Research Article

Leader-Based Consensus of Heterogeneous Nonlinear Multiagent Systems

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This paper considers the leader-based consensus of heterogeneous multiple agents with nonlinear uncertain systems. Based on the information obtained from the following agents' neighbors, leader observers are designed by the following agents to estimate the leader's states and nonlinear dynamics. Then, to achieve leader-based consensus, adaptive distributed controllers are designed for the following agents to track the designed corresponding leader observers. The effectiveness of the leader observers and distributed consensus controllers are illustrated by formal proof and simulation results.

1. Introduction

In the past decades, cooperative control problems of multiagent have attracted more and more attention from researchers for its wide applications in many areas, such as formation control of mobile robots [1], monitoring [2], medical rescue [3], and environmental boundary tracking [4]. Consensus is a fundamental and important problem in the area of multiagent systems where a networked interaction protocol needs to be designed such that the final states of all agents converge to a common value.

Compared with the leaderless consensus in which the agents' states converge to a common constant value [5, 6], the states of multiagent can converge to a leader's dynamic state in leader-based consensus [7]. Though many results on leader-based multiagent consensus have been obtained [7–14], most of the results considered agents with integrator or linear dynamics. However, most real agent systems are inherently nonlinear and uncertainty terms may exist in the systems. The consensus control in integrator or linear dynamics cannot be used directly in consensus of nonlinear agents. Recently, results [15–18] considered the leader-based consensus of

homogeneous nonlinear agents with the assumption that the agents' nonlinear function satisfies the Lipschitz or like-Lipchitz condition. The dynamical differences among agents are neglected in [15–18]. In the paper, we consider the leader-based consensus of heterogeneous agents with different nonlinear dynamics.

NNs and fuzzy techniques are widely used in control design for nonlinear uncertain systems, for their abilities in nonlinear function approximations [19, 20]. In [21–25], the leader-based consensus controllers were designed for heterogeneous nonlinear uncertain multiagent systems. In the consensus control design, the leader's nonlinear dynamics was considered as a bounded disturbance and neural network- (NN-) based consensus control was designed for the following agents.

Distributed estimation via observer design for multiagent consensus is an important topic in the study of multiagent networks, with wide applications especially in sensor networks and robot networks, among many others [26]. Up to now, the results [26–31] have been obtained on multiagent consensus based on the distributed estimation via observer design. However, the considered agents in [26–31] are with

integrator or linear dynamics. In this paper, the considered agents are with nonlinear uncertain dynamics and distributed NN-based leader observers are designed to estimate the leader's state and nonlinear dynamics. Then, to obtain the leader-based consensus, NN controllers are designed for the following agents to track the corresponding leader observers.

2. Preliminaries

2.1. Graph Theory and Some Notations. In this paper, we consider a networked system consisting of N agents $v_1, v_2, \dots, v_N$ and one leader v_0 . The leader's system model and motion are independent of the other agents. Denote $G = (V, E)$ as a general digraph with the nonempty finite set of N nodes or agents $V = \{v_1, v_2, \dots, v_N\}$ and a set of edges or arcs $E \subseteq V \times V$. Denote the connectivity matrix as $A = [a_{ij}]$ with $a_{ij} > 0$ if $(v_j, v_i) \in E$ and $a_{ij} = 0$ otherwise. Note $a_{ii} = 0$. The set of neighbors of a node v_i is $N_i = \{v_j : (v_j, v_i) \in E\}$, that is, the set of nodes with arcs incoming to v_i . Define the in-degree matrix as a diagonal matrix $D = \text{diag}\{d_1, d_2, \dots, d_N\}$ with $d_i = \sum_{j \in N_i} a_{ij}$ for node i . Define the Laplacian matrix associated with graph G as $L = D - A$.

Define an augmented graph as $\bar{G} = \{\bar{V}, \bar{E}\}$, where $\bar{V} = \{v_0, v_1, \dots, v_N\}$ and $\bar{E} \subseteq \bar{V} \times \bar{V}$. Denote the diagonal matrix $B = \text{diag}\{b_1, b_2, \dots, b_N\}$ as the leader adjacency matrix with $b_i = 0$ if the i th agent is a neighbor of the leader and $b_i = 0$ otherwise.

Lemma 1 (see [22–24]). *Let L be irreducible and B has at least one diagonal entry $b_i > 0$. Then $L + B$ is a nonsingular M -matrix. Define $q = [q_1, \dots, q_N]^T = (L + B)^{-1} \mathbf{1}$, $P = \text{diag}\{p_i\} = \text{diag}\{1/q_i\}$, $Q = P(L + B) + (L + B)^T P$. Then P and Q are positive definite matrices.*

2.2. Problem Statement. In the considered leader-based consensus, the leader's nonlinear dynamics is described as follows:

$$\dot{x}_0 = f_0(x_0) \quad (1)$$

with $x_0 \in R$ the state and $f_0(x_0)$ a piecewise continuous function of x_0 .

Consider N ($N \geq 2$) followed heterogenous agents with nonlinear uncertain dynamics described as follows:

$$\dot{x}_i = f_i(x_i) + u_i, \quad i = 1, \dots, N, \quad (2)$$

where $x_i(t) \in R$, u_i denote the state and the control input of the i th agent, respectively; $f_i(x_i)$ is a nonlinear uncertain function, which is assumed to be continuous.

In this paper, we make the following assumptions: the following agent i , $i = 1, \dots, N$, can obtain its neighbors' states x_j , $j \in N_i$, and the leader's state estimation $\hat{x}_{0j}$, only some agents can obtain the leader's states x_0 , and all the following agents do not know the leader's nonlinear dynamics.

The objective of this paper is to design controllers u_i , $i = 1, \dots, N$, such that the consensus errors $|x_i - x_0|$ converge to a small neighborhood of zero.

3. Leader-Based Consensus of Multiple Agents

Since the nonlinear functions $f_i(x_i)$, $f_0(x_0)$ are piecewise continuous and unknown to agent i , they can be estimated by agent i using radial basis function (RBF) NNs on a compact set $\Omega_i \in R$ as follows:

$$\hat{f}_i = W_i^T \phi_i(x_i), \quad (3)$$

$$\hat{f}_{0i} = W_{0i}^T \varphi_i(\hat{x}_{0i}). \quad (4)$$

By ideal estimation the $f_i(x_i)$ and $f_0(x_0)$ can be written as

$$f_i(x_i) = W_i^{*T} \phi_i(x_i) + \epsilon_i(x_i), \quad |\epsilon_i(x_i)| \leq \epsilon_{iM}, \quad (5)$$

$$f_0(x_0) = W_{0i}^{*T} \varphi_i(x_0) + \epsilon_{0i}(x_0), \quad |\epsilon_{0i}(x_0)| \leq \bar{\epsilon}_{iM}, \quad (6)$$

where $\phi_i(x_i) \in R^{n_i}$ and $\varphi_i(x_0) \in R^{m_i}$ are suitable basis sets of n_i functions and m_i functions, respectively; W_i and W_{0i} are current estimation of ideal approximation weights W_i^* and W_{0i}^* , respectively; ξ_i and ζ_i are bounds of ideal estimation errors $\epsilon_i(x_i)$ and $\epsilon_{0i}(x_0)$, respectively; and $\hat{x}_{0i}$ is the estimation of x_0 .

Remark 2. According to the definition of ϕ_i , φ_i , W_i , W_{0i} , it is easy to observe that there exist positive numbers W_M , W_{0M} , ϵ_M , and ϵ_{0M} , such that $\|W_i^*\| \leq W_M$, $\|W_{0i}^*\| \leq W_{0M}$, $|\epsilon_i| \leq \epsilon_M$, $|\epsilon_{0i}| \leq \epsilon_{0M}$, $i = 1, 2, \dots, N$.

3.1. Observers for the Leader Agent. For agent i , the following NN-based observer is designed to estimate the leader's state and nonlinear dynamics:

$$\dot{\hat{x}}_{0i} = \hat{f}_{0i} - c \sum_{j \in N_i} a_{ij} (\hat{x}_{0i} - \hat{x}_{0j}) - cb_i (\hat{x}_{0i} - x_0). \quad (7)$$

Denote $\underline{x}_0 = [x_0, x_0, \dots, x_0]^T \in R^N$, $\hat{\underline{x}}_0 = [\hat{x}_{01}, \dots, \hat{x}_{0N}]^T \in R^N$, $e_0 = \hat{\underline{x}}_0 - \underline{x}_0$, $e_{0i} = \hat{x}_{0i} - x_0$. Taking time derivative of e_0 leads to

$$\dot{e}_0 = \hat{f}_0 - \underline{f}(x_0) - c(L + B)e_0 \quad (8)$$

with $\hat{f}_0 = [\hat{f}_{01}, \hat{f}_{02}, \dots, \hat{f}_{0N}]^T \in R^N$, $\underline{f}(x_0) = [f_0(x_0), \dots, f_0(x_0)]^T \in R^N$.

From (4) and (6), $\hat{f}_{0i} - f_0(x_0)$ can be expressed as

$$\begin{aligned} \hat{f}_{0i} - f_0(x_0) &= W_{0i}^T \varphi_i(\hat{x}_{0i}) - W_{0i}^{*T} \varphi_i(x_0) - \epsilon_{0i} \\ &= -W_{0i}^{*T} \tilde{\varphi}_{0i} - \tilde{W}_{0i}^T \varphi_i(\hat{x}_{0i}) - \epsilon, \end{aligned} \quad (9)$$

where $\tilde{W}_{0i} = W_{0i}^* - W_{0i}$.

Remark 3. From Remark 2, we know that $-W_{0i}^{*T} \tilde{\varphi}_{0i} - \epsilon_{0i}$, $i = 1, 2, \dots, N$, are bounded. Then there exists a positive constant ξ , such that $|-W_{0i}^{*T} \tilde{\varphi}_{0i} - \epsilon_{0i}| \leq \xi$.

The following theorem illustrates the efficiency of the designed NN-observer.

Theorem 4. Consider the leader and its following agents with the dynamics described by (1) and (2). If the NN weights in the observer (7) are updated by

$$\dot{\hat{W}}_{0i} = -\dot{\tilde{W}}_{0i} = -F_i^{-1} e_i p_i \varphi_i(\hat{x}_{0i}) - k F_i^{-1} \tilde{W}_{0i} \quad (10)$$

then the estimation errors e_{0i} , $\tilde{W}_{0i}$, $i = 1, 2, \dots, N$, are uniformly ultimately bounded. Furthermore, the estimation error e_{0i} can be made arbitrarily small by a judicious choice of corresponding gains.

Proof. Consider the following Lyapunov function: $V_0 = (1/2)e_0^T P e_0 + (1/2) \sum_{i=1}^N \tilde{W}_{0i}^T F_i \tilde{W}_{0i}$. Take time derivative of V_0 and substitute (8) and (10). Consider

$$\begin{aligned} \dot{V}_0 &= -c e_0^T P (L + B) e_0 + e_0^T P (\hat{f}_0 - \underline{f}(x_0)) + \sum_{i=1}^N \tilde{W}_{0i}^T F_i \dot{\tilde{W}}_{0i} \\ &= -c e_0^T [P(L + B) + (L + B)^T P] e_0 \\ &\quad + \sum_{i=1}^N p_i e_{0i} (-W_{0i}^{*T} \tilde{\varphi}_{0i} - e_{0i}) \\ &\quad + \sum_{i=1}^N \tilde{W}_{0i}^T [F_i \dot{\tilde{W}}_{0i} - e_i p_i \varphi_i(\hat{x}_{0i})] \\ &\leq -c e_0^T Q e_0 + \xi \sum_{i=1}^N p_i |e_{0i}| - k \sum_{i=1}^N \tilde{W}_{0i}^T \tilde{W}_{0i} \\ &\quad + k \sum_{i=1}^N \|\tilde{W}_i\| W_{0M}. \end{aligned} \quad (11)$$

Since $p_i |e_{0i}| \leq p_{\max} |e_{0i}| \leq p_{\max} ((e_{0i}^2/2\xi_1) + (\xi_1/2))$ with $p_{\max} = \max\{p_i, i = 1, \dots, N\}$ and $\|\tilde{W}_i\| \leq (1/2\xi_2) \tilde{W}_i^T \tilde{W}_i + (\xi_2/2)$ hold for arbitrary positive numbers ξ_1, ξ_2 ,

$$\begin{aligned} \dot{V}_0 &\leq -\left(c\lambda_{\min}(Q) - \frac{\xi p_{\max}}{(2\xi_1)}\right) e_0^T e_0 - \left(k - \frac{kW_{0M}}{(2\xi_2)}\right) \sum_{i=1}^N \tilde{W}_i^T \tilde{W}_i \\ &\quad + 0.5N(p_{\max}\xi_1\xi + \xi_2 W_{0M}). \end{aligned} \quad (12)$$

From (12) we know that the derivative of V_0 is guaranteed to be less than zero, as long as $c\lambda_{\min}(Q) - \xi p_{\max}/(2\xi_1) > 0$, $k - kW_{0M}/(2\xi_2) > 0$, and one of the following conditions holds:

$$\begin{aligned} \|e_0\| &\geq \sqrt{\frac{0.5N(p_{\max}\xi_1\xi + \xi_2 W_{0M})}{c\lambda_{\min}(Q) - \xi p_{\max}/(2\xi_1)}}, \\ \sum_{i=1}^N \tilde{W}_{0i}^T \tilde{W}_{0i} &\geq \frac{0.5N(p_{\max}\xi_1\xi + \xi_2 W_{0M})}{k - kW_{0M}/(2\xi_2)}. \end{aligned} \quad (13)$$

Therefore $\dot{V}_0$ is negative outside a compact set. According to the standard Lyapunov theory extension, the estimation errors e_{0i} , $\tilde{W}_{0i}$, $i = 1, 2, \dots, N$, are uniformly ultimately bounded. Furthermore, the estimation error e_{0i} can be made arbitrarily small by a judicious choice of corresponding gains. $\square$

3.2. Leader-Based Consensus Control Design. Since the effectiveness of the leader observers's estimation of the leader's state and dynamics, the leader-based consensus problem can be solved if some controllers are designed for the following agents to track the designed observers. In this part, NN-based controllers are designed for the agents i , $i = 1, 2, \dots, N$, to track the corresponding observers.

Denote $e_i = x_i - \hat{x}_{0i}$. Take time derivative of e_i and substitute (2) and (8). Consider

$$\dot{e}_i = f_i(x_i) + u_i - \hat{f}_{0i} + c \sum_{j \in N_i} a_{ij} (\hat{x}_{0i} - \hat{x}_{0j}) + cb_i (\hat{x}_{0i} - x_0). \quad (14)$$

If the control law u_i is designed as

$$u_i = -l_i e_i - \hat{f}_i + \hat{f}_{0i} - c \sum_{j \in N_i} a_{ij} (\hat{x}_{0i} - \hat{x}_{0j}) - cb_i (\hat{x}_{0i} - x_0), \quad (15)$$

then we have

$$\begin{aligned} \dot{e}_i &= -l_i e_i + W_i^{*T} \phi_i(x_i) - W_i^T \phi_i(x_i) + \epsilon_i \\ &= -l_i e_i + \tilde{W}_i^T \phi(x_i) + \epsilon_i. \end{aligned} \quad (16)$$

Based on the above analysis, we have the following result.

Theorem 5. Consider the distributed system (1) and the leader (2). If the control law for the following agents is designed as (15) and the NN weights W_i are updated as

$$\dot{W}_i = -\dot{\tilde{W}}_i = \bar{F}_i^{-1} \phi_i(x_i) e_i, \quad (17)$$

then tracking errors e_i , $i = 1, 2, \dots, N$, are uniformly ultimately bounded and can be made arbitrarily small by appropriate choice of corresponding gains; that is, the leader-based consensus is achieved.

Proof. Consider the candidate Lyapunov function $V_1 = (1/2)e_i^2 + (1/2)\tilde{W}_i^T \bar{F}_i \tilde{W}_i$. Take time derivative of V_1 and substitute (16)-(17). Consider

$$\begin{aligned} \dot{V}_1 &= e_i (-l_i e_i + \tilde{W}_i^T \phi(x_i) + \epsilon_i) + \tilde{W}_i^T \bar{F}_i \dot{\tilde{W}}_i \\ &= -l_i e_i^2 + e_i \epsilon_i + \tilde{W}_i^T (\bar{F}_i \dot{\tilde{W}}_i + \phi_i(x_i) e_i) \\ &\leq -l_i e_i^2 + |e_i| \epsilon_N. \end{aligned} \quad (18)$$

Since $|e_i| \epsilon_N \leq (e_i^2/2) + (\epsilon_N^2/2)$ holds,

$$\dot{V}_1 \leq -\left(l_i - \frac{1}{2}\right) e_i^2 + \frac{\epsilon_N^2}{2}. \quad (19)$$

From (19) we know that $\dot{V}_1$ is negative, as long as $l_i - (1/2) > 0$ and $|e_i| \geq (\epsilon_N/\sqrt{2l_i - 1})$. Therefore, according to the standard Lyapunov theory extension, the tracking errors e_i are uniformly ultimately bounded and can be made arbitrarily small by a judicious choice of corresponding gains.

Since $x_i - x_0 = x_i - \hat{x}_{0i} + \hat{x}_{0i} - x_0$, $i = 1, 2, \dots, N$, from Theorem 4 and the uniformly ultimately boundedness of $e_i = x_i - \hat{x}_{0i}$, $i = 1, 2, \dots, N$, we can conclude that the consensus errors $x_i - x_0$, $i = 1, 2, \dots, N$, are uniformly bounded and can be made arbitrarily small by appropriate choice of corresponding gains. $\square$

4. Simulation Results

Consider a group of nonlinear uncertain agents composed of a leader agent 0 and four following agents 1, 2, 3, 4 described in Figure 1. Let the dynamics of the agents be

$$\begin{aligned}\dot{x}_0 &= \sin x_0 \\ \dot{x}_1 &= \cos x_1 + u_1 \\ \dot{x}_2 &= 2x_2 + \sin x_2 + u_2 \\ \dot{x}_4 &= \cos x_4 + u_4.\end{aligned}\quad (20)$$

Simulations are carried out on Pentium(R) Dual-Core CPU and Matlab 2008b environments. In the simulation, the initial values of the five agents are $x_0(0) = 1$, $x_1(0) = 2$, $x_2(0) = 4$, $x_3(0) = 3$, and $x_4(0) = 1$ and the NN weights and the observers are initialized to be zero vectors or zero. Choose the design parameters as $k = 3$, $F_i^{-1} = 100$, $c = 5$, $l_1 = 5$, $l_2 = 8$, $l_3 = 9$, and $l_4 = 5$. The simulation results are present in Figures 2–4, where leader-based consensus errors $x_0 - x_i$, $i = 1, 2, 3, 4$, are presented in Figure 2, and the estimation errors $x_0 - \hat{x}_{0i}$, $i = 1, 2, 3, 4$, in the leader observers are presented in Figures 3 and 4 describing the tracking errors $x_i - x_{0i}$, $i = 1, 2, 3, 4$, between the following agents and their corresponding observers. From the simulation results in Figures 2–4, the errors $x_0 - x_i$, $x_0 - \hat{x}_{0i}$, and $x_i - x_{0i}$, $i = 1, 2, 3, 4$, converge to very small neighborhoods of zero after 3 seconds. So, we can conclude that the leader observers (7) are effective to estimate the leader's states and nonlinear dynamics and the leader-based consensus of multiple nonlinear uncertain agents can be achieved under the controllers (15).

5. Conclusions

This paper addressed the NN-observer-based leader-following consensus of heterogeneous multiagent systems with nonlinear uncertain dynamics. NN-based leader observers were designed to estimate the leader's state and nonlinear dynamics. Then NN-based controllers were designed for the following agents to track the corresponding leader observer so that leader-based consensus can be achieved. The effectiveness of the consensus construction method was illustrated by theoretical analysis and simulation results.

In this paper, the considered leader was with nonlinear time-invariant system. The observer-based leader-following consensus of nonlinear uncertain systems, in which the leader is with nonlinear time-varying system, needs to be considered in the future.

FIGURE 1: Communication topology of five agents.

FIGURE 2: Leader-based consensus errors $x_0 - x_i$, $i = 1, 2, 3, 4$.

FIGURE 3: Leader observers' estimation errors $x_0 - \hat{x}_{0i}$, $i = 1, 2, 3, 4$.

FIGURE 4: The tracking errors $x_i - x_{0i}$, $i = 1, 2, 3, 4$, between the following agents and their corresponding observers.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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