Research Article

Broadband Bioimpedance Spectroscopy Based on a Multifrequency Mixed Excitation and Nuttall Windowed FFT Algorithm

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1. Introduction

Bioimpedance spectroscopy (BIS), which performs measurement of complex electrical impedance of biomaterials over a certain frequency range, has been extensively used in many electrochemical and biomedical applications since it has been shown that BIS can provide physiological and pathological information of biological tissues [1]. In recent years, BIS technology has been increasingly and widely studied and adopted in clinical diagnoses on tissue ischemia [2], mammary cancer [3], and lung cancer [4].

Traditionally, the most commonly used BIS measurement technique is the traditional frequency-sweep (FS) approach which performs measurement of impedance at a single frequency point in a time and sweeps the specific frequencies in the range of interest [5, 6]. To date, the FS method remains popular for its simplicity and ability to describe the stationary properties of linear time-invariant bioimpedance [7]. The main advantage of the FS approach lies in the fact that it can usually obtain high signal-to-noise ratio (SNR) at the cost of the long measuring time, which usually lasts one second to tens of seconds for a complete sweep [8, 9]. However, the physiological status of living body, such as the cardiovascular system, is time-varying and dynamic because of its blood flow, heartbeat, and other factors. Therefore, in the situation of time-varying bioimpedance measurement, the main drawback of the FS method is its sweeping cycle, which can be much longer than the bioimpedance time variations [8]. So the BIS data based on the FS measurement technology cannot accurately represent the instantaneous impedance spectra and may lose important diagnostic information [10, 11].

In recent years, the multifrequency simultaneous (MFS) measurement technique, which applies a broadband excitation and gains its resulting frequency response by means
of spectral analysis using the fast Fourier transform (FFT) algorithm, has been becoming popular for time-varying bioimpedance [8, 12–15]. Thanks to multiple frequencies being excited at the same time, the time to acquire a complete BIS is drastically reduced to tens of milliseconds [4, 16]. Compared to the FS approach, this MFS approach can obtain the impedance frequency response at different frequencies simultaneously and is highly desirable for time-varying biological systems, such as the respiratory or cardiovascular systems [17, 18].

Proper spread-spectrum excitation is crucial for MFS measurement of BIS. Sanchez et al. [15] had a review on several broadband excitations such as maximum length binary sequences (MLBS), chirp. As a binary signal, the MLBS has the advantage of implementation simplicity and has a dense (continuous) amplitude spectrum with plateau envelope where most of the energy is concentrated. But in most BIS applications, only a discrete set of frequencies is required to fit the impedance data to a model like Cole equation [19], and BIS devices usually need several to tens of isolated frequencies (usually with large frequency intervals) over a wide range of frequency (often from 5 kHz to 1 MHz) [20]. Consequently, broadband excitation with sparse spectrum where energy is distributed on finite frequency points with large frequency spacing is ideal for BIS measurements.

The authors previously proposed a multifrequency mixed (MFM) signal whose majority (65% or more) of energy is concentrated on several expected 2nth primary harmonics [21], which provides a proper broadband excitation for MFS measurement BIS. With the broadband excitation, the complex impedance spectrum then can be just calculated as the ratio of the complex Fourier coefficients of the response voltage to the complex Fourier coefficients of the excitation current signal after FFT operations [15, 22].

However, FFT operations usually bring spectral leakage since rigorous integer-period sampling is usually hard to achieve. The digital Fourier transform (DFT) theory always assumes that the input sequence is periodic (assumed period extension), so if the input sequence finishes on an integer number of periods everything is fine. Otherwise, discontinuity occurs in the assumed extended sequences and brings spectral leakage and picket fence effect in FFT operation [23, 24]. Unfortunately, integer-period sampling is usually hard to achieve, since sampling device is hard to begin and ends exactly at the signal's head and tail, respectively. Moreover, as a rectangular wave, the MFM signal contains a great number of undesired harmonic components according to the Fourier series theory [25]. These undesired frequency components, especially those whose frequencies are higher than half of the sampling rate, will lead to spectrum aliasing when performing FFT operations and ultimately degrade the measurement precision [26].

Fortunately, the undesired effects of the spectral leakage can be minimized by weighing the samples by a suitable time window, and the picket fence effect can be reduced by adopting interpolation algorithms [27]. It is proved that windows with big side-lobes attenuation and high side-lobes roll-off rate can sufficiently reduce the spectral leakage [28].

This paper synthesizes a nine-frequency MFM signal \( f(9, t) \) according to the previously proposed method [21] as the broadband excitation, and realizes multifrequency simultaneous (MFS) measurement of BIS based on a Nuttall windowed interpolation FFT Algorithm. The Nuttall window [29], which has excellent characteristics on side-lobe and main-lobe, is adopted to suppress spectral leakage before FFT, and a Nuttall windowed double-spectral-line interpolation FFT algorithm is developed to reduce the error of picket fence effects. A BIS measurement experiment on an RC three-element equivalent circuit is simulated to evaluate the performance of the proposed algorithm.

2. Synthesis of the MFM Signal

According to the MFM signal synthesis principle introduced by the literature [21], let \( N = 9 \); then the nine-frequency MFM signal \( f(9, t) \) is synthesized and the time domain wave form of \( f(9, t) \) within one period is shown in Figure 1. The harmonic amplitude spectra and power percentage spectra of \( f(9, t) \) are shown in Figures 2(a) and 2(b), respectively, where the nine expected 2nth harmonics (the red solid points in Figure 1), namely, the 1st, 2nd, 4th, 8th, 16th, 32nd, 64th, 128th, and 256th harmonics (we called them primary harmonics thereafter) are obviously prominent. The amplitude spectra \( b_k \), power spectra \( p_k \), and initial phase \( \varphi_k \) of \( f(9, t) \) are shown in Table 1, respectively.

As shown in Figure 2 and Table 1, the synthesized MFM signal \( f(9, t) \) has nine 2nth primary harmonic components which occupy up to 65.52% of the total power. The period of one code element width of \( f(9, t) \) is 0.5 \( \mu s \), and its total 512 code elements last a fundamental period \( T_0 = 256 \mu s \), which means that the fundamental frequency of \( f(9, t) \) is \( f_0 = 3.90625 \) kHz. The nine primary harmonic frequencies of \( f(9, t) \) are also listed in Table 1, which expand from 3.90625 kHz to 1 MHz, and cover the main frequency range in most BIS measurements. As a rectangular wave, the MFM signal has a crest factor (CF) of 1.235 (1/\( \sqrt{0.6552} \), lower than the CF of the sinusoid 1.414) according to the computation method mentioned in [30], which is desirable for BIS of living biological systems because a low crest factor may limit the applied current peaks and set biological systems
Table 1: Spectrum characteristics of the nine primary harmonics in the MFM signal $f(9, t)$.

<table>
<thead>
<tr>
<th>$H_k$</th>
<th>$f_0$</th>
<th>$2f_0$</th>
<th>$4f_0$</th>
<th>$8f_0$</th>
<th>$16f_0$</th>
<th>$32f_0$</th>
<th>$64f_0$</th>
<th>$128f_0$</th>
<th>$256f_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_k$</td>
<td>0.3993</td>
<td>0.3988</td>
<td>0.3976</td>
<td>0.3953</td>
<td>0.3909</td>
<td>0.3827</td>
<td>0.3688</td>
<td>0.3482</td>
<td>0.3482</td>
</tr>
<tr>
<td>$p_k$ (%)</td>
<td>7.97</td>
<td>7.95</td>
<td>7.90</td>
<td>7.81</td>
<td>7.64</td>
<td>7.32</td>
<td>6.80</td>
<td>6.06</td>
<td>6.06</td>
</tr>
<tr>
<td>$\varphi_k$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$f$ (kHz)</td>
<td>3.90625</td>
<td>7.8125</td>
<td>15.625</td>
<td>31.25</td>
<td>62.5</td>
<td>125</td>
<td>250</td>
<td>500</td>
<td>1000</td>
</tr>
</tbody>
</table>

Within the linear regime [12, 31]. Furthermore, as a binary-valued function, the MFM signal can be generated by FPGA easily, and the bandwidth of its harmonics can be variable by changing the FPGAs operating clock, and the number of the expected primary harmonics is also adjustable. Based on the features mentioned above, the newly designed wideband excitation source establishes a good foundation for fast measurement of BIS.

3. Nuttall Windowed Interpolation FFT Algorithm

3.1. Nuttall Window. The Nuttall window [29] is a combination of cosine windows and can be expressed in the time domain as follows:

$$w_N(n) = \sum_{m=0}^{M-1} (-1)^m b_m \cos \left( 2\pi n \frac{m}{N} \right),$$

where $M$ is the number of terms of the cosine function; $N$ denotes the length of the window and $n = 0, 1, \ldots, N − 1$; the coefficient $b_m$ should meet the constraint conditions: $\sum_{m=0}^{M-1} b_m = 1$, $\sum_{m=0}^{M-3} (-1)^m b_m = 0$.

The coefficient groups of typical Nuttall windows shown in Tables 2 and 3 list the side-lobe characteristics of typical Nuttall windows and other cosine-combined windows [29].

The side-lobe attenuation level and asymptotic decay rate directly affect the FFT-based spectral analysis results [28]. Among the windows shown in Table 2, the 4-term 3-order Nuttall window has the fastest side-lobe asymptotic decay rate 30 dB/oct and an acceptable side-lobe attenuation level $-82.6$ dB, which is adopted to truncate sampled data and suppress spectrum leakage in FFT.

3.2. Nuttall Windowed Interpolation FFT Algorithm. If $x(t)$ denotes an analog signal with multiple harmonic components, its discrete-time form $x(n)$ can be expressed as

$$x(n) = \sum_{h=1}^{H} A_h \sin \left( 2\pi \frac{hf_1}{f_s} n + \varphi_h \right),$$

where $H$ denotes the number of harmonic components and $h$ is an integer representing the harmonic order. When $h = 1$, $f_1$, $A_1$, $\varphi_1$ are the frequency, amplitude, and initial phase of the fundamental harmonic, respectively. When $h \neq 1$, $hf_1$, $A_h$, $\varphi_h$ denote the frequency, amplitude, and initial phase of the $h$th harmonic respectively. $f_s$ is the digital sampling rate.

If $x(n)$ is truncated by Nuttall window $w_N(n)$, then the windowed signal after discrete Fourier transform (DFT) can be expressed in frequency domain as

$$X(k\Delta f) = \frac{\sum_{h=1}^{H} A_h e^{j\varphi_h} W_N \left( \frac{2\pi (k\Delta f - hf_1)}{f_s} \right)}{\sum_{h=1}^{H} A_h e^{j\varphi_h} W_N(0)},$$

where $W_N(f)$ denotes the continuous spectrum of $w_N(n)$, $\Delta f$ the frequency resolution, and $\Delta f = f_s/N$. $N$ is the truncated
Table 2: Coefficient groups of typical Nuttall windows.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>3-order minimum side-lobe</th>
<th>4-order minimum side-lobe</th>
<th>4-term 1-order</th>
<th>4-term 3-order</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_0$</td>
<td>0.4243801</td>
<td>0.3635819</td>
<td>0.355768</td>
<td>0.338946</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.4973406</td>
<td>0.4891775</td>
<td>0.487396</td>
<td>0.481973</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.1365995</td>
<td>0.144232</td>
<td>0.161054</td>
<td>0.16054</td>
</tr>
<tr>
<td>$b_3$</td>
<td>0.0106411</td>
<td>0.012604</td>
<td></td>
<td>0.018027</td>
</tr>
</tbody>
</table>

Table 3: Side-lobe characteristics of Nuttall windows and other cosine-combined windows.

<table>
<thead>
<tr>
<th>Types of windows</th>
<th>Peak side-lobe (dB)</th>
<th>Asymptotical decay (dB/oct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hann. window</td>
<td>−32</td>
<td>18</td>
</tr>
<tr>
<td>Blackman window</td>
<td>−58</td>
<td>18</td>
</tr>
<tr>
<td>Blackman-Harris window</td>
<td>−92</td>
<td>6</td>
</tr>
<tr>
<td>3-order Nuttall window with minimum side-lobe</td>
<td>−71.49</td>
<td>6</td>
</tr>
<tr>
<td>4-order Nuttall window with minimum side-lobe</td>
<td>−98.2</td>
<td>6</td>
</tr>
<tr>
<td>4-term 1-order Nuttall window</td>
<td>−93.3</td>
<td>18</td>
</tr>
<tr>
<td>4-term 3-order Nuttall window</td>
<td>−82.6</td>
<td>30</td>
</tr>
</tbody>
</table>

Data length. $h f_1$ is the peak spectral of the $h$th harmonic and $h f_1 = k_h \Delta f$.

When signal is sampled asynchronously, its peak frequency of the $h$th harmonic $k_h \Delta f$ can hardly be exactly on the discrete spectral line. As shown in Figure 3, suppose that the largest and the second largest spectral amplitude lines near the peak point $k_h$ are $k_{h1}$ and $k_{h2}$, respectively; they have the relationship

$$k_{h1} \leq k_h \leq k_{h2} = k_{h1} + 1.$$ (4)

And the amplitudes of these two spectral lines, $y_1$ and $y_2$, can be expressed as

$$y_1 = |X(k_{h1}\Delta f)| = \left| A_h \frac{e^{j\varphi_h}}{2j} W_N \left( \frac{2\pi (k_{h1} - k_h)}{N} \right) \right|,$$

$$y_2 = |X(k_{h2}\Delta f)| = \left| A_h \frac{e^{j\varphi_h}}{2j} W_N \left( \frac{2\pi (k_{h2} - k_h)}{N} \right) \right|.$$

Set

$$\beta = \frac{y_2 - y_1}{y_2 + y_1}.$$ (6)

Figure 3: Amplitude spectrum of the $h$th harmonic of $x(n)$ by asynchronous sampling.

And according to (4), set

$$k_{h1} - k_h = -\alpha - 0.5,$$

$$k_{h2} - k_h = -\alpha + 0.5.$$ (7)

Then $\alpha$ is in the range $[-0.5, 0.5]$ and $\beta$ can be expressed as

$$\beta = \frac{W_N \left( \frac{2\pi (-\alpha + 0.5)}{N} \right)}{W_N \left( \frac{2\pi (-\alpha + 0.5)}{N} \right) + W_N \left( \frac{2\pi (-\alpha - 0.5)}{N} \right)}.$$ (8)

Let $\alpha$ be the inverse function of (8); namely, $\alpha = u^{-1}(\beta)$; then the frequency $f_h$ can be corrected by $\alpha$:

$$f_h = k_h \Delta f = (\alpha + k_{h1} + 0.5) \Delta f.$$ (9)
And set the amplitude $A_h$ as the weighted average of the amplitudes of the $k_{h1}$ and $k_{h2}$ spectral lines:

$$A_h = (A_{h2} |W_N (2\pi (k_{h2} - k_{h1})/N)| + A_{h1} |W_N (2\pi (k_{h1} - k_{h2})/N)|)$$

$$\times (|W_N (2\pi (k_{h2} - k_{h1})/N)| + |W_N (2\pi (k_{h1} - k_{h2})/N)|)^{-1}$$

$$= 2 (y_1 + y_2)$$

$$\times (|W_N (2\pi (-\alpha + 0.5)/N)| + |W_N (2\pi (-\alpha - 0.5)/N)|)^{-1}.$$  

According to (3) and (7), the initial phase of the $h$th harmonic can be corrected as follows:

$$\phi_h = \arg [X (k_{h1} \Delta f)] + \frac{\pi}{2} - \arg [W_N \left(\frac{2\pi (k_{h1} - k_{h2})}{N}\right)]$$

$$= \arg [X (k_{h1} \Delta f)] + \frac{\pi}{2} - \arg [W_N \left(\frac{2\pi (-\alpha - 0.5)}{N}\right)].$$

According to (1), the spectrum amplitude function of Nuttall window is

$$W_N (\omega) = \sum_{m=0}^{M-1} (-1)^m b_m \left[ W_R \left(\omega - \frac{2\pi N}{m}\right) + W_R \left(\omega + \frac{2\pi N}{m}\right) \right],$$

where $W_R (\omega)$ denotes the spectrum function of rectangular window and $W_R (\omega) = (\sin (N\omega/2) / \sin (\omega/2)) e^{-jk(N-1)/2\omega}$.

Let $\omega = (2\pi N/k)$; then (12) can be expressed as

$$W_N \left(\frac{2\pi k}{N}\right) = \sin \pi k \cdot e^{-jk}$$

$$\times \left[ \sum_{m=0}^{M-1} (-1)^m b_m \frac{\sin (2\pi m k)}{\sin (\pi (k - m)/N) \sin (\pi (k + m)/N)} \right].$$

Because $| -\alpha \pm 0.5 | \leq 1, k = -\alpha \pm 0.5$, and $N$ is generally very large, (13) can be approximately expressed as

$$\left| W_N \left(\frac{2\pi (-\alpha \pm 0.5)}{N}\right) \right|$$

$$\approx \left| \sin \pi (-\alpha \pm 0.5) \cdot \left[ \sum_{m=0}^{M-1} (-1)^m b_m \frac{N (-\alpha \pm 0.5)}{\pi (-\alpha \pm 0.5)^2 - m^2} \right] \right|.$$  

Substituting (14) into (8), we can obtain the approximation of $\alpha$:

$$\alpha = u^{-1} (\beta) = H (\beta).$$  

Similarly, substituting (14) into (10), we can obtain the approximation of (10):

$$A_h = \frac{(y_1 + y_2)}{N} v(\alpha),$$

where $v(\alpha)$ denotes a function of $\alpha$ and can be expressed as follows:

$$v(\alpha) = \frac{2N}{|W_N (2\pi (-\alpha + 0.5)/N)| + |W_N (2\pi (-\alpha - 0.5)/N)|}.$$  

From (11) and (14), the phase can be corrected as follows:

$$\phi_h = \arg [X (k_{h1} \Delta f)] + \frac{\pi}{2} - \pi (-\alpha - 0.5).$$

As shown in (9), (16), and (18), $f_k, A_h, \phi_k$ are all related to $\alpha$, but $\alpha$ has a complicated relationship with $\beta$ in (8) and with $v(\alpha)$ in (17). However, the value of $\alpha$ and $v(\alpha)$ could hardly be figured out by direct analytical method.

This paper proposes a simple and efficient method based on polynomial approximation to estimate the values of $\alpha$ and $v(\alpha)$. Let $\alpha$ be a series of values in the range $[-0.5, 0.5]$; then the corresponding values of $\beta$ and $v(\alpha)$ can be computed according to (14), (8), and (17). Afterwards, based on the known mapping relationship between $\alpha$ and $\beta$, the inverse function of $\beta$, namely, $\alpha = u^{-1} (\beta)$, can be approximated by a quintic polynomial of $\beta$:

$$\alpha = u^{-1} (\beta) = H (\beta) = 2.95494514\beta$$

$$+ 0.17671943\beta^3 + 0.09230694\beta^5.$$  

Also, based on the known mapping relationship between $\alpha$ and $v(\alpha)$, $v(\alpha)$ can be approximated by a quintic polynomial of $\alpha$:

$$v(\alpha) = 3.20976143 + 0.9187393\alpha^2 + 0.14734229\alpha^4.$$  

For every harmonic, we always can find the largest and the second largest spectral lines $k_{h1}$ and $k_{h2}$ and their amplitudes $y_1$ and $y_2$. Then $\beta$ must have a concrete value according to (6), and the values of $\alpha$ and consequent $v(\alpha)$ can be calculated by (19) and (20), respectively. With the determined $\alpha$ and $v(\alpha)$, the frequency, amplitude, and initial phase of the $h$th harmonic, $f_h, A_h, \phi_h$, are corrected according to (9), (16), and (18).

As to the MFM signal $f(9, t)$, which contains multiple harmonic components, set $h = 1, 2, 4, 8, 16, 32, 64, 128, 256$, and the corresponding primary harmonic parameters $f_h, A_h, \phi_h$, can be computed, respectively.
4. Measurement Experiment

On the basis of Nuttall windowed interpolation FFT algorithm mentioned above, we can establish a BIS multifrequency synchronous measurement scheme as shown in Figure 4.

In Figure 4, the excitation current source $I_O$ is the MFM signal $f(9, t)$, $I_O$ flows through the load $Z_L$, and the voltage drop $V_L$ occurs. In biological tissue, the extracellular and intracellular fluids can be simply equivalent to a resistance, and the cell membrane can be equivalent to a capacitance, which supports the hypothesis that an RC three-element model can represent equivalently the impedance of a biological tissue [1]. This paper adopts an RC three-element equivalent circuit model (dashed box in Figure 4) as the load $Z_L$. In this experiment, we use the typical parameters $R_1 = 330 \Omega$, $R_2 = 590 \Omega$, and $C = 4.7$ nF, since the human body impedance range is 300~1500 $\Omega$, the capacitance value range is 2 pF~22 nF [32].

In order to prove the correctness of the scheme proposed above, we design a BIS measurement simulation experiment. When the RC three-element equivalent circuit is driven by the periodical current source signal $f(9, t)$, the response voltage signal $V_L$ at any time can be simulated according to the method mentioned in the literature [21]. The simultaneous samplings on $I_O$ and $V_L$ are simulated and the corresponding discrete sequence $I(n)$ and $V(n)$ are obtained, respectively. Figure 5(a) shows the simulated $V_L$ in a half-period, and Figure 5(b) gives an example of the simultaneous sampling of the excitation $f(9, t)$ and the response $V_L(t)$.

Like most broadband BIS measurements using periodic excitations, the complex impedance spectrum of the tested RC model can be just calculated as the division of the voltage and current complex Fourier coefficients obtained from their respective FFT operations [15, 22]. $I(n)$ and $V(n)$ are firstly truncated using the Nuttall window; then the Nuttall windowed interpolation FFT algorithm is performed on the two sequences, respectively. Since rigorous integer-period sampling is hard to achieve in practice, ten or more sampling cycles are usually needed for the windowed FFT algorithm. In this paper, we complete the sampling in about ten cycles, which lasts about 2.56 milliseconds (the period of $f(9, t) T_0 = 256 \mu s$). The complex Fourier coefficients of $I(n)$ and $V(n)$, namely, $(i_h, \varphi_h)$ and $(V_h, \Psi_h \ h = 1, 2, 4, 8, \ldots, 256)$ are obtained after FFT. Finally, the tested impedance amplitudes and phases are calculated according to the following formula:

$$|Z_h| = \frac{|V_h|}{|i_h|},$$

$$\theta_h = \Psi_h - \varphi_h.$$  \hspace{1cm} (21)

The BIS measurement results at the nine primary harmonics are shown in Table 4, in which the impedance amplitude relative error $|E_z| \leq 0.3\%$, and the phase absolute error $|E_p| < 0.1^\circ$.

5. Conclusion

Multifrequency simultaneous (MFS) measurement of BIS can greatly reduce measurement time and grasp the transient physiological status of living body compared with the traditional frequency-sweep measurement technology. This paper proposes a BIS multifrequency simultaneous measurement...
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Table 4: Experiment results of BIS measurement under the excitation of the MFM signal $f(9, t)$.

| Primary harmonic | Frequency (kHz) | Standard $Z_s$ | Measured $Z_m$ | Relative errors $|E_z|$ (%) | Standard Ps | Measured Pm | Absolute errors $|E_p|$ (%) |
|------------------|----------------|----------------|----------------|-----------------------------|-------------|-------------|-----------------------------|
| $f_0$            | 3.90625        | 918.1501       | 918.1311       | 0.0021                      | -2.4951     | -2.5098     | 0.0148                       |
| $2f_0$           | 7.8125         | 912.6795       | 912.8680       | 0.0207                      | -4.9562     | -4.9745     | 0.0183                       |
| $4f_0$           | 15.625         | 891.9151       | 891.8473       | 0.0076                      | -9.6518     | -9.6461     | 0.0057                       |
| $8f_0$           | 31.25          | 823.2610       | 822.8110       | 0.0547                      | -17.5164    | -17.5659    | 0.0495                       |
| $16f_0$          | 62.5           | 668.0581       | 667.5680       | 0.0734                      | -26.1026    | -26.1476    | 0.0450                       |
| $32f_0$          | 125            | 487.1455       | 486.5256       | 0.1273                      | -27.3403    | -27.4284    | 0.0881                       |
| $64f_0$          | 250            | 381.8698       | 381.0998       | 0.2016                      | -19.6909    | -19.7728    | 0.0819                       |
| $128f_0$         | 500            | 344.2257       | 343.3310       | 0.2599                      | -11.1973    | -11.2497    | 0.0524                       |
| $256f_0$         | 1000           | 333.6487       | 332.7088       | 0.2817                      | -5.8059     | -5.8323     | 0.0263                       |

(a) The simulated voltage response $V_L(t)$ on the RC three-element load in a half-period

(b) Simultaneous sampling of the excitation $f(9, t)$ and the response $V_L(t)$

Figure 5: The simulated voltage response $V_L(t)$ and simultaneous sampling of the excitation $f(9, t)$ and the response $V_L(t)$.

approach based on an MFM excitation and a Nuttall Windowed interpolation FFT algorithm. On the basis of the nine-frequency MFM signal $f(9, t)$ excitation, a Nuttall window is adopted to truncate the sample data, and an interpolation FFT algorithm based on Nuttall window is developed to perform spectral analysis, in which the parameter correction formula is provided based on polynomial approximation. A BIS measurement simulation experiment is performed on a RC three-element equivalent circuit, and results show a high accuracy with the impedance amplitude relative error.
\[ |E_z| \leq 0.3\% \text{, and the phase absolute error } |E_p| < 0.1^\circ. \]

This paper establishes an algorithm foundation for development of practical broadband BIS measurement systems.

**Conflict of Interests**

The authors declare that they have no conflict of interests.

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