Research Article

$H_{\infty}$ Filtering for Discrete Markov Jump Singular Systems with Mode-Dependent Time Delay Based on T-S Fuzzy Model

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This paper investigates the $H_{\infty}$ filtering problem of discrete singular Markov jump systems (SMJSSs) with mode-dependent time delay based on T-S fuzzy model. First, by Lyapunov-Krasovskii functional approach, a delay-dependent sufficient condition on $H_{\infty}$ disturbance attenuation is presented, in which both stability and prescribed $H_{\infty}$ performance are required to be achieved for the filtering-error systems. Then, based on the condition, the delay-dependent $H_{\infty}$ filter design scheme for SMJSSs with mode-dependent time delay based on T-S fuzzy model is developed in terms of linear matrix inequality (LMI). Finally, an example is given to illustrate the effectiveness of the result.

1. Introduction

Recently, some efforts on filtering and control problem have been put on the Markov jump linear systems with Markovian jump time delays [1–7]. For example, [1] dealt with the robust stabilizability and $H_{\infty}$ disturbance attenuation for a class of uncertain descriptor time-delay systems with jumping parameters. The transition of the jumping parameters in the systems is governed by a finite state Markov process. [2] addressed the problems of delay-dependent robust $H_{\infty}$ control and filtering for Markovian jump linear systems with norm-bounded parameter uncertainties and time-varying delays. The robust $H_{\infty}$ filtering problem for mode-dependent time-delay discrete Markov jump singular systems with parameter uncertainties is discussed in [3]. The energy-to-peak filtering problem of Markov jump systems with interval time-varying delay is investigated in [4]. The $\ell_2-\ell_\infty$ fuzzy control problem was considered for nonlinear stochastic Markov jump systems with neutral time-delays in [5].

In practical applications, to research the nonlinear time-delay system, the scholars considered the T-S fuzzy time-delay model which is a kind of effective representation, and many analysis and synthesis methods for T-S fuzzy time-delay systems have been developed over the past years [8–17]. So, many scholars extended this model to Markov jump systems with time delays [18–28]. Reference [18] was concerned with an $H_{\infty}$ control for a class of T-S fuzzy Markov jump system under unreliable communication links. Reference [19] concerned with the adaptive synchronization for T-S fuzzy neural networks with stochastic noises and Markovian jumping parameters. Reference [20] was concerned with the stability and stabilization problems for a class of nonlinear systems with Markovian jump parameters. The T-S fuzzy model was employed to represent the Markovian jump nonlinear systems with partly unknown transition probabilities. Reference [21] dealt with the delay-dependent asymptotic stability analysis problem for a class of fuzzy bidirectional associative memory neural networks with time-varying interval delays and Markovian jumping parameters by T-S fuzzy model. In [22], the asymptotic stability of fuzzy Markovian jumping genetic regulatory networks with time-varying delays by delay decomposition approach was investigated.

For singular systems, since the regularity, causality, and stability are needed to be considered, this class of systems is more complicated and difficult compared with standard state
space systems. The strict conditions of linear matrix inequalities (LMI) are not easy to obtain. Many references [29–31] discussed the control problems for SMJSs. However, to the best of our knowledge, the $H_{\infty}$ filtering problem of discrete SMJSs with mode-dependent time delay based on T-S fuzzy model has not been addressed, which is the focus of this paper.

This paper is organized as follows: In Section 2, the filter for SMJSs with mode-dependent time delay based on T-S fuzzy model is formulated. In Section 3 we give the sufficient condition to assure asymptotic stability and the $H_{\infty}$ noise-attenuation level bound for the SMJSs filtering-error systems. Based on the condition in Section 3, we present a stable fuzzy filter in term of LMIs. Section 4 provides illustrative examples to demonstrate the effectiveness of the proposed method. Conclusions are given in Section 5.

Notations. The notations used throughout this paper are fairly standard. The superscript “$T$” stands for matrix transpose, and the notation $P > 0$ ($P \geq 0$) means that matrix $P$ is real symmetric and positive (or being positive semi-definite). $I$ and 0 are used to denote appropriate dimensions identity matrix and zero matrix, respectively. The notation $\ast$ in a symmetric always denotes the symmetric block in the matrix. The parameter diag{····} denotes a block-diagonal matrix. Matrices, if not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. System Descriptions and Preliminaries

Given a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, where $\Omega$ is the sample space, $\mathcal{F}$ is the algebra of events and $\mathcal{P}$ is the probability measure define on $\mathcal{F}$. We consider a class of discrete-time uncertain nonlinear SMJSs with time-varying delay over space $(\Omega, \mathcal{F}, \mathcal{P})$. which can be represented by the following T-S model.

Plant Rule I. If $s_{1k} \in F_{i1}$ and $s_{2k} \in F_{i2}$ and ... and $s_{nk} \in F_{in}$ then

\[ Ex_{k+1} = A_1 (r_k) x_k + A_{dl} (r_k) x_{k-d(r_k)} + B_1 (r_k) w_k, \]
\[ y_k = C_1 (r_k) x_k + C_{dl} (r_k) x_{k-d(r_k)} + D_{1l} (r_k) w_k, \]
\[ z_k = L_1 (r_k) x_k + L_{dl} (r_k) x_{k-d(r_k)} + D_{2l} (r_k) w_k, \]
\[ x_k = \phi_k, \]  
\[ k = -d, \ldots, -1, 0, \quad l = 1, 2, \ldots, m, \]  

where, $k \in Z$, $s_{1k}, s_{2k}, \ldots, s_{nk}$ are the premise variables that are measurable and each $F_{ig}$ ($g = 1, 2, \ldots, n$) is fuzzy set. $x_k \in \mathbb{R}^n$ is the state vector. $y_k$ is the measurement. $z_k$ is the signal to be estimated. $w_k$ is the disturbance input vector. $\phi_k$ is the parameter diag{····} block. $m$ is the number of IF-THEN rules, $\phi_k$ is a vector-valued initial function. $[r_k, k \in Z]$ is a Markov chain taking values in finite space $s = \{1, 2, \ldots, N\}$, with a transition probability from mode $i$ at time $k$ to mode $j$ at time $k + 1$ as

\[ p_{ij} = \Pr \{ r_{k+1} = j \mid r_k = i \}, \]  

with $p_{ij} \geq 0$ for $i, j \in s$. $\sum_{j=1}^{N} p_{ij} = 1, d(r_k)$ is a positive integer, denoting the time delay of the system involved in node $r_k$ and $0 < d_i \leq d(i) \leq d_i < \infty$. For each $i \in s$, the matrix $E \in \mathbb{R}^{n \times n}$ is a known constant and singular, and $\text{rank}(E) = r < n$. For each $i \in s$, $A_i(i)$, $A_{dl}(i)$, $B_i(i)$, $C_i(i)$, $C_{dl}(i)$, $D_{1l}(i)$, $L_i(i)$, $L_{dl}(i)$, and $D_{2l}(i)$ are known constant matrices with appropriate dimensions.

Remark 1. In this paper, we consider only the case $d \leq d(i) \leq d_i < \infty$, with $d = \min[d_i, i \in s]$, $d_i = \max[d_i, i \in s]$.

By using center-average defuzzifier, product inference, and singleton fuzzifier, the dynamic fuzzy model (1) can be expressed by the following global model:

\[ \sum_{l=1}^{m} u_l (s_k) E x_{k+1} = \sum_{l=1}^{m} u_l (s_k) A_1 (r_k) x_k + \sum_{l=1}^{m} u_l (s_k) A_{dl} (r_k) x_{k-d(r_k)} + \sum_{l=1}^{m} u_l (s_k) B_1 (r_k) w_k, \]
\[ y_k = \sum_{l=1}^{m} u_l (s_k) C_1 (r_k) x_k + \sum_{l=1}^{m} u_l (s_k) C_{dl} (r_k) x_{k-d(r_k)} + \sum_{l=1}^{m} u_l (s_k) D_{1l} (r_k) w_k, \]
\[ z_k = \sum_{l=1}^{m} u_l (s_k) L_1 (r_k) x_k + \sum_{l=1}^{m} u_l (s_k) L_{dl} (r_k) x_{k-d(r_k)} + \sum_{l=1}^{m} u_l (s_k) D_{2l} (r_k) w_k, \]  

with

\[ u_l (s_k) = \frac{\alpha_l (s_k)}{\sum_{l=1}^{m} \alpha_l (s_k)}, \quad \alpha_l (s_k) = \sum_{g=1}^{n} F_{ig} (s_{gk}). \]  

$F_{ig} (s_{gk})$ is the grade of membership of $s_{gk}$ in $F_{ig}$, and it is assumed that $\alpha_l (s_k) \geq 0, l = 1, 2, \ldots, m, \sum_{l=1}^{m} \alpha_l (s_k) > 0$ for all $k$. Therefore, $u_l (s_k) \geq 0$ and $\sum_{l=1}^{m} u_l (s_k) = 1$ for all $k$.

The system (4) can be represented by the following equation:

\[ Ex_{k+1} = A (r_k) x_k + A_d (r_k) x_{k-d(r_k)} + B (r_k) w_k, \]
\[ y_k = C (r_k) x_k + C_d (r_k) x_{k-d(r_k)} + D_1 (r_k) w_k, \]
\[ z_k = L (r_k) x_k + L_d (r_k) x_{k-d(r_k)} + D_2 (r_k) w_k, \]  

with $\alpha_l (s_k) \geq 0$ for $i, j \in s$. $\sum_{j=1}^{N} p_{ij} = 1$, $d(r_k)$ is a positive integer, denoting the time delay of the system involved in node $r_k$ and $0 < d_i \leq d(i) \leq d_i < \infty$. For each $i \in s$, the matrix $E \in \mathbb{R}^{n \times n}$ is a known constant and singular, and $\text{rank}(E) = r < n$. For each $i \in s$, $A_i(i)$, $A_{dl}(i)$, $B_i(i)$, $C_i(i)$, $C_{dl}(i)$, $D_{1l}(i)$, $L_i(i)$, $L_{dl}(i)$, and $D_{2l}(i)$ are known constant matrices with appropriate dimensions.
with
\[\sum_{l=1}^{m} u_l(s_k) A_l(r_k) = A(k, r_k),\]
\[\sum_{l=1}^{m} u_l(s_k) A_{dl}(r_k) = A_d(k, r_k),\]
\[\sum_{l=1}^{m} u_l(s_k) B_l(r_k) = B(k, r_k),\]
\[\sum_{l=1}^{m} u_l(s_k) C_l(r_k) = C(k, r_k),\]
\[\sum_{l=1}^{m} u_l(s_k) C_{dl}(r_k) = C_d(k, r_k),\]
\[\sum_{l=1}^{m} u_l(s_k) D_{dl}(r_k) = D_d(k, r_k),\]
\[\sum_{l=1}^{m} u_l(s_k) D_{2l}(r_k) = D_2(k, r_k).\]

**Definition 2.** System \( E_{x_{k+1}} = A_0(r_k)x_k \) or the pair \((E, A(r_k))\) is said to be

1. **regular** if \( \det(zE - A(r_k)) \neq 0 \) for any \( r_k = i, i \in \mathbb{S} \),
2. **causal** if it is regular and \( \deg(\det(zE - A(r_k))) = \text{rank}(E) \) for any \( r_k = i, i \in \mathbb{S} \).

**Definition 3.** System (6) is said to be stochastically stable, if for every initial state \((\phi, r_0)\), the following condition
\[ E\{\sum_{k=0}^{\infty} \|x_k(\phi, r_0)\|^2 \mid \phi, r_0\} < \infty \]
is satisfied.

In this paper, employing the idea of PDC, we consider the filter based on T-S fuzzy-model with order \( \hat{n} \) \((\hat{n} = n)\) for full-order filter, and \( 1 \leq \hat{n} \leq n \) for reduced-order filter:
\[
\hat{E}_{x_{k+1}} = \hat{A}(k, r_k) \hat{x}_k + \hat{B}(k, r_k) y_k,
\]
\[
\hat{z}_k = \hat{C}(k, r_k) \hat{x}_k + \hat{D}(k, r_k) y_k,
\]
where,
\[
\hat{A}(k, r_k) = \sum_{l=1}^{m} u_l(s_k) \hat{A}_l(r_k),
\]
\[
\hat{B}(k, r_k) = \sum_{l=1}^{m} u_l(s_k) \hat{B}_l(r_k),
\]
\[
\hat{C}(k, r_k) = \sum_{l=1}^{m} u_l(s_k) \hat{C}_l(r_k).
\]

For each \( r_k = i, i \in \mathbb{S} \), the matrices \( \hat{E} \in \mathbb{R}^{\hat{n} \times \hat{n}}, \hat{A}_i(r_k) \in \mathbb{R}^{\hat{n} \times \hat{n}}, \hat{B}_i(r_k) \in \mathbb{R}^{\hat{n} \times m}, \) and \( \hat{C}_i(r_k) \in \mathbb{R}^{m \times \hat{n}} \) are to be calculated.

**Remark 4.** \( \hat{n} \) may be either equal to \( n \) or less than \( n \).

Let
\[
\hat{x}_k = [x_k^T, \hat{x}_k^T], \quad \hat{z}_k = z_k - \hat{z}_k.
\]

Then, the filtering system from (6) and the filter (8) can be written as
\[
\hat{E}_{\hat{x}_{k+1}} = \hat{A}(k, r_k) \hat{x}_k + \hat{A}_d(k, r_k) \hat{x}_{k-d(r_k)} + \hat{B}(k, r_k) \omega_k,
\]
\[
\hat{z}_k = \hat{L}(k, r_k) \hat{x}_k + \hat{L}_d(k, r_k) \hat{x}_{k-d(r_k)} + \hat{D}(k, r_k) \omega_k,
\]
where, for each \( r_k = i, i \in \mathbb{S} \),
\[
E = \begin{bmatrix} E & 0 \\ 0 & \hat{E} \end{bmatrix},
\]
\[
\hat{A}(k, i) = \begin{bmatrix} A(k, i) & 0 \\ B(k, i) C(k, i) & \hat{A}(k, i) \end{bmatrix},
\]
\[
\hat{A}_d(k, i) = \begin{bmatrix} A_d(k, i) & 0 \\ \hat{B}(k, i) C_d(k, i) & 0 \end{bmatrix},
\]
\[
\hat{B}(k, i) = \begin{bmatrix} A_d(k, i) \\ \hat{B}(k, i) D_1(k, i) \end{bmatrix},
\]
\[
\hat{L}(k, i) = \begin{bmatrix} L(k, i) - \hat{D}(k, i) C(k, i) - \hat{C}(k, i) \\ -\hat{C}(k, i) \end{bmatrix},
\]
\[
\hat{L}_d(k, i) = \begin{bmatrix} L_d(k, i) - \hat{D}(k, i) \omega_k, \end{bmatrix},
\]
\[
\hat{D}(k, i) = D_2(k, i) - \hat{D}(k, i) D_1(k, i).
\]

The \( H_\infty \) filtering problem is formulated as follows. Given system (1) and a scalar \( \gamma > 0 \), determine a filter in the form of (8) such that the filtering error system (11) is regular, causal, stochastically stable and satisfies the following \( H_\infty \) performance:
\[
E \left\{ \sum_{k=0}^{\infty} \|z_k \|^2 \right\} < \gamma \sum_{k=0}^{\infty} \omega_k^T \omega_k.
\]

For the convenience, we use the following notations:
\[
p = \min \left\{ p_{ij}, i, j \in \mathbb{S} \right\}, \quad p = 1 + \left( 1 - p \right) \left( \hat{d} - d \right).
\]

**Lemma 5** (see [32]). Given matrices \( X, Y, \) and \( Z \) with appropriate dimensions, \( Y \) is symmetric and \( Y > 0 \). Then
\[
-Z^T Y^{-1} Z \leq X^T Y X + X^T Z + Z^T X.
\]

**Remark 6.** Without loss of generality, we assume that \( 0 < p < 1 \) and also \( 0 < p < 1 \).

### 3. Main Results

In this section, we proposed the sufficient conditions and design method for the \( H_\infty \) filter in term of LMIs. First of all,
we present a delay-dependent condition such that system (6) is regular, causal, and stochastically stable. Consider system (6) with $\omega_k = 0$

$$E_{x_{k+1}} = A(k, r_k) x_k + A_d(k, r_k) x_{k-d(r_k)} ,$$

$$x_k = \phi_k, \quad k = -d, \ldots, -1, 0.$$ (16)

**Theorem 7.** For a given scalar $\gamma$, the filtering error system (11) is regular, causal, and stochastically stable and satisfies (13), if for each mode $i \in s$, there exist symmetric matrices $X_i > 0$, $Z > 0$, $U > 0$, $S_i$ and matrices $N_{i1}$ and $N_{i2}$ that satisfy the following set of coupled LMIs:

$$\begin{bmatrix}
\Phi_{i11} & \Phi_{i12} & \Phi_{i13} & \bar{d}N_{i1} & L^T(k,i) \\
* & \Phi_{i22} & \Phi_{i23} & \bar{d}N_{i2} & L^T(k,i) \\
* & * & \bar{d}\gamma^2 I & 0 & D^T(k,i) \\
* & * & * & -dZ & 0 \\
* & * & * & * & 0 \\
\end{bmatrix} < 0,$$ (17)

where,

$$\Phi_{i11} = \bar{A}^T(k,i) X_i A(k,i) - \bar{A}^T(k,i) \bar{R}\bar{S} \bar{R} A(k,i)$$

$$- \bar{E}_k^T X_i \bar{E} + \rho U + N_{i1} \bar{E} + \bar{E}_k N_{i1}^T$$

$$+ \bar{d}(\bar{A}(k,i) - \bar{E})^T Z (\bar{A}(k,i) - \bar{E}),$$

$$\Phi_{i12} = \bar{A}^T(k,i) X_i \bar{A}_d(k,i) - \bar{A}^T(k,i) \bar{R}\bar{S} \bar{R} \bar{A}_d(k,i)$$

$$- N_{i1} \bar{E} + \bar{E}_k N_{i2}^T + \bar{d}(\bar{A}(k,i) - \bar{E})^T Z \bar{A}_d(k,i),$$

$$\Phi_{i13} = \bar{A}^T(k,i) X_i \bar{B}(k,i) - \bar{A}^T(k,i) \bar{R}\bar{S} \bar{R} \bar{B}(k,i)$$

$$+ \bar{d}(\bar{A}(k,i) - \bar{E})^T Z \bar{B}(k,i),$$

$$\Phi_{i22} = \bar{A}_d^T(k,i) X_i \bar{A}_d(k,i) - \bar{A}_d^T(k,i) \bar{R}\bar{S} \bar{R} \bar{A}_d(k,i)$$

$$- U - N_{i2} \bar{E} + \bar{E}_k N_{i2}^T + \bar{d}\bar{A}_d^T(k,i) Z \bar{A}_d(k,i),$$

$$\Phi_{i12} = \bar{A}^T(k,i) X_i \bar{A}_d(k,i) - \bar{A}^T(k,i) \bar{R}\bar{S} \bar{R} \bar{A}_d(k,i)$$

$$- U - N_{i2} \bar{E} + \bar{E}_k N_{i2}^T + \bar{d}\bar{A}_d^T(k,i) Z \bar{A}_d(k,i),$$

$$\Phi_{i23} = \bar{A}_d^T(k,i) X_i \bar{B}(k,i) - \bar{A}_d^T(k,i) \bar{R}\bar{S} \bar{R} \bar{B}(k,i)$$

$$+ \bar{d}\bar{A}_d^T(k,i) Z \bar{B}(k,i).$$

Proof. The proof is omitted because it is similar to [3].

Obviously, the filter cannot be obtained from (17). In order to design the filter for system (1) and without loss of generality, we assume that $P = n + \bar{n}$.

**Theorem 8.** Let $\gamma > 0$, $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$, $\varepsilon_4$ and $\varepsilon_5$ be given scalars. There exists a filter in the form of (8) with $\bar{E} = E$ such that the filtering error system (11) is regular, causal, and stochastically stable and satisfies (13); if for each mode $i \in s$, there exist matrices $P_i > 0$, $S_i > 0$, $Z > 0$, $U > 0$, $G_i$, $N_{i1}$, $N_{i2}$, $\bar{A}(k,i)$, $\bar{B}(k,i)$, $\bar{C}(k,i)$, and $\bar{D}(k,i)$ that satisfy the following coupled LMIs:

$$\Phi_i = \begin{bmatrix}
\Phi_{i11} & \Phi_{i12} & \Phi_{i13} & \bar{d}\varepsilon_5 N_{i1} & L^T(k,i) & \bar{A}^T(k,i) & \bar{d}(\bar{A}(k,i) - \bar{E})^T & \varepsilon_1 I & 0 \\
* & \Phi_{i22} & \Phi_{i23} & \bar{d}\varepsilon_2 N_{i2} & L^T(k,i) & \bar{A}_d^T(k,i) & \bar{d}\bar{A}_d^T(k,i) & \varepsilon_1 I & 0 \\
* & * & \Phi_{i33} & 0 & D^T(k,i) & \bar{B}^T(k,i) & \bar{d}\bar{B}^T(k,i) & \varepsilon_1 I & 0 \\
* & * & * & -dZ & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -I & 0 & 0 & 0 & 0 \\
* & * & * & * & G_i^T + G_i & 0 & 0 & \varepsilon_1 W_i & 0 \\
* & * & * & * & 0 & \bar{d}Z - 2\varepsilon_5 I & 0 & 0 & -S_i \\
* & * & * & * & 0 & 0 & -\bar{P} & 0 & 0 \\
* & * & * & * & 0 & 0 & 0 & -\bar{P} & 0 \\
\end{bmatrix} < 0,$$ (19)

where

$$\Phi_{i11} = \rho U + P_i + N_{i1} \bar{E} + \bar{E}_k^T N_{i1}^T + \varepsilon_1 \bar{E}^T + \varepsilon_1 \bar{E}$$

$$+ \varepsilon_2 \bar{A}^T(k,i) \bar{R}^T + \varepsilon_2 \bar{R} \bar{A}(k,i),$$

$$\Phi_{i12} = -N_{i1} \bar{E} + \bar{E}_k^T N_{i2} + \varepsilon_3 \bar{A}_d^T(k,i) \bar{R}^T + \varepsilon_4 \bar{R} \bar{A}_d(k,i),$$

$$\Phi_{i13} = \varepsilon_4 \bar{A}^T(k,i) \bar{R}^T + \varepsilon_5 \bar{R} \bar{B}(k,i),$$

$$\Phi_{i22} = -N_{i2} \bar{E} + \bar{E}_k^T N_{i2} + \varepsilon_5 \bar{A}_d^T(k,i) \bar{R}^T + \varepsilon_4 \bar{R} \bar{A}_d(k,i),$$

$$\Phi_{i23} = \varepsilon_4 \bar{A}^T(k,i) \bar{R}^T + \varepsilon_5 \bar{R} \bar{B}(k,i),$$

$$\Phi_{i33} = \varepsilon_4 \bar{A}_d^T(k,i) \bar{R}^T + \varepsilon_5 \bar{R} \bar{B}(k,i).$$
\[\Phi_{i22} = -U - N_i^T \tilde{E} - \tilde{E}^T N_i + \epsilon_i \tilde{A}_d (k,i) \tilde{R}^T + \epsilon_i \tilde{R} \tilde{A}_d (k,i), \]
\[\Phi_{i33} = -\gamma^2 I + \epsilon_i \tilde{B}^T (k,i) \tilde{R}^T + \epsilon_i \tilde{R} \tilde{B} (k,i), \]
\[W_i = [\sqrt{p_{i1} G_i}, \sqrt{p_{i2} G_i}, \ldots, \sqrt{p_{i2} G_i}], \]
\[\tilde{P} = \text{diag} \{P_1, P_2, \ldots, P_N\} . \]

(20)

Then, based on Lemma 5 and \(X_i > 0, S_i > 0, Z > 0, \) for any matrix \(G_i\) with appropriate dimensions, and scalars \(\epsilon_i, i = 1, \ldots, 5\), the following inequalities hold:

\[-\tilde{X}_i^{-1} \leq G_i \tilde{X}_i G_i^T + G_i, \]
\[-\tilde{E}^T \tilde{X}_i \tilde{E} \leq \epsilon_i \tilde{E}^T + \epsilon_i \tilde{E} + \epsilon_i^2 \tilde{X}_i^{-1}, \]
\[-Z_i^{-1} \leq \epsilon_i^2 Z - 2\epsilon_i I, \]

Let \(P_i = \epsilon_i^2 X_i^{-1}, Z = \epsilon_i^2 Z\). According to the inequalities (23), and applying the Schur complement, it is obtained that if (19) holds, then (21) holds. The proof is completed. □

Then, we readily obtain the following theorem.

**Theorem 9.** Let \(\gamma > 0, \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \) and \(\epsilon_5\) be given scalars. There exists a filter in the form of (8) with \(\tilde{E} = E\) such that the filtering error system (11) is regular, causal, stochastically stable and satisfies (13); if for each mode \(i \in s\), there exist matrices \(P_i > 0, S_i > 0, Z > 0, U > 0, G_i, N_i, A_i(i), B_i(i), C_i(i), D_i(i)\) that satisfy the following coupled LMIs:

\[
\Phi_{ij} < 0, \quad (1 = 1, 2, \ldots, m), \]
\[
\frac{\Phi_{ij} + \Phi_{ji}}{2} < 0, \quad (1 \leq i < j \leq m), \]

(24)

(25)
where
\[
\Phi_{ij11} = \rho U + P_i + N_t E + E^T N_{t1}^T + e_1 E^T + e_j E
\]
\[+ e_2 A_{ij} (i) \overline{R}^T + e_2 \overline{R} \overline{A}_{ij} (i),
\]
\[
\Phi_{ij12} = -N_t E + E^T N_{t2}^T + e_2 A_{ij}^T (i) \overline{R}^T + e_2 \overline{R} \overline{A}_{ij} (i),
\]
\[
\Phi_{ij13} = e_4 A_{ij} (i) \overline{R}^T + e_4 \overline{R} \overline{A}_{ij} (i),
\]
\[
\Phi_{ij22} = -U - N_t E - E^T N_{t2}^T + e_2 A_{ij}^T (i) \overline{R}^T + e_2 \overline{R} \overline{A}_{ij} (i),
\]
\[
\Phi_{ij33} = -\gamma^2 I + e_4 \overline{R} \overline{B}_{ij} (i) \overline{R}^T + e_4 \overline{R} \overline{B}_{ij} (i),
\]
\[
W_i = \left\{ \sqrt{p_{i1} G_{i1}}, \sqrt{p_{i2} G_{i2}}, \ldots, \sqrt{p_{ik} G_{ik}} \right\},
\]
\[
\overline{F} = \text{diag} \{ P_1, P_2, \ldots, P_N \}.
\]

(26)

\[\overline{R} = \text{diag}[R, R]; R \in R^{n \times m} \text{ is any constant matrix satisfying } RE = 0 \text{ with rank}(R) = n - r.\]

**Proof.** Noting that \(u_j(s_k) \geq 0, \sum_{i=1}^{m} u_i(s_k) = 1, \sum_{i=1}^{m} u_j(s_k) = 1\) \(\sum_{j=1}^{m} u_j(s_k) = 1\),

\[E = \begin{bmatrix} E & 0 \\ 0 & \overline{E} \end{bmatrix} = \sum_{i=1}^{m} u_i(s_k) \sum_{j=1}^{m} u_j(s_k) \begin{bmatrix} E & 0 \\ 0 & \overline{E} \end{bmatrix},\]

\[
\overline{A}_d (k, i) = \begin{bmatrix} A_d (k, i) & 0 \\ \overline{B} (k, i) C_d (k, i) & \overline{A}_d (k, i) \end{bmatrix}
\]

\[
= \sum_{i=1}^{m} u_i(s_k) A_d (i)
\]

\[
\sum_{j=1}^{m} u_j(s_k) A_d (i)
\]

\[
= \sum_{i=1}^{m} u_i(s_k) \sum_{j=1}^{m} u_j(s_k) A_d (i)
\]

\[
= \sum_{i=1}^{m} u_i(s_k) \sum_{j=1}^{m} u_j(s_k) \overline{A}_d (i)
\]

\[
\overline{A}_d (k, i) = \begin{bmatrix} A_d (k, i) & 0 \\ \overline{B} (k, i) C_d (k, i) & \overline{A}_d (k, i) \end{bmatrix}
\]

\[= \sum_{i=1}^{m} u_i(s_k) \sum_{j=1}^{m} u_j(s_k) \overline{A}_d (i)
\]

\[= \sum_{i=1}^{m} u_i(s_k) \sum_{j=1}^{m} u_j(s_k) \Phi_{ij} \]

\[+ \sum_{j=1}^{m} u_j(s_k) u_j(s_k) \Phi_{ij}.
\]

(27)

Therefore, condition (19) can guarantee that condition (24) holds. This completes the proof. \[\square\]
4. Example

In this section, we give a numerical example to illustrate the use of the presented method. Consider the system of the form with two fuzzy rules and two modes.

The first rule parameters with 1st mode are as follows:

\[
E = \begin{bmatrix} 3 & 1 \\ 0 & 0 \end{bmatrix}, \quad A_1 (1) = \begin{bmatrix} 1.2 & 0.2 \\ 1.2 & 2.2 \end{bmatrix},
\]

\[
A_{d1} (1) = \begin{bmatrix} 0.1 & -0.1 \\ 0.3 & -0.2 \end{bmatrix}, \quad B_1 (1) = \begin{bmatrix} 0.1 \\ 0.4 \end{bmatrix}, \quad C_1 (1) = \begin{bmatrix} -0.5 & 0.3 \end{bmatrix}, \quad D_{11} (1) = 0.1,
\]

\[
L_1 (1) = \begin{bmatrix} 0.2 & 0.5 \end{bmatrix}, \quad L_{d1} (1) = \begin{bmatrix} 0.3 & 0.1 \end{bmatrix}, \quad D_{21} (1) = 0.2.
\]

The first rule parameters with 2nd mode are as follows:

\[
A_1 (2) = \begin{bmatrix} -0.3 & 1 \\ -2 & 4 \end{bmatrix}, \quad A_{d1} (2) = \begin{bmatrix} 0.1 & 0 \\ -0.2 & -0.1 \end{bmatrix},
\]

\[
B_1 (2) = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \quad C_1 (2) = \begin{bmatrix} -0.4 & 0.6 \end{bmatrix}, \quad D_{11} (2) = 0,
\]

\[
L_1 (2) = \begin{bmatrix} 0.2 & 0.5 \end{bmatrix}, \quad L_{d1} (2) = \begin{bmatrix} 0.3 & 0.4 \end{bmatrix}, \quad D_{21} (2) = 0.2.
\]

The second rule parameters with 1st mode are as follows:

\[
E = \begin{bmatrix} 3 & 1 \\ 0 & 0 \end{bmatrix}, \quad A_2 (1) = \begin{bmatrix} 1.1 & 0.2 \\ 1.1 & 2.2 \end{bmatrix},
\]

\[
A_{d2} (1) = \begin{bmatrix} 0.2 & -0.1 \\ 0.2 & -0.2 \end{bmatrix}, \quad B_2 (1) = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}, \quad C_2 (1) = \begin{bmatrix} -0.4 & 0.3 \end{bmatrix},
\]

\[
C_{d2} (1) = \begin{bmatrix} 0.5 & 0.2 \end{bmatrix}, \quad D_{12} (1) = 0.2,
\]

\[
L_2 (1) = \begin{bmatrix} 0.3 & 0.4 \end{bmatrix}, \quad L_{d2} (1) = \begin{bmatrix} 0.4 & 0.2 \end{bmatrix}, \quad D_{22} (1) = 0.3.
\]

The second rule parameters with 2nd mode are as follows:

\[
A_2 (2) = \begin{bmatrix} -0.5 & 1 \\ -2 & 5 \end{bmatrix}, \quad A_{d2} (2) = \begin{bmatrix} 0.2 & 0 \\ -0.3 & -0.1 \end{bmatrix},
\]

\[
B_2 (2) = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}, \quad C_2 (2) = \begin{bmatrix} -0.5 & 0.6 \end{bmatrix}, \quad C_{d2} (2) = \begin{bmatrix} 0.5 & -0.3 \end{bmatrix}, \quad D_{12} (2) = 0.1,
\]

\[
L_2 (2) = \begin{bmatrix} 0.3 & 0.5 \end{bmatrix}, \quad L_{d2} (2) = \begin{bmatrix} 0.4 & 0.4 \end{bmatrix}, \quad D_{22} (2) = 0.1.
\]

The time-delays \(d(1) = 2, d(2) = 3\). The mode switching is governed by a Markov chain that has the following transition probability:

\[
p_{11} = 0.9, \quad p_{12} = 0.1; \quad p_{21} = 0.2, \quad p_{22} = 0.8.
\]

For the full-order \(H_{∞}\) filter with \(\tilde{E} = E\), we choose that \(\gamma = 1\), and let \(R = \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix}\), \(\tilde{R} = \text{diag}[R, R]\), \(e_1 = -9, e_2 = -4, e_3 = 0.01, e_4 = -0.01, e_5 = 70\). Solving the LMIs (24). The solutions are as follows:

\[
P_1 = \begin{bmatrix} 31.1 & 20.7 & 0 & 0 \\ 20.7 & 54.1 & 0 & 0 \\ 0 & 0 & 3.6 & 1764.1 \\ 0 & 0 & 0 & 1228.4 \end{bmatrix},
\]

\[
P_2 = \begin{bmatrix} 27.9 & -212.1 & 0 & 0 \\ -12.1 & 91.5 & 0 & 0 \\ 0 & 0 & 31.3 & 2.7 \\ 0 & 0 & 2.7 & 1376.5 \end{bmatrix},
\]

\[
S_1 = \begin{bmatrix} 1230.8 & -1.2 & 0 & 0 \\ -1.2 & 1230.2 & 0 & 0 \\ 0 & 0 & 1229.8 & 0 \\ 0 & 0 & 0 & 1228.4 \end{bmatrix},
\]

\[
S_2 = \begin{bmatrix} 1229.9 & 0.3 & 0 & 0 \\ 0.3 & 1228.8 & 0 & 0 \\ 0 & 0 & 1229.5 & 0 \\ 0 & 0 & 0 & 1228.4 \end{bmatrix},
\]

\[
Z = \begin{bmatrix} 74.0485 & -0.2410 & 0 & 0 \\ -0.2410 & 74.1101 & 0 & 0 \\ 0 & 0 & 73.9490 & -0.0228 \\ 0 & 0 & -0.0228 & 60.0563 \end{bmatrix},
\]

\[
U = \begin{bmatrix} 7.3966 & 2.5395 & 0 & 0 \\ 2.5395 & 12.1655 & 0 & 0 \\ 0 & 0 & 9.3139 & 1.3498 \\ 0 & 0 & 1.3498 & 673.6563 \end{bmatrix},
\]

\[
G_1 = \begin{bmatrix} -0.3077 & -0.1549 & 0 & 0 \\ -0.1491 & -0.6274 & 0 & 0 \\ 0 & 0 & -0.4069 & -0.0388 \\ 0 & 0 & -0.0388 & -19.4679 \end{bmatrix},
\]

\[
G_2 = \begin{bmatrix} -0.3639 & -0.2226 & 0 & 0 \\ -0.2226 & -0.66165 & 0 & 0 \\ 0 & 0 & -0.4152 & -0.0422 \\ 0 & 0 & -0.0422 & -20.9454 \end{bmatrix},
\]

\[
N_{11} = \begin{bmatrix} -0.0150 & 0 & 0 & 0 \\ -0.0049 & 0 & 0 & 0 \\ 0 & 0 & -0.0150 & 0 \\ 0 & 0 & -0.0024 & 0 \end{bmatrix},
\]

\[
N_{12} = \begin{bmatrix} -0.0150 & 0 & 0 & 0 \\ -0.0050 & 0 & 0 & 0 \\ 0 & 0 & -0.0149 & -0.0422 \\ 0 & 0 & -0.0020 & 0 \end{bmatrix},
\]

\[
N_{21} = \begin{bmatrix} 0.0150 & 0 & 0 & 0 \\ 0.0049 & 0 & 0 & 0 \\ 0 & 0 & 0.0150 & 0 \\ 0 & 0 & 0.0022 & 0 \end{bmatrix}.
\]
\( N_{22} = \begin{bmatrix} 0.0150 & 0 & 0 & 0 \\ 0.0050 & 0 & 0 & 0 \\ 0 & 0 & 0.0150 & 0 \\ 0 & 0 & 0 & 0.0022 \end{bmatrix} \),

\( \tilde{A}_1 (1) = \begin{bmatrix} 0.0494 & 0.2265 \\ -0.2274 & 104.9215 \end{bmatrix} \),

\( \tilde{B}_1 (1) = \begin{bmatrix} 0.0463 \\ -0.2358 \end{bmatrix}, \quad \tilde{C}_1 (1) = \begin{bmatrix} 2.250 & 0.4325 \end{bmatrix} \),

\( \tilde{D}_1 (1) = 0.4822, \)

\( \tilde{A}_2 (1) = \begin{bmatrix} 0.0463 & 0.2226 \\ -0.2358 & 106.6209 \end{bmatrix}, \quad \tilde{B}_2 (1) = \begin{bmatrix} -0.2546 \\ -0.5478 \end{bmatrix}, \)

\( \tilde{C}_2 (1) = \begin{bmatrix} 2.0125 & 0.5214 \end{bmatrix}, \quad \tilde{D}_2 (1) = 0.0200, \)

\( \tilde{A}_1 (2) = \begin{bmatrix} 0.0487 & 0.1986 \\ -0.2641 & 91.3907 \end{bmatrix}, \quad \tilde{B}_1 (2) = \begin{bmatrix} 2.0125 \\ 0.6578 \end{bmatrix}, \)

\( \tilde{C}_1 (2) = \begin{bmatrix} 3.2156 & 0.6514 \end{bmatrix}, \quad \tilde{D}_1 (2) = 0.0125, \)

\( \tilde{A}_2 (2) = \begin{bmatrix} 0.0487 & 0.1996 \\ -0.2641 & 91.5907 \end{bmatrix}, \quad \tilde{B}_2 (2) = \begin{bmatrix} 0.9854 \\ -1.5278 \end{bmatrix}, \)

\( \tilde{C}_2 (2) = \begin{bmatrix} 0.2145 & 0.8456 \end{bmatrix}, \quad \tilde{D}_2 (2) = 0.0178. \)


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