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Research Article

H_∞ Filtering for Discrete Markov Jump Singular Systems with Mode-Dependent Time Delay Based on T-S Fuzzy Model

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This paper investigates the H_∞ filtering problem of discrete singular Markov jump systems (SMJSs) with mode-dependent time delay based on T-S fuzzy model. First, by Lyapunov-Krasovskii functional approach, a delay-dependent sufficient condition on H_∞-disturbance attenuation is presented, in which both stability and prescribed H_∞ performance are required to be achieved for the filtering-error systems. Then, based on the condition, the delay-dependent H_∞ filter design scheme for SMJSs with mode-dependent time delay based on T-S fuzzy model is developed in term of linear matrix inequality (LMI). Finally, an example is given to illustrate the effectiveness of the result.

1. Introduction

Recently, some efforts on filtering and control problem have been put on the Markov jump linear systems with Markovian jump time delays [1–7]. For example, [1] dealt with the robust stabilizability and H_∞ disturbance attenuation for a class of uncertain descriptor time-delay systems with jumping parameters. The transition of the jumping parameters in the systems is governed by a finite state Markov process. [2] addressed the problems of delay-dependent robust H_∞ control and filtering for Markovian jump linear systems with norm-bounded parameter uncertainties and time-varying delays. The robust H_∞ filtering problem for mode-dependent time-delay discrete Markov jump singular systems with parameter uncertainties is discussed in [3]. The energy-to-peak filtering problem of Markov jump systems with interval time-varying delay is investigated in [4]. The ℓ_2-ℓ_∞ fuzzy control problem was considered for nonlinear stochastic Markov jump systems with neutral time-delays in [5].

In practical applications, to research the nonlinear time-delay system, the scholars considered the T-S fuzzy time-delay model which is a kind of effective representation, and many analysis and synthesis methods for T-S fuzzy time-delay systems have been developed over the past years [8–17]. So, many scholars extended this model to Markov jump systems with time delays [18–28]. Reference [18] was concerned with an H_∞ control for a class of T-S fuzzy Markov jump system under unreliable communication links. Reference [19] concerned with the adaptive synchronization for T-S fuzzy neural networks with stochastic noises and Markovian jumping parameters. Reference [20] was concerned with the stability and stabilization problems for a class of nonlinear systems with Markovian jump parameters. The T-S fuzzy model was employed to represent the Markovian jump nonlinear systems with partly unknown transition probabilities. Reference [21] dealt with the delay-dependent asymptotic stability analysis problem for a class of fuzzy bidirectional associative memory neural networks with time-varying interval delays and Markovian jumping parameters by T-S fuzzy model. In [22], the asymptotic stability of fuzzy Markovian jumping genetic regulatory networks with time-varying delays by delay decomposition approach was investigated.

For singular systems, since the regularity, causality, and stability are needed to be considered, this class of systems is more complicated and difficult compared with standard state

space systems. The strict conditions of linear matrix inequalities (LMI) are not easy to obtain. Many references [29–31] discussed the control problems for SMJSs.

However, to the best of our knowledge, the H_∞ filtering problem of discrete SMJSs with mode-dependent time delay based on T-S fuzzy model has not been addressed, which is the focus of this paper.

This paper is organized as follows: In Section 2, the filter for SMJSs with mode-dependent time delay based on T-S fuzzy model is formulated. In Section 3 we give the sufficient condition to assure asymptotic stability and the H_∞ noise-attenuation level bound for the SMJSs filtering-error systems. Based on the condition in Section 3, we present a stable fuzzy filter in term of LMIs. Section 4 provides illustrative examples to demonstrate the effectiveness of the proposed method. Conclusions are given in Section 5.

Notations. The notations used throughout this paper are fairly standard. The superscript "T" stands for matrix transpose, and the notation $P > 0$ ($P \geq 0$) means that matrix P is real symmetric and positive (or being positive semi-definite). I and 0 are used to denote appropriate dimensions identity matrix and zero matrix, respectively. The notation $*$ in a symmetric always denotes the symmetric block in the matrix. The parameter diag$\{\cdots\}$ denotes a block-diagonal matrix. Matrices, if not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. System Descriptions and Preliminaries

Given a probability space $(\Omega, \mathscr{F}, \mathscr{P})$, where Ω is the sample space, $\mathscr{F}$ is the algebra of events and $\mathscr{P}$ is the probability measure define on $\mathscr{F}$. We consider a class of discrete-time uncertain nonlinear SMJSs with time-varying delay over space $(\Omega, \mathscr{F}, \mathscr{P})$. which can be represented by the following T-S model.

Plant Rule l. If s_{1k} is F_{l1} and s_{2k} is F_{l2} and. . .and s_{nk} is F_{ln} then

$$\begin{aligned} Ex_{k+1} &= A_l(r_k)x_k + A_{dl}(r_k)x_{k-d(r_k)} + B_l(r_k)\omega_k, \\ y_k &= C_l(r_k)x_k + C_{dl}(r_k)x_{k-d(r_k)} + D_{1l}(r_k)\omega_k, \\ z_k &= L_l(r_k)x_k + L_{dl}(r_k)x_{k-d(r_k)} + D_{2l}(r_k)\omega_k, \end{aligned} \tag{1}$$

$$x_k = \phi_k,$$

$$k = -\overline{d}, \ldots, -1, 0, \quad l = 1, 2, \ldots, m, \tag{2}$$

where, $k \in Z$, $s_{1k}, s_{2k}, \ldots, s_{nk}$ are the premise variables that are measurable and each F_{lg} ($g = 1, 2, \ldots, n$) is fuzzy set. $x_k \in R^n$ is the state vector. y_k is the measurement. z_k is the signal to be estimated. ω_k is the disturbance input vector which belongs to $L_2 = \{\{a_k, k \in Z\} \mid \sum_{k=0}^{\infty} a_k^T a_k < \infty\}$. m is the number of IF-THEN rules, ϕ_k is a vector-valued initial function. $\{r_k, k \in Z\}$ is a Markov chain taking values in finite space $s = \{1, 2, \ldots, N\}$, with a transition probability from mode i at time k to mode j at time $k+1$ as

$$p_{ij} = \Pr\{r_{k+1} = j \mid r_k = i\}, \tag{3}$$

with $p_{ij} \geq 0$ for $i, j \in s$. $\sum_{j=1}^{N} p_{ij} = 1$, $d(r_k)$ is a positive integer, denoting the time delay of the system involved in node r_k and $0 < \underline{d}_i \leq d(i) \leq \overline{d}_i < \infty$. For each $i \in s$, the matrix $E \in R^{n\times n}$ is a known constant and singular, and rank$(E) = r < n$.

For each $i \in s$, $A_l(i)$, $A_{dl}(i)$, $B_l(i)$, $C_l(i)$, $C_{dl}(i)$, $D_{1l}(i)$, $L_l(i)$, $L_{1l}(i)$, and $D_{2l}(i)$ are known constant matrices with appropriate dimensions.

Remark 1. In this paper, we consider only the case $\underline{d} \leq d(i) \leq \overline{d} < \infty$, with $\underline{d} = \min\{\underline{d}_i, i \in s\}$, $\overline{d} = \max\{\overline{d}_i, i \in s\}$.

By using center-average defuzzifier, product inference, and singleton fuzzifier, the dynamic fuzzy model (1) can be expressed by the following global model:

$$\begin{aligned} &\sum_{l=1}^{m} u_l(s_k) Ex_{k+1} \\ &\quad = \sum_{l=1}^{m} u_l(s_k) A_l(r_k) x_k + \sum_{l=1}^{m} u_l(s_k) A_{dl}(r_k) x_{k-d(r_k)} \\ &\qquad + \sum_{l=1}^{m} u_l(s_k) B_l(r_k) \omega_k, \\ y_k &= \sum_{l=1}^{m} u_l(s_k) C_l(r_k) x_k + \sum_{l=1}^{m} u_l(s_k) C_{dl}(r_k) x_{k-d(r_k)} \\ &\qquad + \sum_{l=1}^{m} u_l(s_k) D_{1l}(r_k) \omega_k, \\ z_k &= \sum_{l=1}^{m} u_l(s_k) L_l(r_k) x_k + \sum_{l=1}^{m} u_l(s_k) L_{dl}(r_k) x_{k-d(r_k)} \\ &\qquad + \sum_{l=1}^{m} u_l(s_k) D_{2l}(r_k) \omega_k, \end{aligned} \tag{4}$$

with

$$u_l(s_k) = \frac{\alpha_l(s_k)}{\sum_{l=1}^{m} \alpha_l(s_k)}, \quad \alpha_l(s_k) = \prod_{g=1}^{n} F_{lg}(s_{gk}). \tag{5}$$

$F_{lg}(s_{gk})$ is the grade of membership of s_{gk} in F_{lg}, and it is assumed that $\alpha_l(s_k) \geq 0$, $l = 1, 2, \ldots, m$. $\sum_{l=1}^{m} \alpha_l(s_k) > 0$ for all k. Therefore, $u_l(s_k) \geq 0$ and $\sum_{l=1}^{m} u_l(s_k) = 1$ for all k.

The system (4) can be represented by the following equation:

$$\begin{aligned} Ex_{k+1} &= A(k, r_k) x_k + A_d(k, r_k) x_{k-d(r_k)} + B(k, r_k) \omega_k, \\ y_k &= C(k, r_k) x_k + C_d(k, r_k) x_{k-d(r_k)} + D_1(k, r_k) \omega_k, \\ z_k &= L(k, r_k) x_k + L_d(k, r_k) x_{k-d(r_k)} + D_2(k, r_k) \omega_k, \end{aligned} \tag{6}$$

with

$$\begin{aligned}
&\sum_{l=1}^{m} u_l(s_k) A_l(r_k) = A(k, r_k),\\
&\sum_{l=1}^{m} u_l(s_k) A_{dl}(r_k) = A_d(k, r_k),\\
&\sum_{l=1}^{m} u_l(s_k) B_l(r_k) = B(k, r_k),\\
&\sum_{l=1}^{m} u_l(s_k) C_l(r_k) = C(k, r_k),\\
&\sum_{l=1}^{m} u_l(s_k) C_{dl}(r_k) = C_d(k, r_k),\\
&\sum_{l=1}^{m} u_l(s_k) D_{1l}(r_k) = D_1(k, r_k),\\
&\sum_{l=1}^{m} u_l(s_k) L_l(r_k) = L(k, r_k),\\
&\sum_{l=1}^{m} u_l(s_k) L_{dl}(r_k) = L_d(k, r_k),\\
&\sum_{l=1}^{m} u_l(s_k) D_{2l}(r_k) = D_2(k, r_k).
\end{aligned} \tag{7}$$

Definition 2. System $Ex_{k+1} = A(r_k)x_k$ or the pair $(E, A(r_k))$ is said to be

(1) regular if $\det(zE - A(r_k) \not\equiv 0)$ for any $r_k = i, i \in s$,

(2) causal if it is regular and $\text{degree}(\det(zE - A(r_k))) = \text{rank}(E)$ for any $r_k = i, i \in s$.

Definition 3. System (6) is said to be stochastically stable, if for every initial state (ϕ, r_0), the following condition $E\{\sum_{k=0}^{\infty} \|x_k(\phi, r_0)\|^2 \mid \phi, r_0\} < \infty$ is satisfied.

In this paper, employing the idea of PDC, we consider the filter based on T-S fuzzy-model with order $\hat{n}$ ($\hat{n} = n$ for full-order filter, and $1 \le \hat{n} \le n$ for reduced-order filter):

$$\begin{aligned}
\hat{E}\hat{x}_{k+1} &= \hat{A}(k, r_k)\hat{x}_k + \hat{B}(k, r_k) y_k,\\
\hat{z}_k &= \hat{C}(k, r_k)\hat{x}_k + \hat{D}(k, r_k) y_k,
\end{aligned} \tag{8}$$

where,

$$\begin{aligned}
\hat{A}(k, r_k) &= \sum_{l=1}^{m} u_l(s_k)\hat{A}_l(r_k),\\
\hat{B}(k, r_k) &= \sum_{l=1}^{m} u_l(s_k)\hat{B}_l(r_k),\\
\hat{C}(k, r_k) &= \sum_{l=1}^{m} u_l(s_k)\hat{C}_l(r_k).
\end{aligned} \tag{9}$$

For each $r_k = i, i \in s$, the matrices $\hat{E} \in R^{\hat{n}\times\hat{n}}$, $\hat{A}_l(r_k) \in R^{\hat{n}\times\hat{n}}$, $\hat{B}_l(r_k) \in R^{\hat{n}\times m}$, and $\hat{C}_l(r_k) \in R^{q\times\hat{n}}$ are to be calculate.

Remark 4. $\hat{n}$ may be either equal to n or less than n.

Let

$$\bar{x}_k = \left[x_k^T, \hat{x}_k^T\right], \qquad \bar{z}_k = z_k - \hat{z}_k. \tag{10}$$

Then, the filtering system from (6) and the filter (8) can be written as

$$\begin{aligned}
\bar{E}\bar{x}_{k+1} &= \bar{A}(k, r_k)\bar{x}_k + \bar{A}_d(k, r_k)\bar{x}_{k-d(r_k)} + \bar{B}(k, r_k)\omega_k,\\
\bar{z}_k &= \bar{L}(k, r_k)\bar{x}_k + \bar{L}_d(k, r_k)\bar{x}_{k-d(r_k)} + \bar{D}(k, r_k)\omega_k,
\end{aligned} \tag{11}$$

where, for each $r_k = i, i \in s$,

$$\begin{aligned}
\bar{E} &= \begin{bmatrix} E & 0\\ 0 & \hat{E}\end{bmatrix},\\
\bar{A}(k, i) &= \begin{bmatrix} A(k, i) & 0\\ \hat{B}(k, i) C(k, i) & \hat{A}(k, i)\end{bmatrix},\\
\bar{A}_d(k, i) &= \begin{bmatrix} A_d(k, i) & 0\\ \hat{B}(k, i) C_d(k, i) & 0\end{bmatrix},\\
\bar{B}(k, i) &= \begin{bmatrix} A_d(k, i)\\ \hat{B}(k, i) D_1(k, i)\end{bmatrix},\\
\bar{L}(k, i) &= \left[L(k, i) - \hat{D}(k, i) C(k, i), -\hat{C}(k, i)\right],\\
\bar{L}_d(k, i) &= \left[L_d(k, i) - \hat{D}(k, i), 0\right],\\
\bar{D}(k, i) &= D_2(k, i) - \hat{D}(k, i) D_1(k, i).
\end{aligned} \tag{12}$$

The H_∞ filtering problem is formulated as follows. Given system (1) and a scalar $\gamma > 0$, determine a filter in the form of (8) such that the filtering error system (11) is regular, causal, stochastically stable and satisfies the following H_∞ performance:

$$E\left\{\sum_{k=0}^{\infty} \bar{z}_k^T \bar{z}_k\right\} < \gamma^2 \sum_{k=0}^{\infty} \omega_k^T \omega_k. \tag{13}$$

For the convenience, we use the following notations:

$$\underline{p} = \min\left\{p_{ij}, i \in s\right\}, \qquad \rho = 1 + \left(1 - \underline{p}\right)\left(\bar{d} - \underline{d}\right). \tag{14}$$

Lemma 5 (see [32]). *Given matrices X, Y, and Z with appropriate dimensions, Y is symmetric and $Y > 0$. Then*

$$-Z^T Y^{-1} Z \le X^T Y X + X^T Z + Z^T X. \tag{15}$$

Remark 6. Without loss of generality, we assume that $0 < p_{ij} < 1$ and also $0 < \underline{p} < 1$.

3. Main Results

In this section, we proposed the sufficient conditions and design method for the H_∞ filter in term of LMIs. First of all,

we present a delay-dependent condition such that system (6) is regular, causal, and stochastically stable. Consider system (6) with $\omega_k = 0$

$$\begin{aligned} Ex_{k+1} &= A(k, r_k)\, x_k + A_d(k, r_k)\, x_{k-d(r_k)}, \\ x_k &= \phi_k, \quad k = -\overline{d}, \ldots, -1, 0. \end{aligned} \tag{16}$$

Theorem 7. *For a given scalar γ, the filtering error system* (11) *is regular, causal, and stochastically stable and satisfies* (13)*, if, for each mode $i \in s$, there exist symmetric matrices $X_i > 0$, $Z > 0$, $U > 0$, S_i and matrices N_{i1} and N_{i2} that satisfy the following set of coupled LMIs:*

$$\begin{bmatrix} \overline{\Phi}_{i11} & \overline{\Phi}_{i12} & \overline{\Phi}_{i13} & \overline{d}N_{i1} & \overline{L}^T(k,i) \\ * & \overline{\Phi}_{i22} & \overline{\Phi}_{i23} & \overline{d}N_{i2} & \overline{L}_d^T(k,i) \\ * & * & \overline{\Phi}_{i33} - \gamma^2 I & 0 & \overline{D}^T(k,i) \\ * & * & * & -\overline{d}Z & 0 \\ * & * & * & * & I \end{bmatrix}, \tag{17}$$

where,

$$\begin{aligned} \Phi_{i11} &= \overline{A}^T(k,i)\,\overline{X}_i\overline{A}(k,i) - \overline{A}^T(k,i)\,\overline{R}^T\overline{S}_i\overline{R}\,\overline{A}(k,i) \\ &\quad - \overline{E}_k^T X_i \overline{E} + \rho U + N_{i1}\overline{E} + \overline{E}_k^T N_{il}^T \\ &\quad + \overline{d}\left(\overline{A}(k,i) - \overline{E}\right)^T Z\left(\overline{A}(k,i) - \overline{E}\right), \\ \Phi_{i12} &= \overline{A}^T(k,i)\,\overline{X}_i\overline{A}_d(k,i) - \overline{A}^T(k,i)\,\overline{R}^T\overline{S}_i\overline{R}\,\overline{A}_d(k,i) \\ &\quad - N_{i1}\overline{E} + \overline{E}_k^T N_{i2}^T + \overline{d}\left(\overline{A}(k,i) - \overline{E}\right)^T Z\overline{A}_d(k,i), \\ \Phi_{i22} &= \overline{A}_d^T(k,i)\,\overline{X}_i\overline{A}_d(k,i) - \overline{A}_d^T(k,i)\,\overline{R}^T\overline{S}_i\overline{R}\,\overline{A}_d(k,i) \\ &\quad - U - N_{i2}\overline{E} - \overline{E}_k^T N_{i2}^T + \overline{d}\overline{A}_d^T(k,i)\,Z\overline{A}_d(k,i), \\ \overline{X}_i &= \sum_{j=1}^{N} p_{ij}X_j, \qquad \overline{S}_i = \sum_{j=1}^{N} p_{ij}S_j, \\ \overline{\Phi}_{i13} &= \overline{A}^T(k,i)\,\overline{X}_i\overline{B}(k,i) - \overline{A}^T(k,i)\,\overline{R}^T\overline{S}_i\overline{R}\,\overline{B}(k,i) \\ &\quad + \overline{d}\left(\overline{A}(k,i) - \overline{E}\right)^T Z\overline{B}(k,i), \\ \overline{\Phi}_{i23} &= \overline{A}_d^T(k,i)\,\overline{X}_i\overline{B}(k,i) - \overline{A}_d^T(k,i)\,\overline{R}^T\overline{S}_i\overline{R}\,\overline{B}(k,i) \\ &\quad + \overline{d}\overline{A}_d(k,i)\,Z\overline{B}(k,i), \\ \overline{\Phi}_{i33} &= \overline{B}^T(k,i)\,\overline{X}_i\overline{B}(k,i) - \overline{B}^T(k,i)\,\overline{R}^T\overline{S}_i\overline{R}\,\overline{B}(k,i) \\ &\quad + \overline{d}\overline{B}^T(k,i)\,Z\overline{B}(k,i). \end{aligned} \tag{18}$$

$\overline{E} = \mathrm{diag}\{E, \widehat{E}\}$, $\overline{R} = \mathrm{diag}\{R, \widehat{R}\}$, $\widehat{R} \in R^{\widehat{n}\times\widehat{n}}$ is any constant matrix satisfying $\widehat{R}\widehat{E} = 0$ with $\mathrm{rank}(\widehat{R}) = \widehat{n} - \mathrm{rank}(\widehat{E})$.

Proof. The proof is omitted because it is similar to [3]. □

Obviously, the filter cannot be obtained from (17). In order to design the filter for system (1) and without loss of generality, we assume that $P = n + \widehat{n}$.

Theorem 8. *Let $\gamma > 0$, ϵ_1, ϵ_2, ϵ_3, ϵ_4 and ϵ_5 be given scalars. There exists a filter in the form of* (8) *with $\widehat{E} = E$ such that the filtering error system* (11) *is regular, causal, and stochastically stable and satisfies* (13)*; if for each mode $i \in s$, there exist matrices $P_i > 0$, $S_i > 0$, $\overline{Z} > 0$, $U > 0$, G_i, N_{i1}, N_{i2}, $\widehat{A}(k,i)$, $\widehat{B}(k,i)$, $\widehat{C}(k,i)$, and $\widehat{D}(k,i)$ that satisfy the following coupled LMIs:*

$$\Phi_i = \begin{bmatrix} \Phi_{i11} & \Phi_{i12} & \Phi_{i13} & \overline{d}\epsilon_5 N_{i1} & \overline{L}^T(k,i) & \overline{A}^T(k,i) & \overline{d}\left(\overline{A}(k,i) - \overline{E}\right)^T & \epsilon_2 I & 0 \\ * & \Phi_{i22} & \Phi_{i23} & \overline{d}\epsilon_5 N_{i2} & \overline{L}_d^T(k,i) & \overline{A}_d^T(k,i) & \overline{d}\overline{A}_d^T(k,i) & \epsilon_3 I & 0 \\ * & * & \Phi_{i33} & 0 & \overline{D}^T(k,i) & \overline{B}^T(k,i) & \overline{d}\overline{B}^T(k,i) & \epsilon_4 I & 0 \\ * & * & * & -\overline{d}\overline{Z} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & G_i^T + G_i & 0 & 0 & \epsilon_1 W_i \\ * & * & * & * & * & 0 & \overline{d}\overline{Z} - 2\overline{d}\epsilon_5 I & 0 & 0 \\ * & * & * & * & * & 0 & 0 & -\overline{S}_i & 0 \\ * & * & * & * & * & 0 & 0 & 0 & -\overline{P} \end{bmatrix} < 0, \tag{19}$$

where

$$\begin{aligned} \Phi_{i11} &= \rho U + P_i + N_{i1}\overline{E} + \overline{E}^T N_{i1}^T + \epsilon_1\overline{E}^T + \epsilon_1\overline{E} \\ &\quad + \epsilon_2\overline{A}^T(k,i)\,\overline{R}^T + \epsilon_2\overline{R}\,\overline{A}(k,i), \\ \Phi_{i12} &= -N_{i1}\overline{E} + \overline{E}^T N_{i2} + \epsilon_3\overline{A}^T(k,i)\,\overline{R}^T + \epsilon_2\overline{R}\,\overline{A}_d(k,i), \\ \Phi_{i13} &= \epsilon_4\overline{A}^T(k,i)\,\overline{R}^T + \epsilon_2\overline{R}\,\overline{B}(k,i), \\ \Phi_{i23} &= \epsilon_4\overline{A}_d^T(k,i)\,\overline{R}^T + \epsilon_3\overline{R}\,\overline{B}(k,i), \end{aligned}$$

$$\begin{aligned}
\Phi_{i22} &= -U - N_{i2}\overline{E} - \overline{E}^T N_{i2}^T + \epsilon_3 \overline{A}_d^T(k,i)\overline{R}^T + \epsilon_3 \overline{R}\,\overline{A}_d(k,i),\\
\Phi_{i33} &= -\gamma^2 I + \epsilon_4 \overline{B}^T(k,i)\overline{R}^T + \epsilon_4 \overline{R}\,\overline{B}(k,i),\\
W_i &= \left[\sqrt{p_{i1}}G_i, \sqrt{p_{i2}}G_i, \ldots, \sqrt{p_{iN}}G_i\right],\\
\overline{P} &= \operatorname{diag}\{P_1, P_2, \ldots, P_N\}.
\end{aligned} \tag{20}$$

$\overline{R} = \operatorname{diag}\{R, R\}$; $R \in R^{n\times n}$ *is any constant matrix satisfying* $RE = 0$ *with* $\operatorname{rank}(R) = n - r$.

Proof. Since $X_i > 0$, (17) is equivalent to

$$\begin{bmatrix}
\widetilde{\Phi}_{i11} & \widetilde{\Phi}_{i12} & \widetilde{\Phi}_{i13} & \overline{d}N_{i1} & \overline{L}^T(i) & \overline{A}^T(i) & \overline{d}\left(\overline{A}(i) - \overline{E}\right)^T\\
* & \widetilde{\Phi}_{i22} & \widetilde{\Phi}_{i23} & \overline{d}N_{i2} & \overline{L}_d^T(i) & \overline{A}_d^T(i) & \overline{d}\overline{A}_d^T(i)\\
* & * & \widetilde{\Phi}_{i33} & 0 & \overline{D}^T(i) & \overline{B}^T(i) & \overline{d}\overline{B}^T(i)\\
* & * & * & -\overline{d}\overline{Z} & 0 & 0 & 0\\
* & * & * & * & -I & 0 & 0\\
* & * & * & * & * & -\overline{X}_i^{-1} & 0\\
* & * & * & * & * & 0 & -\overline{d}\overline{Z}^{-1}
\end{bmatrix} < 0, \tag{21}$$

where

$$\begin{aligned}
\widetilde{\Phi}_{i11} &= -\overline{A}^T(k,i)\overline{R}^T\overline{S}_i\overline{R}\,\overline{A}(k,i) - \overline{E}^T X_i \overline{E} + \rho U\\
&\quad + N_{i1}\overline{E} + \overline{E}^T N_{i1}^T,\\
\widetilde{\Phi}_{i12} &= -\overline{A}^T(k,i)\overline{R}^T\overline{S}_i\overline{R}\,\overline{A}_d(k,i) - N_{i1}\overline{E} + \overline{E}^T N_{i2}^T,\\
\widetilde{\Phi}_{i13} &= -\overline{A}^T(k,i)\overline{R}^T\overline{S}_i\overline{R}\,\overline{B}(k,i),\\
\widetilde{\Phi}_{i22} &= -\overline{A}_d^T(k,i)\overline{R}^T\overline{S}_i\overline{R}\,\overline{A}_d(k,i) - U - N_{i2}\overline{E} - \overline{E}N_{i2}^T,\\
\widetilde{\Phi}_{i23} &= -\overline{A}_d^T(k,i)\overline{R}^T\overline{S}_i\overline{R}\,\overline{B}(k,i),\\
\widetilde{\Phi}_{i33} &= -\gamma^2 I + \overline{B}^T(k,i)\overline{R}^T\overline{S}_i\overline{R}\,\overline{B}(k,i);
\end{aligned} \tag{22}$$

then, based on Lemma 5 and $X_i > 0$, $S_i > 0$, $Z > 0$, for any matrix G_i with appropriate dimensions, and scalars ϵ_i, $i = 1, \ldots, 5$, the following inequalities hold:

$$\begin{aligned}
-\overline{X}_i^{-1} &\le G_i\overline{X}_i G_i^T + G_i^T + G_i,\\
-\overline{E}^T X_i \overline{E} &\le \epsilon_1 \overline{E}^T + \epsilon_1 \overline{E} + \epsilon_1^2 X_i^{-1},\\
-Z^{-1} &\le \epsilon_5^2 Z - 2\epsilon_5 I,\\
&-\begin{bmatrix}\overline{A}^T(k,i)\\ \overline{A}_d^T(k,i)\\ \overline{B}^T(k,i)\end{bmatrix}\overline{R}^T\overline{S}_i\overline{R}\begin{bmatrix}\overline{A}^T(k,i)\\ \overline{A}_d^T(k,i)\\ \overline{B}^T(k,i)\end{bmatrix}^T\\
&\le \begin{bmatrix}\overline{A}^T(k,i)\\ \overline{A}_d^T(k,i)\\ \overline{B}^T(k,i)\end{bmatrix}\overline{R}^T\begin{bmatrix}\epsilon_2 I\\ \epsilon_3 I\\ \epsilon_4 I\end{bmatrix}^T + \begin{bmatrix}\epsilon_2 I\\ \epsilon_3 I\\ \epsilon_4 I\end{bmatrix}\overline{R}\begin{bmatrix}\overline{A}^T(k,i)\\ \overline{A}_d^T(k,i)\\ \overline{B}^T(k,i)\end{bmatrix}^T\\
&\quad + \begin{bmatrix}\epsilon_2 I\\ \epsilon_3 I\\ \epsilon_4 I\end{bmatrix}\overline{S}_i^{-1}\begin{bmatrix}\epsilon_2 I\\ \epsilon_3 I\\ \epsilon_4 I\end{bmatrix}^T.
\end{aligned} \tag{23}$$

Let $P_i = \epsilon_1^2 X_i^{-1}$, $\overline{Z} = \epsilon_5^2 Z$. According to the inequalities (23), and applying the Schur complement, it is obtained that if (19) holds, then (21) holds. The proof is completed. □

Then, we readily obtain the following theorem.

Theorem 9. *Let $\gamma > 0$, ϵ_1, ϵ_2, ϵ_3, ϵ_4 and ϵ_5 be given scalars. There exists a filter in the form of* (8) *with $\widehat{E} = E$ such that the filtering error system* (11) *is regular, causal, stochastically stable and satisfies* (13)*; if for each mode $i \in s$, there exist matrices $P_i > 0$, $S_i > 0$, $\overline{Z} > 0$, $U > 0$, G_i, N_{i1}, N_{i2}, $\widehat{A}_l(i)$, $\widehat{B}_l(i)$, $\widehat{C}_l(i)$, $\widehat{D}_l(i)$ that satisfy the following coupled LMIs:*

$$\begin{aligned}
&\Phi_{ll} < 0, \quad (1 = 1, 2, \ldots, m),\\
&\frac{\Phi_{lj} + \Phi_{jl}}{2} < 0, \quad (1 \le l < j \le m),
\end{aligned} \tag{24}$$

$$\Phi_{lj} = \begin{bmatrix}
\Phi_{ilj11} & \Phi_{ilj12} & \Phi_{ilj13} & \overline{d}\epsilon_5 N_{i1} & \overline{L}_{lj}^T(i) & \overline{A}_{lj}^T(i) & \overline{d}\left(\overline{A}_{lj}(i) - \overline{E}\right)^T & \epsilon_2 I & 0\\
* & \Phi_{ilj22} & \Phi_{ilj23} & \overline{d}\epsilon_5 N_{i2} & \overline{L}_{dlj}^T(i) & \overline{A}_{dlj}^T(i) & \overline{d}\overline{A}_{dlj}^T(i) & \epsilon_3 I & 0\\
* & * & \Phi_{ilj33} & 0 & \overline{D}_{lj}^T(i) & \overline{B}_{lj}^T(i) & \overline{d}\overline{B}_{lj}^T(i) & \epsilon_4 I & 0\\
* & * & * & -\overline{d}\overline{Z} & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & -I & 0 & 0 & 0 & 0\\
* & * & * & * & * & G_i^T + G_i & 0 & 0 & \epsilon_1 W_i\\
* & * & * & * & * & 0 & \overline{d}\overline{Z} - 2\overline{d}\epsilon_5 I & 0 & 0\\
* & * & * & * & * & 0 & 0 & -\overline{S}_i & 0\\
* & * & * & * & * & 0 & 0 & 0 & -\overline{P}
\end{bmatrix}, \tag{25}$$

where

$$\begin{aligned}
\Phi_{ilj11} &= \rho U + P_i + N_{i1}\overline{E} + \overline{E}^T N_{i1}^T + \epsilon_1 \overline{E}^T + \epsilon_1 \overline{E} \\
&\quad + \epsilon_2 \overline{A}_{lj}^T(i)\,\overline{R}^T + \epsilon_2 \overline{R}\,\overline{A}_{lj}(i), \\
\Phi_{ilj12} &= -N_{i1}\overline{E} + \overline{E}^T N_{i2}^T + \epsilon_3 \overline{A}_{lj}^T(i)\,\overline{R}^T + \epsilon_2 \overline{R}\,\overline{A}_{dlj}(i), \\
\Phi_{ilj13} &= \epsilon_4 \overline{A}_{lj}^T(i)\,\overline{R}^T + \epsilon_2 \overline{R}\,\overline{B}_{lj}(i), \\
\Phi_{i23} &= \epsilon_4 \overline{A}_{dlj}^T(i)\,\overline{R}^T + \epsilon_3 \overline{R}\,\overline{B}_{lj}(i), \\
\Phi_{ilj22} &= -U - N_{i2}\overline{E} - \overline{E}^T N_{i2}^T + \epsilon_3 \overline{A}_{dlj}^T(i)\,\overline{R}^T + \epsilon_3 \overline{R}\,\overline{A}_{dlj}(i), \\
\Phi_{ilj33} &= -\gamma^2 I + \epsilon_4 \overline{B}_{lj}^T(i)\,\overline{R}^T + \epsilon_4 \overline{R}\,\overline{B}_{lj}(i), \\
W_i &= \left[\sqrt{p_{i1}}G_i, \sqrt{p_{i2}}G_i, \ldots, \sqrt{p_{iN}}G_i\right], \\
\overline{P} &= \operatorname{diag}\{P_1, P_2, \ldots, P_N\}.
\end{aligned} \tag{26}$$

$\overline{R} = \operatorname{diag}\{R, R\}$; $R \in R^{n\times n}$ *is any constant matrix satisfying* $RE = 0$ *with* $\operatorname{rank}(R) = n - r$.

Proof. Noting that $u_j(s_k) \geq 0$, $\sum_{l=1}^m u_l(s_k) = 1$, $\sum_{l=1}^m u_l(s_k)$ $\sum_{j=1}^m u_j(s_k) = 1$

$$\begin{aligned}
\overline{E} &= \begin{bmatrix} E & 0 \\ 0 & \widehat{E} \end{bmatrix} = \sum_{l=1}^m u_l(s_k) \sum_{j=1}^m u_j(s_k) \begin{bmatrix} E & 0 \\ 0 & \widehat{E} \end{bmatrix}, \\
\overline{A}(k,i) &= \begin{bmatrix} A(k,i) & 0 \\ \widehat{B}(k,i)\,C(k,i) & \widehat{A}(k,i) \end{bmatrix} \\
&= \begin{bmatrix} \sum_{l=1}^m u_l(s_k)\,A_l(i) & 0 \\ \sum_{j=1}^m u_j(s_k)\,\widehat{B}_j(i) \sum_{l=1}^m u_l(s_k)\,C_l(i) & \sum_{l=1}^m u_l(s_k)\,\widehat{A}_j(i) \end{bmatrix} \\
&= \sum_{l=1}^m u_l(s_k) \sum_{j=1}^m u_j(s_k) \begin{bmatrix} A_l(i) & 0 \\ \widehat{B}_j(i)\,C_l(i) & \widehat{A}_j(i) \end{bmatrix} \\
&= \sum_{l=1}^m u_l(s_k) \sum_{j=1}^m u_j(s_k)\,\overline{A}_{lj}(i), \\
\overline{A}_d(k,i) &= \begin{bmatrix} A_d(k,i) & 0 \\ \widehat{B}(k,i)\,C_d(k,i) & 0 \end{bmatrix} \\
&= \begin{bmatrix} \sum_{l=1}^m u_l(s_k)\,A_{dl}(i) & 0 \\ \sum_{j=1}^m u_j(s_k)\,\widehat{B}_j(i) \sum_{l=1}^m u_l(s_k)\,C_{dl}(i) & 0 \end{bmatrix} \\
&= \sum_{l=1}^m u_l(s_k) \sum_{j=1}^m u_j(s_k) \begin{bmatrix} A_{dl}(i) & 0 \\ \widehat{B}_j(i)\,C_{dl}(i) & 0 \end{bmatrix} \\
&= \sum_{l=1}^m u_l(s_k) \sum_{j=1}^m u_j(s_k)\,\overline{A}_{dlj}(i), \\
\overline{B}(k,i) &= \begin{bmatrix} A_d(k,i) \\ \widehat{B}(k,i)\,D_1(k,i) \end{bmatrix} \\
&= \sum_{l=1}^m u_l(s_k) \sum_{j=1}^m u_j(s_k) \begin{bmatrix} A_{dl}(i) \\ \widehat{B}_j(i)\,D_{1l}(i) \end{bmatrix} \\
&= \sum_{l=1}^m u_l(s_k) \sum_{j=1}^m u_j(s_k)\,\overline{B}_{lj}(i), \\
\overline{L}(k,i) &= \left[L(k,i) - \widehat{D}(k,i)\,C(k,i), -\widehat{C}(k,i)\right] \\
&= \sum_{l=1}^m u_l(s_k) \sum_{j=1}^m u_j(s_k) \left[L_l(i) - \widehat{D}_j(i)\,C_l(i), -\widehat{C}_j(i)\right] \\
&= \sum_{l=1}^m u_l(s_k) \sum_{j=1}^m u_j(s_k)\,\overline{L}_{lj}(i), \\
\overline{L}_d(k,i) &= \left[L_d(k,i) - \widehat{D}(k,i)\,C_d(k,i), 0\right] \\
&= \sum_{l=1}^m u_l(s_k) \sum_{j=1}^m u_j(s_k) \left[L_{dl}(i) - \widehat{D}_j(i)\,C_{dl}(i), 0\right] \\
&= \sum_{l=1}^m u_l(s_k) \sum_{j=1}^m u_j(s_k)\,\overline{L}_{dlj}(i), \\
\overline{D}(k,i) &= \left[D_2(k,i) - \widehat{D}(k,i)\,D_1(k,i)\right] \\
&= \sum_{l=1}^m u_l(s_k) \sum_{j=1}^m u_j(s_k) \left[D_{2l}(i) - \widehat{D}_j(i)\,D_{1l}(i)\right] \\
&= \sum_{l=1}^m u_l(s_k) \sum_{j=1}^m u_j(s_k)\,\overline{D}_{lj}(i), \\
\sum_{l=1}^m u_l(s_k) \sum_{j=1}^m u_j(s_k)\,\Phi_{ij} &= \sum_{l=1}^m u_l(s_k)\,\Phi_{ll} \\
&\quad + \sum_{l<j}^m u_l(s_k)\,u_j(s_k)\left(\Phi_{lj} + \Phi_{jl}\right).
\end{aligned} \tag{27}$$

Therefore, condition (19) can guarantee that condition (24) holds. This completes the proof. □

4. Example

In this section, we give a numerical example to illustrate the use of the presented method. Consider the system of the form with two fuzzy rules and two modes.

The first rule parameters with 1st mode are as follows:

$$\begin{gathered} E = \begin{bmatrix} 3 & 1 \\ 0 & 0 \end{bmatrix}, \qquad A_1(1) = \begin{bmatrix} 1.2 & 0.2 \\ 1.2 & 2.2 \end{bmatrix}, \\ A_{d1}(1) = \begin{bmatrix} 0.1 & -0.1 \\ 0.3 & -0.2 \end{bmatrix}, \qquad B_1(1) = \begin{bmatrix} 0.1 \\ 0.4 \end{bmatrix}, \\ C_1(1) = \begin{bmatrix} -0.5 & 0.3 \end{bmatrix}, \\ C_{d1}(1) = \begin{bmatrix} 0.4 & 0.1 \end{bmatrix}, \qquad D_{11}(1) = 0.1, \\ L_1(1) = \begin{bmatrix} 0.2 & 0.5 \end{bmatrix}, \qquad L_{d1}(1) = \begin{bmatrix} 0.3 & 0.1 \end{bmatrix}, \\ D_{21}(1) = 0.2. \end{gathered} \tag{28}$$

The first rule parameters with 2nd mode are as follows:

$$\begin{gathered} A_1(2) = \begin{bmatrix} -0.3 & 1 \\ -2 & 4 \end{bmatrix}, \qquad A_{d1}(2) = \begin{bmatrix} 0.1 & 0 \\ -0.2 & -0.1 \end{bmatrix}, \\ B_1(2) = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \qquad C_1(2) = \begin{bmatrix} -0.4 & 0.6 \end{bmatrix}, \\ C_{d1}(2) = \begin{bmatrix} 0.4 & -0.3 \end{bmatrix}, \qquad D_{11}(2) = 0, \\ L_1(2) = \begin{bmatrix} 0.2 & 0.5 \end{bmatrix}, \qquad L_{d1}(2) = \begin{bmatrix} 0.3 & 0.4 \end{bmatrix}, \\ D_{21}(2) = 0.2. \end{gathered} \tag{29}$$

The second rule parameters with 1st mode are as follows:

$$\begin{gathered} E = \begin{bmatrix} 3 & 1 \\ 0 & 0 \end{bmatrix}, \qquad A_2(1) = \begin{bmatrix} 1.1 & 0.2 \\ 1.1 & 2.2 \end{bmatrix}, \\ A_{d2}(1) = \begin{bmatrix} 0.2 & -0.1 \\ 0.2 & -0.2 \end{bmatrix}, \qquad B_2(1) = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}, \\ C_2(1) = \begin{bmatrix} -0.4 & 0.3 \end{bmatrix}, \\ C_{d2}(1) = \begin{bmatrix} 0.5 & 0.2 \end{bmatrix}, \qquad D_{12}(1) = 0.2, \\ L_2(1) = \begin{bmatrix} 0.3 & 0.4 \end{bmatrix}, \qquad L_{d2}(1) = \begin{bmatrix} 0.4 & 0.2 \end{bmatrix}, \\ D_{22}(1) = 0.3. \end{gathered} \tag{30}$$

The second rule parameters with 2nd mode are as follows:

$$\begin{gathered} A_2(2) = \begin{bmatrix} -0.5 & 1 \\ -2 & 5 \end{bmatrix}, \qquad A_{d2}(2) = \begin{bmatrix} 0.2 & 0 \\ -0.3 & -0.1 \end{bmatrix}, \\ B_2(2) = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}, \qquad C_2(2) = \begin{bmatrix} -0.5 & 0.6 \end{bmatrix}, \\ C_{d2}(2) = \begin{bmatrix} 0.5 & -0.3 \end{bmatrix}, \qquad D_{12}(2) = 0.1, \\ L_2(2) = \begin{bmatrix} 0.3 & 0.5 \end{bmatrix}, \qquad L_{d2}(2) = \begin{bmatrix} 0.4 & 0.4 \end{bmatrix}, \\ D_{22}(2) = 0.1. \end{gathered} \tag{31}$$

The time-delays $d(1) = 2, d(2) = 3$. The mode switching is governed by a Markov chain that has the following transition probability:

$$p_{11} = 0.9, \qquad p_{12} = 0.1; \qquad p_{21} = 0.2, \qquad p_{22} = 0.8. \tag{32}$$

For the full-order H_∞ filter with $\widehat{E} = E$, we choose that $\gamma = 1$, and let $R = \left[\begin{smallmatrix} 0 & 0 \\ 0 & 5 \end{smallmatrix}\right]$, $\overline{R} = \operatorname{diag}\{R, R\}$, $\epsilon_1 = -9$, $\epsilon_2 = -4$, $\epsilon_3 = 0.01$, $\epsilon_4 = -0.01$, $\epsilon_5 = 70$. Solving the LMIs (24). The solutions are as follows:

$$P_1 = \begin{bmatrix} 31.1 & 20.7 & 0 & 0 \\ 20.7 & 54.1 & 0 & 0 \\ 0 & 0 & 34 & 3.6 \\ 0 & 0 & 3.6 & 1764.1 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 27.9 & -212.1 & 0 & 0 \\ -12.1 & 91.5 & 0 & 0 \\ 0 & 0 & 31.3 & 2.7 \\ 0 & 0 & 2.7 & 1376.5 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 1230.8 & -1.2 & 0 & 0 \\ -1.2 & 1230.2 & 0 & 0 \\ 0 & 0 & 1229.8 & 0 \\ 0 & 0 & 0 & 1228.4 \end{bmatrix},$$

$$S_2 = \begin{bmatrix} 1229.9 & 0.3 & 0 & 0 \\ 0.3 & 1228.8 & 0 & 0 \\ 0 & 0 & 1229.5 & 0 \\ 0 & 0 & 0 & 1228.4 \end{bmatrix},$$

$$Z = \begin{bmatrix} 74.0485 & -0.2410 & 0 & 0 \\ -0.2410 & 74.1101 & 0 & 0 \\ 0 & 0 & 73.9490 & -0.0228 \\ 0 & 0 & -0.0228 & 60.0563 \end{bmatrix},$$

$$U = \begin{bmatrix} 7.3966 & 2.5395 & 0 & 0 \\ 2.5395 & 12.1655 & 0 & 0 \\ 0 & 0 & 9.3139 & 1.3498 \\ 0 & 0 & 1.3498 & 675.6563 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} -0.3077 & -0.1549 & 0 & 0 \\ -0.1491 & -0.6274 & 0 & 0 \\ 0 & 0 & -0.4069 & -0.0388 \\ 0 & 0 & -0.0388 & -19.4679 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} -0.3639 & -0.2226 & 0 & 0 \\ -0.2226 & -0.66165 & 0 & 0 \\ 0 & 0 & -0.4152 & -0.0422 \\ 0 & 0 & -0.0422 & -20.9454 \end{bmatrix},$$

$$N_{11} = \begin{bmatrix} -0.0150 & 0 & 0 & 0 \\ -0.0049 & 0 & 0 & 0 \\ 0 & 0 & -0.0150 & 0 \\ 0 & 0 & -0.0024 & 0 \end{bmatrix},$$

$$N_{12} = \begin{bmatrix} -0.0150 & 0 & 0 & 0 \\ -0.0050 & 0 & 0 & 0 \\ 0 & 0 & -0.0149 & -0.0422 \\ 0 & 0 & -0.0020 & 0 \end{bmatrix},$$

$$N_{21} = \begin{bmatrix} 0.0150 & 0 & 0 & 0 \\ 0.0049 & 0 & 0 & 0 \\ 0 & 0 & 0.0150 & 0 \\ 0 & 0 & 0.0022 & 0 \end{bmatrix},$$

$$N_{22} = \begin{bmatrix} 0.0150 & 0 & 0 & 0 \\ 0.0050 & 0 & 0 & 0 \\ 0 & 0 & 0.0150 & 0 \\ 0 & 0 & 0.0022 & 0 \end{bmatrix},$$

$$\widehat{A}_1(1) = \begin{bmatrix} 0.0494 & 0.2265 \\ -0.2274 & 104.9215 \end{bmatrix},$$

$$\widehat{B}_1(1) = \begin{bmatrix} 0.0463 \\ -0.2358 \end{bmatrix}, \qquad \widehat{C}_1(1) = \begin{bmatrix} 1.2250 & 0.4325 \end{bmatrix},$$

$$\widehat{D}_1(1) = 0.4822,$$

$$\widehat{A}_2(1) = \begin{bmatrix} 0.0463 & 0.2226 \\ -0.2358 & 106.6209 \end{bmatrix}, \qquad \widehat{B}_2(1) = \begin{bmatrix} -0.2546 \\ -0.5478 \end{bmatrix},$$

$$\widehat{C}_2(1) = \begin{bmatrix} 2.0125 & 0.5214 \end{bmatrix}, \qquad \widehat{D}_2(1) = -0.0200,$$

$$\widehat{A}_1(2) = \begin{bmatrix} 0.0487 & 0.1986 \\ -0.2641 & 91.3907 \end{bmatrix}, \qquad \widehat{B}_1(2) = \begin{bmatrix} 2.0125 \\ 0.6578 \end{bmatrix},$$

$$\widehat{C}_1(2) = \begin{bmatrix} 3.2156 & 0.6514 \end{bmatrix}, \qquad \widehat{D}_1(2) = 0.0125,$$

$$\widehat{A}_2(2) = \begin{bmatrix} 0.0487 & 0.1996 \\ -0.2641 & 91.5907 \end{bmatrix}, \qquad \widehat{B}_2(2) = \begin{bmatrix} 0.9854 \\ -1.5278 \end{bmatrix},$$

$$\widehat{C}_2(2) = \begin{bmatrix} 0.2145 & 0.8456 \end{bmatrix}, \qquad \widehat{D}_2(2) = 0.0178. \tag{33}$$

5. Conclusion

The problem on H_∞ filter design has been addressed for T-S fuzzy-model-based SMJSs. Based on the Lyapunov-Krasovskii functional approach, a sufficient condition for the existence of H_∞ filter, which stabilizes the T-S fuzzy-model-based filtering-error systems and guarantees a prescribed level on disturbance attenuation, has been obtained in terms of LMI form. The numerical example has shown the effectiveness of the proposed method. In addition, the ℓ_2-ℓ_∞ filtering problems for T-S fuzzy delayed systems are also challenging, which could be our further work.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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