Research Article

Consensus of the Multiagent System with a Dynamic Leader Based on Directed Topology Using Laplace Transform

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The consensus of the multiagent system with directed topology and a leader is investigated, in which the leader is dynamic. Based on Laplace transform method, the accurate upper error bound between the leader and the followers can be obtained. It is also proved that all agents of the system will aggregate and eventually form a cohesive cluster following the leader if the leader is globally reachable. Finally, some simulation examples are given to illustrate the theoretical results.

1. Introduction

In recent years, the consensus problem of multiagent systems has become a hot topic due to their broad applications, such as cooperative unmanned air vehicles, automated highway systems, air traffic control, and autonomous underwater vehicles [1–22].

Various algorithms and models about multiagent systems have been discussed based on different tasks or interests. The leader-following system is one of the most interesting topics in the motion control of the multiagent systems. Vicsek et al. [1] proposed a simple model about autonomous agents moving with a constant identical speed and tending to the average direction of its neighbors. It is demonstrated numerically that all the agents will move in the same direction at the same speed eventually. Jadbabaie et al. [2] gave the theoretical explanation for the numerical results of the Vicsek model by algebraic graph theory. Based on the Vicsek model, some researchers designed the leader-following model, where the leader agent can be regarded as the control input used to control the other agents in such system. Shi et al. [5] regarded the reference signal as a virtual leader for guiding the agent

group to move at the desired constant velocity. Hong et al. [7] discussed the leader-following system with variable coupling topology. Olfati-Saber [3] used virtual leaders to accomplish obstacle avoidance.

The coupling topology plays an important role in the studies of multiagent systems. Because of the complexity in the consensus analysis with directed topology, most researchers focus on the undirected topology or balance topology to simplify the problem [7–22]. In this paper, we will discuss the consensus problem with directed topology and give a weak condition for reaching network consensus. According to Laplace transform, the accurate upper error bound between the leader and followers can be calculated and some assumptions needed in Lyapunov approach can be simplified. Finally, some simulation examples are provided to illustrate the theoretical results.

This paper is organized as follows. Section 2 proposes the model formulation and some preliminaries of graph theory. Section 3 gives the analysis of the convergence of the model. Section 4 gives simulations of the theoretical results. Finally, the main contribution of this paper is summarized in Section 5.

2. Model and Preliminaries

2.1. Model. Consider a multiagent system of $n + 1$ agents, where the leader is labeled with 0 and the followers are labeled with $1, 2, \dots, n$. The motion of the system is described by

$$\dot{x}_i = - \sum_{j \in \mathcal{N}_i} a_{ij} (x_i - x_j) + b_i (x_0 - x_i), \quad i = 1, 2, \dots, n, \quad (1)$$

$$\dot{x}_0 = v(t),$$

where $x_i \in \mathbb{R}^m$ represents the state of agent i ; $\mathcal{N}_i$ is the neighbor set of agent i . $A = [a_{ij}] \in \mathbb{R}^{n \times n}$ is the coupling matrix, where a_{ij} is the weight parameter with $a_{ij} \geq 0$ and $a_{ii} = 0$. $b_i(x_0 - x_i)$ is the term performing attractive effect from leader agent 0 to agent i . Define $B = \text{diag}\{b_1, b_2, \dots, b_n\} \in \mathbb{R}^{n \times n}$. $b_i > 0$ means agent i can measure the information of the leader with strength b_i , while $b_i = 0$ means there is no information from the leader to agent i . $v(t)$ is the velocity of the leader, which is a bounded function.

2.2. Preliminaries. To discuss the coordinated control among the agents, graph theory is a very effective tool. Regard the agent as a node and the connection link between any two agents as an edge; the coupling topology is conveniently described by a directed graph. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ be a weighted digraph of order n with the set of nodes $\mathcal{V} = \{1, 2, 3, \dots, n\}$ and set of arcs $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. $A = [a_{ij}]$ is adjacency matrix of graph $\mathcal{G}$. An arc of $\mathcal{G}$ is denoted by (i, j) , which starts from i and ends at j . The set of neighbors of node i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$. A digraph $\mathcal{G}$ is strongly connected if there exists a path between any two distinct nodes. For a node j , if there exists at least a path from every other node i in $\mathcal{G}$ to node j , we say that node j is globally reachable.

A diagonal matrix $D = \text{diag}\{d_1, d_2, \dots, d_n\} \in \mathbb{R}^{n \times n}$ is a degree matrix of $\mathcal{G}$ with its diagonal elements $d_i = \sum_{j \in \mathcal{N}_i} a_{ij}$, $i = 1, 2, \dots, n$. Then the Laplacian of the weighted digraph $\mathcal{G}$ (or matrix A) is defined as

$$L = D - A \in \mathbb{R}^{n \times n}. \quad (2)$$

In order to study the leader-following problem, a multiagent system with directed topology is considered, which is consisting of n agents and one leader. In $\overline{\mathcal{G}}$, if there exists at least one path from every node in $\mathcal{G}$ to the leader, we say that the leader is globally reachable in $\overline{\mathcal{G}}$.

It is easy to see that L has a zero eigenvalue corresponding to the right eigenvector $\mathbf{1} = (1, 1, \dots, 1)^T$.

Lemma 1 (see [9]). *A digraph $\mathcal{G}$ has a globally reachable node if and only if 0 is a simple eigenvalue of L (i.e., $\text{rank}(L) = n - 1$).*

Lemma 2. *The nonzero eigenvalues of L have positive real parts.*

Proof. The proof is similar to that of [3]; it is omitted here. $\square$

Lemma 3. *The eigenvalues of $L + B$ have positive real parts if and only if the leader is a globally reachable node in digraph $\overline{\mathcal{G}}$.*

Proof. If node 0 is globally reachable, the Laplacian matrix of $\mathcal{G}$ can be written as

$$\overline{L} = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 \\ l_{10} & l_{11} & l_{12} & \dots & l_{1n} \\ l_{20} & l_{21} & l_{22} & \dots & l_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ l_{n0} & l_{n1} & l_{n2} & \dots & l_{nn} \end{pmatrix} \in \mathbb{R}^{(n+1) \times (n+1)}. \quad (3)$$

According to Lemma 1, then $\text{rank}(\overline{L}) = n$.

Notice that the block matrix of $\overline{L}$ is

$$\begin{pmatrix} l_{11} & l_{12} & \dots & l_{1n} \\ l_{21} & l_{22} & \dots & l_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ l_{n1} & l_{n2} & \dots & l_{nn} \end{pmatrix} = L + B \in \mathbb{R}^{n \times n}. \quad (4)$$

Then, we can have $\text{rank}(L + B) = n$, where L is the Laplacian matrix of $\mathcal{G}$. $\square$

3. Coordinated Control Analysis

In this section, we focus on the coordinated control problem of model (1). Denote $X = (x_1, x_2, \dots, x_n)^T$, $X_0 = (x_0, x_0, \dots, x_0)^T$, and $V = (v, v, \dots, v)^T = \mathbf{1}v$; then the model can be rewritten into matrix form

$$\dot{X} = -LX - B(X - X_0). \quad (5)$$

Based on system (5), we can derive the following theorem.

Theorem 4. *If leader 0 is globally reachable in digraph $\overline{\mathcal{G}}$, all the agents described by (1) will converge and form a cohesive cluster following the leader asymptotically. Moreover, the errors between the leader and followers will be included in a fixed bound.*

Proof. To solve this problem, we introduce the Laplace transform, succinctly denoted by $\mathcal{L}(\cdot)$ in this paper. System (5) can be transformed as

$$\dot{X} - \dot{X}_0 = -(L + B)(X - X_0) - V. \quad (6)$$

Define $X - X_0 = Y$ as the error vector; then (6) can be written as

$$\dot{Y} = -(L + B)Y - V. \quad (7)$$

Applying $\mathcal{L}$ to (7), then we have

$$\mathcal{L}(\dot{Y}) = \mathcal{L}(-(L + B)Y - V). \quad (8)$$

That is,

$$[sI_n + (L + B)] \mathcal{L}(Y) = Y(0) - \mathcal{L}(V). \quad (9)$$

For simplicity, denote $\mathcal{L}(Y) = \mathcal{Y}(s)$ and $\mathcal{L}(V) = \mathcal{V}(s)$, where s is the Laplace variable; then

$$[sI_n + (L + B)] \mathcal{Y}(s) = Y(0) - \mathcal{V}(s). \quad (10)$$

According to *Cramer rule*, we can get solutions of (10):

$$\mathcal{Y}_i(s) = \frac{\Delta[sI_n + (L+B)]^i}{\Delta[sI_n + (L+B)]}, \quad i = 1, 2, \dots, n, \quad (11)$$

where I_n is the identity matrix of order n ; $\Delta[sI_n + (L+B)]^i$ is the determinant of matrix $sI_n + (L+B)$ in which the i th column has been replaced by $Y(0) - \mathcal{V}(s)$; and $\Delta[sI_n + (L+B)]$ is the determinant of matrix $sI_n + (L+B)$. We can separate (11) into two parts as follows:

$$\mathcal{Y}_i(s) = \frac{\Delta[sI_n + (L+B)]_1^i}{\Delta[sI_n + (L+B)]} - \frac{\Delta[sI_n + (L+B)]_2^i}{\Delta[sI_n + (L+B)]}, \quad (12)$$

where $\Delta[sI_n + (L+B)]_1^i$ is the determinant of matrix $sI_n + (L+B)$ in which the i th column has been replaced by $Y(0)$; $\Delta[sI_n + (L+B)]_2^i$ is the determinant of matrix $sI_n + (L+B)$ in which the i th column has been replaced by $\mathcal{V}(s)$. Because $\mathcal{V}(s) = \mathcal{L}[v(t)]\mathbf{1}$, determinant $\Delta[sI_n + (L+B)]_2^i$ can be transformed as

$$\Delta[sI_n + (L+B)]_2^i = \mathcal{L}[v(t)] \cdot \Delta[sI_n + (L+B)]_3^i, \quad (13)$$

where $\Delta[sI_n + (L+B)]_3^i$ is the determinant of matrix $sI_n + (L+B)$ in which the i th column has been replaced by $\mathbf{1}$. Since $\Delta[sI_n + (L+B)]_1^i$, $\Delta[sI_n + (L+B)]_3^i$, and $\Delta[sI_n + (L+B)]$ are polynomials about s , for simplicity, we denote $P_{i1}(s) = \Delta[sI_n + (L+B)]_1^i$, $P_{i3}(s) = \Delta[sI_n + (L+B)]_3^i$, and $Q(s) = \Delta[sI_n + (L+B)]$, respectively. Then (12) can be rewritten as

$$\begin{aligned} \mathcal{Y}_i(s) &= \frac{\Delta[sI_n + (L+B)]_1^i}{\Delta[sI_n + (L+B)]} - \frac{\mathcal{L}[v(t)] \Delta[sI_n + (L+B)]_3^i}{\Delta[sI_n + (L+B)]} \\ &= \frac{P_{i1}(s)}{Q(s)} - \frac{\mathcal{L}[v(t)] P_{i3}(s)}{Q(s)}. \end{aligned} \quad (14)$$

According to Lemma 3, the eigenvalues of $L+B$ have positive real parts, denoted as

$$\lambda_1, \lambda_2, \dots, \lambda_j, \quad (15)$$

with multiplicity as $p, q, \dots, r$. So we have

$$\Delta(sI + L + B) = (s + \lambda_1)^p (s + \lambda_2)^q \cdots (s + \lambda_j)^r. \quad (16)$$

If the fractional expressions in (14) are reducible, we should make reduction of the fraction expressions. In the following discussion, we assume that the fractional expressions

in (14) are irreducible. Applying *Heaviside's Method*, (14) can be expanded into

$$\begin{aligned} \mathcal{Y}_i(s) &= \frac{c_{1p}^i}{(s + \lambda_1)^p} + \frac{c_{1(p-1)}^i}{(s + \lambda_1)^{(p-1)}} + \cdots + \frac{c_{11}^i}{(s + \lambda_1)} \\ &\quad + \frac{c_{2q}^i}{(s + \lambda_2)^q} + \frac{c_{2(q-1)}^i}{(s + \lambda_2)^{(q-1)}} + \cdots + \frac{c_{21}^i}{(s + \lambda_2)} \\ &\quad + \cdots + \frac{c_{jr}^i}{(s + \lambda_j)^r} + \frac{c_{j(r-1)}^i}{(s + \lambda_j)^{(r-1)}} \\ &\quad + \cdots + \frac{c_{j1}^i}{(s + \lambda_j)} - \mathcal{L}[v(t)] \\ &\quad \times \left(\frac{d_{1p}^i}{(s + \lambda_1)^p} + \frac{d_{1(p-1)}^i}{(s + \lambda_1)^{(p-1)}} + \cdots + \frac{d_{11}^i}{(s + \lambda_1)} \right. \\ &\quad \left. + \frac{d_{2q}^i}{(s + \lambda_2)^q} + \frac{d_{2(q-1)}^i}{(s + \lambda_2)^{(q-1)}} + \cdots \right. \\ &\quad \left. + \frac{d_{21}^i}{(s + \lambda_2)} + \cdots + \frac{d_{jr}^i}{(s + \lambda_j)^r} \right. \\ &\quad \left. + \frac{d_{j(r-1)}^i}{(s + \lambda_j)^{(r-1)}} + \cdots + \frac{d_{j1}^i}{(s + \lambda_j)} \right), \end{aligned} \quad (17)$$

where

$$\begin{aligned} c_{1k}^i &= \frac{d^{p-k}}{ds^{p-k}} \left[(s + \lambda_1)^{(p-k)} \frac{P_{i1}(s)}{Q(s)} \right] \Big|_{s=\lambda_1}, \quad k = 1, 2, \dots, p, \\ c_{2k}^i &= \frac{d^{q-k}}{ds^{q-k}} \left[(s + \lambda_2)^{(q-k)} \frac{P_{i1}(s)}{Q(s)} \right] \Big|_{s=\lambda_2}, \quad k = 1, 2, \dots, q, \\ &\vdots \\ c_{jk}^i &= \frac{d^{r-k}}{ds^{r-k}} \left[(s + \lambda_j)^{(r-k)} \frac{P_{i1}(s)}{Q(s)} \right] \Big|_{s=\lambda_j}, \quad k = 1, 2, \dots, r, \\ d_{1k}^i &= \frac{d^{p-k}}{ds^{p-k}} \left[(s + \lambda_1)^{(p-k)} \frac{P_{i3}(s)}{Q(s)} \right] \Big|_{s=\lambda_1}, \quad k = 1, 2, \dots, p, \\ d_{2k}^i &= \frac{d^{q-k}}{ds^{q-k}} \left[(s + \lambda_2)^{(q-k)} \frac{P_{i3}(s)}{Q(s)} \right] \Big|_{s=\lambda_2}, \quad k = 1, 2, \dots, q, \\ &\vdots \\ d_{jk}^i &= \frac{d^{r-k}}{ds^{r-k}} \left[(s + \lambda_j)^{(r-k)} \frac{P_{i3}(s)}{Q(s)} \right] \Big|_{s=\lambda_j}, \quad k = 1, 2, \dots, r. \end{aligned} \quad (18)$$

Applying Inverse Laplace transform to (17), we obtain

$$\begin{aligned}
 y_i(t) = & \left(c_{11}^i + \frac{c_{12}^i}{1!}t + \cdots + \frac{c_{1p}^i}{(p-1)!}t^{(p-1)} \right) e^{(-\lambda_1 t)} \\
 & + \left(c_{21}^i + \frac{c_{22}^i}{1!}t + \cdots + \frac{c_{2q}^i}{(q-1)!}t^{(q-1)} \right) e^{(-\lambda_2 t)} + \cdots \\
 & + \left(c_{j1}^i + \frac{c_{j2}^i}{1!}t + \cdots + \frac{c_{jr}^i}{(r-1)!}t^{(r-1)} \right) e^{(-\lambda_j t)} \\
 & + \int_0^t \left[\left(d_{11}^i + \frac{d_{12}^i}{1!}\xi + \cdots + \frac{d_{1p}^i}{(p-1)!}\xi^{(p-1)} \right) e^{(-\lambda_1 \xi)} \right. \\
 & \quad + \left(d_{21}^i + \frac{d_{22}^i}{1!}\xi + \cdots + \frac{d_{2q}^i}{(q-1)!}\xi^{(q-1)} \right) \\
 & \quad \times e^{(-\lambda_2 \xi)} + \cdots \\
 & \quad + \left(d_{j1}^i + \frac{d_{j2}^i}{1!}\xi + \cdots + \frac{d_{jr}^i}{(r-1)!}\xi^{(r-1)} \right) \\
 & \quad \times e^{(-\lambda_j \xi)} \left. \right] v(t-\xi) d\xi.
 \end{aligned} \tag{19}$$

According to integral property, it is easy to have

$$\begin{aligned}
 y_i(t) \leq & \left(c_{11}^i + \frac{c_{12}^i}{1!}t + \cdots + \frac{c_{1p}^i}{(p-1)!}t^{(p-1)} \right) e^{(-\lambda_1 t)} \\
 & + \left(c_{21}^i + \frac{c_{22}^i}{1!}t + \cdots + \frac{c_{2q}^i}{(q-1)!}t^{(q-1)} \right) e^{(-\lambda_2 t)} + \cdots \\
 & + \left(c_{j1}^i + \frac{c_{j2}^i}{1!}t + \cdots + \frac{c_{jr}^i}{(r-1)!}t^{(r-1)} \right) e^{(-\lambda_j t)} + |v(t)| \\
 & \times \left| \int_0^t \left[\left(d_{11}^i + \frac{d_{12}^i}{1!}\xi + \cdots + \frac{d_{1p}^i}{(p-1)!}\xi^{(p-1)} \right) e^{(-\lambda_1 \xi)} \right. \right. \\
 & \quad + \left(d_{21}^i + \frac{d_{22}^i}{1!}\xi + \cdots + \frac{d_{2q}^i}{(q-1)!}\xi^{(q-1)} \right) \\
 & \quad \times e^{(-\lambda_2 \xi)} + \cdots \\
 & \quad + \left(d_{j1}^i + \frac{d_{j2}^i}{1!}\xi + \cdots + \frac{d_{jr}^i}{(r-1)!}\xi^{(r-1)} \right) \\
 & \quad \times e^{(-\lambda_j \xi)} \left. \right] d\xi \Big|.
 \end{aligned} \tag{20}$$

FIGURE 1: Errors between leader and followers about system (1), in which 0 is globally reachable in $\bar{\mathcal{G}}$ and $v(t) = 1.3$.

Since as $t \rightarrow +\infty$, we have $e^{(-\lambda t)} \rightarrow 0$, then we can get the following inequality:

$$\begin{aligned}
 |y_i(t)| & \leq \varepsilon + H \left| \frac{d_{11}^i}{\lambda_1} + \frac{d_{12}^i}{\lambda_1^2} + \cdots + \frac{d_{1p}^i}{\lambda_1^p} + \frac{d_{21}^i}{\lambda_2} + \frac{d_{22}^i}{\lambda_2^2} \right. \\
 & \quad + \cdots + \frac{d_{2q}^i}{\lambda_2^q} + \cdots + \frac{d_{j1}^i}{\lambda_j} + \frac{d_{j2}^i}{\lambda_j^2} + \cdots + \frac{d_{jr}^i}{\lambda_j^r} \Big| \\
 & = \varepsilon + HM_i,
 \end{aligned} \tag{21}$$

as $t \rightarrow +\infty$ where ε is a function close to 0, H is the upper bound of $v(t)$, and $M_i = |(d_{11}^i/\lambda_1) + (d_{12}^i/\lambda_1^2) + \cdots + (d_{1p}^i/\lambda_1^p) + (d_{21}^i/\lambda_2) + (d_{22}^i/\lambda_2^2) + \cdots + (d_{2q}^i/\lambda_2^q) + \cdots + (d_{j1}^i/\lambda_j) + (d_{j2}^i/\lambda_j^2) + \cdots + (d_{jr}^i/\lambda_j^r)|$. It shows that the upper bound of $y_i(t)$ is influenced by $v(t)$ and the coupling matrix A . Define $M = \max_{i=1,2,\dots,n} \{M_i\}$ and choose $\varepsilon = 0$; we have

$$|x_i(t) - x_0(t)| = |y_i(t)| \leq HM, \tag{22}$$

as $t \rightarrow +\infty$. Then, any agent of the system will enter into a region with the leader as the center bounded by HM within sufficient time.

This completes the proof. $\square$

Remark 1. In Theorem 4, the error between the leader 0 and any agent is bounded. If the velocity of leader is zero, we can get that the error will tend to zero, which implies that all the agents will asymptotically approach the same states of the leader. If the velocity of the leader is nonzero and bounded, it is easy to see that the states and velocities of all the followers will keep in a bounded region following the leader.

FIGURE 2: Errors between leader and followers about system (1), in which 0 is globally reachable in $\bar{\mathcal{G}}$ and $v(t) = 1.5 + 2 \sin(4t)$.

FIGURE 3: Errors between leader and followers about system (1), in which 0 is globally reachable in $\bar{\mathcal{G}}$ and $v(t) = 0.7$.

4. Numerical Simulations

In order to verify the above theoretical analysis, we present some numerical simulations to illustrate the systems. These simulations are performed with ten followers and one leader, and the initial positions of the agents are chosen randomly. The coupling matrix A is given based on certain conditions.

Figures 1, 2, and 3 show the errors between leader and followers about system (1), under different coupling topology and control. It is easy to see that the states of the error are contained in a bounded region. Figures 4 and 5 depict the trajectories of the agents in 2-dimensional Euclidian space. If leader agent in the coupling topology is globally reachable, then the followers in the system will reach synchronization with the leader within sufficient time. Figures 6 and 7 show the trajectories of the agents in 3-dimensional Euclidian space, in which the states of the followers are influenced by

FIGURE 4: Trajectories of the agents about system (1), in which 0 is globally reachable in $\bar{\mathcal{G}}$ and $v(t) = 0.7 + \sin(t)$.

FIGURE 5: Trajectories of the agents about system (1), in which 0 is globally reachable in $\bar{\mathcal{G}}$ and $v(t) = 0.7$.

the state of leader, and all agents in the system can achieve consensus eventually.

5. Conclusion

In this paper, we have investigated the coordinated control of leader-following multiagent systems. It is proved that the agents of the systems will aggregate and form a cluster following the leader asymptotically. Meanwhile, we have studied the coupling topology among the agents in the general case. The systems considered in this paper can better reflect the collective behavior in practice. It is clear that the ideas and approaches about graph theory and linear dynamical system theory will play an important role in the analysis of the multiagent system. Finally, simulations give an effective demonstration of the leader-following systems.

FIGURE 6: Trajectories of the agents about system (1), in which 0 is globally reachable in $\overline{\mathcal{E}}$.

FIGURE 7: Trajectories of the agents about system (1), in which 0 is globally reachable in $\overline{\mathcal{E}}$.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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