Research Article

Ultimate Capacity Analysis and Determination of the Position of Failure Surface for Uplift Piles

Qian Su,¹ Xiaoxi Zhang,¹ Pingbao Yin,² and Wenhui Zhao¹

¹ School of Civil Engineering, Southwest Jiaotong University, Chengdu, Sichuan 610031, China

² School of Civil Engineering and Architecture, Changsha University of Science & Technology, Changsha, Hunan 410076, China

Correspondence should be addressed to Xiaoxi Zhang; zhangxiaoxi2529@163.com

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Ultimate capacity and failure surface position of uplift piles are dependent on soil parameters. In this paper, the horizontal slice method is used to discuss the relation among the ultimate uplift capacity, the failure surface position, and soil parameters with Mohr-Coulomb failure criterion. According to the limit equilibrium analysis, the ultimate uplift capacity is calculated by dividing soil around the pile into slices with considering the potential failure surface as a group of several sectional planes. Then the multivariate function used to calculate ultimate capacity is established and optimized by the sequential quadratic programming. Through the numerical calculation and comparison with the previous research, the results show that the method is reasonable and effective and can be used to determine the failure surface and the magnitude of the ultimate capacity of uplift piles.

1. Introduction

Uplift piles are commonly used as foundation systems for structures requiring uplift resistance such as high-rise structures and transmission lines. However, the problems of how to determine the position of failure surface and how to evaluate ultimate uplift capacity are difficult in the designing of the geotechnical engineering. The pile-soil interaction mechanism is not well understood, and very few papers are available to analyse the ultimate uplift capacity and failure mechanism.

So far, there are three main approaches for studying the uplift piles: laboratory model test, field test, and theoretic analysis. Deshmukh et al. [1] estimated the uplift capacity of pile anchors in cohesionless soil using Kotter's equation and assuming the cone frustum failure surface. Shelke et al. [2, 3] proposed to predict the ultimate uplift capacity of the single bent pile and pile group with a bent embedded in sand considering arching effect. Shanker et al. [4] presented a simple semiempirical model for predicting the uplift capacity of piles embedded in sand. But they are only applicable for sand and cohesionless soil. For clayey soil, Mohan and Chandra [5], Turner [6], and Sowa [7] reported the ultimate uplift capacity of vertical cast in situ concrete piles in clay subjected to axial uplifting force. Shin et al. [8] presented laboratory model test results for estimation of the ultimate

uplift capacity of rigid metal piles embedded in a compacted near-saturated clayey soil.

In previous researches, line and curve failure surfaces were assumed; however, ultimate capacity and failure surface shape of the uplift piles influenced each other and they were dependent on soil parameters such as internal frictional angle, cohesion, pile-soil friction angle, and unit weight of the soil. In this paper, we study the uplift capacity and failure surface position with c - ϕ soil using horizontal slice method. The c - ϕ soil around uplift piles was divided into multiple slices, and the ultimate uplift capacity and failure surface were considered as the combination of multiple slices. It can be known that when the number of segments was infinite uplift capacity takes its accurate value. Based on the limit equilibrium approach, we analyzed the forces balance of the c - ϕ soil. Then the multivariate function to calculate uplift capacity was established and optimized by the sequential quadratic programming. Lastly, we report net uplift capacity of the pile compared with previous researches.

2. Load Transfer Mechanism of Uplift Pile

The load transfer process and failure mechanism are shown in Figure 1. The relative displacement between the pile and

FIGURE 1: The load transfer mechanism of uplift pile.

FIGURE 2: The model of the single pile under the uplift load.

the soil occurs as the pile is acted on by the uplift load, where the uplift load would be transferred to the soil by the shaft resistance. With the uplift load on the pile, the shaft resistance will increase fast. When it reaches a certain critical value, tensile-shear failure of soil will occur and then the pile and the soil will be extracted. In this case, the shaft resistance equals the net uplift capacity value.

3. Theoretical Model

The model of calculation for uplift capacity of single piles with $c-\phi$ soil is shown in Figure 2. The failure surface of soil around the pile was usually considered as a straight line or curves, but, in fact, the shape and position of failure surface are unknown and are determined by some parameters. The $c-\phi$ soil around the pile is divided into masses with considering the failure sliding surface as a group of several sectional planes; the angle between every sectional failure surface and horizontal plane is α_i . If the number of sections is large enough, we can get

FIGURE 3: The calculation model of multiple slices.

smooth and continuous failure surface and the reasonable uplift capacity.

Figure 3 shows the plane calculation model of multiple masses with horizontal slice method. The following assumptions are also made: (1) backfill material obeys Mohr-Coulomb failure criterion. (2) The $c-\phi$ soil is homogeneous and isotropic. (3) The failure surface is tangent to a point of tip of pile or the pile surface. (4) The horizontal interslices force is ignored.

During uplift of a vertical circular pile embedded in sand, an axisymmetric solid body of revolution of soil along with the pile is assumed to be initiated to move up along the resulting surface.

In the failure surface, the forces R_i and T_i are related as

$$T_i = R_i \tan \varphi + cl_i, \quad (1)$$

where l_i is the length of i th sectional failure surface; R_i is the normal force in the i th sectional failure surface; T_i is the tangential force in the i th sectional failure surface. Consider

$$l_i = \frac{h}{\sin \alpha_i} = \frac{H}{n} \frac{1}{\sin \alpha_i}. \quad (2)$$

The equilibrium of forces in the horizontal direction for the i th mass gives

$$\sum X = 0, \quad \pi d F_{xi} = 2\pi \left(\frac{d}{2} + \frac{t_i + t_{i-1}}{2} \right) (T_i \cos \alpha_i + N_i \sin \alpha_i), \quad (3)$$

where t_i is the length of the i th slice surface and the expression is as follows:

$$t_i = \frac{1}{2}d + h \sum_{j=1}^i \cot \alpha_j. \quad (4)$$

The equilibrium of forces in the vertical direction for the i th mass gives

$$\begin{aligned} \sum Y = 0, \\ \pi d F_{yi} = W_i + 2\pi \left(\frac{d}{2} + \frac{t_i + t_{i-1}}{2} \right) (T_i \sin \alpha_i - N_i \cos \alpha_i) \\ + \pi q_i \left[\left(\frac{d}{2} + t_i \right)^2 - \left(\frac{d}{2} \right)^2 \right] \\ - \pi q_{i-1} \left[\left(\frac{d}{2} + t_{i-1} \right)^2 - \left(\frac{d}{2} \right)^2 \right], \end{aligned} \quad (5)$$

where q_i is soil pressure in the i th slice and the expression is as follows:

$$q_i = \gamma h(n - i), \quad (6)$$

and W_i is the weight of soil slice and the expression is as follows:

$$W_i = \pi \left[\left(\frac{d}{2} + \frac{t_i - t_{i-1}}{2} \right)^2 - \left(\frac{d}{2} \right)^2 \right] h\gamma. \quad (7)$$

The forces F_{xi} and F_{yi} and P_a are related as

$$\frac{F_{yi}}{\sin \delta} = \frac{F_{xi}}{\cos \delta}. \quad (8)$$

Solving (11) and (13), the uplift pile frictional resistance can be expressed as

$$\begin{aligned} F_{yi} = \frac{1}{d} & \left(A_i W_i + q_i \left[\left(\frac{d}{2} + t_i \right)^2 - \left(\frac{d}{2} \right)^2 \right] A_i \right. \\ & - q_{i-1} \left[\left(\frac{d}{2} + t_{i-1} \right)^2 - \left(\frac{d}{2} \right)^2 \right] A_i \\ & \left. + c l_i (A_i \sin \alpha_i - B_i \cos \alpha_i) (d + t_i + t_{i-1}) \right) \\ & \times (A_i - B_i \cot \delta)^{-1}. \end{aligned} \quad (9)$$

The uplift capacity component can be reduced as

$$P_u = \pi d \sum_{i=1}^n F_{yi} = f(\alpha_1, \alpha_2, \dots, \alpha_n). \quad (10)$$

The uplift capacity of single piles is obtained by minimizing P_u with respect to α_i . To this end, the following set of equations has to be solved:

$$\begin{aligned} \frac{\partial P_u}{\partial \alpha_1} &= 0 \\ \frac{\partial P_u}{\partial \alpha_2} &= 0 \\ &\vdots \\ \frac{\partial P_u}{\partial \alpha_n} &= 0. \end{aligned} \quad (11)$$

Knowing the length of the single pile (L), unit weight (γ), cohesion (c), internal friction angle (ϕ) of soil, the pile-soil friction angle (δ), and the pile diameter (d), we employ the sequential quadratic programming to optimize the object function to get the limit uplift capacity of single piles. This is an iterative method for nonlinear optimization and has become the most successful method for solving nonlinearly constrained optimization problems [9].

4. Verification and Numerical Result

4.1. Existing Predictive Models. In the following, the commonly applied predictive models are described in brief. Standard model method, the earliest method, assumed that the failure takes place on a cylindrical surface. Net uplift capacity of a vertical pile can be estimated as follows:

$$P_{nu} = \frac{1}{2} \pi K_s d \gamma L^2 \tan \delta, \quad (12)$$

where K_s is the lateral earth pressure coefficient, d is the diameter of the pile, γ is the unit weight of the soil, L is the length of the pile, and δ is the pile-soil friction angle. Levacher and Sieffert [10] and Das [11] suggested that K_s can be taken as equal to $K_0 = (1 - \sin \phi)$ for bored piles. Field engineers generally estimate the uplift capacity of the pile by assuming a slip surface as a truncated inverted cone with the enveloping sides rising at $\phi/2$ degrees from the vertical. Dead weight within the frustum is usually considered as the ultimate uplift capacity of the pile. Consider

$$P_{nu} = \frac{1}{3} \pi \gamma L^3 \tan^2 \frac{\phi}{2}. \quad (13)$$

Meyerhof's model method [12] ignored the weight of the pile. The uplift capacity can be expressed as

$$P_{nu} = \frac{1}{3} \pi K_u d \gamma L^2 \tan \delta, \quad (14)$$

where K_u is the uplift coefficient and can vary within wide limits and can depend not only on the soil properties, but also on the type of pile and method of installation. Das [13] reported that the unit skin friction at the soil pile interface increases linearly with depth up to a critical embedment ratio. The critical embedment ratio is dependent on the relative density (D_r) and is expressed as

$$\begin{aligned} \left(\frac{H}{d} \right)_{cr} &= 0.156 D_r + 3.58, \quad D_r \leq 0.7, \\ \left(\frac{H}{d} \right)_{cr} &= 14.5, \quad D_r \geq 0.7. \end{aligned} \quad (15)$$

Uplift capacity of piles in sand can be estimated as

$$\begin{aligned} P_u &= \frac{1}{2} p \gamma \pi K_u L^2 \tan \delta, \quad \frac{L}{d} \leq \left(\frac{L}{d} \right)_{cr}, \\ P_u &= \frac{1}{2} p \gamma \pi K_u L_{cr}^2 \tan \delta + p \gamma L_{cr} K_u \tan \delta (L - L_{cr}), \quad \frac{L}{d} \geq \left(\frac{L}{d} \right)_{cr}. \end{aligned} \quad (16)$$

Chattopadhyay and Pise [14] assumed the failure surface to be curved and proposed a generalized theory to evaluate uplift resistance of a circular vertical pile embedded in sand. Consider

$$P_u = A_1 \pi d \gamma L^2, \quad (17)$$

where A_1 is the uplift capacity factor. The above theories differ mainly in their assumptions with regard to the shape and extent of the failure surface.

TABLE 2: Error between the proposed method and model experimental values of the ultimate uplift capacity.

Method	Maximum relative error	Average relative error
Meyerhof method	69.4%	21.6%
Das method	69.3%	17.7%
Chattopadhyay method	106.4%	15.6%
Shanker method	51.6%	22.1%
Truncated method	133.2%	51.7%
Proposed method	33.0%	14.8%

TABLE 3: Comparison of uplift capacity calculated using different methods and field test result.

Parameters	Gross uplift capacity (kN)		
Diameter (m)	1.2	Ismael and Klym full-scale test	888.8
Length (m)	6.4	Chattopadhyay method [14]	968.8
Internal friction angle (Deg.)	34	Shanker method [4]	714.3
Pile-soil friction angle (Deg.)	27	Truncated method [17]	463.2
Unit weight (kN/m ³)	11	Meyerhof method [12]	325.1
		Proposed method	935.2

4.2. Comparisons with Different Methods. Values of proposed net uplift capacity have been compared with different predictive methods and model experimental results conducted by several scholars. As can be seen from Tables 1 and 2, the maximum relative error between the proposed method and model experimental values of the ultimate uplift capacity is within 33.0%, and the average relative error is 14.8%. However, the relative error of the Meyerhof method is within 69.4%, and the average relative error is 21.6%; the relative error of the Das method is within 69.3%, and the average relative error is 17.7%; the relative error of the Chattopadhyay method is within 106.4%, and the average relative error is 15.6%; the relative error of the Shanker method is within 51.6%, and the average relative error is 22.1%; the relative error of the Truncated method is within 133.2%, and the average relative error is 51.7%. From the above study it is found that the proposed method can give more reasonable predictions.

Ismael and Klym [15] reported a full-scale test under uplift of cylindrical pier of diameter 1.2 m and a length of 6.4 m, embedded in a compact fine to medium sand with some silt and traces of clay. The internal friction angle equals 34°, and soil unit weight is 11 kN/m³. The pile-soil friction angle is 80% of the value of ϕ for theoretical prediction [16]. The predicted gross uplift capacity is 935.2 kN, very close to the field test measured value of 888.8 kN. A comparison of the results obtained by different theoretical method is investigated in Table 3. It is seen that the predicted gross uplift capacity is in agreement with the values reported by Chattopadhyay and Pise [14] and Shanker et al. [4].

FIGURE 4: Failure surface shape and position with different internal friction angles.

FIGURE 5: Failure surface shape and position with different cohesions.

Meanwhile, the results obtained by Meyerhof [12] method and Truncated [17] method underestimate the experimental results.

4.3. Failure Surface Comparison. The ultimate uplift capacity can be significantly influenced by the shape and position of failure surfaces. Figures 4 and 5 show the influence of the cohesion and internal friction angle on the shape and position of failure surfaces with $L/d = 8$, $d = 1$ m, $\gamma = 15.79$ kN/m³, and $n = 16$. The results show that with the increase of the value of cohesion and internal friction angle, the ultimate uplift capacity can improve effectively. The failure surface moves to the pile gradually with decreasing cohesion and internal friction angle.

5. Summary and Conclusions

A theoretical method for predicting the uplift capacity and failure surface of piles embedded in c - ϕ soil has been proposed based on the horizontal slice method in the framework of limit equilibrium analysis. The relation among the ultimate

uplift capacity, the failure surface position, and soil parameters was discussed. It enables a reasonably logical analysis and the quantitative estimates to be made of the effects of soil parameters. The results obtained by the proposed method are found to be in close agreement with model experimental and full-scale test results. With the increase of the value of cohesion and internal friction angle, the ultimate uplift capacity can improve effectively. The failure surface moves to the pile gradually with decreasing cohesion and internal friction angle.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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