Research Article

Synchronization Control of Time-Varying Complex Dynamic Network with Nonidentical Nodes and Coupling Time-Delay

Youjian Zhang, Wenjun Yan, and Qiang Yang

College of Electrical Engineering, Zhejiang University, Hangzhou 310027, China

Correspondence should be addressed to Wenjun Yan; yanwenjun_zju@163.com

Received 15 January 2014; Accepted 18 April 2014; Published 7 May 2014

Academic Editor: Guoqiang Hu

Copyright © 2014 Youjian Zhang et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper addresses the synchronization problem for a class of complex networks with time-varying topology as well as nonidentical nodes and coupling time-delay and presents two efficient control schemes to synchronize the network onto any given smooth goal dynamics. The time-varying network is supposed to be bounded within a certain range, which cannot be controlled. Through the adoption of hybrid control with linear static feedback control and adaptive feedback control, two control schemes are derived to guarantee such complex networks to reach the global synchronization. Finally, a set of numerical simulation experiments are carried out and the results demonstrate the effectiveness of the suggested control solutions.

1. Introduction

In the past decades, complex dynamic networks have been considered to be one of the challenging areas and received increasing attention from research and industry community due to the wide applications in various fields, for example, biological systems [1], wireless communication networks [2], artificial intelligent systems [3, 4], and so forth. In general, a complex network consists of a collection of interconnected nodes, where individual nodes are basic units representing specific contents or dynamics and the connections (edges) represent the interactions amongst them. It is virtually universally agreed that many collective behaviors and complex phenomena observed in the nature and society are related to the issue of network synchronization [1]. As a response, much research effort has been made to exploit the pattern and regulation of the network synchronization, and many control tools, for example, pinning control [5, 6], adaptive control [7], sliding mode control [8], and robust control [9, 10], have been applied to address the technical challenges.

In the literature, the existing solutions are mainly derived for the networks with constant topologies; that is, topologies are not varying over time (e.g., [6–8]). In fact, the studies on networks with time-varying topologies are more realistic

as the dynamical characteristics of the complex networks can be better modeled and investigated. In parallel, there are some existing works concerning synchronization in dynamic networks; that is, network coupling structure or strength dynamically changes over time (e.g., [11–13]). Xiao et al. [11] studied the synchronization of complex switched networks subject to network delay as well as switching behaviors exhibited in both nodes and topological structure. The synchronization in a complex dynamical network with randomly switching topology was studied by Lee et al. [12]. Lee et al. [13] addressed the issue in an uncertain complex dynamical network, where the norm-bounded uncertainties imposed on the complex dynamical network in a random fashion.

In addition, the behavior of a complex dynamical network can be determined by both coupling configuration amongst nodes and node dynamics. For the sake of simplicity, previous studies often assume that the dynamics of all network nodes are identical, which makes the control solutions not able to be directly applied to the realistic networks as the individual network nodes often demonstrate diverse physical parameters [14, 15]. The synchronization of the complex networks with nonidentical nodes is still an open problem to be further addressed and some solutions are available [14–20]. In [14], the authors designed a high gain

integral controller for synchronization of complex dynamical networks with unknown nonidentical nodes. In [15], the authors investigated the issue for complex networks with nonidentical nodes. Shi et al. [16] presented an adaptive synchronization control solution for a novel complex dynamical network model with nonidentical nodes and nonderivative and derivative couplings. Fan et al. [17] investigated the synchronization in the context of a class of complex dynamical network with the similar nodes and coupling time-delay. Wang et al. [18] investigated the global bounded synchronization problem of the complex dynamical networks with coupled nonidentical nodes and time-varying topology.

In this paper, we address the synchronization problem with the explicit consideration of the dynamical complex networks with the time-varying topology as well as nonidentical nodes and coupling time-delay simultaneously. To our best knowledge, little research outcome tackling this issue has been reported. The main technical contributions of this paper can be summarized twofold: (1) two synchronization control laws are proposed for time-varying complex networks with nonidentical nodes and coupling time-delay and (2) the assessment verifies that a kind of complex network with nonidentical nodes and time-varying topological structures can be well synchronized through applying the suggested control solution onto any given smooth goal dynamics, for example, a periodic orbit or a chaotic trajectory.

The remainder of this paper is organized as follows: the problem formulation and preliminaries are introduced in Section 2; in Section 3, two synchronization control laws for time-varying complex networks with nonidentical nodes and coupling time-delay are presented, followed by Section 4 carrying out the numerical simulations and presenting a set of key experimental results; finally, the conclusive remarks are given in Section 5.

Notations. In this study, a real symmetric matrix $P > 0$ (≥ 0) denotes that P is a positive definite (or positive semidefinite) matrix, and $A > (\geq) B$ indicating that $A - B > (\geq) 0$. The superscript “ T ” represents the transpose of a matrix. The matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. Problem Formulation and Preliminaries

Consider a class of complex network consisting of N nonidentical nodes with linearly diffusive time-delay couplings, and each node is an n -dimensional dynamic system. The controlled complex network can be described as the following form:

$$\begin{aligned} \dot{x}_i(t) = & f_i(x_i(t)) + c \sum_{j=1}^N a_{ij}(t) H x_j(t - \tau(t)) \\ & + u_i(t), \quad i = 1, 2, \dots, N, \end{aligned} \quad (1)$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \dots, x_{in}(t)]^T \in R^n$ is the state variable of node i ; c is the coupling strength ($c > 0$); $u_i(t)$ is the control action applied to node i ; $f_i : [0, \infty) \times R^n \rightarrow$

R^n is a vector-valued continuous function which defines the dynamics of node i ; $H \in R^{n \times n}$ is a positive definite diagonal matrix which describes the individual couplings between node i and node j ; and the coupling configuration matrix $A(t) = [a_{ij}(t)]_{n \times n} \in R^{N \times N}$ represents the time-varying topological structure of network (1) at time t . If there is a connection from node j to node i , $a_{ij}(t) = a_{ji}(t) > 0$ ($i \neq j$); otherwise, $a_{ij}(t) = a_{ji}(t) = 0$, ($i \neq j$). Finally, the row sum of $A(t)$ is zero; for example, $a_{ii}(t) = -\sum_{j=1, j \neq i}^N a_{ij}(t) < 0$, $i, j = 1, 2, \dots, N$. The term of time-varying delay $\tau(t)$ is subject to

$$\tau(t) \geq 0, \quad \dot{\tau}(t) \leq \mu < 1, \quad \forall t \geq 0. \quad (2)$$

Assumption 1. The weighted connection between any two nodes is bounded; that is,

$$0 \leq \underline{a}_{ij} \leq a_{ij}(t) \leq \bar{a}_{ij} < \infty, \quad i \neq j. \quad (3)$$

Lemma 2 (see [7]). *The eigenvalues of an irreducible matrix $H = (h_{ij})_{N \times N}$ with $h_{ij} \geq 0$, $1 \leq i, j \leq N$, and $i \neq j$ and $h_{ii} = -\sum_{j=1, j \neq i}^N h_{ij}$, $i = 1, 2, \dots, N$, have the following properties: all the eigenvalues of H are less than or equal to 0; namely $H \leq 0$.*

Corollary 3. Denote $\underline{A} = \{\underline{a}_{ij}\}_{n \times n}$, $\bar{A} = \{\bar{a}_{ij}\}_{n \times n}$, and $A(t) = \{a_{ij}(t)\}$; then the following properties can be derived:

$$0 \geq \underline{A} \geq A(t) \geq \bar{A}. \quad (4)$$

Proof. Denote $c_{ij} = \bar{a}_{ij} - a_{ij}(t)$, $C = \{c_{ij}\}$, and based on Assumption 1, we have $c_{ij} \geq 0$, $1 \leq i, j \leq N$, $i \neq j$, and $c_{ii} = \bar{a}_{ii} - a_{ii}(t) = -\sum_{j=1, j \neq i}^N \bar{a}_{ij} - \sum_{j=1, j \neq i}^N a_{ij}(t) = -\sum_{j=1, j \neq i}^N c_{ij}$, $i = 1, 2, \dots, N$. Thus, from Lemma 2, one can derive that $\underline{A}(t) \leq 0$, $\bar{A} = A(t) + C \leq 0$, and $C \leq 0$, and one can derive $\bar{A} \leq A(t)$. In a similar way, one can obtain $A(t) \leq \underline{A} \leq 0$. This completes the proof. $\square$

Definition 4 (see [15]). Assuming $s(t) \in R^n$ is any smooth dynamics, the controlled complex network described in (1) is considered to be globally asymptotically synchronized onto the homogenous state $s(t)$ if its solution satisfies $\lim_{t \rightarrow \infty} \|x_i(t) - s(t)\| = 0$, $i = 1, 2, \dots, N$, for any initial conditions.

To analyze the synchronization problem of dynamical networks, the problem can be effectively transformed into the stabilization analysis of the corresponding error networks, where the synchronization error vectors are defined as $e_i(t) = x_i(t) - s(t)$, $i = 1, \dots, N$. As the row sum of the coupling matrix is zero, one can derive $\sum_{j=1}^N a_{ij}(t) H s(t - \tau(t)) = 0$; thus the following dynamical error equation can be obtained:

$$\begin{aligned} \dot{e}_i(t) = & \dot{x}_i(t) - \dot{s}(t) \\ = & f_i(x_i(t)) - \dot{s}(t) + u_i(t) + c \sum_{j=1}^N a_{ij}(t) H x_j(t - \tau(t)) \\ = & f_i(x_i(t)) - \dot{s}(t) + u_i(t) \end{aligned}$$

$$\begin{aligned}
& + c \sum_{j=1}^N a_{ij}(t) H(x_j(t - \tau(t)) - s(t - \tau(t))) \\
& = f_i(x_i(t)) - \dot{s}(t) + c \sum_{j=1}^N a_{ij}(t) H e_j(t - \tau(t)) + u_i(t).
\end{aligned} \tag{5}$$

For synchronizing the dynamics of network (1) onto the given homogenous state $s(t)$, the suitable control laws $u_i(t)$ need to be designed such that the synchronization errors converge to zero in the norm sense. From the error system expressed in (5), the hybrid control scheme can be designed and described as follows:

$$u_i(t) = u_{i1}(t) + u_{i2}(t), \tag{6}$$

$$\begin{aligned}
u_{i1}(t) &= \dot{s}(t) - f_i(s(t)), \\
u_{i2}(t) &= -cd_i(x_i(t) - s(t)).
\end{aligned} \tag{7}$$

Remark 5. The control input $u_{i1}(t)$ can be considered as an open-loop control or entrainment control. The entrainment control based approaches provide an efficient tool to entrain nonlinear dynamical systems to any desired goal dynamics [21]. However, the open-loop control cannot guarantee the system stability during the control process. Thus, additional control is required to ensure the globally asymptotical system synchronization. In this paper, the open-plus-closed-loop (OPCL) [15] control method is adopted for the synchronization of complex networks given in (1). As a result, the entrainment control, static feedback control, and adaptive control are incorporated to regulate the network synchronization: the entrainment control $u_{i1}(t)$ provides the basis for developing the synchronization schemes for a network with nonidentical nodes and compensates the nonidentity; and the feedback control and adaptive control $u_{i2}(t)$ are used to ensure the global stability of the synchronization process.

Remark 6. Due to the fact that the network nodes are nonidentical, a common equilibrium solution of all isolated nodes can be hardly determined. Thus, when the complex network (1) has been synchronized onto an expected state $s(t)$, the input control (6) still needs to provide control signal to compensate the nonidentity, implying that the input control (6) is not zero when the network (1) is synchronized. In the case that all nonidentical network nodes have the same equilibrium, the input control (6) will be zero when the network (1) is synchronized.

Before presenting the derivation of the main results, the assumptions and lemmas are introduced as follows.

Assumption 7 (see [15]). For system (1), there exist the constants ω_i , $i = 1, 2, \dots, N$, such that

$$\begin{aligned}
(x - y)^T (f_i(x) - f_i(y)) &\leq \omega_i (x - y)^T (x - y), \\
\forall x, y \in R^n, \quad 1 \leq i \leq N.
\end{aligned} \tag{8}$$

It has been recognized that many typical benchmark chaotic systems, such as the Lorenz system, Chen system, Lü system, and the unified chaotic system, satisfy Assumption 7.

Assumption 8. The network (1) is considered to be always a connected network during the network dynamical changes (e.g., coupling strength variation) over time in this study; that is, the network has no isolated clusters. Otherwise, one may consider the synchronization on each connected component of the network separately.

Lemma 9 (see [22]). Let Q and R be two symmetric matrices, and matrix S has the compatible dimension. Then

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} < 0 \tag{9}$$

if and only if $R < 0$ and $Q - SR^{-1}S^T < 0$.

3. Main Results

In this section, we design and present two feedback control strategies (7) based on the hybrid control scheme (6). The sufficient conditions are derived to ensure the synchronization of time-varying complex network with nonidentical nodes and constant coupling time-delay. For the sake of clarity, denote $\psi_i^T(t) = [\bar{e}_i^T(t) \bar{e}_i^T(t - \tau)]$ and $\bar{e}_i(t) = [e_{1i}(t), e_{2i}(t), \dots, e_{N_i}(t)]^T$, which will be used in the rest of this section.

3.1. Synchronization Scheme Combining Static Feedback Control. In the control scheme (7), if $d_1, d_2, \dots, d_n$ are positive constants, the controller $u_{i2}(t) = -cd_i(x_i(t) - s(t))$ is considered as the linear static feedback controller and the error dynamical system can be written as the following form:

$$\begin{aligned}
\dot{e}_i(t) &= f_i(x_i(t)) - f_i(s(t)) \\
&+ c \sum_{j=1}^N a_{ij}(t) H e_j(t - \tau(t)) - cd_i(x_i(t) - s(t)).
\end{aligned} \tag{10}$$

Thus, we need to prove that, if $d_1, d_2, \dots, d_n$ are appropriately designed, the controller (7) can synchronize the network (1) to the given goal state $s(t)$.

Theorem 10. Suppose Assumptions 1, 7, and 8 and (2) hold, if $d_1, d_2, \dots, d_n$ are nonnegative constants and satisfy the following condition:

$$2W - 2cD + I + (1 - \mu)^{-1} c^2 h^2 \underline{A} \underline{A}^T < 0, \tag{11}$$

where $W = \text{diag}(\omega_1, \dots, \omega_N)$, $D = \text{diag}(d_1, d_2, \dots, d_n)$, and $h = \max(h_i)$, $i = 1, \dots, N$, and the hybrid control controller with static feedback control (7) guarantees that the controlled network (1) can be asymptotically synchronized to the given goal state $s(t)$.

Proof. We define the Lyapunov-Krasovskii function as

$$V(t) = \sum_{i=1}^N e_i^T(t) e_i(t) + \sum_{i=1}^N \int_{t-\tau(t)}^t e_i^T(s) e_i(s) ds. \tag{12}$$

Differentiating the function $V(t)$, we have

$$\begin{aligned}
 \dot{V}(t) &= 2 \sum_{i=1}^N e_i^T(t) \left[f_i(x_i(t)) - f_i(s(t)) \right. \\
 &\quad \left. + c \sum_{j=1}^N a_{ij}(t) H e_j(t - \tau(t)) - c d_i e_i(t) \right] \\
 &\quad + \sum_{i=1}^N \left[e_i^T(t) e_i(t) - (1 - \dot{\tau}(t)) \right. \\
 &\quad \left. \times e_i^T(t - \tau(t)) e_i(t - \tau(t)) \right] \\
 &\leq 2 \sum_{i=1}^N e_i^T(t) \omega_i e_i(t) \\
 &\quad + 2c \sum_{i=1}^N e_i^T(t) \sum_{j=1}^N a_{ij}(t) H e_j(t - \tau(t)) \\
 &\quad - 2c \sum_{i=1}^N e_i^T(t) d_i e_i(t) \\
 &\quad + \sum_{i=1}^N e_i^T(t) e_i(t) - (1 - \mu) \sum_{i=1}^N e_i^T(t - \tau(t)) e_i(t - \tau(t)) \\
 &= 2 \sum_{i=1}^n \bar{e}_i^T(t) W \bar{e}_i(t) \\
 &\quad + 2c \sum_{i=1}^n h_i \bar{e}_i^T(t) A(t) \bar{e}_i(t - \tau(t)) - 2c \sum_{i=1}^n \bar{e}_i^T(t) D \bar{e}_i(t) \\
 &\quad + \sum_{i=1}^n \bar{e}_i^T(t) \bar{e}_i(t) - (1 - \mu) \sum_{i=1}^n \bar{e}_i^T(t - \tau(t)) \bar{e}_i(t - \tau(t)) \\
 &= \sum_{i=1}^n \psi_i^T(t) \Omega_i \psi_i(t), \tag{13}
 \end{aligned}$$

where

$$\Omega_i = \begin{bmatrix} 2W - 2cD + I & c h_i A(t) \\ c h_i A^T(t) & -(1 - \mu) I \end{bmatrix}. \tag{14}$$

From Corollary 3, one can have $0 \leq A(t)A^T(t) \leq \underline{A}\underline{A}^T$, and hence the following can be derived

$$\begin{aligned}
 2W - 2cD + I + (1 - \mu)^{-1} c^2 h_i^2 A(t) A^T(t) \\
 \leq 2W - 2cD + I + (1 - \mu)^{-1} c^2 h^2 \underline{A}\underline{A}^T. \tag{15}
 \end{aligned}$$

Therefore, if inequality (11) holds, the inequality $2W - 2cD + I + (1 - \mu)^{-1} c^2 h_i^2 A(t) A^T(t) < 0$ holds, which guarantees $\dot{V}(t) < 0$. Based on LaSalle's invariance principle [23], starting from any initial condition, we have $e_i(t) \rightarrow 0$ as $t \rightarrow \infty$. Therefore, the controlled network (1) is globally asymptotically synchronized; that is, $x_i(t) \rightarrow s(t)$ as $t \rightarrow \infty$, $i = 1, \dots, N$. This completes the proof. $\square$

Remark 11. Suppose that the feedback gain matrix is $D = \text{diag}(d_1, d_2, \dots, d_n)$, $d_i > 0$, $i = 1, 2, \dots, N$. A Cost Function is defined as $CF = \sum_{i=1}^N d_i$. Then we can get the most efficient control strategy at the meaning of the defined Lyapunov function. The smaller the CF is, the more efficient a control strategy will be to achieve synchronization and the more easily the implementation will be realized. Thus we can get the most efficient control strategy in a certain meaning by minimizing the sum of elements of D , which satisfies condition (11), to make the system individually synchronized, and the pseudooptimal control cost is $CF_{\text{best}} = \min(\text{sum}(d_i))$.

3.2. Synchronization Scheme Combining Adaptive Feedback Control. In the control scheme (7), if $d_1, d_2, \dots, d_n$ are defined as

$$\dot{d}_i = \rho_i e_i^T(t) e_i(t), \quad i = 1, 2, \dots, N, \tag{16}$$

where ρ_i are positive constants, we say that the controller $u_{i2}(t) = -c d_i(x_i(t) - s(t))$ is an adaptive feedback controller. In the following, we will prove that the system (1) would get global synchronized to the given homogenous state $s(t)$ under the controller (6) with adaptive feedback control (17).

Theorem 12. Suppose that Assumptions 1, 7, and 8 and (2) hold, and if $d_1, d_2, \dots, d_n$ are defined as

$$\dot{d}_i = \rho_i e_i^T(t) e_i(t), \quad i = 1, 2, \dots, N, \tag{17}$$

where ρ_i are positive constants, the system can reach the global synchronization under the control scheme (6) with adaptive control (17). Also, the adaptive feedback gains converge to the certain bounded constants; that is, $d_i(t) \rightarrow l_i$ as $t \rightarrow \infty$, where $l_i \geq 0$, $i = 1, 2, \dots, N$.

Proof. Define a Lyapunov-Krasovskii function as follows:

$$\begin{aligned}
 V(t) &= \sum_{i=1}^N e_i^T(t) e_i(t) + \sum_{i=1}^N \int_{t-\tau(t)}^t e_i^T(s) e_i(s) ds \\
 &\quad + \frac{c}{\rho_i} \sum_{i=1}^N (d_i(t) - l_i)^2. \tag{18}
 \end{aligned}$$

Differentiating the function $V(t)$, we have

$$\begin{aligned}
 \dot{V}(t) &= 2 \sum_{i=1}^N e_i^T(t) \left[f_i(x_i(t)) - f_i(s(t)) \right. \\
 &\quad \left. + c \sum_{j=1}^N a_{ij}(t) H e_j(t - \tau(t)) - c d_i e_i(t) \right] \\
 &\quad + \sum_{i=1}^N \left[e_i^T(t) e_i(t) - (1 - \dot{\tau}(t)) \right. \\
 &\quad \left. \times e_i^T(t - \tau(t)) e_i(t - \tau(t)) \right] \\
 &\quad + 2c \sum_{i=1}^N (d_i(t) - l_i) e_i^T(t) e_i(t)
 \end{aligned}$$

$$\begin{aligned}
&\leq 2 \sum_{i=1}^N e_i^T(t) \omega_i e_i(t) \\
&\quad + 2c \sum_{i=1}^N e_i^T(t) \sum_{j=1}^N a_{ij}(t) H e(t - \tau(t)) \\
&\quad - 2c \sum_{i=1}^N e_i^T(t) l_i e_i(t) \\
&\quad + \sum_{i=1}^N e_i^T(t) e_i(t) - (1 - \mu) \sum_{i=1}^N e_i^T(t - \tau(t)) e_i(t - \tau(t)) \\
&= 2 \sum_{i=1}^n \bar{e}_i^T(t) W \bar{e}_i(t) + 2c \sum_{i=1}^n h_i \bar{e}_i^T(t) A(t) \bar{e}_i(t - \tau(t)) \\
&\quad - 2c \sum_{i=1}^n \bar{e}_i^T(t) L \bar{e}_i(t) \\
&\quad + \sum_{i=1}^n \bar{e}_i^T(t) \bar{e}_i(t) - (1 - \mu) \sum_{i=1}^n \bar{e}_i^T(t - \tau(t)) \bar{e}_i(t - \tau(t)) \\
&= \sum_{i=1}^n \psi_i^T(t) \Omega_i \psi_i(t),
\end{aligned} \tag{19}$$

$$\text{where } \Omega_i = \begin{bmatrix} 2W - 2cL + I & ch_i A(t) \\ ch_i A^T(t) & -(1 - \mu)I \end{bmatrix}.$$

According to Corollary 3, one can easily get that $0 \leq A(t)A^T(t) \leq \underline{A}\underline{A}^T$. Then we can derive the following inequality:

$$\begin{aligned}
&2W - 2cL + I + (1 - \mu)^{-1} c^2 h_i^2 A(t) A^T(t) \\
&\leq 2W - 2cL + I + (1 - \mu)^{-1} c^2 h^2 \underline{A}\underline{A}^T.
\end{aligned} \tag{20}$$

$$A(t) = \begin{bmatrix} -(3 + 0.5 \sin t + 2 \sin 10t + 0.5 \cos t) & 1 + \sin 10t & 0.5 + 0.5 \sin t & 0.5 + 0.5 \cos t & 1 + \sin 10t \\ * & -(1 + \sin 10t) & 0 & 0 & 0 \\ * & * & -(0.5 + 0.5 \sin t) & 0 & 0 \\ * & * & * & -(0.5 + 0.5 \cos t) & 0 \\ * & * & * & * & -(1 + \sin 10t) \end{bmatrix}. \tag{23}$$

The state-space functions and parameters of these nonidentical nodes are chosen as

$$f_1(x_1) = \begin{cases} 10(x_{12} - x_{11} - F(x_{11})) \\ x_{11} - x_{12} + x_{13} \\ -14.87x_{12}, \end{cases} \tag{24}$$

where $F(x_{11}) = -0.68x_{11} + 0.5(-1.27 + 0.68)(|x_{11} + 1| + |x_{11} - 1|)$,

$$f_2(x_2) = \begin{cases} 10(x_{22} - x_{21}) \\ 28x_{21} - x_{22} - x_{21}x_{23} \\ x_{21}x_{22} - \left(\frac{8}{3}\right)x_{23}, \end{cases}$$

Thus, the diagonal matrix L can be found to make inequality (21) hold,

$$2W - 2cL + I + (1 - \mu)^{-1} c^2 h^2 \underline{A}\underline{A}^T < 0, \tag{21}$$

which can guarantee that inequality $\Omega_i < 0$ holds. Then we have $\dot{V}(t) < 0$, so the controlled network (1) is globally asymptotically synchronized according to Lyapunov stability theory [23]. This completes the proof. $\square$

4. Numerical Simulation

In this section, two numerical simulation experiments are carried out to verify the effectiveness of the proposed synchronization control solutions.

4.1. Synchronizing Network by Hybrid Controller with Static Feedback Control. This numerical example aims to illustrate the effectiveness of the Corollary 3 (Theorem 10) presented in Section 3. Consider a network consisting 5 nonidentical nodes with time-varying network topology and coupling time-delay, as described by Chua's system, Lorenz system, and linear system. The complex network is given by

$$\dot{x}_i(t) = f_i(x_i(t)) + c \sum_{j=1}^5 a_{ij}(t) H x_j(t - \tau(t)), \tag{22}$$

where $c = 1$, $H = \text{diag}(1, 1, 1)$, $\tau = 0.6 + 0.4 \sin(t)$, and the time-varying connection matrix

$$\begin{aligned}
f_3(x_3) &= \begin{cases} -3x_{31} + x_{32} + 2x_{33} \\ x_{32} + 3x_{33} \\ 0.5x_{31} + 4x_{32} - 2x_{33}, \end{cases} \\
f_4(x_4) &= \begin{cases} 5x_{42} - 10x_{41} + 3(|x_{41} + 1| + |x_{41} - 1|) \\ 2x_{41} - 4x_{42} + 3x_{43} \\ -5x_{42}, \end{cases} \\
f_5(x_5) &= \begin{cases} x_{52} \\ x_{53} \\ -x_{51} - x_{52} - x_{53} + 4 \ln(x_{53} + \sqrt{1 + x_{53}^2}). \end{cases}
\end{aligned} \tag{25}$$

FIGURE 1: The trajectories of the network state without control.

To demonstrate the effectiveness of Theorem 10, we use the synchronization scheme (6) which includes static feedback control (7) to achieve the synchronization between network (22) and the solution $s(t)$ of the following Rössler system:

$$\begin{aligned}\dot{s}_1(t) &= -(s_2 + s_3), \\ \dot{s}_2(t) &= s_1 + 0.2s_2, \\ \dot{s}_3(t) &= s_3(s_1 - 5.7) + 0.2.\end{aligned}\quad (26)$$

The initial conditions of each node are chosen as follows: $x_1(0) = [0.6, -0.4, 0.2]$, $x_2(0) = [-0.5, 0.3, -0.3]$, $x_3(0) = [0.4, 0.7, 1]$, $x_4(0) = [-1, 0.5, 0.9]$, $x_5(0) = [0.2, -0.3, -0.8]$, and $s(0) = [0.06, 0.16, 0.52]$. In order to derive the static control gain, the parameters need to be determined to satisfy Assumption 1. Based on the work in [15], we can derive that $\omega_1 = 9.1$, $\omega_2 = 27.5$, $\omega_3 = 3.5$, $\omega_4 = 0.4$, and $\omega_5 = 3.1$. By using MATLAB LMI Toolbox, it can be calculated that $D = [86.3, 54.7, 15.7, 12.6, 30.3]$ and $CF_{\text{best}} = 199.6$ based on Theorem 10.

In Figure 1, the trajectories of the uncontrolled state of nodes are depicted which demonstrate the original behavior of the complex network (26). The simulation result with control inputs which are calculated by Corollary 3 is illustrated in Figure 2. It can be seen that the trajectories of system state are stabilized and approach to the same state orbits.

4.2. Synchronizing Network by Hybrid Controller with Adaptive Feedback Control. Here, we adopt the control scheme (6) which contains adaptive control (17) to achieve the synchronization between network (22) and a chaotic trajectory given by (26), that is, verifying the effectiveness of the control scheme by synchronizing the system and target trajectory given in Section 4.1.

The simulation result with control inputs which are calculated by Theorem 12 is presented in Figure 3, and the adaptive control gains are presented in Figure 4. It can be observed from Figure 3 that the trajectories of system state can be stabilized and converged to the given system trajectory.

FIGURE 2: The trajectories of the network state with the feedback control.

FIGURE 3: The trajectories of the network state with the adaptive control.

In addition, the adaptive gains reach some certain constant values and the complex network is synchronized.

5. Conclusions

In this paper, the synchronization of time-varying complex dynamical networks with nonidentical nodes and coupling time-delay is investigated where the coupling matrix is assumed to be uncontrollable and bounded in a certain range. By combining the compensation approach for the nonidentity and error feedback effects, we presented two efficient control approaches that can successfully synchronize the complex network onto any given smooth goal dynamics. In addition, the sufficient conditions have been derived to guarantee the global asymptotic stability throughout the network synchronization process. Finally, the simulation experiments are carried out to assess the performance and the

FIGURE 4: The adaptive gains.

numerical results confirm the effectiveness of the suggested control solutions. The conservation of the design approach for the controller needs to be further investigated and evaluated in future work, and more research effort needs to be made to improve the controller design with the minimum conservation.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

The authors would like to thank the anonymous reviewers for their invaluable comments. This work is supported in part by the National High-Tech R&D Program (863 plan) under the Grant no. 2012AA051704, the National Natural Science Foundation of China (under Grant no. 60804045 and no. 51107113), and the Science and Technology Plan of Zhejiang Province (no. 2013C31005).

References

- [1] L. D. F. Costa, O. N. Oliveira, G. Travieso et al., "Analyzing and modeling real-world phenomena with complex networks: a survey of applications," *Advances in Physics*, vol. 60, no. 3, pp. 329–412, 2011.
- [2] L. Lin, L. Chen, X. Fu, and S. Liu, "An improved wireless sensor network mode based on complex network theory," *Procedia Engineering*, vol. 24, pp. 360–364, 2011.
- [3] Y. Zhao, Z. Duan, G. Wen, and Y. Zhang, "Distributed finite-time tracking control for multi-agent systems: an observer-based approach," *Systems & Control Letters*, vol. 62, no. 1, pp. 22–28, 2013.
- [4] G. Wen, G. Hu, W. Yu, J. Cao, and G. Chen, "Consensus tracking for higher-order multi-agent systems with switching directed topologies and occasionally missing control inputs," *Systems & Control Letters*, vol. 62, no. 12, pp. 1151–1158, 2013.
- [5] W. Yu, G. Chen, and J. Lü, "On pinning synchronization of complex dynamical networks," *Automatica*, vol. 45, no. 2, pp. 429–435, 2009.
- [6] L. Deng, Z. Wu, and Q. Wu, "Pinning synchronization of complex network with non-derivative and derivative coupling," *Nonlinear Dynamics*, vol. 73, no. 1-2, pp. 775–782, 2013.
- [7] W. Yu, P. DeLellis, G. Chen, M. di Bernardo, and J. Kurths, "Distributed adaptive control of synchronization in complex networks," *IEEE Transactions on Automatic Control*, vol. 57, no. 8, pp. 2153–2158, 2012.
- [8] L. Lü, M. Yu, C. Li et al., "Projective synchronization of a class of complex network based on high-order sliding mode control," *Nonlinear Dynamics*, vol. 73, no. 1-2, pp. 411–416, 2013.
- [9] Y. Zhao, Z. Duan, G. Wen, and G. Chen, "Robust consensus tracking of multi-agent systems with uncertain Lur'e-type nonlinear dynamics," *IET Control Theory & Applications*, vol. 7, no. 9, pp. 1249–1260, 2013.
- [10] G. Hu, "Robust consensus tracking of a class of second-order multi-agent dynamic systems," *Systems & Control Letters*, vol. 61, no. 1, pp. 134–142, 2012.
- [11] J.-W. Xiao, Y. Huang, Y.-W. Wang, and J. O. Yi, "Synchronization of complex switched networks with two types of delays," *Neurocomputing*, vol. 74, no. 17, pp. 3151–3157, 2011.
- [12] T. H. Lee, D. H. Ji, J. H. Park, and H. Y. Jung, "Decentralized guaranteed cost dynamic control for synchronization of a complex dynamical network with randomly switching topology," *Applied Mathematics and Computation*, vol. 219, no. 3, pp. 996–1010, 2012.
- [13] T. H. Lee, J. H. Park, and Z. Wu, "Robust H_∞ decentralized dynamic control for synchronization of a complex dynamical network with randomly occurring uncertainties," *Nonlinear Dynamics*, vol. 70, no. 1, pp. 559–570, 2012.
- [14] D. W. Lee, W. J. Yoo, D. H. Ji, and J. H. Park, "Integral control for synchronization of complex dynamical networks with unknown non-identical nodes," *Applied Mathematics and Computation*, vol. 224, pp. 140–149, 2013.
- [15] Q. Song, J. Cao, and F. Liu, "Synchronization of complex dynamical networks with nonidentical nodes," *Physics Letters A*, vol. 374, no. 4, pp. 544–551, 2010.
- [16] M. Shi, J. Li, and C. He, "Synchronization of complex dynamical networks with nonidentical nodes and derivative coupling via distributed adaptive control," *Mathematical Problems in Engineering*, vol. 2013, Article ID 172608, 11 pages, 2013.
- [17] Y.-Q. Fan, Y.-H. Wang, Y. Zhang, and Q.-R. Wang, "The synchronization of complex dynamical networks with similar nodes and coupling time-delay," *Applied Mathematics and Computation*, vol. 219, no. 12, pp. 6719–6728, 2013.
- [18] L. Wang, W. Qian, and Q. Wang, "Bounded synchronization of a time-varying dynamical network with nonidentical nodes," *International Journal of Systems Science*, vol. 44, no. 12, Article ID 815825, 2013.
- [19] J. Zhao, D. J. Hill, and T. Liu, "Global bounded synchronization of general dynamical networks with nonidentical nodes," *IEEE Transactions on Automatic Control*, vol. 57, no. 10, pp. 2656–2662, 2012.
- [20] J. Zhao, D. J. Hill, and T. Liu, "Stability of dynamical networks with non-identical nodes: a multiple V -Lyapunov function method," *Automatica*, vol. 47, no. 12, pp. 2615–2625, 2011.
- [21] F. Liu, Q. Song, and J. Cao, "Improvements and applications of entrainment control for nonlinear dynamical systems," *Chaos*, vol. 18, no. 4, Article ID 043120, 13 pages, 2008.

- [22] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, vol. 15 of *SIAM Studies in Applied Mathematics*, SIAM, Philadelphia, Pa, USA, 1994.
- [23] J. Hale, *Theory of Functional Differential Equations*, Springer, New York, NY, USA, 1993.