Research Article

H_∞ Control of Supply Chain Based on Switched Model of Stock Level

Junzhi Luo and Wanly Yang

School of Department of Fundamental Courses, Academy of Armored Force Engineering, Beijing 100072, China

Correspondence should be addressed to Junzhi Luo; zgyljz@126.com

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This paper is concerned with the problem of H_∞ control for a class of discrete supply chain systems. A new method based on network control technique is presented to address this issue. Supply chain systems are modeled as networked systems with stochastic time delay. Sufficient conditions for H_∞ controller design are given in terms of a set of linear matrix inequalities, based on which the mean-square asymptotic stability as well as H_∞ performance is satisfied for such systems. Simulation results are provided to demonstrate the effectiveness of the proposed method.

1. Introduction

A supply chain is a network of facilities and distribution entities (suppliers, manufacturers, distributors, and retailers) that manufactures raw materials into intermediate and finished products and distributes finished products to customers [1]. Nowadays, with the globalization of business, the phenomena of increasing competition among entities and more customer demanding lead to more complex dynamic behaviors of supply chain, such as demand fluctuation and lead-time delay. The static model is insufficient to model such systems with the dynamic characteristics; as a result, the dynamic analysis and control of supply chain systems have attracted more and more interests and attentions; see, for example, [2–13].

Various dynamic models have been employed for modeling and analyzing supply chain systems, which are mainly classified into continuous differential equations and discrete difference equations. For example, the piecewise-linear H_∞ controller was designed based on continuous differential equation for a production system in [5], the discrete difference models were provided to analyze system performance in [6–8, 10, 11], and the robust control strategy based on discrete models was given with uncertainty in [12, 13]. It should be noted that the information exchanging of production schedule, inventory positions, and procurement plan and customer demand, making decision, and implementing decision are

usually realized by network. In essence, supply chain systems are network control systems; the introduction of network facilitates the analysis and synthesis of supply chain systems.

As far as the network control systems (NCS) are concerned, they are spatially distributed systems in which the communication between sensors, actuators, and controllers occurs through a shared band-limited digital communication network. In the past decades, due to attractive features such as increased system flexibility, simple installation and maintenance, lower cost, and reduced weight and power, the study of NCSs has attracted considerable attention; see, for example, [14–18]. It is well recognized that the presence of network usually leads to the signal transmission delays. A common assumption in most of existing results on network-based control is that the network-induced delay has upper bounded or both lower and upper bounded. This assumption has been used in a great number of papers, such as [19, 20]. In fact, due to the unpredictability of the network environment, network-induced delay is random by nature. Furthermore, the disturbance of external factors or network congestion will make the induced delay become big, even bigger than upper bound of time delay. For these reasons, some researchers are devoted to the study of the problem of random delays in stochastic control systems [15, 21–25]. On the other hand, the switching among the different agents is the important feature of a supply chain system. It is essential

to consider the switching behavior when modeling a supply chain system. The great advance in the study of switched delay systems [26, 27] helps us to the modeling, analysis, and synthesis of a supply chain system. However, to the authors' best knowledge, the influence of the network factors and the switching behavior on supply chain systems is scarcely investigated in the existing literature. This motivates the present study.

In this paper, we are interested in the study of H_∞ control for a class of discrete supply chain systems. The following aspects are well addressed. Firstly, supply chain systems are modeled as switched network model with stochastic delay when considering the existence of switching and the random of network-induced delay. Discrete difference equations are utilized to describe the supply chain system, which is distinct from existed results that are concerned with the models of such systems. Secondly, sufficient conditions are obtained based on Lyapunov stability theory, which guarantee mean-square stability, and the bullwhip effect of supply chain system is studied in the sense of the stochastic H_∞ performance. Lastly, simulation results are provided to demonstrate the effectiveness of the proposed method.

The organization of this paper is as follows. In Section 2, the switched network model with stochastic delay is given to describe supply chain. In Section 3, the mean-square stability with H_∞ performance for supply chain systems is analyzed based on network model. Then, an example is provided to illustrate the effectiveness of the proposed method in Section 4. Finally, Section 5 concludes this paper.

Notations. In this paper, we use $P > 0$ ($\geq, <, \leq 0$) to denote a positive definite (semidefinite, negative definite, and seminegative definite) matrix P . “ E ” stands for mathematic expectation of random vector. The superscript “ T ” stands for matrix transpose and the symmetric terms in a matrix are denoted by “ $*$ ”; “ $\|\cdot\|$ ” denotes the L_2 -norm which is given by $\|\omega(k)\| = (\sum_{k=0}^{\infty} \omega^T(k)\omega(k))^{1/2}$.

2. Problem Formulating

Consider the following supply chain system described by

$$\begin{aligned} x(k+1) &= \sum_{i=1}^3 \sigma_i(k) A_i x(k) + \sum_{i=1}^3 \sigma_i(k) B_i u(k) \\ &\quad + \sum_{i=1}^3 \sigma_i(k) F_i d(k), \\ z(k) &= \sum_{i=1}^3 \sigma_i(k) C_i x(k) + \sum_{i=1}^3 \sigma_i(k) D_i u(k), \end{aligned} \quad (1)$$

where $x(k) \in \mathbb{R}$, $u(k) \in \mathbb{R}$, and $d(k) \in \mathbb{R}$ denote the stock level, manufactured level, and customers demand, respectively. From the point view of control system, they are state variable, control input, and exogenous disturbance. $z(k) \in \mathbb{R}$ is output. $\sigma_i(k)$ is a switching signal which takes its value in the finite set $M = \{1, 2, 3\}$. Moreover, $\sigma_i(k) = i$ denotes that the i th subsystem is activated.

Before giving the switching rules, we define the switching function

$$\text{switch}(s) = \begin{cases} 1 & s > 0 \\ 0 & s \leq 0. \end{cases} \quad (2)$$

Now, the switched systems are given as follows.

- (i) If $\text{switch}[x(k) - L] = 1$, where L is a positive scalar, that is, the stock level is more than the value L of warning, some of the produced parts are being stored, the first subsystem is active.
- (ii) If $\text{switch}[x(k)(L - x(k))] = 1$, that is, the stock level is between 0 and the value L of warning, the second subsystem is active; otherwise, the third subsystem is active if $\text{switch}[-x(k)] = 1$.

In this paper, state feedback H_∞ controller based on networked control will be designed to stabilize the system (1), which takes the following form:

$$u(k) = K_o x(k - d_k), \quad (3)$$

where d_k is network-induced delay.

Substituting the controllers (3) with system (1), the augmented system (1) with delays gives rise to the following system:

$$\begin{aligned} x(k+1) &= \sum_{i=1}^3 \sigma_i(k) A_i x(k) + \sum_{i=1}^3 \sigma_i(k) B_i K_i x(k - d_k) \\ &\quad + \sum_{i=1}^3 \sigma_i(k) F_i d(k), \\ z(k) &= \sum_{i=1}^3 \sigma_i(k) C_i x(k) + \sum_{i=1}^3 \sigma_i(k) D_i K_i x(k - d_k). \end{aligned} \quad (4)$$

Here, we assume that the network-induced delay d_k takes values in a finite set, that is, $d_k \in \{\tau_1, \tau_2, \dots, \tau_q\}$, and the occurrence probability of the delays τ_i is α_j , that is, $\text{Prob}\{d_k = \tau_j\} = \alpha_j$, where α_j is a positive scalar and $\sum_{j=1}^q \alpha_j = 1$.

Due to the random of d_k , the random vector $\Pi_{d_k=\tau_j}$ is defined as follows:

$$\Pi_{d_k=\tau_j} = \begin{cases} 1 & d_k = \tau_j \\ 0 & d_k \neq \tau_j. \end{cases} \quad (5)$$

Then, it is easy to get

$$E\{\Pi_{d_k=\tau_j}\} = \text{Prob}\{d_k = \tau_j\} = \alpha_j, \quad j = 1, 2, \dots, q. \quad (6)$$

By introducing random delay parameters, the supply chain system is transformed into a multiple delay system. The closed-loop supply chain system (1) can be rewritten as

$$\begin{aligned}
 x(k+1) &= \sum_{i=1}^3 \sigma_i(k) A_i x(k) \\
 &+ \sum_{i=1}^3 \sigma_i(k) \sum_{j=1}^q \Pi_{d_k=\tau_j} B_i K_i x(k-\tau_j) \\
 &+ \sum_{i=1}^3 \sigma_i(k) F_i d(k), \\
 z(k) &= \sum_{i=1}^3 \sigma_i(k) C_i x(k) \\
 &+ \sum_{i=1}^3 \sigma_i(k) \sum_{j=1}^q \Pi_{d_k=\tau_j} D_i K_i x(k-\tau_j).
 \end{aligned} \quad (7)$$

Before proceeding further, some definitions are given in the following.

Definition 1. The closed-loop supply chain system (7) with $d(k) = 0$ is said to be mean-square stable if, for any $\epsilon > 0$, there is $\delta(\epsilon) > 0$, such that $E\{\|x(k)\|^2\} < \epsilon$, $k > 0$ when $E\{\|x(0)\|^2\} < \delta(\epsilon)$.

In addition, if

$$\lim_{k \rightarrow \infty} E\{\|x(k)\|^2\} = 0 \quad (8)$$

for any initial conditions, the closed-loop system (7) is said to be globally mean-square asymptotically stable (GMSAS).

Definition 2. The closed-loop supply chain system (7) is said to satisfy the H_∞ performance, if

- (1) the augmented closed-loop system (7) is mean-square asymptotically stable with $d(k) = 0$,
- (2) the closed-loop system (7) satisfies $E\{\|z(k)\|^2\} \leq \gamma^2 \|d(k)\|^2$ for all nonzero $d(k) \neq 0$ under the zero initial condition.

Definition 3. The bullwhip effect is the amplification phenomenon which is described by the sum of the ratio of $z(k)$ to the customer's demand fluctuation $d(k)$; that is,

$$\gamma^2 = \sum_k \frac{\|z(k)\|^2}{\|d(k)\|^2}, \quad (9)$$

where $z(k)$ refers to production and inventory vector, $d(k)$ refers to demand fluctuation vector, time serial is $k = 1, 2, \dots, N$, and γ refers to the bullwhip effect.

Remark 4. For the existence of $d(k)$, the effect of $d(k)$ to $z(k)$ is also transformed into the problem of H_∞ performance analysis for the closed-loop supply chain systems; that is, the effect of bullwhip can be studied by the problem of H_∞ performance analysis.

Our purpose is to design a controller (3) such that the effect of $d(k)$ to the output $z(k)$ is under a desired level in the H_∞ performance.

3. Analysis on H_∞ Performance of Supply Chain

In this section, we will give the sufficient conditions such that the H_∞ performance can be guaranteed for system (7). The following theorem will play an important role in the controller design.

Theorem 5. For given scalars $\gamma > 0$, if there exist scholar $\mu > 1$ and symmetric positive definite matrices $P_i > 0$ ($i = 1, 2, 3$), $Q_j > 0$ ($j = 1, 2, \dots, q$), such that the following matrix inequalities simultaneously hold

$$\Omega_{ij} = \begin{bmatrix} \Xi_1 & \Xi_2 & \Xi_4 \\ * & \Xi_3 & 0 \\ * & * & \Xi_5 \end{bmatrix} < 0, \quad (10)$$

$$P_i \leq \mu P_i, \quad \forall (i, l) \in M \times M, \quad (11)$$

where

$$\begin{aligned}
 \Xi_1 &= \text{diag} \left\{ -P_i + \sum_{j=1}^q Q_j, -Q_1, -Q_2, \dots, -Q_q, -\gamma^2 I \right\}, \\
 \Xi_2 &= \begin{bmatrix} \sqrt{\alpha_1} A_i^T & \sqrt{\alpha_2} A_i^T & \cdots & \sqrt{\alpha_q} A_i^T & 0 \\ \sqrt{\alpha_1} K_i^T B_i^T & 0 & \cdots & 0 & 0 \\ * & \sqrt{\alpha_2} K_i^T B_i^T & \cdots & 0 & 0 \\ * & * & \ddots & \vdots & \vdots \\ * & * & * & \sqrt{\alpha_q} K_i^T B_i^T & 0 \\ * & * & * & * & F_i^T \end{bmatrix}, \\
 \Xi_3 &= \text{diag} \{ -P_i^{-1}, -P_i^{-1}, \dots, -P_i^{-1}, -P_i^{-1} \}, \\
 \Xi_4 &= \begin{bmatrix} \sqrt{\alpha_1} C_i^T & \sqrt{\alpha_2} C_i^T & \cdots & \sqrt{\alpha_q} C_i^T \\ \sqrt{\alpha_1} K_i^T D_i^T & 0 & \cdots & 0 \\ * & \sqrt{\alpha_2} K_i^T D_i^T & \cdots & 0 \\ * & * & \ddots & \vdots \\ * & * & * & \sqrt{\alpha_q} K_i^T D_i^T \end{bmatrix}, \\
 \Xi_5 &= \text{diag} \{ -I, -I, \dots, -I \},
 \end{aligned} \quad (12)$$

then system (7) is mean-square stable and satisfies H_∞ performance.

Proof. First, we consider system (7) with $d(k) = 0$.

Construct a Lyapunov function in the form of

$$V_{\sigma_i(k)}(x_k) = V_1(x_k) + V_2(x_k), \quad (13)$$

in which

$$\begin{aligned} V_1(x_k) &= x^T(k) P_o x(k), \\ V_2(x_k) &= \sum_{j=1}^q \sum_{l=k-\tau_j}^{k-1} x^T(l) Q_j x(l). \end{aligned} \quad (14)$$

It is easy to see that

$$\begin{aligned} E(\Delta V_1 | x_k) &= E\{x^T(k+1) P_i x(k+1)\} - x^T(k) P_i x(k) \\ &= x^T(k) (A_i^T P_i A_i - P_i) x(k) \\ &\quad + 2 \sum_{j=1}^q \alpha_j x^T(k) A_i^T P_i K_i x(k - \tau_j) \\ &\quad + \sum_{j=1}^q \alpha_j x^T(k - \tau_j) K_i^T B_i^T P_i B_i K_i x(k - \tau_j), \\ E(\Delta V_2 | x_k) &= \sum_{j=1}^q [x^T(k) Q_j x(k) - x^T(k - \tau_j) Q_j x(k - \tau_j)] \\ E(\Delta V | x_k) &= x^T(k) \left(A_i^T P_i A_i - P_i + \sum_{j=1}^q Q_j \right) x(k) \\ &\quad + 2 \sum_{j=1}^q \alpha_j x^T(k) A_i^T P_i K_i x(k - \tau_j) \\ &\quad + \sum_{j=1}^q \alpha_j x^T(k - \tau_j) K_i^T B_i^T P_i B_i K_i x(k - \tau_j) \\ &\quad - \sum_{j=1}^q x^T(k - \tau_j) Q_j x(k - \tau_j) \\ &= \eta^T(k) \Gamma_{ij} \eta(k), \end{aligned} \quad (15)$$

where

$$\begin{aligned} \eta(k) &= [x^T(k) \ x^T(k - \tau_1) \ x^T(k - \tau_2) \ \cdots \ x^T(k - \tau_q)]^T, \\ \Gamma_{ij} &= \begin{bmatrix} \Gamma_{ij}^{11} & \Gamma_{ij}^{12} & \Gamma_{ij}^{13} & \cdots & \Gamma_{ij}^{15} \\ * & \Gamma_{ij}^{22} & \Gamma_{ij}^{23} & \cdots & \Gamma_{ij}^{25} \\ * & * & \Gamma_{ij}^{33} & \cdots & \Gamma_{ij}^{35} \\ * & * & * & \ddots & \vdots \\ * & * & * & \cdots & \Gamma_{ij}^{55} \end{bmatrix}, \\ \Gamma_{ij}^{11} &= A_i^T P_i A_i - P_i + \sum_{j=1}^q Q_j, \end{aligned}$$

$$\begin{aligned} \Gamma_{ij}^{12} &= \alpha_1 K_i^T P_i B_i K_i, \\ \Gamma_{ij}^{13} &= \alpha_2 K_i^T P_i B_i K_i, \\ \Gamma_{ij}^{15} &= \alpha_q K_i^T P_i B_i K_i, \\ \Gamma_{ij}^{22} &= \alpha_1 K_i^T B_i^T P_i B_i K_i - Q_1, \\ \Gamma_{ij}^{23} &= 0, \\ \Gamma_{ij}^{25} &= 0, \\ \Gamma_{ij}^{33} &= \alpha_2 K_i^T B_i^T P_i B_i K_i - Q_2, \\ \Gamma_{ij}^{35} &= 0, \\ \Gamma_{ij}^{55} &= \alpha_q K_i^T B_i^T P_i B_i K_i - Q_q. \end{aligned} \quad (16)$$

According to Schur complement, we can derive that $\Gamma_{ij} < 0$ which is equivalent to $\Lambda_{ij} < 0$, where

$$\begin{aligned} \Lambda_{ij} &= \begin{bmatrix} \bar{\Xi}_1 & \bar{\Xi}_2 \\ * & \bar{\Xi}_3 \end{bmatrix} < 0, \\ \bar{\Xi}_1 &= \text{diag} \left\{ -P_i + \sum_{j=1}^q Q_j, -Q_1, -Q_2, \dots, -Q_q \right\}, \\ \bar{\Xi}_2 &= \begin{bmatrix} \sqrt{\alpha_1} A_i^T & \sqrt{\alpha_2} A_i^T & \cdots & \sqrt{\alpha_q} A_i^T \\ \sqrt{\alpha_1} K_i^T B_i^T & 0 & \cdots & 0 \\ * & \sqrt{\alpha_2} K_i^T B_i^T & \cdots & 0 \\ * & * & \ddots & \vdots \\ * & * & * & \sqrt{\alpha_q} K_i^T B_i^T \end{bmatrix}, \\ \bar{\Xi}_3 &= \text{diag} \{ -P_i^{-1}, -P_i^{-1}, \dots, -P_i^{-1} \}. \end{aligned} \quad (17)$$

At the same time, we can know $\Lambda_{ij} < 0$ if $\Omega_{ij} < 0$. $\square$

Let $0 = k_0 < k_1 < \cdots < k_l < \cdots < +\infty$, k_i ($i = 0, 1, 2, \dots, l$) be switching times over the interval $[0, k]$, $N_\sigma(s, t)$ be the switching numbers of switching signal $\sigma_i(k)$ over the interval $[s, t]$.

If $E[\Delta V | k] < 0$, there exists $0 < \alpha < 1$, such that the following inequality holds:

$$E(V_{\sigma_i(k+1)}(x_{k+1}) | x_k) \leq \alpha V_{\sigma_i(k)}(x_k), \quad \forall k \in [k_i, k_{i+1}). \quad (18)$$

Then

$$\begin{aligned} E(V_{\sigma_i(k)}(x_k)) &\leq \alpha^{k-k_i} V_{\sigma_i(k_i)}(x_{k_i}) \\ &\leq \alpha^{k-k_i} \mu V_{\sigma_i(k_{i-1})}(x_{k_i}) \\ &\leq \alpha^{k-k_i} \mu \alpha^{(k_i-k_{i-1})} V_{\sigma_i(k_{i-1})}(x_{k_{i-1}}) \\ &\leq \cdots \end{aligned}$$

$$\begin{aligned}
&\leq \alpha^{k-k_0} \mu^i V_{\sigma_i(k_0)}(x_{k_0}) \\
&= \alpha^k \mu^{N_\sigma(0,k)} V_{\sigma_i(0)}(x_0), \\
E \left(\sum_{k=0}^N V_{\sigma_i(k)}(x_k) \right) \\
&\leq V_{\sigma_i(0)}(x_0) + \alpha \mu^{N_\sigma(0,1)} V_{\sigma_i(0)}(x_0) \\
&\quad + \alpha^2 \mu^{N_\sigma(0,2)} V_{\sigma_i(0)}(x_0) + \cdots + \alpha^N \mu^{N_\sigma(0,N)} V_{\sigma_i(0)}(x_0) \\
&\leq \mu^{T_{\max}} (1 + \alpha + \alpha^2 + \cdots + \alpha^N) V_{\sigma_i(0)}(x_0) \\
&\leq \mu^{T_{\max}} \frac{1 - \alpha^{N+1}}{1 - \alpha} V_{\sigma_i(0)}(x_0),
\end{aligned} \tag{19}$$

where $T_{\max} = N_\sigma(0, N)$. Since $Q_j > 0$, $P_i > 0$, it can be given that

$$\begin{aligned}
&\lim_{k \rightarrow \infty} E \left\{ \sum_{k=0}^N x(k)^T x(k) \mid x_0 \right\} \\
&\leq \mu^{T_{\max}} \frac{1}{(1 - \alpha) \lambda_{\min}(P_0)} V_{\sigma_i(0)}(x_0).
\end{aligned} \tag{20}$$

Furthermore, we can get

$$\lim_{k \rightarrow \infty} E \{ \|x(k)\|^2 \} = 0. \tag{21}$$

Based on Definition 1, we can derive that system (7) is mean-square asymptotically stable.

Second, we consider system (7) with $d(k) \neq 0$:

$$\begin{aligned}
&E [z(k)^T z(k)] - \gamma^2 d(k)^T d(k) + E [\Delta V(k)] \\
&= E \left[C_i x(k) + \sum_{j=1}^q \Pi_{d_k=\tau_j} D_i K_i x(k - \tau_j) \right]^T \\
&\quad \times \left[C_i x(k) + \sum_{j=1}^q \Pi_{d_k=\tau_j} D_i K_i x(k - \tau_j) \right] \\
&\quad + x(k)^T A_i^T P_i F_i d(k) - \gamma^2 d(k)^T d(k) \\
&\quad + d(k)^T F_i^T P_i F_i d(k) \\
&\quad + x^T(k) \left[A_i^T P_i A_i - P_i + \sum_{j=1}^q Q_j \right] x(k) \\
&\quad + 2 \sum_{j=1}^q \alpha_j x^T(k) A_i^T P_i K_i x(k - \tau_j) \\
&\quad + \sum_{j=1}^q \alpha_j x^T(k - \tau_j) K_i^T B_i^T P_i B_i K_i x(k - \tau_j) \\
&\quad + \sum_{j=1}^q \alpha_j x^T(k - \tau_j) K_i^T B_i^T P_i F_i d(k)
\end{aligned}$$

$$\begin{aligned}
&- \sum_{j=1}^q x^T(k - \tau_j) Q_j x(k - \tau_j) \\
&= \bar{\eta}^T(k) \Delta_{ij} \bar{\eta}(k),
\end{aligned} \tag{22}$$

where

$$\begin{aligned}
&\bar{\eta}(k) \\
&= [x^T(k) \quad x^T(k - \tau_1) \quad x^T(k - \tau_2) \cdots x^T(k - \tau_q) \quad d^T(k)]^T,
\end{aligned}$$

$$\Delta_{ij} = \begin{bmatrix} \Delta_{ij}^{11} & \Delta_{ij}^{12} & \Delta_{ij}^{13} & \cdots & \Delta_{ij}^{15} & \Delta_{ij}^{16} \\ * & \Delta_{ij}^{22} & \Delta_{ij}^{23} & \cdots & \Delta_{ij}^{25} & \Delta_{ij}^{26} \\ * & * & \Delta_{ij}^{33} & \cdots & \Delta_{ij}^{35} & \Delta_{ij}^{36} \\ * & * & * & \ddots & \vdots & \vdots \\ * & * & * & * & \Delta_{ij}^{55} & \Delta_{ij}^{56} \\ * & * & * & * & * & \Delta_{ij}^{66} \end{bmatrix},$$

$$\Delta_{ij}^{11} = -P_i + \sum_{j=1}^q Q_j + A_i^T P_i A_i + C_i^T C_i,$$

$$\Delta_{ij}^{12} = \alpha_1 A_i^T P_i B_i K_i + \alpha_1 C_i^T D_i K_i,$$

$$\Delta_{ij}^{13} = \alpha_2 A_i^T P_i B_i K_i + \alpha_2 C_i^T D_i K_i,$$

$$\Delta_{ij}^{15} = \alpha_q A_i^T P_i B_i K_i + \alpha_q C_i^T D_i K_i,$$

$$\Delta_{ij}^{16} = A_i^T P_i F_i,$$

$$\Delta_{ij}^{22} = \alpha_1 K_i^T B_i^T P_i B_i K_i - Q_1 + \alpha_1 K_i^T D_i^T D_i K_i,$$

$$\Delta_{ij}^{23} = \alpha_1 \alpha_2 K_i^T B_i^T P_i B_i K_i + \alpha_1 \alpha_2 K_i^T D_i^T D_i K_i,$$

$$\Delta_{ij}^{25} = \alpha_1 \alpha_q K_i^T B_i^T P_i B_i K_i + \alpha_1 \alpha_q K_i^T D_i^T D_i K_i,$$

$$\Delta_{ij}^{26} = \alpha_1 K_i^T B_i^T P_i F_i,$$

$$\Delta_{ij}^{33} = \alpha_2 K_i^T B_i^T P_i B_i K_i - Q_2 + \alpha_2 K_i^T D_i^T D_i K_i,$$

$$\Delta_{ij}^{35} = \alpha_2 \alpha_q K_i^T B_i^T P_i B_i K_i + \alpha_2 \alpha_q K_i^T D_i^T D_i K_i,$$

$$\Delta_{ij}^{36} = \alpha_q K_i^T B_i^T P_i F_i,$$

$$\Delta_{ij}^{55} = \alpha_q K_i^T B_i^T P_i B_i K_i - Q_q + \alpha_q K_i^T D_i^T D_i K_i,$$

$$\Delta_{ij}^{56} = \alpha_q K_i^T B_i^T P_i F_i,$$

$$\Delta_{ij}^{66} = F_i^T P_i F_i - \gamma^2 I.$$

(23)

By Schur complement, we can obtain

$$\Delta_{ij} < 0 \tag{24}$$

which is equivalent to

$$\Omega_{ij} < 0. \quad (25)$$

Let $x(0) = 0$, for $E[\Delta V(k)] < 0$; then we have

$$E \left\{ \sum_{k=1}^{\infty} z(k)^T z(k) \right\} \leq \sum_{k=1}^{\infty} \gamma^2 d(k) d(k); \quad (26)$$

that is,

$$E \{ \|z(k)\|^2 \} \leq \gamma^2 \|d(k)\|^2. \quad (27)$$

According to the definition of H_{∞} performance, we can know that system (7) with $d(k) \neq 0$ satisfies H_{∞} performance.

It is important to note that the conditions of Theorem 5 are not linear matrix inequalities; the following Theorem 6 will give another equivalent condition.

Theorem 6. For given scalars $\gamma > 0$, if there exist scholar $\mu > 1$ and symmetric positive definite matrices $X_i > 0$, $Z_{ij} > 0$, $i = 1, 2, 3$, $j = 1, 2, \dots, q$, such that the following linear matrix inequalities simultaneously hold

$$\Pi_{ij} = \begin{bmatrix} \Lambda_1 & \Lambda_2 & \Lambda_4 \\ * & \Lambda_3 & 0 \\ * & * & \Lambda_5 \end{bmatrix} < 0, \quad (28)$$

$$X_l \leq \mu X_i, \quad \forall (i, l) \in M \times M, \quad (29)$$

where

$$\Lambda_1 = \text{diag} \left\{ -X_i + \sum_{i=1}^q Z_{ij}, -Z_{i1}, -Z_{i2}, \dots, -Z_{iq}, -\gamma^2 I \right\},$$

$$\Lambda_2 = \begin{bmatrix} \sqrt{\alpha_1} X_i A_i^T & \sqrt{\alpha_2} X_i A_i^T & \cdots & \sqrt{\alpha_q} X_i A_i^T & 0 \\ \sqrt{\alpha_1} Y_i^T B_i^T & 0 & \cdots & 0 & 0 \\ * & \sqrt{\alpha_2} Y_i^T B_i^T & \cdots & 0 & 0 \\ * & * & \ddots & 0 & 0 \\ * & * & * & \sqrt{\alpha_q} Y_i^T B_i^T & 0 \\ * & * & * & * & Y_i^T F_i^T \end{bmatrix},$$

$$\Lambda_3 = \text{diag} \{ -X_i, -X_i, \dots, -X_i, -X_i \},$$

$$\Lambda_4 = \begin{bmatrix} \sqrt{\alpha_1} X_i C_i^T & \sqrt{\alpha_2} X_i C_i^T & \cdots & \sqrt{\alpha_q} X_i C_i^T \\ \sqrt{\alpha_1} Y_i^T D_i^T & 0 & \cdots & 0 \\ * & \sqrt{\alpha_2} Y_i^T D_i^T & \cdots & 0 \\ * & * & \ddots & 0 \\ * & * & * & \sqrt{\alpha_q} Y_i^T D_i^T \end{bmatrix},$$

$$\Lambda_5 = \text{diag} \{ -I, -I, \dots, -I \},$$

(30)

then, system (7) satisfies H_{∞} performance. Furthermore, the gain of controller can be given by $K_i = Y_i^T X_i^{-1}$.

Proof. Left and right multiplying (10) by the following matrix

$$\text{diag} \{ -P_i^{-1}, -P_i^{-1}, \dots, -P_i^{-1}, -P_i^{-1}, I, I, \dots, I, I, \dots, I \} \quad (31)$$

and letting

$$X_i = P_i^{-1}, \quad Y_i = P_i^{-1} K_i^T, \quad Z_{ij} = P_i^{-1} Q_j^T P_i^{-1}, \quad (32)$$

we have

$$\Pi_{ij} < 0. \quad (33)$$

Meanwhile, left and right multiplying (11) by P_i^{-1} and P_l^{-1} gives rise to $X_l \leq \mu X_i$. According to condition (28) and (29) of Theorem 6, it is obvious that system (7) maintains H_{∞} performance. The proof is completed. $\square$

Remark 7. Let $q = 1$; the stochastic model of system (7) can be transformed into deterministic model. The stochastic model of system (7) is more universal than deterministic model. Moreover, the conditions of Theorems 5 and 6 have more applications than the one given based on the deterministic model.

4. Numerical Example

In this section, we will give a numerical example to demonstrate the effectiveness of the provided method for supply chain system modeled as network control system.

Example 8. We take the coefficient matrixes of system as follows:

$$\begin{aligned} A_1 &= [0.8], & A_2 &= [0.9], & A_3 &= [0]; \\ B_1 &= [0], & B_2 &= [0.1], & B_3 &= [0.1]; \\ C_1 &= [-0.05], & C_2 &= [-0.05], & C_3 &= [0]; \\ D_1 &= [0], & D_2 &= [-0.25], & D_3 &= [-0.32]; \\ F_1 &= [-1], & F_2 &= [-1], & F_3 &= [-1]. \end{aligned} \quad (34)$$

Let $\tau_1 = 1$, $\tau_2 = 2$, and $\tau_3 = 3$; $\alpha_1 = 0.1$, $\alpha_2 = 0.8$, and $\alpha_3 = 0.1$; and $\gamma = 1$. Solving the LMIs (28) with the help of the Matlab LMI Toolbox, we can obtain

$$\begin{aligned} X_1 &= [18.7701], & X_2 &= [20.1826], \\ X_3 &= [21.2242]; \\ Z_{11} &= Z_{12} = Z_{13} = [1.5447], \\ Z_{21} &= Z_{22} = Z_{23} = [0.6837], \\ Z_{31} &= Z_{32} = Z_{33} = [5.7842]; \\ Y_1 &= [13.002]; & Y_2 &= [14.0007]; \\ Y_3 &= [14.9991]. \end{aligned} \quad (35)$$

FIGURE 1: The state variable of system (7).

FIGURE 2: The control variable of system (7).

By further calculating, we obtain the following gain matrices for the H_∞ controller (3):

$$K_1 = [0.6926]; \quad K_2 = [0.6937]; \quad K_3 = [0.7067]. \quad (36)$$

Choosing the initial state as $x(0) = 3$ and the customer demand disturbance with $d(k) = \sin k$, the state response and the control input are illustrated in Figures 1 and 2. We find that the state is maintained at zero when the time is trending to infinity from Figure 1. That is to say, the stock level of supply chain can be maintained at the state of equilibrium and the

bullwhip effect is limited by scholar $\gamma = 1$; that is, the effect of $d(k)$ on $z(k)$ is limited by scholar $\gamma = 1$.

5. Conclusion

The problem of H_∞ control for a class of discrete supply chain systems is addressed in this paper. With the help of network control technique, supply chain systems are modeled as network control systems with stochastic time delay. The mean-square stability and H_∞ performance are studied based on the theory of switched and network control system. Sufficient conditions for the H_∞ controllers design are given in terms of a set of linear matrix inequalities. Finally, simulation examples are provided to illustrate the effectiveness of the developed results. If network-induced delay is random with probability distribution function, the problem of analysis and synthesis of such system will be considered in future work.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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