Research Article

A Delay Decomposition Approach to the Stability Analysis of Singular Systems with Interval Time-Varying Delay

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This paper investigates delay-dependent stability problem for singular systems with interval time-varying delay. An appropriate Lyapunov-Krasovskii functional is constructed by decomposing the delay interval into multiple equidistant subintervals, where both the information of every subinterval and time-varying delay have been taken into account. Employing the Lyapunov-Krasovskii functional, improved delay-dependent stability criteria for the considered systems to be regular, impulse-free, and stable are established. Finally, two numerical examples are presented to show the effectiveness and less conservativeness of the proposed method.

1. Introduction

During the last few decades, there has been a growing research interest in the analysis and synthesis of time-delay systems, which widely exist in various practical systems such as biological systems, chemical systems, electronic systems, and network control systems [1]. Usually, the range of delays considered in most of existing references is from zero to an upper bound [2–6]. In practice, however, the delay range may have a nonzero lower bound, and such systems are referred to as interval time-varying delay systems. For this reason, the stability of the systems with such interval time-varying delays has attracted considerable attention. As we know, in order to reduce conservatism of the stability criteria, many approaches were developed; for example, by using free-weighting matrices, [7, 8] present some stability conditions for systems with interval time-varying delays. In [9], via a Lyapunov-Krasovskii functional with fewer matrix variables whose derivative is estimated using the convex analysis method, a simple stability criterion was obtained; this result is improved in [10] by using the reciprocally convex approach. Recently, some new methods were proposed to the stability analysis of interval time-varying delay systems. New classes of Lyapunov-Krasovskii functionals and augmented Lyapunov-Krasovskii functionals were introduced in [11–15], where some multi-integral terms were introduced in

Lyapunov-Krasovskii functionals. In [16–18], combining with a decomposition approach, the upper bound of the derivative of Lyapunov-Krasovskii functional was estimated tightly and new stability results were obtained. It has been shown that these new methods can be applied efficiently to derive less conservative stability results for systems with interval time-varying delay.

On the other hand, singular systems have been extensively studied in the past years due to the fact that singular systems can better describe the behavior of some physical systems than regular ones. Singular systems are also referred to as descriptor systems, differential algebraic systems, or semistate systems. A great number of fundamental results based on the theory of regular systems have been extended to the area of singular systems [19]. It is well known that the stability analysis for singular systems is much more complicated than that for regular systems because it requires considering not only stability but also regularity and absence of impulse (for continuous singular systems) [20–30] or causality (for discrete singular systems) [31–34]. Recently, more and more attention has been paid to singular systems with delay. To obtain delay-dependent conditions, many efforts have been made in the literature. Via different Lyapunov-Krasovskii functionals, some stability criteria were obtained in [20, 21], and the results in [20, 21] were improved in [22, 23]

using the free-weighting matrices and discretized Lyapunov-Krasovskii functional, respectively. However, the involved time delays in [20–23] are all time invariant, which limits the scope of applications of the given results. In the case where time-varying delays appear in singular systems, some stability results were proposed in [24–27]. The range of time-varying delay considered in [24–27] is from zero to an upper bound. For singular systems with interval time-varying delay, there are fewer results [28–30] and there still exists some room for further investigation.

In this paper, our purpose is to present some new delay-dependent stability criteria for singular systems with interval time-varying delay. An appropriate Lyapunov-Krasovskii functional is constructed by using the idea of “delay decomposition.” Employing this Lyapunov-Krasovskii functional, some new delay-dependent sufficient conditions ensuring the stability for the considered systems are obtained in terms of LIMs. The main contribution of this paper is in two aspects. First, the new results will be less conservative than the existing ones, which will be demonstrated by two numerical examples. Second, the new results will involve fewer decision variables than some existing ones; hence, they are mathematically less complex and more computationally efficient.

Notations. Throughout this paper, $\mathbf{R}^n$ denotes the n -dimensional Euclidean space, while $\mathbf{R}^{m \times n}$ refers to the set of all real matrices with m rows and n columns. A^T represents the transpose of the matrix A . For real symmetric matrices X and Y , the notation $X \geq Y$ (resp., $X > Y$) means matrix $X - Y$ is positive-semidefinite (resp., positive-definite). I is the identity matrix with appropriate dimensions. $\|x\|$ refers to the Euclidean norm of the vector x ; that is, $\|x\| = \sqrt{x^T x}$.

2. Problem Formulation and Preliminaries

Consider the following singular system with interval time-varying delay described by

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + A_d x(t - d(t)), \\ x(t) &= \varphi(t), \quad t \in [-d_2, 0], \end{aligned} \quad (1)$$

where $x(t) \in \mathbf{R}^n$ is the state vector and the initial condition $\varphi(t) \in \mathbf{R}^n$ is a continuously differentiable vector-valued function. The matrix $E \in \mathbf{R}^{n \times n}$ may be singular and ranks $E = r \leq n$, $A, A_d \in \mathbf{R}^{n \times n}$ are known real constant matrices. $d(t)$ is the time-varying delay satisfying

$$d_1 \leq d(t) \leq d_2, \quad \dot{d}(t) \leq \mu, \quad (2)$$

where $0 < d_1 < d_2$ and $0 \leq \mu < 1$ are known constant scalars.

In this paper, our objective is to establish new delay-dependent stability conditions for singular time-delay system (1). The following definitions and lemma will be used in the proof of our main results.

Definition 1 (see [19]). (i) The pair (E, A) is said to be regular if $\det(sE - A)$ is not identically zero. (ii) The pair (E, A) is said to be impulse-free if $\deg(\det(sE - A)) = \text{rank } E$.

Definition 2 (see [28, 30]). (i) The singular time-delay system (1) is said to be regular and impulse-free if the pair (E, A) is regular and impulse-free. (ii) The singular time-delay system (1) is said to be stable if, for any $\varepsilon > 0$, there exists a scalar $\delta(\varepsilon) > 0$ such that, for any compatible initial conditions $\varphi(t)$ satisfying $\sup_{-d_2 \leq t \leq 0} \|\varphi(t)\| \leq \delta(\varepsilon)$, the solution $x(t)$ of system (1) satisfies $\|x(t)\| \leq \varepsilon$ for any $t \geq 0$; moreover, $\lim_{t \rightarrow \infty} x(t) = 0$.

Lemma 3 (see [1]). For any symmetric positive defined matrix $R > 0$, scalars $\gamma_2 > \gamma_1 \geq 0$, and vector function $x : [\gamma_1, \gamma_2] \mapsto \mathbf{R}^n$ such that the following integrations are well defined, the following inequality holds

$$-(\gamma_2 - \gamma_1) \int_{\gamma_1}^{\gamma_2} x^T(s) R x(s) ds \leq - \int_{\gamma_1}^{\gamma_2} x^T(s) ds R \int_{\gamma_1}^{\gamma_2} x(s) ds. \quad (3)$$

3. Main Results

In this section, we consider the stability problem of singular time-delay system (1). We first present a delay-dependent stability criterion for singular time-delay system (1) as follows.

Theorem 4. Given scalars $0 < d_1 < d_2$ and $0 \leq \mu < 1$ for any delay $d(t)$ satisfying (2), singular time-delay system (1) is regular, impulse-free, and stable if there exist the following matrices:

$$\begin{aligned} P > 0, \quad W > 0, \quad Q &= \begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix} > 0, \\ R &= \begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix} > 0, \quad S_i > 0, \\ Z_i &> 0, \quad M_i \quad (i = 1, 2), \end{aligned} \quad (4)$$

and U , such that the LMIs

$$\Xi + \Xi_k < 0, \quad (5)$$

$$\begin{bmatrix} Z_k & M_k \\ M_k^T & Z_k \end{bmatrix} > 0 \quad (6)$$

hold for any $k \in \{1, 2\}$, where

$$\begin{aligned} \Xi &= e_1 (E^T P + U \Psi^T) e_0^T + e_0 (P E + \Psi U^T) e_1^T \\ &+ e_0 \left(\left(\frac{d_1}{2} \right)^2 (S_1 + S_2) + \left(\frac{d_2 - d_1}{2} \right)^2 (Z_1 + Z_2) \right) e_0^T \\ &+ e_3 W e_3^T - (1 - \mu) e_6 W e_6^T + [e_1 \quad e_2] Q [e_1 \quad e_2]^T \end{aligned}$$

$$\begin{aligned}
& -[e_2 \ e_3] Q [e_2 \ e_3]^T + [e_3 \ e_4] R [e_3 \ e_4]^T \\
& -[e_4 \ e_5] R [e_4 \ e_5]^T - (e_1 - e_2) E^T S_1 E (e_1 - e_2)^T \\
& - (e_2 - e_3) E^T S_2 E (e_2 - e_3)^T, \\
\Xi_1 = & - (e_4 - e_5) E^T Z_2 E (e_4 - e_5)^T \\
& - \begin{bmatrix} e_6^T - e_4^T \\ e_3^T - e_6^T \end{bmatrix}^T \begin{bmatrix} E^T Z_1 E & E^T M_1 E \\ E^T M_1^T E & E^T Z_1 E \end{bmatrix} \begin{bmatrix} e_6^T - e_4^T \\ e_3^T - e_6^T \end{bmatrix}, \\
\Xi_2 = & - (e_3 - e_4) E^T Z_1 E (e_3 - e_4)^T \\
& - \begin{bmatrix} e_6^T - e_5^T \\ e_4^T - e_6^T \end{bmatrix}^T \begin{bmatrix} E^T Z_2 E & E^T M_2 E \\ E^T M_2^T E & E^T Z_2 E \end{bmatrix} \begin{bmatrix} e_6^T - e_5^T \\ e_4^T - e_6^T \end{bmatrix},
\end{aligned} \tag{7}$$

and $\Psi \in \mathbf{R}^{n \times (n-r)}$ is any full-column rank matrix satisfying $E^T \Psi = 0$, $e_i = [0_{n \times (i-1)n} \ I_n \ 0_{n \times (6-i)n}]^T$ ($i = 1, 2, \dots, 6$) are block entry matrices, and $e_0 = (Ae_1^T + A_d e_6^T)^T$.

Proof. The proof is divided into two parts. The first one deals with the regularity and impulse-free property, and the second one treats the stability property of the studied class of systems. Let us first of all show that the singular time-delay system (1) is regular and impulse-free. Using (5), it is easy to see that the following inequality holds:

$$\begin{aligned}
& E^T P A + A^T P E + U \Psi^T A + A^T \Psi U^T + \left(\frac{d_1}{2}\right)^2 A^T (S_1 + S_2) A \\
& + \left(\frac{d_2 - d_1}{2}\right)^2 A^T (Z_1 + Z_2) A + Q_1 - E^T S_1 E < 0.
\end{aligned} \tag{8}$$

Using the fact that $S_i > 0$, $Z_i > 0$ ($i = 1, 2$), and $Q_1 > 0$, from (8), we have

$$E^T P A + A^T P E + U \Psi^T A + A^T \Psi U^T - E^T S_1 E < 0. \tag{9}$$

It follows from (9) and Lemma 3.5 in [32] that the pair (E, A) is regular and impulse-free. Thus, according to Definition 2, singular time-delay system (1) is regular and impulse-free.

Next, we will show the stability of the singular time-delay system (1). To the end, the following Lyapunov-Krasovskii functional for system (1) is considered:

$$V(x_t) = \sum_{i=1}^4 V_i(x_t), \tag{10}$$

where

$$V_1(x_t) = x^T(t) E^T P E x(t) + \int_{t-d(t)}^{t-d_1} x^T(s) W x(s) ds,$$

$$\begin{aligned}
V_2(x_t) = & \int_{t-(d_1/2)}^t \xi_1^T(s) Q \xi_1(s) ds \\
& + \int_{t-((d_1+d_2)/2)}^{t-d_1} \xi_2^T(s) R \xi_2(s) ds,
\end{aligned}$$

$$\begin{aligned}
V_3(x_t) = & \frac{d_1}{2} \left(\int_{-d_1/2}^0 \int_{t+\alpha}^t \dot{x}^T(\beta) E^T S_1 E \dot{x}(\beta) d\beta d\alpha \right. \\
& \left. + \int_{-d_1}^{-d_1/2} \int_{t+\alpha}^t \dot{x}^T(\beta) E^T S_2 E \dot{x}(\beta) d\beta d\alpha \right),
\end{aligned}$$

$$\begin{aligned}
V_4(x_t) = & \frac{d_2 - d_1}{2} \\
& \times \left(\int_{-(d_1+d_2)/2}^{-d_1} \int_{t+\alpha}^t \dot{x}^T(\beta) E^T Z_1 E \dot{x}(\beta) d\beta d\alpha \right. \\
& \left. + \int_{-d_2}^{-(d_1+d_2)/2} \int_{t+\alpha}^t \dot{x}^T(\beta) E^T Z_2 E \dot{x}(\beta) d\beta d\alpha \right),
\end{aligned} \tag{11}$$

with

$$\begin{aligned}
\xi_1(s) = & \begin{bmatrix} x^T(s) & x^T\left(s - \frac{d_1}{2}\right) \end{bmatrix}^T, \\
\xi_2(s) = & \begin{bmatrix} x^T(s) & x^T\left(s - \frac{d_2 - d_1}{2}\right) \end{bmatrix}^T.
\end{aligned} \tag{12}$$

Define $\xi(t) = [x^T(t) \ x^T(t - (d_1/2)) \ x^T(t - d_1) \ x^T(t - ((d_1 + d_2)/2)) \ x^T(t - d_2) \ x^T(t - d(t))]^T$. Now calculating the derivative of $V(x_t)$ along the trajectory of the system (1), we derive

$$\begin{aligned}
\dot{V}_1(x_t) = & 2x^T(t) E^T P E \dot{x}(t) + x^T(t - d_1) W x(t - d_1) \\
& - (1 - \dot{d}(t)) x^T(t - d(t)) W x(t - d(t)) \\
\leq & \xi^T(t) (e_1 E^T P e_0^T + e_0 P E e_1^T + e_3 W e_3^T \\
& - (1 - \mu) e_6 W e_6^T) \xi(t),
\end{aligned}$$

$$\begin{aligned}
\dot{V}_2(x_t) = & \xi_1^T(t) Q \xi_1(t) - \xi_1^T\left(t - \frac{d_1}{2}\right) Q \xi_1\left(t - \frac{d_1}{2}\right) \\
& + \xi_2^T(t - d_1) R \xi_2(t - d_1) - \xi_2^T\left(t - \frac{d_1 + d_2}{2}\right) \\
& \times R \xi_2\left(t - \frac{d_1 + d_2}{2}\right)
\end{aligned}$$

$$\begin{aligned}
&= \xi^T(t) \left([e_1 \ e_2] Q [e_1 \ e_2]^T - [e_2 \ e_3] \right. \\
&\quad \times Q [e_2 \ e_3]^T + [e_3 \ e_4] R [e_3 \ e_4]^T \\
&\quad \left. - [e_4 \ e_5] R [e_4 \ e_5]^T \right) \xi(t),
\end{aligned}$$

$$\begin{aligned}
\dot{V}_3(x_t) &= \left(\frac{d_1}{2} \right)^2 \dot{x}^T(t) E^T (S_1 + S_2) E \dot{x}(t) \\
&\quad - \frac{d_1}{2} \int_{t-(d_1/2)}^t \dot{x}^T(s) E^T S_1 E \dot{x}(s) ds \\
&\quad - \frac{d_1}{2} \int_{t-d_1}^{t-(d_1/2)} \dot{x}^T(s) E^T S_2 E \dot{x}(s) ds.
\end{aligned} \tag{13}$$

By Lemma 3, it can be shown that

$$\begin{aligned}
\dot{V}_3(x_t) &\leq \left(\frac{d_1}{2} \right)^2 \dot{x}^T(t) E^T (S_1 + S_2) E \dot{x}(t) \\
&\quad - \left(x(t) - x\left(t - \frac{d_1}{2}\right) \right)^T \\
&\quad \times E^T S_1 E \left(x(t) - x\left(t - \frac{d_1}{2}\right) \right) \\
&\quad - \left(x\left(t - \frac{d_1}{2}\right) - x(t - d_1) \right)^T E^T S_2 E \\
&\quad \times \left(x\left(t - \frac{d_1}{2}\right) - x(t - d_1) \right) \\
&= \xi^T(t) \left(\left(\frac{d_1}{2} \right)^2 e_0^T (S_1 + S_2) e_0^T \right. \\
&\quad \left. - (e_1 - e_2) E^T S_1 E (e_1 - e_2)^T \right. \\
&\quad \left. - (e_2 - e_3) E^T S_2 E (e_2 - e_3)^T \right) \xi(t),
\end{aligned} \tag{14}$$

$$\begin{aligned}
\dot{V}_4(x_t) &= \left(\frac{d_2 - d_1}{2} \right)^2 \dot{x}^T(t) E^T (Z_1 + Z_2) E \dot{x}(t) \\
&\quad - \frac{d_2 - d_1}{2} \int_{t-((d_1+d_2)/2)}^{t-d_1} \dot{x}^T(s) E^T Z_1 E \dot{x}(s) ds \\
&\quad - \frac{d_2 - d_1}{2} \int_{t-d_2}^{t-((d_1+d_2)/2)} \dot{x}^T(s) E^T Z_2 E \dot{x}(s) ds.
\end{aligned} \tag{15}$$

When $d_1 \leq d(t) \leq (d_1 + d_2)/2$, we have

$$\begin{aligned}
\dot{V}_4(x_t) &= \left(\frac{d_2 - d_1}{2} \right)^2 \dot{x}^T(t) E^T (Z_1 + Z_2) E \dot{x}(t) \\
&\quad - \frac{d_2 - d_1}{2} \int_{t-d_2}^{t-((d_1+d_2)/2)} \dot{x}^T(s) E^T Z_2 E \dot{x}(s) ds
\end{aligned}$$

$$\begin{aligned}
&- \frac{d_2 - d_1}{2} \int_{t-((d_1+d_2)/2)}^{t-d(t)} \dot{x}^T(s) E^T Z_1 E \dot{x}(s) ds \\
&- \frac{d_2 - d_1}{2} \int_{t-d(t)}^{t-d_1} \dot{x}^T(s) E^T Z_1 E \dot{x}(s) ds.
\end{aligned} \tag{16}$$

Using Lemma 3 again, we get

$$\begin{aligned}
\dot{V}_4(x_t) &\leq \left(\frac{d_2 - d_1}{2} \right)^2 \dot{x}^T(t) E^T (Z_1 + Z_2) E \dot{x}(t) \\
&\quad - \left(x\left(t - \frac{d_1 + d_2}{2}\right) - x(t - d_2) \right)^T E^T Z_2 E \\
&\quad \times \left(x\left(t - \frac{d_1 + d_2}{2}\right) - x(t - d_2) \right) \\
&\quad - \left(1 + \frac{2(d(t) - d_1)}{d_1 + d_2 - 2d(t)} \right) \\
&\quad \times \left(x(t - d(t)) - x\left(t - \frac{d_1 + d_2}{2}\right) \right)^T E^T Z_1 E \\
&\quad \times \left(x(t - d(t)) - x\left(t - \frac{d_1 + d_2}{2}\right) \right) \\
&\quad - \left(1 + \frac{d_1 + d_2 - 2d(t)}{2(d(t) - d_1)} \right) \\
&\quad \times (x(t - d_1) - x(t - d(t)))^T E^T Z_1 E \\
&\quad \times (x(t - d_1) - x(t - d(t))).
\end{aligned} \tag{17}$$

Define

$$\begin{aligned}
J &= \left[\sqrt{\frac{2(d(t) - d_1)}{d_1 + d_2 - 2d(t)}} \right. \\
&\quad \times \left(x(t - d(t)) - x\left(t - \frac{d_1 + d_2}{2}\right) \right)^T E^T \\
&\quad \left. - \sqrt{\frac{d_1 + d_2 - 2d(t)}{2(d(t) - d_1)}} (x(t - d_1) - x(t - d(t)))^T E^T \right],
\end{aligned} \tag{18}$$

with the condition (6); then, premultiplying and postmultiplying $\begin{bmatrix} Z_1 & M_1 \\ M_1^T & Z_1 \end{bmatrix} > 0$ by J and J^T , respectively, we can obtain

$$\begin{aligned}
&- \frac{2(d(t) - d_1)}{d_1 + d_2 - 2d(t)} \left(x(t - d(t)) - x\left(t - \frac{d_1 + d_2}{2}\right) \right)^T \\
&\quad \times E^T Z_1 E \left(x(t - d(t)) - x\left(t - \frac{d_1 + d_2}{2}\right) \right) \\
&- \frac{d_1 + d_2 - 2d(t)}{2(d(t) - d_1)} (x(t - d_1) - x(t - d(t)))^T
\end{aligned}$$

$$\begin{aligned}
& \times E^T Z_1 E (x(t-d_1) - x(t-d(t))) \\
& \leq - \left(x(t-d(t)) - x\left(t - \frac{d_1+d_2}{2}\right) \right)^T \\
& \quad \times E^T M_1 E (x(t-d_1) - x(t-d(t))) \\
& \quad - (x(t-d_1) - x(t-d(t)))^T E^T M_1^T E \\
& \quad \times \left(x(t-d(t)) - x\left(t - \frac{d_1+d_2}{2}\right) \right); \quad (19)
\end{aligned}$$

hence,

$$\begin{aligned}
\dot{V}_4(x_t) & \leq \left(\frac{d_2-d_1}{2} \right)^2 \dot{x}^T(t) E^T (Z_1 + Z_2) E \dot{x}(t) \\
& \quad - \left(x\left(t - \frac{d_1+d_2}{2}\right) - x(t-d_2) \right)^T E^T Z_2 E \\
& \quad \times \left(x\left(t - \frac{d_1+d_2}{2}\right) - x(t-d_2) \right) \\
& \quad - \begin{bmatrix} x(t-d(t)) - x\left(t - \frac{d_1+d_2}{2}\right) \\ x(t-d_1) - x(t-d(t)) \end{bmatrix}^T \\
& \quad \times \begin{bmatrix} E^T Z_1 E & E^T M_1 E \\ E^T M_1^T E & E^T Z_1 E \end{bmatrix} \\
& \quad \times \begin{bmatrix} x(t-d(t)) - x\left(t - \frac{d_1+d_2}{2}\right) \\ x(t-d_1) - x(t-d(t)) \end{bmatrix} \\
& = \xi^T(t) \left(\left(\frac{d_2-d_1}{2} \right)^2 e_0^T (Z_1 + Z_2) e_0^T \right. \\
& \quad - (e_4 - e_5)^T E^T Z_2 E \\
& \quad \times (e_4 - e_5)^T - \begin{bmatrix} e_6^T - e_4^T \\ e_3^T - e_6^T \end{bmatrix}^T \\
& \quad \times \begin{bmatrix} E^T Z_1 E & E^T M_1 E \\ E^T M_1^T E & E^T Z_1 E \end{bmatrix} \\
& \quad \times \begin{bmatrix} e_6^T - e_4^T \\ e_3^T - e_6^T \end{bmatrix} \left. \right) \xi(t). \quad (20)
\end{aligned}$$

Note that when $d(t) = d_1$ or $d(t) = (d_1 + d_2)/2$, we have $\xi^T(t)(e_3 - e_6) = 0$ or $\xi^T(t)(e_6 - e_4) = 0$, respectively. So relation (20) still holds.

When $(d_1 + d_2)/2 \leq d(t) \leq d_2$, (15) can be rewritten as

$$\begin{aligned}
\dot{V}_4(x_t) & = \left(\frac{d_2-d_1}{2} \right)^2 \dot{x}^T(t) E^T (Z_1 + Z_2) E \dot{x}(t) \\
& \quad - \frac{d_2-d_1}{2} \int_{t-((d_1+d_2)/2)}^{t-d_1} \dot{x}^T(s) E^T Z_1 E \dot{x}(s) ds \\
& \quad - \frac{d_2-d_1}{2} \int_{t-d_2}^{t-d(t)} \dot{x}^T(s) E^T Z_2 E \dot{x}(s) ds \\
& \quad - \frac{d_2-d_1}{2} \int_{t-d(t)}^{t-((d_1+d_2)/2)} \dot{x}^T(s) E^T Z_2 E \dot{x}(s) ds. \quad (21)
\end{aligned}$$

From condition (6) and Lemma 3, proceeding in a similar manner as above, we can get

$$\begin{aligned}
\dot{V}_4(x_t) & \leq \xi^T(t) \left(\left(\frac{d_2-d_1}{2} \right)^2 e_0^T (Z_1 + Z_2) e_0^T \right. \\
& \quad - (e_3 - e_4)^T E^T Z_1 E (e_3 - e_4)^T \\
& \quad - \begin{bmatrix} e_6^T - e_5^T \\ e_4^T - e_6^T \end{bmatrix}^T \begin{bmatrix} E^T Z_2 E & E^T M_2 E \\ E^T M_2^T E & E^T Z_2 E \end{bmatrix} \\
& \quad \times \left. \begin{bmatrix} e_6^T - e_5^T \\ e_4^T - e_6^T \end{bmatrix} \right) \xi(t). \quad (22)
\end{aligned}$$

Noting $E^T \Psi = 0$, we can deduce

$$0 = 2x^T(t) U \Psi^T E \dot{x}(t) = 2\xi^T(t) e_1 U \Psi^T e_0^T \xi(t), \quad (23)$$

where U is any matrix with appropriate dimensions.

Combining (10)–(14), (20)–(23) yield

$$\dot{V}(x_t) \leq \begin{cases} \xi^T(t) (\Xi + \Xi_1) \xi(t), & d_1 \leq d(t) \leq \frac{d_1+d_2}{2}, \\ \xi^T(t) (\Xi + \Xi_2) \xi(t), & \frac{d_1+d_2}{2} \leq d(t) \leq d_2. \end{cases} \quad (24)$$

Therefore, we can see if LMIs (5) and (6) hold for any $k \in \{1, 2\}$; then $\dot{V}(x_t) < 0$ holds for any $d(t) \in [d_1, d_2]$. Hence, there exist a sufficiently small scalar $\varepsilon > 0$, such that

$$\dot{V}(t) \leq -\varepsilon \|x(t)\|^2. \quad (25)$$

By (25), the following steps are similar to the proofs of Theorem 3.1 in [25] and Proposition 1 in [28]; we deduce that singular time-delay system (1) is stable. This completes the proof. $\square$

Remark 5. Based on the Lyapunov-Krasovskii functional (10), Theorem 4 proposed a new delay-dependent criterion guaranteeing the singular time-delay system (1) to be regular, impulse-free, and stable. Lyapunov-Krasovskii functional

(10) is constructed by using the idea of “delay decomposition” [28–30]. We decompose the delay intervals $[0, d_1]$ and $[d_1, d_2]$ into two equidistant subintervals, respectively, such that the information of delay states $x(t - (d_1/2))$ and $x(t - ((d_1 + d_2)/2))$ is all taken into account. Compared with [29], in which the Lyapunov-Krasovskii functional was constructed by decomposing delay intervals $[0, d_1]$ and $[0, d_2]$ into two equidistant subintervals, respectively, our Lyapunov-Krasovskii functional contains the delay state $x(t - ((d_1 + d_2)/2))$. Thus, by checking the variation of $\dot{V}(x_t)$ for the case when $d(t) \in [d_1, (d_1 + d_2)/2]$ or $d(t) \in [(d_1 + d_2)/2, d_2]$, respectively, we introduce a new estimation on the upper bound of $\dot{V}(x_t)$. That is to say, the delay state $x(t - ((d_1 + d_2)/2))$ plays a key role in the further reduction of conservatism. Furthermore, our Lyapunov-Krasovskii functional can be easily extended to the case as the number of delay partitions grows, which can be seen from the following discussion.

In the following, by decomposing the delay intervals $[0, d_1]$ and $[d_1, d_2]$ into $N(N \geq 2)$ equidistant subintervals, respectively, we will derive a more general stability result than Theorem 4. For this purpose, the following Lyapunov-Krasovskii functional for singular time-delay system (1) is considered:

$$\begin{aligned} \bar{V}(x_t) = & x^T(t) E^T P E x(t) + \int_{t-d(t)}^{t-d_1} x^T(s) W x(s) ds \\ & + \sum_{i=1}^{N-1} \left(\int_{t-(i/N)d_1}^{t-((i-1)/N)d_1} \eta_1^T(s) Q_i \eta_1(s) ds \right. \\ & \quad \left. + \int_{t-d_1-(i/N)(d_2-d_1)}^{t-d_1-((i-1)/N)(d_2-d_1)} \eta_2^T(s) R_i \eta_2(s) ds \right) \\ & + \sum_{i=1}^N \left(\frac{d_1}{N} \int_{-(i/N)d_1}^{-(i-1)/N)d_1} \int_{t+\alpha}^t \dot{x}^T(\beta) E^T \right. \\ & \quad \times S_i E \dot{x}(\beta) d\beta d\alpha \\ & \quad + \frac{d_2-d_1}{N} \int_{-d_1-(i/N)(d_2-d_1)}^{-d_1-((i-1)/N)(d_2-d_1)} \\ & \quad \times \int_{t+\alpha}^t \dot{x}^T(\beta) E^T Z_i E \dot{x}(\beta) d\beta d\alpha \Big), \end{aligned} \quad (26)$$

where

$$\begin{aligned} \eta_1(s) &= \begin{bmatrix} x^T(s) & x^T\left(s - \frac{1}{N}d_1\right) \end{bmatrix}^T, \\ \eta_2(s) &= \begin{bmatrix} x^T(s) & x^T\left(s - \frac{1}{N}(d_2-d_1)\right) \end{bmatrix}^T, \end{aligned}$$

$$P > 0, \quad W > 0, \quad Q_i = \begin{bmatrix} Q_{i1} & Q_{i2} \\ Q_{i2}^T & Q_{i3} \end{bmatrix} > 0,$$

$$R_i = \begin{bmatrix} R_{i1} & R_{i2} \\ R_{i2}^T & R_{i3} \end{bmatrix} > 0 \quad (i = 1, 2, \dots, N-1),$$

$$S_j > 0, \quad Z_j > 0 \quad (j = 1, 2, \dots, N). \quad (27)$$

Using Lyapunov-Krasovskii functional (26), the following theorem can be obtained.

Theorem 6. Given an integer $N \geq 2$ and scalars $0 < d_1 < d_2$, $0 \leq \mu < 1$, for any delay $d(t)$ satisfying (2), singular time-delay system (1) is regular, impulse-free, and stable if there exist matrices

$$\begin{aligned} P > 0, \quad W > 0, \quad Q_i &= \begin{bmatrix} Q_{i1} & Q_{i2} \\ Q_{i2}^T & Q_{i3} \end{bmatrix} > 0, \\ R_i &= \begin{bmatrix} R_{i1} & R_{i2} \\ R_{i2}^T & R_{i3} \end{bmatrix} > 0 \quad (i = 1, 2, \dots, N-1), \end{aligned} \quad (28)$$

$$S_j > 0, \quad Z_j > 0, \quad M_j \quad (j = 1, 2, \dots, N)$$

and U , such that LMIs

$$\Gamma + \Gamma_k < 0, \quad (29)$$

$$\begin{bmatrix} Z_k & M_k \\ M_k^T & Z_k \end{bmatrix} > 0 \quad (30)$$

hold for any $k \in \{1, 2, \dots, N\}$, where

$$\begin{aligned} \Gamma = & e_1 (E^T P + U \Psi^T) e_0^T + e_0 (P E + \Psi U^T) e_1^T \\ & + e_{N+1} W e_{N+1}^T - (1 - \mu) e_{2N+2} W e_{2N+2}^T \\ & + \sum_{i=1}^{N-1} \left([e_i \ e_{i+1}] Q_i [e_i \ e_{i+1}]^T \right. \\ & \quad - [e_{i+1} \ e_{i+2}] Q_i [e_{i+1} \ e_{i+2}]^T \\ & \quad + [e_{N+i} \ e_{N+i+1}] R_i [e_{N+i} \ e_{N+i+1}]^T \\ & \quad \left. - [e_{N+i+1} \ e_{N+i+2}] R_i [e_{N+i+1} \ e_{N+i+2}]^T \right) \\ & + \sum_{i=1}^N \left(e_0 \left(\left(\frac{d_1}{N} \right)^2 S_i + \left(\frac{d_2-d_1}{N} \right)^2 Z_i \right) e_0^T \right. \\ & \quad - (e_i - e_{i+1}) E^T S_i E (e_i - e_{i+1})^T \\ & \quad \left. - (e_{N+i} - e_{N+i+1}) E^T Z_i E (e_{N+i} - e_{N+i+1})^T \right), \end{aligned}$$

$$\begin{aligned} \Gamma_k &= (e_{N+k} - e_{N+k+1}) E^T Z_k E (e_{N+k} - e_{N+k+1})^T \\ &\quad - \begin{bmatrix} e_{2N+2}^T - e_{N+k+1}^T \\ e_{N+k}^T - e_{2N+2}^T \end{bmatrix}^T \begin{bmatrix} E^T Z_k E & E^T M_k E \\ E^T M_k^T E & E^T Z_k E \end{bmatrix} \\ &\quad \times \begin{bmatrix} e_{2N+2}^T - e_{N+k+1}^T \\ e_{N+k}^T - e_{2N+2}^T \end{bmatrix}, \end{aligned} \quad (31)$$

and $\Psi \in \mathbf{R}^{n \times (n-r)}$ is any full-column rank matrix satisfying $E^T \Psi = 0$, $e_i = [0_{n \times (i-1)n} \quad I_n \quad 0_{n \times (2N-i+2)n}]^T$ ($i = 1, 2, \dots, 2N+2$) are block entry matrices, and $e_0 = (Ae_1^T + A_d e_{2N+2}^T)^T$.

Proof. The proof of Theorem 6 can be carried out using methods in the proof of Theorem 4. Hence, it is omitted. $\square$

Remark 7. Based on the Lyapunov-Krasovskii functional (26), Theorem 6 proposed an improved delay-dependent criterion guaranteeing the singular time-delay system (1) to be regular, impulse-free, and stable. Lyapunov-Krasovskii functional (26) is constructed by decomposing the delay intervals $[0, d_1]$ and $[d_1, d_2]$ into N equidistant subintervals, respectively, such that the information of delay states $x(t - (i/N)d_1)$ and $x(t - d_1 - (i/N)(d_2 - d_1))$ ($i = 1, 2, \dots, N$) is all taken into account. Compared with [28, 30], the augmented vectors $\eta_1(s)$ and $\eta_2(s)$ were introduced in Lyapunov-Krasovskii functional (26), so (26) is more general than the ones in [28, 30].

Remark 8. If the integer N is set to 2, then Theorem 6 reduces to Theorem 4. That is, Theorem 6 is an extension of Theorem 4. It should be pointed out that the conservatism of Theorem 6 lies in the parameter N , which refers to the number of delay partitioning; that is, the conservatism is reduced as the partitions grow. On the other hand, the computational complexity also depends on the partition number N ; that is, the computational complexity is increased as the partitions become thinner. Generally taking the small value of N such as $N = 2$ and $N = 3$, we can obtain less conservative and simple results. This can be seen from the simulation results in the sequel.

For system (1) with the routine delay case described by

$$0 \leq d \leq (t) d, \quad \dot{d}(t) \leq \mu, \quad (32)$$

where $d > 0$ and $0 \leq \mu < 1$ are known constant scalars, the corresponding Lyapunov-Krasovskii reduces to

$$\begin{aligned} \bar{V}(x_t) &= x^T(t) E^T P E x(t) + \int_{t-d(t)}^t x^T(s) W x(s) ds \\ &\quad + \sum_{i=1}^{N-1} \int_{t-(i/N)d}^{t-((i-1)/N)d} \zeta^T(s) R_i \zeta(s) ds \\ &\quad + \frac{d}{N} \sum_{i=1}^N \int_{t-(i/N)d}^{t-((i-1)/N)d} \int_{t+\alpha}^t \dot{x}^T(\beta) E^T Z_i E \dot{x}(\beta) d\beta d\alpha, \end{aligned} \quad (33)$$

where

$$\begin{aligned} \zeta(s) &= \begin{bmatrix} x^T(s) & x^T\left(s - \frac{1}{N}d\right) \end{bmatrix}^T, \quad P > 0, \quad W > 0, \\ R_i &= \begin{bmatrix} R_{i1} & R_{i2} \\ R_{i2}^T & R_{i3} \end{bmatrix} > 0 \quad (i = 1, 2, \dots, N-1), \\ Z_j &> 0 \quad (j = 1, 2, \dots, N). \end{aligned} \quad (34)$$

Similar to the proof of Theorem 4, we can obtain the following delay-dependent stability criterion for singular time-delay system (1) with $d(t)$ satisfying (32).

Corollary 9. Given an integer $N \geq 2$ and scalars $d > 0$, $0 \leq \mu < 1$, for any delay $d(t)$ satisfying (32), singular time-delay system (1) is regular, impulse-free, and stable if there exist matrices

$$\begin{aligned} P &> 0, \quad W > 0, \\ R_i &= \begin{bmatrix} R_{i1} & R_{i2} \\ R_{i2}^T & R_{i3} \end{bmatrix} > 0 \quad (i = 1, 2, \dots, N-1), \\ Z_j &> 0, \quad M_j \quad (j = 1, 2, \dots, N), \end{aligned} \quad (35)$$

and U , such that the LMI (30) and LMI

$$Y + Y_k < 0 \quad (36)$$

hold for any $k \in \{1, 2, \dots, N\}$, where

$$\begin{aligned} Y &= e_1 (E^T P + U \Psi^T) e_0^T + e_0 (P E + \Psi U^T) e_1^T \\ &\quad + e_1 W e_1^T - (1 - \mu) e_{N+2} W e_{N+2}^T \\ &\quad + \sum_{i=1}^{N-1} \left([e_i \quad e_{i+1}] R_i [e_i \quad e_{i+1}]^T \right. \\ &\quad \left. - [e_{i+1} \quad e_{i+2}] R_i [e_{i+1} \quad e_{i+2}]^T \right) \\ &\quad + \sum_{i=1}^N \left(\left(\frac{d}{N} \right)^2 e_0 Z_i e_0^T - (e_i - e_{i+1}) E^T Z_i E (e_i - e_{i+1})^T \right), \\ Y_k &= (e_k - e_{k+1}) E^T Z_k E (e_k - e_{k+1})^T - \begin{bmatrix} e_{N+2}^T - e_{k+1}^T \\ e_k^T - e_{N+2}^T \end{bmatrix}^T \\ &\quad \times \begin{bmatrix} E^T Z_k E & E^T M_k E \\ E^T M_k^T E & E^T Z_k E \end{bmatrix} \begin{bmatrix} e_{N+2}^T - e_{k+1}^T \\ e_k^T - e_{N+2}^T \end{bmatrix}, \end{aligned} \quad (37)$$

and $\Psi \in \mathbf{R}^{n \times (n-r)}$ is any full-column rank matrix satisfying $E^T \Psi = 0$, $e_i = [0_{n \times (i-1)n} \quad I_n \quad 0_{n \times (N-i+2)n}]^T$ ($i = 1, 2, \dots, N+2$) are block entry matrices, and $e_0 = (Ae_1^T + A_d e_{N+2}^T)^T$.

Remark 10. Theorem 6 and Corollary 9 give stability criteria of system (1) with $d(t)$ satisfying (2) and (32), respectively. It is noted that the conditions in Theorem 6 and

Corollary 9 are both delay-dependent and rate-dependent. However, the information of delay rate may not be known in many cases, or $d(t)$ even may not be differentiable; then Theorem 6 and Corollary 9 fail to work. Regarding these circumstances, delay-dependent and rate-independent criteria can be derived by choosing $W = 0$ in Theorem 6 and Corollary 9.

For the case when $d(t)$ satisfies $0 < d_1 \leq d(t) \leq d_2$ and μ is unknown, the following result can be obtained from Theorem 6 by setting $W = 0$.

Corollary 11. *Given an integer $N \geq 2$ and scalars $0 < d_1 < d_2$, for any delay $d(t)$ satisfying $d_1 \leq d(t) \leq d_2$, singular time-delay system (1) is regular, impulse-free, and stable if there exist matrices*

$$\begin{aligned} P > 0, \quad Q_i &= \begin{bmatrix} Q_{i1} & Q_{i2} \\ Q_{i2}^T & Q_{i3} \end{bmatrix} > 0, \\ R_i &= \begin{bmatrix} R_{i1} & R_{i2} \\ R_{i2}^T & R_{i3} \end{bmatrix} > 0 \quad (i = 1, 2, \dots, N-1), \\ S_j &> 0, \quad Z_j > 0, \quad M_j \quad (j = 1, 2, \dots, N), \end{aligned} \quad (38)$$

and U , such that the LMI (30) and LMI

$$\Gamma_0 + \Gamma_k < 0 \quad (39)$$

hold for any $k \in \{1, 2, \dots, N\}$, where

$$\begin{aligned} \Gamma_0 &= e_1 \left(E^T P + U \Psi^T \right) e_0^T + e_0 \left(P E + \Psi U^T \right) e_1^T \\ &+ \sum_{i=1}^{N-1} \left([e_i \ e_{i+1}] Q_i [e_i \ e_{i+1}]^T \right. \\ &\quad - [e_{i+1} \ e_{i+2}] Q_i [e_{i+1} \ e_{i+2}]^T \\ &\quad + [e_{N+i} \ e_{N+i+1}] R_i [e_{N+i} \ e_{N+i+1}]^T \\ &\quad \left. - [e_{N+i+1} \ e_{N+i+2}] R_i [e_{N+i+1} \ e_{N+i+2}]^T \right) \\ &+ \sum_{i=1}^N \left(e_0 \left(\left(\frac{d_1}{N} \right)^2 S_i + \left(\frac{d_2 - d_1}{N} \right)^2 Z_i \right) e_0^T \right. \\ &\quad - (e_i - e_{i+1}) E^T S_i E (e_i - e_{i+1})^T \\ &\quad \left. - (e_{N+i} - e_{N+i+1}) E^T Z_i E (e_{N+i} - e_{N+i+1})^T \right), \end{aligned} \quad (40)$$

and Γ_k, Ψ, e_i ($i = 0, 1, 2, \dots, 2N + 2$) are the same as those defined in Theorem 6.

For the case when $d(t)$ satisfies $0 \leq d(t) \leq d$ and μ is unknown, the following result can be obtained from Corollary 9 by setting $W = 0$.

Corollary 12. *Given an integer $N \geq 2$ and scalar $d > 0$, for any delay $d(t)$ satisfying $0 \leq d(t) \leq d$, singular time-delay system (1) is regular, impulse-free, and stable if there exist matrices*

$$\begin{aligned} P > 0, \quad R_i &= \begin{bmatrix} R_{i1} & R_{i2} \\ R_{i2}^T & R_{i3} \end{bmatrix} > 0 \quad (i = 1, 2, \dots, N-1), \\ Z_j &> 0, \quad M_j \quad (j = 1, 2, \dots, N), \end{aligned} \quad (41)$$

and U , such that the LMI (30) and LMI

$$\Upsilon_0 + \Upsilon_k < 0 \quad (42)$$

hold for any $k \in \{1, 2, \dots, N\}$, where

$$\begin{aligned} \Upsilon_0 &= e_1 \left(E^T P + U \Psi^T \right) e_0^T + e_0 \left(P E + \Psi U^T \right) e_1^T \\ &+ \sum_{i=1}^{N-1} \left([e_i \ e_{i+1}] R_i [e_i \ e_{i+1}]^T - [e_{i+1} \ e_{i+2}] \right. \\ &\quad \left. \times R_i [e_{i+1} \ e_{i+2}]^T \right) \\ &+ \sum_{i=1}^N \left(\left(\frac{d}{N} \right)^2 e_0 Z_i e_0^T - (e_i - e_{i+1}) E^T Z_i E (e_i - e_{i+1})^T \right), \end{aligned} \quad (43)$$

and Υ_k, Ψ, e_i ($i = 0, 1, 2, \dots, N + 2$) are the same as those defined in Corollary 9.

Remark 13. Compared with the results in [20–25], the LMIs in Theorems 4 and 6 and Corollaries 9 and 11, 12 are all strict LMIs; thus, they can be directly solved by using any LMI toolbox like the one of Matlab or the one of Scilab.

When the matrix E is nonsingular, the stability problem of singular time-delay system (1) is reduced to analyzing the stability of the following regular system:

$$\begin{aligned} \dot{x}(t) &= A x(t) + A_d x(t - d(t)), \\ x(t) &= \varphi(t), \quad t \in [-d_2, 0]. \end{aligned} \quad (44)$$

This problem has been widely studied in the recent literature (see, e.g., [7–15]). Choose the following Lyapunov-Krasovskii functional:

$$\begin{aligned} \widetilde{V}(x_t) &= x^T(t) P x(t) + \int_{t-d(t)}^{t-d_1} x^T(s) W x(s) ds \\ &+ \sum_{i=1}^{N-1} \left(\int_{t-(i/N)d_1}^{t-((i-1)/N)d_1} \eta_1^T(s) Q_i \eta_1(s) ds \right. \\ &\quad \left. + \int_{t-d_1-(i/N)(d_2-d_1)}^{t-d_1-((i-1)/N)(d_2-d_1)} \eta_2^T(s) R_i \eta_2(s) ds \right) \end{aligned}$$

TABLE 1: Allowable upper bound d_2 with $d_1 = 0$ for different μ .

Methods	$\mu = 0$	$\mu = 0.3$	$\mu = 0.7$
Zhong and Yang [20]	0.5567	—	—
Wu and Zhou [22]	1.0660	—	—
Li and Jia [24]	1.0660	1.0130	0.9510
Lin and Fei [28]	1.0660	1.0260	1.0220
Corollary 9 ($N = 2$)	1.0937	1.0597	1.0597
Corollary 9 ($N = 3$)	1.1000	1.0689	1.0689

TABLE 2: Allowable upper bound d_2 with given d_1 for different μ .

d_1	Methods	$\mu = 0$	$\mu = 0.3$	$\mu = 0.7$
0.3	Lin and Fei [28]	1.0660	1.0310	1.0290
	Jiao [29]	1.1075	1.0607	1.0607
	Theorem 4	1.1095	1.0704	1.0704
	Theorem 6 ($N = 3$)	1.1122	1.0729	1.0729
0.6	Lin and Fei [28]	1.0660	1.0420	1.0420
	Jiao [29]	1.1075	1.0679	1.0679
	Theorem 4	1.1166	1.0727	1.0727
	Theorem 6 ($N = 3$)	1.1184	1.0748	1.0748
1.1	Lin and Fei [28]	—	—	—
	Jiao [29]	1.1075	1.1066	1.1066
	Theorem 4	1.1078	1.1066	1.1066
	Theorem 6 ($N = 3$)	1.1155	1.1133	1.1133

$$\begin{aligned}
& + \sum_{i=1}^N \left(\frac{d_1}{N} \int_{-(i/N)d_1}^{-(i-1)/N d_1} \int_{t+\alpha}^t \dot{x}^T(\beta) S_i \dot{x}(\beta) d\beta d\alpha \right. \\
& \quad \left. + \frac{d_2 - d_1}{N} \int_{-d_1 - (i/N)(d_2 - d_1)}^{-d_1 - ((i-1)/N)(d_2 - d_1)} \right. \\
& \quad \left. \times \int_{t+\alpha}^t \dot{x}^T(\beta) Z_i \dot{x}(\beta) d\beta d\alpha \right), \quad (45)
\end{aligned}$$

where vectors $\eta_1(s), \eta_2(s)$ and matrices P, W, Q_i, R_i ($i = 1, 2, \dots, N-1$), S_j, Z_j ($j = 1, 2, \dots, N$) are defined the same as those defined in (26).

By employing the Lyapunov-Krasovskii functional (45) and using the similar proof of Theorem 4, we can obtain the following delay-dependent stability criterion for time-delay system (44).

Corollary 14. *Given an integer $N \geq 2$ and scalars $0 < d_1 < d_2$, $0 \leq \mu < 1$, for any delay $d(t)$ satisfying (2), time-delay system (44) is stable if there exist matrices*

$$\begin{aligned}
P &> 0, \quad W > 0, \quad Q_i = \begin{bmatrix} Q_{i1} & Q_{i2} \\ Q_{i2}^T & Q_{i3} \end{bmatrix} > 0, \\
R_i &= \begin{bmatrix} R_{i1} & R_{i2} \\ R_{i2}^T & R_{i3} \end{bmatrix} > 0 \quad (i = 1, 2, \dots, N-1), \\
S_j &> 0, \quad Z_j > 0, \quad M_j \quad (j = 1, 2, \dots, N),
\end{aligned} \quad (46)$$

such that the LMI (30) and LMI (47)

$$\tilde{\Gamma} + \tilde{\Gamma}_k < 0 \quad (47)$$

hold for any $k \in \{1, 2, \dots, N\}$, where

$$\begin{aligned}
\tilde{\Gamma} &= e_1 P e_0^T + e_0 P e_1^T + e_{N+1} W e_{N+1}^T - (1 - \mu) e_{2N+2} W e_{2N+2}^T \\
&+ \sum_{i=1}^{N-1} \left([e_i \ e_{i+1}] Q_i [e_i \ e_{i+1}]^T - [e_{i+1} \ e_{i+2}] \right. \\
&\quad \times Q_i [e_{i+1} \ e_{i+2}]^T + [e_{N+i} \ e_{N+i+1}] R_i \\
&\quad \times [e_{N+i} \ e_{N+i+1}]^T - [e_{N+i+1} \ e_{N+i+2}] R_i \\
&\quad \times [e_{N+i+1} \ e_{N+i+2}]^T \Big) \\
&+ \sum_{i=1}^N \left(e_0 \left(\left(\frac{d_1}{N} \right)^2 S_i + \left(\frac{d_2 - d_1}{N} \right)^2 Z_i \right) e_0^T \right. \\
&\quad \left. - (e_i - e_{i+1}) S_i (e_i - e_{i+1})^T \right. \\
&\quad \left. - (e_{N+i} - e_{N+i+1}) Z_i (e_{N+i} - e_{N+i+1})^T \right),
\end{aligned}$$

$$\begin{aligned}
\tilde{\Gamma}_k &= (e_{N+k} - e_{N+k+1}) Z_k (e_{N+k} - e_{N+k+1})^T \\
&- \begin{bmatrix} e_{2N+2}^T - e_{N+k+1}^T \\ e_{N+k}^T - e_{2N+2}^T \end{bmatrix}^T \\
&\times \begin{bmatrix} Z_k & M_k \\ M_k^T & Z_k \end{bmatrix} \begin{bmatrix} e_{2N+2}^T - e_{N+k+1}^T \\ e_{N+k}^T - e_{2N+2}^T \end{bmatrix}, \quad (48)
\end{aligned}$$

and e_i ($i = 0, 1, 2, \dots, 2N+2$) are the same as those defined in Theorem 6.

Based on Corollary 9, we can get the following stability criterion for system (44) with delay $d(t)$ satisfying (32).

Corollary 15. *Given an integer $N \geq 2$ and scalars $d > 0$, $0 \leq \mu < 1$, for any delay $d(t)$ satisfying (32), time-delay system (44) is stable if there exist matrices*

$$P > 0, \quad W > 0,$$

$$\begin{aligned}
R_i &= \begin{bmatrix} R_{i1} & R_{i2} \\ R_{i2}^T & R_{i3} \end{bmatrix} > 0 \quad (i = 1, 2, \dots, N-1), \\
Z_j &> 0, \quad M_j \quad (j = 1, 2, \dots, N),
\end{aligned} \quad (49)$$

such that the LMI (30) and LMI

$$\tilde{Y} + \tilde{Y}_k < 0 \quad (50)$$

hold for any $k \in \{1, 2, \dots, N\}$, where

$$\begin{aligned} \tilde{Y} &= e_1 P e_0^T + e_0 P e_1^T + e_1 W e_1^T - (1 - \mu) e_{N+2} W e_{N+2}^T \\ &\quad + \sum_{i=1}^{N-1} \left([e_i \ e_{i+1}] R_i [e_i \ e_{i+1}]^T \right. \\ &\quad \left. - [e_{i+1} \ e_{i+2}] R_i [e_{i+1} \ e_{i+2}]^T \right) \\ &\quad + \sum_{i=1}^N \left(\left(\frac{d}{N} \right)^2 e_0 Z_i e_0^T - (e_i - e_{i+1}) Z_i (e_i - e_{i+1})^T \right), \\ \tilde{Y}_k &= (e_k - e_{k+1}) Z_k (e_k - e_{k+1})^T - \begin{bmatrix} e_{N+2}^T - e_{k+1}^T \\ e_k^T - e_{N+2}^T \end{bmatrix}^T \\ &\quad \times \begin{bmatrix} Z_k & M_k \\ M_k^T & Z_k \end{bmatrix} \begin{bmatrix} e_{N+2}^T - e_{k+1}^T \\ e_k^T - e_{N+2}^T \end{bmatrix}, \end{aligned} \quad (51)$$

and e_i ($i = 0, 1, 2, \dots, N+2$) are the same as those defined in Corollary 9.

When the information of μ is unknown, the following results can be obtained directly from Corollaries 11 and 12 for system (44) for two cases: $0 < d_1 \leq d(t) \leq d_2$ and $0 \leq d(t) \leq d$.

Corollary 16. Given an integer $N \geq 2$ and scalars $0 < d_1 < d_2$, for any delay $d(t)$ satisfying $d_1 \leq d(t) \leq d_2$, time-delay system (44) is stable if there exist matrices

$$\begin{aligned} P &> 0, \quad Q_i = \begin{bmatrix} Q_{i1} & Q_{i2} \\ Q_{i2}^T & Q_{i3} \end{bmatrix} > 0, \\ R_i &= \begin{bmatrix} R_{i1} & R_{i2} \\ R_{i2}^T & R_{i3} \end{bmatrix} > 0 \quad (i = 1, 2, \dots, N-1), \\ S_j &> 0, \quad Z_j > 0, \quad M_j \quad (j = 1, 2, \dots, N), \end{aligned} \quad (52)$$

such that the LMI (30) and LMI

$$\tilde{\Gamma}_0 + \tilde{\Gamma}_k < 0 \quad (53)$$

hold for any $k \in \{1, 2, \dots, N\}$, where

$$\begin{aligned} \tilde{\Gamma}_0 &= e_1 P e_0^T + e_0 P e_1^T \\ &\quad + \sum_{i=1}^{N-1} \left([e_i \ e_{i+1}] Q_i [e_i \ e_{i+1}]^T \right. \\ &\quad \left. - [e_{i+1} \ e_{i+2}] Q_i [e_{i+1} \ e_{i+2}]^T \right. \\ &\quad \left. + [e_{N+i} \ e_{N+i+1}] R_i [e_{N+i} \ e_{N+i+1}]^T \right. \\ &\quad \left. - [e_{N+i+1} \ e_{N+i+2}] R_i [e_{N+i+1} \ e_{N+i+2}]^T \right) \\ &\quad + \sum_{i=1}^N \left(e_0 \left(\left(\frac{d_1}{N} \right)^2 S_i + \left(\frac{d_2 - d_1}{N} \right)^2 Z_i \right) e_0^T \right. \\ &\quad \left. - (e_i - e_{i+1}) S_i (e_i - e_{i+1})^T \right. \\ &\quad \left. - (e_{N+i} - e_{N+i+1}) Z_i (e_{N+i} - e_{N+i+1})^T \right), \\ \tilde{\Gamma}_k &= (e_{N+k} - e_{N+k+1}) Z_k (e_{N+k} - e_{N+k+1})^T \\ &\quad - \begin{bmatrix} e_{2N+2}^T - e_{N+k+1}^T \\ e_{N+k}^T - e_{2N+2}^T \end{bmatrix}^T \begin{bmatrix} Z_k & M_k \\ M_k^T & Z_k \end{bmatrix} \begin{bmatrix} e_{2N+2}^T - e_{N+k+1}^T \\ e_{N+k}^T - e_{2N+2}^T \end{bmatrix}, \end{aligned} \quad (54)$$

and e_i ($i = 0, 1, 2, \dots, 2N+2$) are the same as those defined in Theorem 6.

Corollary 17. Given an integer $N \geq 2$ and scalar $d > 0$, for any delay $d(t)$ satisfying $0 \leq d(t) \leq d$, time-delay system (44) is stable if there exist matrices

$$\begin{aligned} P &> 0, \quad R_i = \begin{bmatrix} R_{i1} & R_{i2} \\ R_{i2}^T & R_{i3} \end{bmatrix} > 0 \quad (i = 1, 2, \dots, N-1), \\ Z_j &> 0, \quad M_j \quad (j = 1, 2, \dots, N), \end{aligned} \quad (55)$$

such that the LMI (30) and LMI

$$\tilde{Y}_0 + \tilde{Y}_k < 0 \quad (56)$$

hold for any $k \in \{1, 2, \dots, N\}$, where

$$\begin{aligned} \tilde{Y}_0 &= e_1 P e_0^T + e_0 P e_1^T \\ &\quad + \sum_{i=1}^{N-1} \left([e_i \ e_{i+1}] R_i [e_i \ e_{i+1}]^T \right. \\ &\quad \left. - [e_{i+1} \ e_{i+2}] R_i [e_{i+1} \ e_{i+2}]^T \right) \end{aligned} \quad (55)$$

TABLE 3: Allowable upper bound d_2 with given d_1 for different μ .

d_1	Methods	$\mu = 0.3$	$\mu = 0.5$	$\mu = 0.9$
2	Zhang et al. [8]	2.81	2.61	2.61
	Sun et al. [12]	3.01	2.56	2.56
	Qian and Liu [15]	3.02	2.69	2.69
	Corollary 14 ($N = 2$)	3.18	2.75	2.75
	Corollary 14 ($N = 3$)	3.21	2.77	2.77
3	Zhang et al. [8]	3.31	3.31	3.31
	Sun et al. [12]	3.34	3.34	3.34
	Qian and Liu [15]	3.41	3.41	3.41
	Corollary 14 ($N = 2$)	3.46	3.46	3.46
	Corollary 14 ($N = 3$)	3.48	3.48	3.48
4	Zhang et al. [8]	4.09	4.09	4.09
	Sun et al. [12]	4.16	4.16	4.16
	Qian and Liu [15]	4.20	4.20	4.20
	Corollary 14 ($N = 2$)	4.25	4.25	4.25
	Corollary 14 ($N = 3$)	4.28	4.28	4.28
5	Zhang et al. [8]	—	—	—
	Sun et al. [12]	5.02	5.02	5.02
	Qian and Liu [15]	5.03	5.03	5.03
	Corollary 14 ($N = 2$)	5.09	5.09	5.09
	Corollary 14 ($N = 3$)	5.12	5.12	5.12

$$+ \sum_{i=1}^N \left(\left(\frac{d}{N} \right)^2 e_0 Z_i e_0^T - (e_i - e_{i+1}) Z_i (e_i - e_{i+1})^T \right),$$

$$\begin{aligned} \tilde{Y}_k &= (e_k - e_{k+1}) Z_k (e_k - e_{k+1})^T \\ &\quad - \begin{bmatrix} e_{N+2}^T - e_{k+1}^T \\ e_k^T - e_{N+2}^T \end{bmatrix}^T \begin{bmatrix} Z_k & M_k \\ M_k^T & Z_k \end{bmatrix} \begin{bmatrix} e_{N+2}^T - e_{k+1}^T \\ e_k^T - e_{N+2}^T \end{bmatrix}, \end{aligned} \quad (57)$$

and e_i ($i = 0, 1, 2, \dots, N + 2$) are the same as those defined in Corollary 9.

4. Numerical Examples

In the section, numerical examples are given to illustrate the effectiveness and the less conservatism of obtained results in this paper.

Example 18. Consider the singular time-delay system (1) with

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0.5 & 0 \\ 0 & -1 \end{bmatrix}, \quad A_d = \begin{bmatrix} -1.1 & 1 \\ 0 & 0.5 \end{bmatrix}. \quad (58)$$

For $d_1 = 0$, the allowable upper bound d_2 , which guarantees regular, impulse-free, and stable systems (1) for different μ , is listed in Table 1. From Table 1, it can be seen that the stability criteria in Corollary 9 are less conservative than those in [20, 22, 24, 28].

For systems with interval time-varying delay, the allowable upper bound d_2 , which guarantees regular, impulse-free,

TABLE 4: Number of decision variables.

Methods	Number of decision variables
Zhang et al. [8]	$13n^2 + 5n$
Sun et al. [12]	$18n^2 + 8n$
Qian and Liu [15]	$22n^2 + 8n$
Corollary 14 ($N = 2$)	$9n^2 + 5n$
Corollary 14 ($N = 3$)	$15n^2 + 8n$

and stable systems (1) with given lower bound d_1 for different μ , is listed in Table 2. From Table 2, it can be seen that the stability criteria in Theorems 4 and 6 are less conservative than those in [28, 29]. Especially, when $d_1 = 1.1$, the result in [28] is not feasible while the allowable upper bound d_2 can also be obtained from Theorems 4 and 6 in this paper.

Example 19. Consider the time-delay system (44) with

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}. \quad (59)$$

This example is used in many recent papers such as [7–15, 29]. The purpose is to compare the allowable upper bound d_2 which guarantees the stability of the considered system for given lower bound d_1 .

For various μ , the allowable upper bound d_2 is listed in Table 3. Moreover, the number of decision variables involved in the stability criteria is given in Table 4. From Tables 3 and 4, it can be seen that Corollary 14 ($N = 2$) in this paper has fewer decision variables and less conservatism than those results in [8, 12, 15].

For unknown μ , the allowable upper bound d_2 is listed in Table 5. From Table 5, it can be seen that Corollary 16 in this paper is less conservative than those results in [7–15, 29].

5. Conclusion

This paper deals with the problem of stability for singular systems with interval time-varying delay. By using a delay decomposition approach, new stability criteria for singular time-delay systems to be regular, impulse-free, and stable are proposed in terms of LMIs. Based on the obtained criterion, some improved stability results for the regular systems with interval time-varying delay are also given. The obtained results in this paper have been shown to be less conservative than recently reported results. Moreover, the proposed method decreases the computational complexity comparable to some existing methods. Two numerical examples are given to illustrate the applicability of the results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

TABLE 5: Allowable upper bound d_2 with given d_1 for unknown μ .

Methods	$d_1 = 1$	$d_1 = 2$	$d_1 = 3$	$d_1 = 4$
He et al. [7]	1.74	2.43	3.22	4.06
Shao [9]	1.76	2.50	3.25	4.07
Zhang et al. [8] and Park et al. [10]	2.06	2.61	3.31	4.09
Sun et al. [12]	1.90	2.56	3.34	4.16
Qian et al. [14]	1.90	2.56	3.34	4.17
Sun et al. [11]	1.90	2.56	3.41	4.17
Qian and Liu [15]	2.09	2.69	3.41	4.20
Liu et al. [13]	2.09	2.69	3.41	4.20
Jiao [29]	2.12	2.72	3.45	4.25
Corollary 16 ($N = 2$)	2.21	2.75	3.46	4.26
Corollary 16 ($N = 3$)	2.25	2.77	3.48	4.28

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