cdots & l_{3n} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ l_{n1} & l_{n2} & \cdots & l_{n(j+1)} & l_{n(j+2)} & \cdots & l_{nn} \end{vmatrix}$$

$$= M_{1j}.$$

Since j is arbitrary, we can obtain $M_{11} = M_{12} = \cdots = M_{1n}$. Similarly, we can have $M_{21} = M_{22} = \cdots = M_{2n}, \dots, M_{n1} = M_{n2} = \cdots = M_{nn}$.

This completes the proof. $\square$

3. Model Formulation and Analysis

Consider a multiagent system consisting of n agents in m -dimensional Euclidian space whose motion is governed by the following delay differential equations:

$$\dot{x}_i(t) = - \sum_{j \in \mathcal{N}_i} a_{ij} (x_i(t - \tau) - x_j(t - \tau)), \quad i = 1, 2, \dots, n, \quad (5)$$

where $x_i \in \mathbb{R}^m$ represents the state of agent i , $\mathcal{N}_i$ is the neighbor set of agent i , $A = (a_{ij})$ is the coupling weight matrix, and a_{ij} is weight parameter with $a_{ij} > 0$, if agent i has information with agent j ; otherwise, $a_{ii} = 0$; the time delay $\tau > 0$ is a constant.

For simplicity of discussion, we take $m = 1$ in model (5). Then, model (5) can be rewritten into a matrix form:

$$\dot{X}(t) = -LX(t - \tau), \quad (6)$$

where $X(t) = (x_1(t), x_2(t), \dots, x_n(t))^T \in \mathbb{R}^n$ and L is the Laplacian matrix.

For system (6), we can have the following main results by Laplace transform.

Theorem 4. For system (6), if there exists at least one globally reachable node in $\mathcal{G}$ and the time delay parameter τ satisfies

$$\tau < \min_{i=2,3,\dots,n} \frac{\pi/2 - |\arg(\lambda_i)|}{\|\lambda_i\|}, \quad (7)$$

then the states of all agents of the system will asymptotically converge to a constant value; that is,

$$c = \frac{\begin{bmatrix} x_1(0) & l_{12} & \dots & l_{1n} \\ x_2(0) & l_{22} & \dots & l_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_n(0) & l_{n2} & \dots & l_{nn} \end{bmatrix}}{\lambda_1 \lambda_2 \dots \lambda_n}, \quad (8)$$

where $\lambda_1 \lambda_2 \dots \lambda_n$ are the nonzero eigenvalues of L and $\arg(\lambda_i)$ and $\|\lambda_i\|$ are the argument and modulus of λ_i , respectively.

Proof. Applying Laplace transform on (6), succinctly denoted as $\mathcal{L}(\cdot)$, we can obtain

$$\begin{aligned} s\mathcal{L}(X(t)) - X(0) &= -L \int_0^\infty e^{-st} X(t-\tau) dt \\ &= -L \int_\tau^\infty e^{-st} X(t-\tau) dt \\ &= -L \int_0^\infty e^{-s(\xi+\tau)} X(t) d\xi \\ &= -e^{-s\tau} L \mathcal{L}(X(t)); \end{aligned} \quad (9)$$

that is,

$$(sI + e^{-s\tau} L) \mathcal{L}(X(t)) = X(0), \quad (10)$$

where s is the Laplace variable. For simplicity, $\mathcal{L}X(t)$ is denoted as $\mathcal{X}(s)$.

According to Cramer rule, we can get the solutions of (10):

$$\mathcal{X}_i(s) = \frac{\Delta(sI + e^{-s\tau} L)^i}{\Delta(sI + e^{-s\tau} L)}, \quad i = 1, 2, \dots, n, \quad (11)$$

where $\Delta(sI + e^{-s\tau} L)^i$ is the determinant of $sI + e^{-s\tau} L$ in which the i th column has been replaced by $X(0)$; $\Delta(sI + e^{-s\tau} L)$ is the determinant of matrix $(sI + e^{-s\tau} L)$.

By calculating, we can obtain

$$\mathcal{X}_i(s) = \frac{e^{-(n-1)s\tau} \Delta(se^{s\tau} I + L)^i}{e^{-ns\tau} \Delta(se^{s\tau} I + L)} = \frac{e^{s\tau} \Delta(se^{s\tau} I + L)^i}{\Delta(se^{s\tau} I + L)}. \quad (12)$$

That is fractional expression about variable $se^{s\tau}$. If there are common factors between $\Delta(se^{s\tau} I + L)^i$ and $\Delta(se^{s\tau} I + L)$, we can reduce them and make them irreducible. In the following, assume that $\Delta(se^{s\tau} I + L)^i$ and $\Delta(se^{s\tau} I + L)$ are irreducible. For simplicity, we denote $P_i(s) = \Delta(se^{s\tau} I + L)^i$ and $Q(s) = \Delta(se^{s\tau} I + L)$.

According to Lemma 1, the zero eigenvalue of L is simple and the eigenvalues of L are denoted as

$$0, \lambda_2, \lambda_3, \dots, \lambda_h, \lambda_j, \dots, \lambda_l, \quad (13)$$

where $\lambda_j, \dots, \lambda_l$ are the multiple eigenvalues corresponding with the multiplicity as $q, \dots, p$, respectively. Then, it follows that

$$\begin{aligned} \Delta(se^{s\tau} I + L) &= se^{s\tau} (se^{s\tau} + \lambda_2) \dots (se^{s\tau} + \lambda_h) (se^{s\tau} + \lambda_j)^p \dots (s + \lambda_l)^q. \end{aligned} \quad (14)$$

By Heaviside's method, (12) can be expanded into

$$\begin{aligned} \mathcal{X}_i(s) &= e^{s\tau} \left(\frac{c_1^i}{se^{s\tau}} + \frac{c_2^i}{se^{s\tau} + \lambda_2} + \dots + \frac{c_h^i}{se^{s\tau} + \lambda_h} \right. \\ &\quad + \frac{c_{jp}^i}{(se^{s\tau} + \lambda_j)^p} + \frac{c_{j(p-1)}^i}{(se^{s\tau} + \lambda_j)^{(p-1)}} \\ &\quad + \dots + \frac{c_{j1}^i}{(se^{s\tau} + \lambda_j)} + \dots \\ &\quad + \frac{c_{lq}^i}{(se^{s\tau} + \lambda_l)^q} + \frac{c_{l(q-1)}^i}{(se^{s\tau} + \lambda_l)^{(q-1)}} \\ &\quad \left. + \dots + \frac{c_{l1}^i}{(se^{s\tau} + \lambda_l)} \right) \\ &\triangleq \frac{c_1^i}{s} + \frac{c_2^i}{s + \lambda_2 e^{-s\tau}} + \dots + \frac{c_h^i}{s + \lambda_h e^{-s\tau}} \\ &\quad + \frac{c_{jp}^i}{(s + \lambda_j e^{-s\tau})^p} + \frac{c_{j(p-1)}^i}{(s + \lambda_j e^{-s\tau})^{(p-1)}} \\ &\quad + \dots + \frac{c_{j1}^i}{(s + \lambda_j e^{-s\tau})} + \dots \\ &\quad + \frac{c_{lq}^i}{(s + \lambda_l e^{-s\tau})^q} + \frac{c_{l(q-1)}^i}{(s + \lambda_l e^{-s\tau})^{(q-1)}} \\ &\quad + \dots + \frac{c_{l1}^i}{(s + \lambda_l e^{-s\tau})}, \end{aligned} \quad (15)$$

where

$$\begin{aligned} c_1^i &= s \frac{P_i(s)}{Q(s)} \Big|_{s=0}, \quad c_e^i = (s + \lambda_e) \frac{P_i(s)}{Q(s)} \Big|_{s=\lambda_e}, \quad e = 2, \dots, h, \\ c_{jf}^i &= \frac{d^{p-f}}{ds^{p-f}} \left[(s + \lambda_j)^{(p-f)} \frac{P_i(s)}{Q(s)} \right] \Big|_{s=\lambda_j}, \quad f = 1, 2, \dots, p, \\ &\vdots \\ c_{lg}^i &= \frac{d^{q-g}}{ds^{q-g}} \left[(s + \lambda_l)^{(q-g)} \frac{P_i(s)}{Q(s)} \right] \Big|_{s=\lambda_l}, \quad g = 1, 2, \dots, q. \end{aligned} \quad (16)$$

Equation (15) is the transfer function of $x_i(t)$; the Laplace reverse transform of the first term is a constant value c_1^i . In order to make $x_i(t)$ asymptotically stable, the real part of the denominators' roots must be negative, which implies that the roots of transcendental equation $s + \lambda_k e^{-s\tau} = 0$ must have negative real part. Then, the roots of

$$s + \lambda_k e^{-s\tau} = 0 \quad (17)$$

will be analyzed in the following. To solve (17), we let $s = x + iy$, and $\lambda_k = u + iv$, where $u > 0$, and $i = \sqrt{-1}$ is the plural unit. Then,

$$x + iy + ue^{-(x+iy)\tau} + ive^{-(x+iy)\tau} = 0, \quad (18)$$

which can be expanded as

$$\begin{aligned} x + iy + ue^{-x\tau} (\cos(-y\tau) + i \sin(-y\tau)) \\ + ive^{-x\tau} (\cos(-y\tau) + i \sin(-y\tau)) = 0; \end{aligned} \quad (19)$$

that is,

$$\begin{aligned} x + ue^{-x\tau} \cos(-y\tau) - ve^{-x\tau} \sin(-y\tau) \\ + i(y + ue^{-x\tau} \sin(-y\tau) + ve^{-x\tau} \cos(-y\tau)) = 0. \end{aligned} \quad (20)$$

Separating the real and imaginary parts, then we can have

$$\begin{aligned} x + ue^{-x\tau} \cos(-y\tau) - ve^{-x\tau} \sin(-y\tau) &= 0, \\ y + ue^{-x\tau} \sin(-y\tau) + ve^{-x\tau} \cos(-y\tau) &= 0. \end{aligned} \quad (21)$$

Solving the above equations, we can get

$$\begin{aligned} x + \sqrt{u^2 + v^2} e^{-x\tau} \cos(-y\tau + \varphi) &= 0, \\ y + \sqrt{u^2 + v^2} e^{-x\tau} \sin(-y\tau + \varphi) &= 0, \end{aligned} \quad (22)$$

where $\varphi = \arg(\lambda_k) = \arctan(v/u)$ is the argument of λ and $\varphi \in (-\pi/2, \pi/2)$.

According to (22), we can know that

$$\begin{aligned} x^2 + y^2 &= (u^2 + v^2) e^{-2x\tau}, \\ y\tau &= \pm \tau \sqrt{(u^2 + v^2) e^{-2x\tau} - x^2}. \end{aligned} \quad (23)$$

Substituting those into the first equation of (22), we have

$$x + \sqrt{u^2 + v^2} e^{-x\tau} \cos\left(\mp \tau \sqrt{(u^2 + v^2) e^{-2x\tau} - x^2} + \varphi\right) = 0; \quad (24)$$

that is,

$$x + \sqrt{u^2 + v^2} e^{-x\tau} \cos\left(\tau \sqrt{(u^2 + v^2) e^{-2x\tau} - x^2} \pm \varphi\right) = 0. \quad (25)$$

Let x be equal to the critical value; that is, $x = 0$; then

$$\tau \sqrt{(u^2 + v^2)} \pm \varphi = \frac{\pi}{2} + m\pi, \quad m = 0, \pm 1, \pm 2, \dots \quad (26)$$

In order to get the smallest positive value of τ , we have $m = 0$, and $\tau \sqrt{(u^2 + v^2)} + |\varphi| = \pi/2$, since $\tau \sqrt{(u^2 + v^2)} \geq 0$ and the sign of φ is decided by u and v . So, we can get the critical value of the time delay, denoted as τ_0 , and

$$\tau_0 = \frac{\pi/2 - |\varphi|}{\sqrt{u^2 + v^2}}. \quad (27)$$

In the following, we will show that the roots of (17) have negative real part when $\tau < \tau_0$.

Since the complex eigenvalues of L are conjugated, their imaginary signs arguments are opposite. According to (25), we define the function as

$$\begin{aligned} f(x) \\ = x + \sqrt{u^2 + v^2} e^{-x\tau} \cos\left(\tau \sqrt{(u^2 + v^2) e^{-2x\tau} - x^2} + |\varphi|\right). \end{aligned} \quad (28)$$

For arbitrary $\tau_1 > (\pi/2 - |\varphi|)/(\sqrt{u^2 + v^2})$, let $x = 0$, and if $\tau = \tau_1$, then

$$f(0) = \sqrt{u^2 + v^2} \cos\left(\tau_1 \sqrt{u^2 + v^2} + |\varphi|\right) < 0. \quad (29)$$

For function $\sqrt{(u^2 + v^2) e^{-2x\tau_1} - x^2}$, there exists a positive root $b > 0$, and it decreases monotonically in the interval $[0, b]$.

Taking

$$\tau_1 \sqrt{(u^2 + v^2) e^{-2x\tau_1} - x^2} + |\varphi| = \frac{\pi}{2}, \quad (30)$$

there exists only one solution in the interval $(0, b)$, denoted as x_1 .

Then

$$f(x_1) = x_1 > 0. \quad (31)$$

From (29) and (31), there exists a root of $f(x)$ between 0 and x_1 . Moreover, if $\tau = \tau_1$, the same result holds.

For arbitrary $\tau_2 < \tau_0$, then

$$f(0) = \sqrt{u^2 + v^2} \cos\left(\tau_2 \sqrt{u^2 + v^2} + |\varphi|\right) > 0. \quad (32)$$

FIGURE 1: States of the agents about model (5) with the coupling matrix A_1 , when $\tau = 0$ and 0.6, respectively.

FIGURE 2: States of the agents about model (5) with the coupling matrix A_1 , when $\tau = \pi/2$ and 1.8, respectively.

FIGURE 3: States of the agents about model (38) with the coupling matrix A_2 and the feedback gain $a = 0.5$, when $\tau = 0.3$ and 0.5089, respectively.

Matrix A_1 is the associated Laplacian matrix with only one zero eigenvalue, and the other eigenvalues are all equal to 1. And the critical value of delay is $h_1 = \pi/2$.

For matrix A_2 , the associated digraph $\mathcal{G}$ is strongly connected, and the eigenvalues of L are

$$\begin{aligned} &0.0000, 2.0000, 1.8090 + 0.5878i, 1.8090 - 0.5878i; \\ &1.3090 + 0.9511i, 1.3090 - 0.9511i, 0.6910 + 0.9511i; \\ &0.6910 - 0.9511i, 0.1910 + 0.5878i, 0.1910 - 0.5878i. \end{aligned}$$

The critical value of delay $h_3 = \min_{i=2,3,\dots,n}((\pi/2 - |\arg(\lambda_i)|)/a\|\lambda_i\|) = 0.5089$.

Figures 1 and 2 show the simulation results, where the coupling matrix is A_1 . We can find that, from Figure 1, the states of all agents of model (5) will asymptotically converge to a constant value when $\tau = 0, 0.6$ ($\tau < \pi/2$), respectively; however, from Figure 2, the states of all agents of the system will diverge when $\tau = \pi/2, 1.8$ ($\tau \geq \pi/2$), respectively. Figure 3 shows, for model (38), the states of all agents of the system with the coupling matrix A_2 , and the feedback gain will asymptotically converge to a constant value when $\tau = 0.3, 0.5089$ ($\tau < \pi/2$), respectively.

5. Conclusion

In this paper, we have considered the consensus of multiagent systems with directed topology and communication time delay. We have proved that the system aggregates and forms a cluster in finite time if the time delays are smaller than the critical value. The methods and results of this paper can be extended to discuss the leader-follower second-order multiagent system with time delay.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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