Research Article

Application of Base Force Element Method to Mesomechanics Analysis for Concrete

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The base force element method (BFEM) on damage mechanics is used to analyze the compressive strength, the size effects of compressive strength, and fracture process of concrete at mesolevel. The concrete is taken as three-phase composites consisting of coarse aggregate, hardened cement mortar, and interfacial transition zone (ITZ) on mesolevel. The random aggregate model is used to simulate the mesostructure of concrete. The mechanical properties and fracture process of concrete under uniaxial compression loading are simulated using the BFEM on damage mechanics. The simulation results agree with the test results. This analysis method is the new way for investigating fracture mechanism and numerical simulation of mechanical properties for concrete.

1. Introduction

Concrete is considered as heterogeneous composites whose mechanical performance is much related with the microstructure of material. The composite behavior of concrete is exceedingly complex and up to now many details such as strain softening, microcrack propagation, failure mechanisms, and size effects are still far from being fully understood. Since it is difficult to look inside concrete to observe the actual crack propagation or to experimentally determine the microscopic stress field, it has become obvious that further progress based exclusively on experimental studies will be limited [1].

In order to overcome this defect, the concept of numerical concrete was presented by Wittmann et al. [2] based on micromechanics. Subsequently, some scholars did some creative works in this field and made a number of models. Among them, the two important models are the lattice model and the random aggregate model. For example, Schlangen et al. [3, 4] applied the lattice model to simulate the failure mechanism of concrete. Liu and Wang [5] adopted the random aggregate model to simulate cracking process of concrete using FEM. Peng et al. [6] adopted the random aggregate model to simulate the mechanical properties of

rolled compacted concrete on mesolevel using FEM. Du et al. [7–11] researched a mesoelement equivalent method for the simulation of macromechanical properties of concrete and a mesoscale analysis method for the simulation of nonlinear damage and failure behavior of reinforced concrete members.

The finite element method (FEM) is one of the most important numerical methods developed from 1950s, and it has been the most popular and widely used numerical analysis tool for problems in engineering and science. Over the past 50 years, numerous efforts techniques have been proposed for developing finite element models [12–15] and some other improvement and alternative methods have been proposed and developed, such as boundary element methods [16, 17] and meshless methods [18, 19]. In recent years, a new type of finite element method, the base force element method (BFEM), has been developed by Peng et al. [20–25] based on the concept of the base forces by Gao [26]. Further, the base force element method (BFEM) on potential energy principle was used to analyze recycled aggregate concrete on mesolevel [27].

In this paper, the base force element method (BFEM) on damage mechanics is used to analyze the size effect on compressive strength for concrete at mesolevel.

FIGURE 1: The base forces.

2. Basic Formula of Base Force Theory

2.1. Base Forces. Consider a two-dimensional material domain. $\mathbf{x}^\alpha$ ($\alpha = 1, 2$) denote the Lagrangian coordinates. Let $\mathbf{Q}$ denote the position vector of a point after deformation. The tangent basis vectors can be written in the form

$$\mathbf{Q}_1 = \frac{\partial \mathbf{Q}}{\partial x^1}, \quad \mathbf{Q}_2 = \frac{\partial \mathbf{Q}}{\partial x^2}. \quad (1)$$

The position vector of a point $\mathbf{Q}$ can be written as

$$\mathbf{Q} = x^1 \mathbf{Q}_1 + x^2 \mathbf{Q}_2. \quad (2)$$

In order to describe the stress state at a point $\mathbf{Q}$, a parallelogram with the edges $dx^1 \mathbf{Q}_1, dx^2 \mathbf{Q}_2$ is shown in Figure 1. Let $d\mathbf{T}^\alpha$ denote the force acting on the α edge. We calculate the limit

$$\mathbf{t}^\alpha = \lim_{dx^\alpha \rightarrow 0} \frac{d\mathbf{T}^\alpha}{dx^{\alpha+1}}, \quad (3)$$

where we promise $3 = 1$ for indexes. Quantities $\mathbf{t}^\alpha$ ($\alpha = 1, 2$) are called the base forces at point $\mathbf{Q}$ in the two-dimensional coordinate system x^α .

In order to further explain the meaning of $\mathbf{t}^\alpha$, let us compare $\mathbf{t}^\alpha$ with the stress vector $\boldsymbol{\sigma}^\alpha$ which represent the forces per unit area in the deformed body. That is,

$$\mathbf{t}^\alpha = A_Q \boldsymbol{\sigma}^\alpha, \quad (4)$$

where A_Q is called base area $A_Q = |\mathbf{Q}_1 \times \mathbf{Q}_2|$.

The base forces $\mathbf{t}^\alpha$ can also be understood as stress flux.

According to the definitions of various stress tensors, the relation between the base forces and various stress tensors can be given. For example, the Cauchy stress is

$$\boldsymbol{\sigma} = \frac{1}{A_Q} \mathbf{t}^\alpha \otimes \mathbf{Q}_\alpha, \quad (5)$$

where $\otimes$ is the dyadic symbol and the summation rule is implied.

When the body force per unit mass is zero, the equilibrium equation can be written as

$$\frac{\partial \mathbf{t}^\alpha}{\partial x^\alpha} = 0. \quad (6)$$

FIGURE 2: A triangular element.

2.2. Conjugate Variable of Base Forces. Let $\mathbf{P}$ denote the position vector of a point before deformation; the displacement $\mathbf{u}$ of a point is

$$\mathbf{u} = \mathbf{Q} - \mathbf{P}. \quad (7)$$

The gradient of displacement $\mathbf{u}_\alpha$ can be written as

$$\mathbf{u}_\alpha = \frac{\partial \mathbf{u}}{\partial x^\alpha} = \mathbf{Q}_\alpha - \mathbf{P}_\alpha. \quad (8)$$

Further, the elastic law can be given as follows:

$$\mathbf{t}^\alpha = \rho A_Q \frac{\partial W}{\partial \mathbf{u}_\alpha}, \quad (9)$$

in which W is the strain energy density and ρ is the mass density after deformation.

Equation (9) expresses the $\mathbf{t}^\alpha$ by strain energy directly. Thus, $\mathbf{u}_\alpha$ is just the conjugate variable of $\mathbf{t}^\alpha$. It can be seen that the mechanics problem can be completely established by means of $\mathbf{t}^\alpha$ and $\mathbf{u}_\alpha$.

For the small deformation case, the Green strain $\boldsymbol{\varepsilon}$ can be written as

$$\boldsymbol{\varepsilon} = \frac{1}{2} (\mathbf{u}_\alpha \otimes \mathbf{P}^\alpha + \mathbf{P}^\alpha \otimes \mathbf{u}_\alpha), \quad (10)$$

where $\mathbf{P}^\alpha$ is the conjugate of $\mathbf{P}_\alpha$.

3. Model of the BFEM

3.1. Stiffness Matrix. Consider a triangular element as shown in Figure 2; the stiffness matrix of a base force element can be obtained [27] as

$$\mathbf{K}^{IJ} = \frac{E}{2A(1+\nu)} \left[\frac{2\nu}{1-2\nu} \mathbf{m}^I \otimes \mathbf{m}^J + m^{IJ} \mathbf{U} + \mathbf{m}^J \otimes \mathbf{m}^I \right] \quad (11)$$

($I = 1, 2, 3; J = 1, 2, 3$),

in which E is Young's modulus, ν is Poisson's ratio, A is the area of an element, $\mathbf{U}$ is the unit tensor $\mathbf{U} = \mathbf{P}_\alpha \otimes \mathbf{P}^\alpha = \mathbf{P}^\alpha \otimes \mathbf{P}_\alpha$, $m^{IJ} = \mathbf{m}^I \cdot \mathbf{m}^J$, and $\mathbf{m}^I$ can be calculated from

$$\mathbf{m}^I = m_\alpha^I \mathbf{P}_\alpha = \frac{1}{2} (L_{IJ} \mathbf{n}^{JJ} + L_{IK} \mathbf{n}^{IK}), \quad (12)$$

where L_{IJ} and L_{IK} are the lengths of edges IJ and IK of an element and $\mathbf{n}^{IJ}$ and $\mathbf{n}^{IK}$ denote the external normals of edges IJ and IK , respectively.

Further, the stiffness matrix of a base force element can be written as

$$\mathbf{K}^{IJ} = \frac{E}{2A(1+\nu)} \left[\frac{2\nu}{1-2\nu} m_\alpha^I \mathbf{P}_\alpha \otimes m_\beta^J \mathbf{P}_\beta + (m_\alpha^I \mathbf{P}_\alpha \cdot m_\beta^J \mathbf{P}_\beta) \mathbf{U} + m_\beta^J \mathbf{P}_\beta \otimes m_\alpha^I \mathbf{P}_\alpha \right], \quad (13)$$

where the summation rule is implied.

For the Cartesian coordinate system, we introduce the unit basic vectors $\mathbf{e}_\alpha$ ($\alpha = 1, 2$) and have

$$\mathbf{P}_\alpha = \mathbf{e}_\alpha, \quad \mathbf{P}_\beta = \mathbf{e}_\beta, \quad \mathbf{U} = \mathbf{e}_y \otimes \mathbf{e}_y \quad (14)$$

or

$$\begin{aligned} \mathbf{P}_1 &= \mathbf{e}_x, & \mathbf{P}_2 &= \mathbf{e}_y; \\ \mathbf{P}_1 &= \mathbf{e}_x, & \mathbf{P}_2 &= \mathbf{e}_y, & \mathbf{U} &= \mathbf{e}_x \otimes \mathbf{e}_x + \mathbf{e}_y \otimes \mathbf{e}_y. \end{aligned} \quad (15)$$

Substitute (15) into (13) and consider $\mathbf{e}_x \cdot \mathbf{e}_x = 1, \mathbf{e}_x \cdot \mathbf{e}_y = 0$; we can obtain

$$\begin{aligned} \mathbf{K}^{IJ} &= \frac{E}{2A(1+\nu)} \left[\frac{2\nu}{1-2\nu} (m_x^I m_x^J \mathbf{e}_x \otimes \mathbf{e}_x + m_x^I m_y^J \mathbf{e}_x \otimes \mathbf{e}_y \right. \\ &\quad \left. + m_y^I m_x^J \mathbf{e}_y \otimes \mathbf{e}_x + m_y^I m_y^J \mathbf{e}_y \otimes \mathbf{e}_y) \right. \\ &\quad \left. + (m_x^I m_x^J + m_y^I m_y^J) (\mathbf{e}_x \otimes \mathbf{e}_x + \mathbf{e}_y \otimes \mathbf{e}_y) \right. \\ &\quad \left. + (m_x^I m_x^J \mathbf{e}_x \otimes \mathbf{e}_x + m_x^I m_y^J \mathbf{e}_y \otimes \mathbf{e}_x \right. \\ &\quad \left. + m_y^I m_x^J \mathbf{e}_x \otimes \mathbf{e}_y + m_y^I m_y^J \mathbf{e}_y \otimes \mathbf{e}_y) \right] \quad (16) \end{aligned}$$

or

$$\begin{aligned} [K_{IJ}]^e &= \frac{E}{2A(1+\nu)} \\ &\times \begin{bmatrix} \frac{2-2\nu}{1-2\nu} m_x^I m_x^J + m_y^I m_y^J & \frac{2\nu}{1-2\nu} m_x^I m_y^J + m_y^I m_x^J \\ \frac{2\nu}{1-2\nu} m_y^I m_x^J + m_x^I m_y^J & \frac{2-2\nu}{1-2\nu} m_y^I m_y^J + m_x^I m_x^J \end{bmatrix}. \end{aligned} \quad (17)$$

For the plane stress problem, it is necessary to replace $E/(1-\nu^2)$ by E and $\nu/(1-\nu)$ by ν in (16) and (17).

Further, the stiffness matrix of a base force element for the plane stress problem can be written as

$$\begin{aligned} [K_{IJ}]^e &= \frac{E}{A(1-\nu^2)} \\ &\times \begin{bmatrix} m_x^I m_x^J + \frac{1-\nu}{2} m_y^I m_y^J & \nu m_x^I m_y^J + \frac{1-\nu}{2} m_y^I m_x^J \\ \nu m_y^I m_x^J + \frac{1-\nu}{2} m_x^I m_y^J & m_y^I m_y^J + \frac{1-\nu}{2} m_x^I m_x^J \end{bmatrix}, \end{aligned} \quad (18)$$

in which $\mathbf{m}^I$ and $\mathbf{m}^J$ can be calculated from

$$\begin{aligned} \mathbf{m}^I &= m_i^I \mathbf{e}_i = \frac{1}{2} (L_{IJ} \mathbf{n}^{IJ} + L_{LI} \mathbf{n}^{LI}) \\ &= \frac{1}{2} (L_{IJ} n_i^{IJ} \mathbf{e}_i + L_{LI} n_i^{LI} \mathbf{e}_i) = \frac{1}{2} (L_{IJ} n_i^{IJ} + L_{LI} n_i^{LI}) \mathbf{e}_i, \\ \mathbf{m}^J &= m_i^J \mathbf{e}_i = \frac{1}{2} (L_{JK} \mathbf{n}^{JK} + L_{IJ} \mathbf{n}^{IJ}) \\ &= \frac{1}{2} (L_{JK} n_i^{JK} \mathbf{e}_i + L_{IJ} n_i^{IJ} \mathbf{e}_i) = \frac{1}{2} (L_{JK} n_i^{JK} + L_{IJ} n_i^{IJ}) \mathbf{e}_i \end{aligned} \quad (19)$$

or

$$\begin{aligned} \begin{Bmatrix} m_x^I \\ m_y^I \end{Bmatrix} &= \frac{1}{2} \left(L_{IJ} \begin{Bmatrix} n_x^{IJ} \\ n_y^{IJ} \end{Bmatrix} + L_{LI} \begin{Bmatrix} n_x^{LI} \\ n_y^{LI} \end{Bmatrix} \right) \\ &= \frac{1}{2} \left(L_{IJ} \begin{Bmatrix} \frac{y_J - y_I}{L_{IJ}} \\ -\frac{x_J - x_I}{L_{IJ}} \end{Bmatrix} + L_{LI} \begin{Bmatrix} \frac{y_I - y_L}{L_{LI}} \\ -\frac{x_I - x_L}{L_{LI}} \end{Bmatrix} \right) \\ &= \frac{1}{2} \begin{Bmatrix} y_J - y_L \\ x_L - x_J \end{Bmatrix}, \\ \begin{Bmatrix} m_x^J \\ m_y^J \end{Bmatrix} &= \frac{1}{2} \left(L_{JK} \begin{Bmatrix} n_x^{JK} \\ n_y^{JK} \end{Bmatrix} + L_{IJ} \begin{Bmatrix} n_x^{IJ} \\ n_y^{IJ} \end{Bmatrix} \right) \\ &= \frac{1}{2} \left(L_{JK} \begin{Bmatrix} \frac{y_K - y_J}{L_{JK}} \\ -\frac{x_K - x_J}{L_{JK}} \end{Bmatrix} + L_{IJ} \begin{Bmatrix} \frac{y_J - y_I}{L_{IJ}} \\ -\frac{x_J - x_I}{L_{IJ}} \end{Bmatrix} \right) \\ &= \frac{1}{2} \begin{Bmatrix} y_K - y_I \\ x_I - x_K \end{Bmatrix}. \end{aligned} \quad (20)$$

3.2. Strain Tensor of an Element. When the element is small enough, the real strain ε can be replaced by the average strain $\bar{\varepsilon}$. We can obtain the average strain $\bar{\varepsilon}$ in the element as

$$\bar{\varepsilon} = \frac{1}{A} \int_A \varepsilon dA, \quad (21)$$

in which A is the area of an element.

FIGURE 3: Attribute recognition figure.

Substituting (10) into (21), we can easily derive the following equation:

$$\bar{\varepsilon} = \frac{1}{2A} (\mathbf{u}_I \otimes \mathbf{m}^I + \mathbf{m}^I \otimes \mathbf{u}_I), \quad (22)$$

where $I = (1, 2, 3)$ is the local code number for the nodes of a triangular element and the summation rule is implied.

Substitute $\mathbf{u}_I = u_{Ii} \mathbf{e}_i$ and $\mathbf{m}^I = m_j^I \mathbf{e}_j$ into (22); we can obtain the explicit expression of the tensor formula as

$$\begin{aligned} \bar{\varepsilon} = \frac{1}{2A} \sum_{I=1}^3 [& (2u_{Ix}m_x^I) \mathbf{e}_x \otimes \mathbf{e}_x + (u_{Ix}m_y^I + u_{Iy}m_x^I) \mathbf{e}_x \otimes \mathbf{e}_y \\ & + (u_{Ix}m_y^I + u_{Iy}m_x^I) \mathbf{e}_y \otimes \mathbf{e}_x + (2u_{Iy}m_y^I) \mathbf{e}_y \otimes \mathbf{e}_y] \end{aligned} \quad (23)$$

or

$$\begin{aligned} \bar{\varepsilon}_x &= \frac{1}{A} \sum_{I=1}^3 (u_{Ix}m_x^I), \\ \bar{\varepsilon}_y &= \frac{1}{A} \sum_{I=1}^3 (u_{Iy}m_y^I), \\ \bar{\gamma}_{xy} &= \frac{1}{A} \sum_{I=1}^3 (u_{Ix}m_y^I + u_{Iy}m_x^I). \end{aligned} \quad (24)$$

3.3. Stress Tensor of an Element. When the element is small enough, the real stress $\boldsymbol{\sigma}$ can be replaced by the average stress $\bar{\boldsymbol{\sigma}}$. According to the generalized Hooke law, the stress component expressions of an element can be obtained for the plane stress problem as follows:

$$\begin{aligned} \sigma_x &= \frac{E}{1-\nu^2} (\varepsilon_x + \nu \varepsilon_y), \\ \sigma_y &= \frac{E}{1-\nu^2} (\varepsilon_y + \nu \varepsilon_x), \\ \tau_{xy} &= \frac{E}{2(1+\nu)} \gamma_{xy}. \end{aligned} \quad (25)$$

For the plane strain problem, it is necessary to replace E by $E/(1-\nu^2)$ and ν by $\nu/(1-\nu)$ in (25).

FIGURE 4: Bilinear damage model.

4. Random Aggregate Model

Based on the Fuller grading curve, Walraven and Reinhardt [28] put the three-dimensional grading curve into the probability of any point which located in the sectional plane of specimens, and its expression is as follows:

$$\begin{aligned} P_c (D < D_0) &= P_k \left(1.065 \left(\frac{D_0}{D_{\max}} \right)^{1/2} - 0.053 \left(\frac{D_0}{D_{\max}} \right)^4 - 0.012 \left(\frac{D_0}{D_{\max}} \right)^6 \right. \\ &\quad \left. - 0.0045 \left(\frac{D_0}{D_{\max}} \right)^8 - 0.0025 \left(\frac{D_0}{D_{\max}} \right)^{10} \right), \end{aligned} \quad (26)$$

where P_k is the volume percentage of aggregate volume among the specimens, in general $P_k = 0.75$, D_0 is the diameter of sieve pore, and $D_{\max}$ is the maximum aggregate size.

According to (26), the numbers of coarse aggregate particles with various sizes can be obtained. By Monte Carlo method, random to create the centroid coordinates of all kinds of coarse aggregate particles, namely, to generate random aggregate model.

According to the projection method, we dissect the specimens of concrete with different phases of materials. Then, the phase of coarse aggregate, the phase of hardened cement, and the phase of interfacial transition zone (ITZ) can be judged by a computer code as in Figure 3.

5. Damage Model of Materials

Components of concrete such as coarse aggregate, cement mortar, and interfacial transition zone (ITZ) are basically quasibrittle material, whose failure patterns are mainly brittle failure.

In this paper, according to the characteristics of concrete on mesostructure, the damage degradation of concrete is described by the bilinear damage model, and the failure principal is the criterion of maximum tensile stress. Damage constitutive model is defined as $\tilde{E} = E_0(1-D)$ ($0 \leq D \leq 1$),

FIGURE 5: Random aggregate model with 150 mm × 150 mm × 150 mm.

FIGURE 6: Stress-strain curve of specimen with 150 mm × 150 mm × 150 mm.

FIGURE 7: Random aggregate model with 100 mm × 100 mm × 100 mm.

TABLE 1: Material parameters of uniaxial compressive tests.

Materials	Elastic modulus/GPa	Poisson's ratio	Tensile strength/MPa	λ	η	ξ
Coarse aggregate	70	0.16	10	0.1	5	10
Cement mortar	30	0.22	4	0.1	4	10
Interfacial transition zone	15	0.2	2.5	0.1	3	10

FIGURE 8: Stress-strain curve of concrete with 100 mm × 100 mm × 100 mm.

FIGURE 9: Random aggregate model with 300 mm × 300 mm × 300 mm.

shown in Figure 4, where E_0 is the initial Young modulus; the damage factor D can be expressed as follows:

$$D = \begin{cases} 0 & \varepsilon < \varepsilon_0 \\ 1 - \frac{\eta - \lambda}{\eta - 1} \frac{\varepsilon_0}{\varepsilon} + \frac{1 - \lambda}{\eta - 1} & \varepsilon_0 < \varepsilon \leq \varepsilon_r \\ 1 - \lambda \frac{\varepsilon_0}{\varepsilon} & \varepsilon_r < \varepsilon \leq \varepsilon_u \\ 1 & \varepsilon > \varepsilon_u \end{cases} \quad (27)$$

where f_t is the tensile strength of material, the residual tensile strength is defined as $f_{tr} = \lambda f_t$, the residual strength coefficient λ ranges from 0 to 1, the residual strain is $\varepsilon_r = \eta \varepsilon_0$,

ε_0 is the peak strain, η is the residual strain coefficient, the ultimate strain is defined as $\varepsilon_u = \xi \varepsilon_0$, where ξ is ultimate strain coefficient, and ε is principal tensile strain of element.

6. Numerical Example

According to the test results obtained from the experiment, material parameters of concrete are selected, which is completely coherent with model material used in the experiment. Material parameters of numerical simulation are shown in Table 1. The concrete specimens were loaded by displacement steps.

FIGURE 10: Stress-strain curve of specimen with 300 mm $\times$ 300 mm $\times$ 300 mm.

FIGURE 11: Propagation process of cracks of the specimen with 100 mm $\times$ 100 mm $\times$ 100 mm.

FIGURE 12: Propagation process of cracks of the specimen with 150 mm × 150 mm × 150 mm.

6.1. Compressive Strength of Concrete Specimen. For the size 150 mm × 150 mm × 150 mm of compression specimen, the numbers of coarse aggregate particles can be obtained according to (26). The random aggregate model is generated as in Figure 5.

The uniaxial compressive stress-strain curve of concrete is got as shown in Figure 6. The compressive strengths of the three specimens were 24.28 MPa, 24.40 MPa, and 24.41 MPa. The uniaxial compressive strength average of the specimen group is 24.36 MPa. The result BFEM on mesodamage analysis for concrete is consistent with the test results [29].

6.2. Size Effects of Compressive Strength. In order to study the size effects of compressive strength for concrete specimen, we used three different sizes of specimens in the numerical simulations, which are 100 mm × 100 mm × 100 mm, 150 mm × 150 mm × 150 mm, and 300 mm × 300 mm × 300 mm.

6.2.1. Compressive Strength of 100 mm × 100 mm × 100 mm Specimens. For the size 100 mm × 100 mm × 100 mm of compression specimen, the numbers of coarse aggregate particles can be obtained according to (26). The random aggregate model is as in Figure 7.

The uniaxial compressive stress-strain curve of concrete is got as shown in Figure 8. The compressive strengths of the four specimens were 25.85 MPa, 25.33 MPa, and 25.67 MPa. The uniaxial compressive strength average of the specimen group is 25.62 MPa.

6.2.2. Compressive Strength of 300 mm × 300 mm × 300 mm Specimens. For the size 300 mm × 300 mm × 300 mm of compression specimen, the numbers of coarse aggregate particles can be obtained according to (26). The generated random aggregate model is as in Figure 9.

The uniaxial compressive stress-strain curve of concrete is got as shown in Figure 10. The compressive strengths of the four specimens were 22.59 MPa, 22.72 MPa, and 22.70 MPa. The uniaxial compressive strength average of the specimen group is 22.67 MPa.

The results of 150 mm × 150 mm × 150 mm specimens are shown in Figure 6. The size effects of mechanical properties of concrete under uniaxial compression loading are shown in Table 2.

6.3. Fracture Process of Concrete. In the calculation, we use the incremental method and the displacement loading steps. According to the first strength theory, the element

FIGURE 13: Propagation process of cracks of the RAC specimen with 300 mm × 300 mm × 300 mm.

TABLE 2: Different sizes of concrete compressive strength under uniaxial compression loading.

Different sizes of concrete	Compressive strength/MPa
100 mm × 100 mm × 100 mm	25.62
150 mm × 150 mm × 150 mm	24.36
300 mm × 300 mm × 300 mm	22.67

causes damage when the first principal stress exceeds the allowable value of the element. All the damaged elements were represented by the black color.

The propagation process of cracks of the specimen of concrete with 100 mm × 100 mm × 100 mm by uniaxial compression is shown in Figure 11.

The propagation process of cracks of the specimen with 150 mm × 150 mm × 150 mm by uniaxial compression is shown in Figure 12.

The propagation process of cracks of the specimen with 300 mm × 300 mm × 300 mm by uniaxial compression is shown in Figure 13.

7. Conclusions

In this paper, the base force element method (BFEM) on damage mechanics is used to analyze the size effect on compressive strength for concrete at mesolevel. The characteristics of the BFEM compared with the traditional FEM are as follows. The expression of the stiffness matrix is a precise expression, and it is not necessary to introduce Gauss' integral for calculating the stiffness coefficient at a point. This expression of stiffness matrix can be used for calculating the stiffness of various elements with a unified method. This expression of stiffness matrix can be used in any coordinate system. The method of constructing the stiffness matrix does not regulate the introduction of interpolation. The model of the base force element method was used to analyze the damage problem for concrete and was used to analyze the relationships of mesostructure and macroscopic mechanical performance of concrete in this paper. The concrete is taken as three-phase composites consisting of coarse aggregate, hardened cement mortar, and interfacial transition zone (ITZ) on mesolevel. The random aggregate model is used for the numerical simulation of uniaxial compressive performance of concrete. The results by the BFEM show that the uniaxial compressive

strengths of specimens are approximately coincident with the experiment results and the size effect of specimens is in agreement with the common rule. The following conclusions can be drawn.

- (1) It shows that the BFEM with the mesodamage model is feasible and effective to study failure process and mechanical parameter of concrete.
- (2) When specimens are under the same conditions, the compressive strength of concrete decreases with increasing specimen size.
- (3) Macrofailure of concrete apparently lags behind the growth of microdamage; that is, macrofailure can be considered as the accumulation of microdamage.
- (4) The interface is the weakest part of concrete, and the selection of mechanical parameters of interface plays an important role on the numerical results of uniaxial compression for specimens. More future studies should be done on researching the mechanical property of the interface.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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