Research Article

Fitness Estimation Based Particle Swarm Optimization Algorithm for Layout Design of Truss Structures

Ayang Xiao,¹ Benli Wang,¹ Chaoli Sun,² Shijie Zhang,¹ and Zhenguo Yang¹

¹ Research Centre of Satellite Technology, Harbin Institute of Technology, Harbin 150001, China

² Complex System and Computational Intelligence Laboratory, Taiyuan University of Science and Technology, Taiyuan 030024, China

Correspondence should be addressed to Ayang Xiao; ayang.xiao@gmail.com

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Due to the fact that vastly different variables and constraints are simultaneously considered, truss layout optimization is a typical difficult constrained mixed-integer nonlinear program. Moreover, the computational cost of truss analysis is often quite expensive. In this paper, a novel fitness estimation based particle swarm optimization algorithm with an adaptive penalty function approach (FEPSO-AP) is proposed to handle this problem. FEPSO-AP adopts a special fitness estimate strategy to evaluate the similar particles in the current population, with the purpose to reduce the computational cost. Further more, a laconic adaptive penalty function is employed by FEPSO-AP, which can handle multiple constraints effectively by making good use of historical iteration information. Four benchmark examples with fixed topologies and up to 44 design dimensions were studied to verify the generality and efficiency of the proposed algorithm. Numerical results of the present work compared with results of other state-of-the-art hybrid algorithms shown in the literature demonstrate that the convergence rate and the solution quality of FEPSO-AP are essentially competitive.

1. Introduction

As a typical real world project, truss structural analysis is considered to be computationally expensive [1]. Moreover, truss layout optimization, which solves truss sizing variables (e.g., cross-sectional areas of elements) and shape variables (e.g., coordinates of nodes) simultaneously, is known as a typical multimodal and highly nonlinear problem [2]. The various differences (e.g., physical nature, magnitude, continuity, etc.) between the two types of variables make the truss layout optimization problem a difficult task [3]. Hence, in the past decades, researchers mainly focused on solving truss layout optimization via multilevel methods [4].

Vanderplaats and Moses [5] proposed an alternating gradient method to decompose truss layout optimization into a number of subproblems, each of which optimized a subset of the design variables. The subproblems are solved iteratively until a converged optimal solution is found. Zhou [6] used a similar two-level approximation concept to optimize the cross-sectional areas of the members and the coordinates

of the joints. Gil and Andreu [7] used fully stressed design method and conjugate gradient method to optimize the sizing and shape parameters of bridges, respectively.

Due to the strong coupling between the variables, the search efficiency of multilevel methods is often limited [8]. Aiming for addressing this issue and also benefiting from recent rapid advances in computational power, single-level methods which optimize all design variables simultaneously are becoming popular and competitive. Wang et al. [9] presented an optimality criteria (OC) algorithm for spatial truss layout optimization. Fourie and Groenwold [10] used new operators, namely, the elite velocity and the elite particle, in the standard particle swarm optimization (PSO) algorithm to optimize the truss layout, with the purpose of increasing the probability of migration to regions with high fitness. In fact, various metaheuristic algorithms including simulated annealing (SA) [11], genetic algorithm (GA) [12], charged system search (CSS) [13], and artificial bee colony algorithm (ABC) [14] have been introduced to address truss layout optimization problems.

More recently, researchers turn to hybridizing different techniques to further enhance the searching efficiency of metaheuristic algorithms. Lingyun et al. [18] proposed a niche hybrid genetic algorithm to solve the truss shape and sizing optimization in a simple and effective manner. Kaveh and Zolghadr [23] developed a hybridized CSS-BBBC algorithm with trap recognition capability for truss layout optimization. Zuo et al. [24] proposed a hybrid OC-GA approach for fast and global truss layout optimization; Kaveh and Javadi [25] used harmony search and ray optimizer to enhance the PSO algorithm to optimize truss layout under multiple frequency constraints; Liu and Ye [26] designed a genetic simulated annealing algorithm for domes layout optimization. Gholizadeh [17] proposed a hybridized cellular automata and PSO algorithm for truss layout optimization.

However, the common weakness of metaheuristic algorithms based structural optimization is that a huge number of structural analyses are required, which is quite time-consuming. In this paper, we propose a new hybridized algorithm, termed FEPSO-AP for truss layout optimization that aims to enhance the optimal efficiency by using the fitness estimations to partly substitute the computationally expensive fitness calculations. The finite element method (FEM) is adopted to evaluate the structural performance. Empirical results demonstrate that the proposed method is highly promising for truss layout optimization.

2. Statement of Truss Layout Optimization Problem

The main aim of truss layout optimization can be formulated as follows:

$$\text{Minimize: } w(\mathbf{X}) = \sum_{i=1}^{ne} \rho_i \cdot A_i \cdot l_i, \quad (1)$$

$$\text{Subject to: } g(\mathbf{X}) \leq 0,$$

where w and g denote the weight of truss structure and the maximum violated structural constraint; $\mathbf{X}$ is the vector of all design variables; ρ_i , l_i , and A_i represent the material density, length, and cross-sectional area of the i th truss element, respectively.

Different types of constraints might be considered simultaneously depending on the problem to be solved. Four typical design constraints involved in this work can be stated by

$$\begin{aligned} g_i^\sigma(\mathbf{X}) &= \frac{\sigma_i(\mathbf{X})}{\sigma_{i,\text{all}}} - 1 \leq 0, \quad i = 1, 2, \dots, ne, \\ g_j^d(\mathbf{X}) &= \frac{d_j(\mathbf{X})}{d_{j,\text{all}}} - 1 \leq 0, \quad j = 1, 2, \dots, nj, \\ g_i^b(\mathbf{X}) &= \frac{|\sigma_i(\mathbf{X})|}{\sigma_{i,er}} - 1 \leq 0, \quad \text{if } \sigma_i(\mathbf{X}) < 0, \\ g_k^f(\mathbf{X}) &= \frac{f_k(\mathbf{X})}{f_{k,\text{all}}} - 1 \leq 0, \quad k = 1, 2, \dots, nk, \end{aligned} \quad (2)$$

in which g_i^σ is the truss element stress constraint, while σ_i and $\sigma_{i,\text{all}}$ stand for the actual largest stress of the i th truss element and its stress limit, respectively; g_j^d is the nodal displacement constraint, while d_j and $d_{j,\text{all}}$ stand for the actual largest nodal displacement of the j th node and its displacement limit, respectively; g_i^b is the local buckling constraint, while $\sigma_{i,er}$ stands for the Euler critical stress of the i th element; g_k^f is the structural natural frequency constraint, while f_k and $f_{k,\text{all}}$ stand for the k th structural natural frequency and its frequency limit; ne , nj , and nk represent the number of structural elements, nodes, and constrained natural frequencies, respectively.

3. FEPSO-AP Algorithm

This section describes the fitness estimation based PSO algorithm with an adaptive penalty function approach (FEPSO-AP) developed in this research. As FEPSO-AP integrates PSO, FE, and AP, this section recalls the basic concept of PSO algorithm, fitness estimation strategy, and adaptive constraint handling approach. Finally, a framework of FEPSO-AP algorithm is presented.

3.1. The PSO Algorithm. As one of the most popular metaheuristic algorithms, PSO has found a wide application in real world projects for its structural concision and searching efficiency [17]. In a standard PSO [27], it is assumed that each of the particles has a position and a certain velocity. The position of particle represents a candidate solution to the optimization problem, and the velocity of particle determines the particle's movement. Hence, the flying of particles can be considered as the swarm searching of design domain.

If the particle flies from its current position to the next position, its velocity and position are updated by

$$\begin{aligned} \vec{v}_i(t+1) &= \omega \vec{v}_i(t) + c_1 \mathbf{r}_1 (\vec{p}_i(t) - \vec{x}_i(t)) \\ &\quad + c_2 \mathbf{r}_2 (\vec{p}_g(t) - \vec{x}_i(t)), \\ \vec{x}_i(t+1) &= \vec{x}_i(t) + \vec{v}_i(t+1), \end{aligned} \quad (3)$$

in which $\vec{v}_i(t)$ and $\vec{x}_i(t)$ indicate the velocity and position of the i th particle at iteration t ; $\vec{p}_i(t)$ and $\vec{p}_g(t)$ represent the historical best position of the i th particle and the global best position of all particles till iteration t , respectively. ω stands for the inertia weight; c_1 and c_2 are cognitive and social parameters, respectively. $\mathbf{r}_1$ and $\mathbf{r}_2$ are diagonal matrixes, of which the diagonal elements are uniformly distributed random numbers among the range of (0, 1).

3.2. The Fitness Estimation Strategy. Sun et al. [28] proposed a novel fitness estimation strategy for PSO to solve computationally expensive problems.

According to (3), the positions of any two arbitrary particles selected from the swarm can be formulated by

$$\begin{aligned} \vec{x}_i(t+1) &= (1 + \omega - \varphi_1 - \varphi_2) \vec{x}_i(t) + \varphi_1 \vec{p}_i(t) \\ &\quad + \varphi_2 \vec{p}_g(t) - \omega \vec{x}_i(t-1), \end{aligned}$$

$$\begin{aligned}\vec{x}_j(t+1) = & (1 + \omega - \varphi'_1 - \varphi'_2) \vec{x}_j(t) + \varphi'_1 \vec{p}_j(t) \\ & + \varphi'_2 \vec{p}_g(t) - \omega \vec{x}_j(t-1),\end{aligned}\quad (4)$$

in which $\varphi_1 = c_1 \mathbf{r}_1$, $\varphi_2 = c_2 \mathbf{r}_2$, $\varphi'_1 = c_1 \mathbf{r}'_1$, and $\varphi'_2 = c_2 \mathbf{r}'_2$. It can be found that the above two equations are both related to the global best position $\vec{p}_g(t)$. Hence, (4) can be abbreviated to

$$\begin{aligned}\varphi_2 \vec{x}_j(t+1) + \varphi'_2 (1 + \omega - \varphi_1 - \varphi_2) \vec{x}_i(t) \\ + \varphi'_2 \varphi_1 \vec{p}_i(t) + \varphi_2 \omega \vec{x}_j(t-1) \\ = \varphi'_2 \vec{x}_i(t+1) + \varphi_2 (1 + \omega - \varphi'_1 - \varphi'_2) \vec{x}_j(t) \\ + \varphi_2 \varphi'_1 \vec{p}_j(t) + \varphi'_2 \omega \vec{x}_i(t-1).\end{aligned}\quad (5)$$

Use a virtual position $\vec{x}_v(t+1)$ to present the value of (5). Supposing the influences of all coefficients are similar, Figure 1 describes the geometrical relationship among the virtual position $\vec{x}_v(t+1)$ and all subitems contained by (5). As shown in Figure 1, the fitness of virtual position $\vec{x}_v(t+1)$ can be estimated by either the fitness of $\vec{x}_j(t+1)$, $\vec{x}_i(t)$, $\vec{p}_i(t)$, and $\vec{x}_j(t-1)$ or the fitness of $\vec{x}_i(t+1)$, $\vec{x}_j(t)$, $\vec{p}_j(t)$, and $\vec{x}_i(t-1)$. Hence, if the current fitness value and the historical best fitness of every particle at iteration t and $t-1$ are obtained already, the explicit relationship between the fitness of particle i and j at iteration $t+1$ can be expressed in the following formulations:

$$\begin{aligned}f^e(\vec{x}_j(t+1)) \\ = d_v^j(t+1) \\ \cdot \left\{ \alpha \cdot \left[\frac{f(\vec{x}_i(t+1))}{d_v^i(t+1)} + \frac{f(\vec{x}_i(t-1))}{d_v^i(t-1)} \right. \right. \\ \left. \left. + \frac{f(\vec{x}_j(t))}{d_v^j(t)} + \frac{f(\vec{p}_j(t))}{d_v^{pj}(t)} \right] \right. \\ \left. - \frac{f(\vec{x}_j(t-1))}{d_v^j(t-1)} + \frac{f(\vec{x}_i(t))}{d_v^i(t)} + \frac{f(\vec{p}_i(t))}{d_v^{pi}(t)} \right\},\end{aligned}\quad (6)$$

$$\begin{aligned}\alpha \\ = \frac{(1/d_v^j(t+1)) + (1/d_v^j(t-1)) + (1/d_v^i(t)) + (1/d_v^{pi}(t))}{(1/d_v^i(t+1)) + (1/d_v^i(t-1)) + (1/d_v^j(t)) + (1/d_v^{pj}(t))},\end{aligned}\quad (7)$$

in which $d_v^i(t+1)$, $d_v^i(t)$, $d_v^i(t-1)$, $d_v^j(t+1)$, $d_v^j(t)$, $d_v^j(t-1)$, $d_v^{pi}(t)$, and $d_v^{pj}(t)$ represent the distance between the virtual position $\vec{x}_v(t+1)$ and the real positions $\vec{x}_i(t+1)$, $\vec{x}_i(t)$, $\vec{x}_i(t-1)$, $\vec{x}_j(t+1)$, $\vec{x}_j(t)$, $\vec{x}_j(t-1)$, $\vec{p}_i(t)$, and $\vec{p}_j(t)$, respectively.

FIGURE 1: Illustration of the virtual position.

3.3. Adaptive Constraint Handling Approach. As mentioned in Section 2, truss layout optimization problem is often taken as a multiple constrained optimization problem. Coello Coello [29] pointed out that the constraint handling procedure plays an important role in the exploration and exploitation of constrained optimization problems. Similar to other metaheuristic algorithms, FEPSo is designed for unconstrained problems. Hence, it is necessary to incorporate constraint handling approach into FEPSo to solve truss layout optimization problems.

In this work, we use penalty function methods (PEM) to transfer a constrained problem into an unconstrained problem by adding the influence of violated constraints to the initial fitness function. A pseudoobjective function is stated as follows:

$$\Phi(\mathbf{X}) = f(\mathbf{X}) + W \times G(\mathbf{X}), \quad (8)$$

where W and $G(\mathbf{X})$ are positive penalty factor and penalty function. The common difficulty existing in standard PEM is to set the constant penalty factor W properly. Runarsson and Yao [30] pointed out that if the penalty factor W turns out to be too large, the searching landscape would be quite rough, and thus qualified exploitation and exploration of the design domain are hard to be achieved; on the contrary, if the penalty factor W is too small, it would be quite possible to lose feasible solutions.

As summarized in [29], the common target of adaptive penalty factors is to make a good balance between the objective function and the constraints violation. In this work, we developed a laconic adaptive penalty factor as follows:

$$W = 2^{1-\rho}, \quad (9)$$

in which ρ is the ratio of feasible solutions in current population. It can be concluded that if W is too small, the obtained unfeasible solutions will increase the value of ρ , and thus, the algorithm would be pushed to explore more feasible solutions. On the other hand, if the feasible solutions congregate in the population, the value of W will approach zero, and thus a detailed exploitation would be performed.

3.4. The Framework of FEPSO-AP Algorithm. The penalty function $G(\mathbf{X})$ of truss layout optimization problem is determined by

$$G(\mathbf{X}) = w(\mathbf{X}) \times V(\mathbf{X}), \quad (10)$$

$$V(\mathbf{X}) = \max \{0, g(\mathbf{X})\}, \quad (11)$$

in which $\mathbf{X}$ is the structural design vector, $V(\mathbf{X})$ is the violated constraints, and $w(\mathbf{X})$ and $g(\mathbf{X})$ are defined by (1).

According to (8)–(10), the pseudoobjective function can be abbreviated to

$$\Phi(\mathbf{X}) = w(\mathbf{X}) \times (1 + 2^{1-\rho} \times V(\mathbf{X})). \quad (12)$$

To transfer the formulated truss layout optimization problem into an unconstrained optimization problem, the adaptive penalty function, the fitness estimation strategy, and the PSO algorithm are assembled as follows.

Step 1. Set the termination condition of optimization and the initial values of ω , c_1 , and c_2 . Set the iteration times $\text{iter} = 1$.

Step 2. Generate the initial positions $\mathbf{X} = \{\vec{x}_i(t)\}$ and the initial velocities $\mathbf{V} = \{\vec{v}_i(t)\}$ of particles population randomly, $i = 1, 2, 3, \dots, \text{popsize}$.

Step 3. Calculate the fitness values $f(\vec{x}_i(t))$ of all particles and update every particle's historical best position $\vec{p}_i(t)$ and the global best position $\vec{p}_g(t)$ by

$$f(\vec{x}_i(t)) = \Phi(\vec{x}_i(t)), \quad (13)$$

$$\vec{p}_i(t) = \min \{f(\vec{x}_i(t'))\}, \quad t' = 1, 2, 3, \dots, t, \quad (14)$$

$$\vec{p}_g(t) = \min \{f(\vec{x}_i(t))\}, \quad i = 1, 2, 3, \dots, \text{popsize}.$$

Step 4. Update the velocity $\vec{v}_i(t+1)$ and the position $\vec{x}_i(t+1)$ of every particle based on (3). Set the iteration times $\text{iter} = \text{iter} + 1$.

Step 5. Calculate the distances between every pair of particles.

- (5.1) Select particle i randomly and calculate its fitness value $f(\vec{x}_i(t+1))$ based on (13).
- (5.2) Choose a particle j nearby particle i and calculate the virtual position $\vec{x}_j(t+1)$ by using the historical information of particle j based on (5).
- (5.3) If $\vec{x}_j(t+1)$ does not coincide with $\vec{x}_i(t+1)$, $\vec{x}_j(t)$, $\vec{p}_j(t)$, $\vec{x}_i(t-1)$, $\vec{x}_j(t+1)$, $\vec{x}_i(t)$, $\vec{p}_i(t)$, or $\vec{x}_j(t-1)$, estimate the fitness value $f^e(\vec{x}_j(t+1))$ of particle j by (6); otherwise, calculate the fitness value of particle j based on (13).
- (5.4) Repeat (5.1)–(5.4) till the fitness values of all particles are evaluated.

Step 6. Update every particle's historical best position $\vec{p}_i(t+1)$ and the global best position $\vec{p}_g(t+1)$. If $\vec{p}_g(t+1)$ is obtained by estimation, recalculate the corresponding particle's fitness value by (13), and rechoose the global best position $\vec{p}_g(t+1)$.

TABLE 1: Loading condition acting on the planar 15-bar truss.

Case	Node	F_x (kips)	F_y (kips)
1	8	0	−10.0

Repeat this step till a nonestimated value of $\vec{p}_g(t+1)$ is obtained.

Step 7. Repeat Steps 4–6 till the termination condition is reached. Output the global best position $\vec{p}_g(t+1)$ and its objective value.

4. Benchmark Examples

The following four benchmark examples have been used to demonstrate the generality and efficiency of the FEPSO-AP algorithm:

- (i) a planar 15-bar truss subjected to a single load condition and stress constraints,
- (ii) a spatial 25-bar truss subjected to a single load condition under stress and displacement constraints,
- (iii) a planar 37-bar truss subjected to multiple frequency constraints,
- (iv) a planar 47-bar truss subjected to three load conditions under stress and local buckling constraints.

Programs of FEPSO-AP algorithm and structural finite element method (FEM) algorithm are developed by using MATLAB R2013a. A personal computer with a Pentium E5700 processor and 2 GB memory under the Microsoft Windows 7 operating system has been used to run the optimization software.

For all benchmarks examined in this study, the FEPSO-AP algorithm parameters are set as the usual constants of standard PSO which are obtained by [31]: $\omega = 0.7298$, $c_1 = 2.05$, and $c_2 = 2.05$. According to the design dimensions of four benchmarks, the population sizes are set as 46, 26, 38, and 88, while the maximum numbers of FEM analyses are set as 4000, 4500, 8000, and 20000, respectively.

Twenty-five independent runs are performed with the best one being selected for each problem.

4.1. Planar 15-Bar Truss. The original geometry of planar 15-bar truss is shown in Figure 2(a). Two nodes (ID: 1 and 5) are totally fixed and the x -coordinates of other two nodes (ID: 4 and 8) are fixed as well. The single loading condition is listed in Table 1.

All design variables are classified into 23 groups:

sizing variables: A_i , $i = 1, 2, \dots, 15$;

shape variables: $x_2 = x_6$; $x_3 = x_7$; y_2 ; y_3 ; y_4 ; y_6 ; y_7 ; y_8 .

Material parameters and design constraints are listed in Table 2.

Figure 2(b) shows the optimum design found by this work, of which two nodes (ID: 4 and 8) are highly overlapped.

TABLE 2: Material parameters, design constraints, and search range of the planar 15-bar truss optimization problem.

Category	Values
Material Parameters	
Density	0.1 lb/in ³
Modulus of elasticity	1×10^4 ksi
Constraints	
Stress	The allowable elements stress interval: [-25 ksi, 25 ksi]
Search range	
Shape variables	100 in. $\leq x_2 \leq 140$ in.; 220 in. $\leq x_3 \leq 260$ in.; 100 in. $\leq y_2 \leq 140$ in.; 100 in. $\leq y_3 \leq 140$ in.; 50 in. $\leq y_4 \leq 90$ in.; -20 in. $\leq y_6 \leq 20$ in.; -20 in. $\leq y_7 \leq 20$ in.; 20 in. $\leq y_8 \leq 60$ in.; $x_{1,5} = 0$ in.; $y_1 = 120$ in.; $x_{4,8} = 360$ in.
Sizing variables	$S = \{0.111, 0.141, 0.174, 0.220, 0.270, 0.287, 0.347, 0.440, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.800, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.300, 10.850, 13.330, 14.290, 17.170, 19.180\}$ in. ² $A_i \in S, i = 1, 2, \dots, 15$

TABLE 3: Comparison of optimized designs found for the planar 15-bar truss.

No.	Variable	FA [15]	FM-GA [16]	PSO [17]	CPSO [17]	SCPSO [17]	FEPSO-AP
1	A_1	0.954	1.081	0.954	1.174	0.954	1.081
2	A_2	0.539	0.539	1.081	0.539	0.539	0.539
3	A_3	0.220	0.287	0.270	0.347	0.270	0.270
4	A_4	0.954	0.954	1.081	0.954	0.954	0.954
5	A_5	0.539	0.539	0.539	0.954	0.539	0.539
6	A_6	0.220	0.141	0.287	0.141	0.174	0.111
7	A_7	0.111	0.111	0.141	0.141	0.111	0.111
8	A_8	0.111	0.111	0.111	0.111	0.111	0.111
9	A_9	0.287	0.539	0.347	1.174	0.287	0.347
10	A_{10}	0.440	0.440	0.440	0.141	0.347	0.347
11	A_{11}	0.440	0.539	0.270	0.440	0.347	0.440
12	A_{12}	0.220	0.270	0.111	0.440	0.220	0.287
13	A_{13}	0.220	0.220	0.347	0.141	0.220	0.287
14	A_{14}	0.270	0.141	0.440	0.141	0.174	0.111
15	A_{15}	0.220	0.287	0.220	0.347	0.270	0.270
16	x_2	114.9670	101.5775	106.052	102.287	137.222	100.009
17	x_3	247.0400	227.9112	239.025	240.505	259.909	248.078
18	y_2	125.9190	134.7986	130.356	112.584	123.501	131.524
19	y_3	111.0670	128.2206	114.273	108.043	110.002	123.211
20	y_4	58.2980	54.8630	51.987	57.795	59.936	54.077
21	y_6	-17.5640	-16.4484	1.814	-6.430	-5.180	-9.039
22	y_7	-5.8210	-13.3007	9.183	-1.801	4.219	-14.905
23	y_8	31.4650	54.8572	46.909	57.799	57.883	54.084
Best weight (lb)		75.5473	76.6854	82.2344	77.6153	72.6153	74.1673
Maximum displacement (in.)		24.9993	24.9992	24.9999	24.9909	24.9912	24.9999
Number of structural analyses		8,000	8,000	4,500	4,500	4,500	4,000

Table 3 compares the best design found by this work with those reported in the literature. It can be seen that the results achieved by the proposed algorithm are quite close to the best results reported in the literature.

4.2. Spatial 25-Bar Truss. The original geometry of spatial 25-bar truss is shown in Figure 3(a). Two nodes (ID: 1 and 2) are totally fixed and the z -coordinates of four nodes (ID: 7, 8, 9,

and 10) are fixed as well. The loading condition is listed in Table 4.

To ensure the structural symmetries, all design variables are classified into 13 groups:

sizing variables: $A_1; A_2 = A_3 = A_4 = A_5; A_6 = A_7 = A_8 = A_9; A_{10} = A_{11}; A_{12} = A_{13}; A_{14} = A_{15} = A_{16} = A_{17}; A_{18} = A_{19} = A_{20} = A_{21}; A_{22} = A_{23} = A_{24} = A_{25};$

FIGURE 2: Layout optimization of the planar 15-bar truss.

FIGURE 3: Layout optimization of the spatial 25-bar truss.

TABLE 4: Loading condition acting on the spatial 25-bar truss.

Case	Node	F_x (kips)	F_y (kips)	F_z (kips)
1	1	1.0	-10.0	-10.0
	2	0.0	-10.0	-10.0
	3	0.5	0.0	0.0
	6	0.6	0.0	0.0

shape variables: $x_4 = x_5 = -x_3 = -x_6$; $x_8 = x_9 = -x_7 = -x_{10}$; $y_3 = y_4 = -y_5 = -y_6$; $y_7 = y_8 = -y_9 = -y_{10}$; $z_3 = z_4 = z_5 = z_6$.

Material parameters and design constraints are listed in Table 5.

Figure 3(b) shows the optimum design found by this work. Table 6 compares the best design found by this work with those reported in the literature. It can be concluded that by using the proposed algorithm it is possible to achieve the best feasible results at a low computational cost.

4.3. Planar 37-Bar Truss. The original geometry of planar 37-bar truss is shown in Figure 4(a). Eleven nodes (ID: 1, 2, 4, 6, 8, 10, 12, 14, 16, 18, and 20) are totally fixed and the x -coordinates of all other nine nodes (ID: 3, 5, 7, 9, 11, 13, 15, 17, and 19) are fixed as well. A nonstructural mass of 10 kg is attached to all free nodes on the lower chord.

To ensure the structural symmetric about the y -axis, all design variables are classified into 19 groups:

sizing variables: $A_1 = A_{27}$; $A_2 = A_{26}$; $A_3 = A_{24}$; $A_4 = A_{25}$; $A_5 = A_{23}$; $A_6 = A_{21}$; $A_7 = A_{22}$; $A_8 = A_{20}$; $A_9 = A_{18}$; $A_{10} = A_{19}$; $A_{11} = A_{17}$; $A_{12} = A_{15}$; $A_{13} = A_{16}$; A_{14} ;

shape variables: x_3 ; x_5 ; x_7 ; x_9 ; $y_3 = y_{19}$; $y_5 = y_{17}$; $y_7 = y_{15}$; $y_9 = y_{13}$; y_{11} .

Material parameters and design constraints are listed in Table 7.

Figure 4(b) shows the optimum design explored by this work. Table 8 compares the best design presented by this work with those reported in the literature. It can be seen that

TABLE 5: Material parameters, design constraints, and search range of the spatial 25-bar truss optimization problem.

Category	Values
Material parameters	
Density	0.1 lb/in ³
Modulus of elasticity	1×10^4 ksi
Constraints	
Stress	The allowable elements stress interval: [-40 ksi, 40 ksi]
Displacement	The allowable nodal displacement interval: [-0.35 in., 0.35 in.]
Search range	
Shape variables	20 in. $\leq x_4 \leq 60$ in.; 40 in. $\leq x_8 \leq 80$ in.; 40 in. $\leq y_4 \leq 80$ in.; 100 in. $\leq y_8 \leq 140$ in.; 90 in. $\leq z_4 \leq 130$ in.; $-x_1 = x_2 = 37.5$ in.; $y_{1,2} = 0$ in.; $z_{1,2} = 200$ in.
Sizing variables	$S = \{0.1, 0.2, 0.3, \dots, 3.2, 3.3, 3.4\}$ in. ² $A_i \in S, i = 1, 2, \dots, 25$

TABLE 6: Comparison of optimized designs found for the spatial 25-bar truss.

No.	Variable	FA [15]	FM-GA [16]	PSO [17]	CPSO [17]	SCPSO [17]	FEPSO-AP
1	A_1	0.1	0.1	0.1	0.3	0.1	0.1
2	A_2	0.1	0.1	0.1	0.1	0.1	0.1
3	A_6	0.9	1.1	1.1	1.0	1.0	1.0
4	A_{10}	0.1	0.1	0.1	0.1	0.1	0.1
5	A_{12}	0.1	0.1	0.4	0.1	0.1	0.1
6	A_{14}	0.1	0.1	0.1	0.1	0.1	0.1
7	A_{18}	0.1	0.2	0.4	0.2	0.1	0.1
8	A_{22}	1.0	0.8	0.7	0.9	0.9	0.9
9	x_4	37.3200	33.0487	27.6169	33.4976	36.9520	36.8958
10	y_4	55.7400	53.5663	51.6196	62.3735	54.5786	54.1337
11	z_4	126.6200	129.9092	129.9071	114.5945	129.9758	130.0000
12	x_8	50.1400	43.7826	42.5526	40.0531	51.7317	51.9924
13	y_8	136.4000	136.8381	132.7241	133.6695	139.5316	140.0000
	Best weight (lb)	118.83	120.1149	129.2076	123.5403	117.2271	117.3022
	Maximum displacement (in.)	0.3500	0.3500	0.3503	0.3505	0.3518	0.3500
	Maximum stress (ksi)	18.8302	17.1574	16.4391	15.5913	19.9702	20.0182
	Minimum stress (ksi)	-9.4017	-6.4822	-10.9931	-6.4102	-9.4049	-9.4024
	Number of structural analyses	6,000	10,000	4,500	4,500	4,500	4,500

TABLE 7: Material parameters, design constraints, and search range of the planar 37-bar truss optimization problem.

Category	Values
Material parameters	
Density	7800 kg/m ³
Modulus of elasticity	2.1×10^{11} N/m ²
Constraints	
Natural frequencies	$f_1 \geq 20$ Hz; $f_2 \geq 40$ Hz; $f_3 \geq 60$ Hz;
Search range	
Shape variables	$x_1 = 0$ m; $x_{2,3} = 1$ m; $x_{4,5} = 2$ m; $x_{6,7} = 3$ m; $x_{8,9} = 4$ m; $x_{10,11} = 5$ m; $x_{12,13} = 6$ m; $x_{14,15} = 7$ m; $x_{16,17} = 8$ m; $x_{18,19} = 9$ m; $x_{20} = 10$ m; $0 \leq y_{3,5,7,9,11} \leq 3$ m;
Sizing variables	$1 \times 10^{-4} \text{ m}^2 \leq A_i \leq 10 \times 10^{-4} \text{ m}^2, i = 1, 2, \dots, 14$ $A_j = 4 \times 10^{-3} \text{ m}^2, i = 28, 29, \dots, 30$

TABLE 8: Comparison of optimized designs found for the planar 37-bar truss.

No.	Variable	OC [9]	GA [18]	PSO [19]	RO [20]	FEPSO-AP
1	A_1	3.2508	2.8932	2.6797	3.0124	3.4197
2	A_2	1.2364	1.1201	1.1568	1.0623	0.9766
3	A_3	1.0000	1.0000	2.3476	1.0005	0.8313
4	A_4	2.5386	1.8655	1.7182	2.2647	2.8073
5	A_5	1.3714	1.5962	1.2751	1.6339	1.2997
6	A_6	1.3681	1.2642	1.4819	1.6717	1.6483
7	A_7	2.4290	1.8254	4.6850	2.0591	2.4972
8	A_8	1.6522	2.0009	1.1246	1.6607	1.5379
9	A_9	1.8257	1.9526	2.1214	1.4941	1.7590
10	A_{10}	2.3022	1.9705	3.8600	2.4737	2.7069
11	A_{11}	1.3103	1.8294	2.9817	1.5260	1.3046
12	A_{12}	1.4067	1.2358	1.2021	1.4823	1.4004
13	A_{13}	2.1896	1.4049	1.2563	2.4148	3.0476
14	A_{14}	1.0000	1.0000	3.3276	1.0034	0.5947
15	y_3	1.2086	1.1998	0.9637	1.0010	0.8756
16	y_5	1.5788	1.6553	1.3978	1.3909	1.2546
17	y_7	1.6719	1.9652	1.5929	1.5893	1.4446
18	y_9	1.7703	2.0737	1.8812	1.7507	1.5889
19	y_{11}	1.8502	2.3050	2.0856	1.8336	1.6480
Natural frequencies (Hz)	f_1	20.0850	20.0013	20.0001	20.056	20.020
	f_2	42.0743	40.0305	40.0003	40.035	40.022
	f_3	62.9383	60.0000	60.0001	60.030	60.233
	f_4	74.4539	73.0444	73.0440	74.387	72.137
	f_5	90.0576	89.8244	89.8240	85.929	84.065
Best weight (kg)		366.50	368.84	377.20	364.04	362.8812
Number of structural analyses		—	—	20,000	32,000	8,000

FIGURE 4: Layout optimization of the planar 37-bar truss.

TABLE 9: Loading conditions acting on the planar 47-bar truss.

Case	Node	F_x (kips)	F_y (kips)
1	17 and 22	6.0	-14.0
2	17	6.0	-14.0
3	22	6.0	-14.0

the results achieved by the proposed algorithm are even better than the best results reported in the literature.

4.4. Planar 47-Bar Truss. The original geometry of planar 47-bar truss is shown in Figure 5(a). Four nodes (ID: 15, 16, 17, and 22) are totally fixed and the y -coordinates of other two nodes (ID: 1 and 2) are fixed as well. The loading conditions are listed in Table 9.

To ensure the structural symmetric about the y -axis, all design variables are classified into 44 groups:

sizing variables: $A_1 = A_3$; $A_2 = A_4$; $A_5 = A_6$; A_7 ; $A_8 = A_9$; $A_{10} = A_{11}$; $A_{12} = A_{13}$; $A_{14} = A_{15}$; $A_{16} = A_{17}$; $A_{18} = A_{19}$; $A_{20} = A_{21}$; $A_{22} = A_{23}$; $A_{24} = A_{25}$; $A_{26} = A_{27}$; $A_{28} = A_{29}$; $A_{30} = A_{31}$; $A_{32} = A_{33}$; $A_{34} = A_{35}$; $A_{36} = A_{37}$; $A_{38} = A_{39}$; $A_{40} = A_{41}$; $A_{42} = A_{43}$; $A_{44} = A_{45}$; $A_{46} = A_{47}$;

shape variables: $x_1 = -x_2$; $x_3 = -x_4$; $y_3 = y_4$; $x_5 = -x_6$; $y_5 = y_6$; $x_7 = -x_8$; $y_7 = y_8$; $x_9 = -x_{10}$; $y_9 = y_{10}$; $x_{11} = -x_{12}$; $y_{11} = y_{12}$; $x_{13} = -x_{14}$; $y_{13} = y_{14}$; $x_{19} = -x_{20}$; $y_{19} = y_{20}$; $x_{18} = -x_{21}$; $y_{18} = y_{21}$.

Material parameters and design constraints are listed in Table 10.

TABLE 10: Material parameters, design constraints, and search range of the planar 47-bar truss optimization problem.

Category	Values
Material parameters	
Density	0.3 lb/in ³
Modulus of elasticity	3×10^4 ksi
Constraints	
Stress	The allowable elements stress interval: $[-15 \text{ ksi}, 20 \text{ ksi}]$
Local buckling	$ (\sigma_c)_i \leq \beta EA_i / l_i^2, \quad i = 1, 2, \dots, 47, \quad \beta = 3.96$
Search range	
Shape variables	$0 \text{ in.} \leq x_i \leq 120 \text{ in.}, (i = 2, 4, 6, 8); -30 \text{ in.} \leq x_j \leq 90 \text{ in.} (j = 10, 12, 14, 20); 30 \text{ in.} \leq x_{21} \leq 150 \text{ in.};$ $60 \text{ in.} \leq y_4 \leq 180 \text{ in.}; 180 \text{ in.} \leq y_6 \leq 300 \text{ in.}; 300 \text{ in.} \leq y_8 \leq 420 \text{ in.}; 360 \text{ in.} \leq y_{10} \leq 480 \text{ in.}; 420 \text{ in.}$ $\leq y_{12} \leq 540 \text{ in.}; 480 \text{ in.} \leq y_{14} \leq 600 \text{ in.}; 540 \text{ in.} \leq y_{20} \leq 660 \text{ in.}; 540 \text{ in.} \leq y_{21} \leq 660 \text{ in.}$
Sizing variables	$S = \{0.1, 0.2, 0.3, \dots, 4.8, 4.9, 5.0\} \text{ in.}^2 \quad A_i \in S, \quad i = 1, 2, \dots, 47$

FIGURE 5: Layout optimization of the planar 47-bar truss.

Figure 5(b) shows the optimum design identified by this work. Table 11 compares the best design of this work with those reported in the literature. It can be seen that by using the proposed algorithm it is possible to achieve better results at a lower computational cost.

5. Conclusion

In this work, a new hybrid PSO algorithm is proposed to solve a quite challenging task in truss optimization area: truss layout optimization with multiple constraints.

Two computational techniques are adopted to further enhance the performance of PSO algorithm. In the first fitness estimation strategy, the evaluation of particles is partly substituted by the estimation of similar particles, with the purpose to reduce the computational cost of real world optimization problem. In the second adaptive penalty function approach, the iteration information is merged into the penalty function

to find a good balance between the exploration and exploitation of the constrained design domain. The resulted algorithm is termed as FEPSO-AP.

Four benchmark truss layout optimization problems, subject to nodal displacement constraints, element stress constraints, natural frequency constraints, and local buckling constraints, are used to verify the performance of FEPSO-AP. Numerical results demonstrate that three out of four benchmarks, to which the FEPSO-AP based optimization is applied, delivered the best feasible designs to the author's knowledge. Moreover, the convergence rate of the FEPSO-AP algorithm is quite competitive comparing to other state-of-the-art hybrid algorithms published in the former literatures.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

TABLE II: Comparison of optimized designs found for the planar 47-bar truss.

No.	Variable	GA [21]	FSD-ES [22]	PSO [17]	CPSO [17]	SCPSO [17]	FEPSO-AP
1	A_3	2.50	2.70	2.80	2.60	2.50	3.20
2	A_4	2.20	2.50	2.70	2.50	2.50	2.80
3	A_5	0.70	0.70	0.80	0.70	0.80	0.70
4	A_7	0.10	0.10	1.10	0.30	0.10	0.10
5	A_8	1.30	0.90	0.80	1.20	0.70	0.60
6	A_{10}	1.30	1.10	1.30	1.10	1.40	1.60
7	A_{12}	1.80	1.80	1.80	1.60	1.70	1.90
8	A_{14}	0.50	0.70	0.90	0.80	0.80	0.90
9	A_{15}	0.80	0.90	1.20	1.10	0.90	1.00
10	A_{18}	1.20	1.30	1.40	1.30	1.30	2.00
11	A_{20}	0.40	0.30	0.30	0.30	0.30	0.10
12	A_{22}	1.20	1.10	1.40	0.80	0.90	0.40
13	A_{24}	0.90	1.00	1.10	1.00	1.00	1.40
14	A_{26}	1.00	0.90	1.20	1.00	1.10	1.50
15	A_{27}	3.60	0.80	1.60	0.90	5.00	1.20
16	A_{28}	0.10	0.10	1.00	0.10	0.10	0.70
17	A_{30}	2.40	2.70	2.80	2.70	2.50	5.00
18	A_{31}	1.10	0.80	0.80	0.90	1.00	1.00
19	A_{33}	0.10	0.10	0.10	0.10	0.10	0.10
20	A_{35}	2.70	3.00	3.00	3.00	2.80	3.00
21	A_{36}	0.80	0.90	0.90	1.00	0.90	0.50
22	A_{38}	0.10	0.00	0.10	0.20	0.10	0.10
23	A_{40}	2.80	3.20	3.30	3.30	3.00	3.30
24	A_{41}	1.30	1.00	0.90	0.90	1.00	0.30
25	A_{43}	0.20	0.10	0.10	0.10	0.10	0.10
26	A_{45}	3.00	3.30	3.30	3.30	3.20	3.50
27	A_{46}	1.20	1.10	1.20	1.10	1.20	0.40
28	x_2	114.0000	100.9724	98.8628	99.3630	101.3393	87.7275
29	x_4	97.0000	80.4772	78.6595	83.4439	85.9111	72.4352
30	y_4	125.0000	136.8699	146.7331	126.3845	135.9645	162.6451
31	x_6	76.0000	64.3908	66.5231	69.5148	74.7969	67.2113
32	y_6	261.0000	247.0491	239.0901	218.2013	237.7447	218.2041
33	x_8	69.0000	55.2589	55.6936	58.0004	64.3115	50.6507
34	y_8	316.0000	338.4534	327.7882	322.2272	321.3416	375.4549
35	x_{10}	56.0000	48.7333	48.8641	51.4015	53.3345	36.6525
36	y_{10}	414.0000	409.7380	398.6775	401.5626	414.3025	408.7230
37	x_{12}	50.0000	43.4742	43.1400	46.8605	46.0277	36.9960
38	y_{12}	463.0000	472.1479	464.7831	458.3021	489.9216	483.4295
39	x_{14}	54.0000	44.8349	37.8993	46.8885	41.8353	37.9558
40	y_{14}	524.0000	512.1901	511.0450	527.8575	522.4161	535.7644
41	x_{20}	1.0000	3.8414	18.2341	16.2354	1.0005	4.6875
42	y_{20}	587.0000	591.1449	594.0710	610.8496	598.3905	599.7416
43	x_{21}	99.0000	84.5040	90.9369	98.3239	97.8696	101.4535
44	y_{21}	631.0000	630.3472	621.3943	624.9580	624.0552	605.4302
Best weight (lb)		1925.7897	1842.6609	1975.8393	1908.8301	1864.0985	1799.7037
Maximum stress (ksi)		19.9528	20.0000	19.0636	19.3351	19.4735	19.9808
Minimum stress (ksi)		-14.9973	-15.0000	-14.9999	-14.9986	-15.0000	-14.9986
Number of buckling elements		0	0	0	0	0	0
Number of structural analyses		100,000	55,802	25,000	25,000	25,000	20,000

References

- [1] S. Shan and G. G. Wang, "Survey of modeling and optimization strategies to solve high-dimensional design problems with computationally-expensive black-box functions," *Structural and Multidisciplinary Optimization*, vol. 41, no. 2, pp. 219–241, 2010.
- [2] A. Kaveh and A. Zolghadr, "Democratic PSO for truss layout and size optimization with frequency constraints," *Computers and Structures*, vol. 130, pp. 10–21, 2014.
- [3] A. Xiao, B. Wang, and Y. Jin, "Evolutionary truss layout optimization using the vectorized structure approach," in *Proceedings of the IEEE Congress on Evolutionary Computation (CEC '13)*, pp. 2879–2886, June 2013.
- [4] M. W. Dobbs and L. P. Felton, "Optimization of truss geometry," *Journal of the Structural Division*, vol. 95, pp. 2105–2118, 1969.
- [5] G. N. Vanderplaats and F. Moses, "Structural optimization by methods of feasible directions," *Computers and Structures*, vol. 3, no. 4, pp. 739–755, 1973.
- [6] M. Zhou, "Geometrical optimization of trusses by a two-level approximation concept," *Structural Optimization*, vol. 1, no. 4, pp. 235–240, 1989.
- [7] L. Gil and A. Andreu, "Shape and cross-section optimization of a truss structure," *Computers & Structures*, vol. 79, no. 7, pp. 681–689, 2001.
- [8] K. Deb and S. Gulati, "Design of truss-structures for minimum weight using genetic algorithms," *Finite Elements in Analysis and Design*, vol. 37, no. 5, pp. 447–465, 2001.
- [9] D. Wang, W. H. Zhang, and J. S. Jiang, "Truss optimization on shape and sizing with frequency constraints," *AIAA Journal*, vol. 42, no. 3, pp. 622–630, 2004.
- [10] P. C. Fourie and A. A. Groenwold, "The particle swarm optimization algorithm in size and shape optimization," *Structural and Multidisciplinary Optimization*, vol. 23, no. 4, pp. 259–267, 2002.
- [11] O. Hasançebi and F. Erbatur, "Layout optimisation of trusses using simulated annealing," *Advances in Engineering Software*, vol. 33, no. 7-10, pp. 681–696, 2002.
- [12] H. Kawamura, H. Ohmori, and N. Kito, "Truss topology optimization by a modified genetic algorithm," *Structural and Multidisciplinary Optimization*, vol. 23, no. 6, pp. 467–472, 2002.
- [13] A. Kaveh and S. Talatahari, "Optimal design of skeletal structures via the charged system search algorithm," *Structural and Multidisciplinary Optimization*, vol. 41, no. 6, pp. 893–911, 2010.
- [14] M. Sonmez, "Artificial Bee Colony algorithm for optimization of truss structures," *Applied Soft Computing Journal*, vol. 11, no. 2, pp. 2406–2418, 2011.
- [15] L. F. F. Miguel, R. H. Lopez, and L. F. F. Miguel, "Multimodal size, shape, and topology optimisation of truss structures using the Firefly algorithm," *Advances in Engineering Software*, vol. 56, pp. 23–37, 2013.
- [16] H. Rahami, A. Kaveh, and Y. Gholipour, "Sizing, geometry and topology optimization of trusses via force method and genetic algorithm," *Engineering Structures*, vol. 30, no. 9, pp. 2360–2369, 2008.
- [17] S. Gholizadeh, "Layout optimization of truss structures by hybridizing cellular automata and particle swarm optimization," *Computers and Structures*, vol. 125, pp. 86–99, 2013.
- [18] W. Lingyun, Z. Mei, W. Guangming, and M. Guang, "Truss optimization on shape and sizing with frequency constraints based on genetic algorithm," *Computational Mechanics*, vol. 35, no. 5, pp. 361–368, 2005.
- [19] H. M. Gomes, "Truss optimization with dynamic constraints using a particle swarm algorithm," *Expert Systems with Applications*, vol. 38, no. 1, pp. 957–968, 2011.
- [20] A. Kaveh and M. Khayatizad, "Ray optimization for size and shape optimization of truss structures," *Computers and Structures*, vol. 117, pp. 82–94, 2013.
- [21] O. Hasançebi and F. Erbatur, "Layout optimization of trusses using improved GA methodologies," *Acta Mechanica*, vol. 146, no. 1-2, pp. 87–107, 2001.
- [22] A. Ahrari, A. A. Atai, and K. Deb, "Simultaneous topology, shape and size optimization of truss structures by fully stressed design based on evolution strategy," *Engineering Optimization*, 2013.
- [23] A. Kaveh and A. Zolghadr, "Truss optimization with natural frequency constraints using a hybridized CSS-BBBC algorithm with trap recognition capability," *Computers & Structures*, vol. 102-103, pp. 14–27, 2012.
- [24] W. Zuo, J. Bai, and B. Li, "A hybrid OC-GA approach for fast and global truss optimization with frequency constraints," *Applied Soft Computing Journal*, vol. 14, pp. 528–535, 2014.
- [25] A. Kaveh and S. M. Javadi, "Shape and size optimization of trusses with multiple frequency constraints using harmony search and ray optimizer for enhancing the particle swarm optimization algorithm," *Acta Mechanica*, vol. 225, no. 6, pp. 1595–1605, 2014.
- [26] W. Liu and J. Ye, "Collapse optimization for domes under earthquake using a genetic simulated annealing algorithm," *Journal of Constructional Steel Research*, vol. 97, pp. 59–68, 2014.
- [27] R. Eberhart and J. Kennedy, "A new optimizer using particle swarm theory," in *Proceedings of the 6th International Symposium on Micro Machine and Human Science*, vol. 1, pp. 39–43, October 1995.
- [28] C. Sun, J. Zeng, J. Pan, S. Xue, and Y. Jin, "A new fitness estimation strategy for particle swarm optimization," *Information Sciences*, vol. 221, pp. 355–370, 2013.
- [29] C. A. Coello Coello, "Theoretical and numerical constraint-handling techniques used with evolutionary algorithms: a survey of the state of the art," *Computer Methods in Applied Mechanics and Engineering*, vol. 191, no. 11-12, pp. 1245–1287, 2002.
- [30] T. P. Runarsson and X. Yao, "Stochastic ranking for constrained evolutionary optimization," *IEEE Transactions on Evolutionary Computation*, vol. 4, no. 3, pp. 284–294, 2000.
- [31] M. Clerc and J. Kennedy, "The particle swarm-explosion, stability, and convergence in a multidimensional complex space," *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 1, pp. 58–73, 2002.