Research Article

Stability Analysis of Pulse-Width-Modulated Feedback Systems with Time-Varying Delays

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The stability problem of pulse-width-modulated feedback systems with time-varying delays and stochastic perturbations is studied. With the help of an improved functional construction method, we establish a new Lyapunov-Krasovskii functional and derive several stability criteria about p th moment exponential stability.

1. Introduction

PWM has been widely used in many fields, such as attitude control systems, adaptive control systems, signal processing, and modeling of neuron behavior [1–3]. In actual progress, it has always operated in all kinds of disturbances. At the same time, time-varying delays inevitably occur owing to the unavoidable finite switching speed of amplifiers. For some systems, the effect of time-varying delays can be ignored. How to keep the scheduled operation or work of the state running well, especially in engineering applications, is becoming more and more significant.

A growing number of scholars are devoting time to the PWM feedback systems; meanwhile a set of stability results has been established by a variety of methods [4–12]. Also, lots of scholars have researched some systems with delays [7, 13–19]. In [15], the authors investigated robust exponential stability and delayed-state-feedback stabilization of uncertain impulsive stochastic systems with time-varying delays. Sun and Cao [16] gave some definitions on the p th moment exponential stability in mean and established several p th moment globally stability criteria in mean. By using the Lyapunov technique and Razumikhin method, the authors in [17] investigated impulsive effects on stability analysis of high-order BAM neural networks with time delays. And in [18], they established the global exponential stability of the neural networks with its estimated exponential convergence

rate. The authors in [19] gave a time-varying delay-dependent criterion for impulsive synchronization to ensure the delayed discrete complex networks switching topology tending to a synchronous state.

To the best of the author's knowledge, there are a few (if any) results for the stability analysis of the critical case of PWM systems with time-varying delays and stochastic perturbations, most of the existing work only considers one condition [7, 8, 15]. In the present paper, we try to make a contribution to this issue. It is noted that the linear plant in this paper has one and only one pole at origin, and the rest of the poles are in the left side of the complex plane, which is more representative and more universal. Based on the references [7, 13], the present paper will further study the stability of PWM feedback systems with time-varying delays and establish several new stability criteria. In Section 2, we give some definitions and lemmas. In Section 3, firstly, a criterion on mean square exponential stability of stochastic feedback systems with time-varying delays is given. Secondly, by introducing new variables, we will establish a new Lyapunov-Krasovskii functional with the help of an improved functional construction method. Then, associating with linear matrix inequalities, we will establish criteria for the p th moment exponential stability and the p th moment exponential asymptotic stability. Finally, we demonstrate the applicability of our results by means of an example.

2. Notations and Definitions

A pulse-width modulator is described by

$$\begin{aligned} u(t) &= m(e(t)) \\ &= \begin{cases} M \operatorname{sgn}(e(kT)), & t \in [kT, kT + T_k), \\ 0, & t \in [kT + T_k, kT + T), \end{cases} \end{aligned} \quad (1)$$

with

$$T_k = \begin{cases} \beta |e(kT)|, & |e(kT)| \leq \frac{T}{\beta}, \\ 0, & |e(kT)| > \frac{T}{\beta}, \end{cases} \quad (2)$$

where $e(t) = r(t) - f(t)$, $r(t)$ is the external input and $f(t)$ is the system output and T_k is the pulse-width for $k = 0, 1, 2, \dots$. The sampling period T , the amplitude of the pulse M , and β are all assumed to be constants.

Consider the following stochastic PWM feedback system with time-varying delays:

$$\begin{aligned} dx(t) &= Ax(t)dt + A_d x(t-d)dt + Hu(t)dt \\ &\quad + D\vartheta(t, x(t), x(t-d))dW_t, \\ z(t) &= C_1 x(t) + C_2 x(t-d), \end{aligned} \quad (3)$$

where $x \in R^n$, $u, \vartheta \in R$, u is the output of the pulse-width modulator, ϑ is the nonlinear stochastic perturbations, A, A_d, H, D are the matrices of appropriate dimensions, H is the pulse-width control matrix, $0 < d \leq T_k$, and W_t is a scalar wiener process which is defined on the probability space (Ω, F, P) .

Note that $x_e = 0$ is an equilibrium point of stochastic PWM feedback systems.

Definition 1. Let (X, d) be a metric space, $X \subset R^n$, $A \subset X$, and $T \subset R^+$. For any fixed $a \in A$ (a is called the initial state), $t_0 \in T$, a stochastic process $\{x(t, \omega, a, t_0), t \in T_{a, t_0}\}$ with domain X is called a stochastic motion if $x(t_0, \omega, a, t_0) = a$ for all $\omega \in \Omega$, where $T_{a, t_0} = [t_0, t_1) \cap T$, $t_1 > t_0$, and t_1 is finite or infinite.

Definition 2. Let S be a family of stochastic motions with domain X given by

$$S \subset \{x(\cdot, \cdot, a, t_0) : x(t_0, \omega, a, t_0) = a, \omega \in \Omega, a \in A, t_0 \in T\}. \quad (4)$$

We call the four-tuple $\{T, X, A, S\}$ a stochastic dynamical system.

Definition 3 (see [20, 21]). For any fixed functional V , the infinitesimal generation operator $\mathcal{L}V(x(t), t)$ is

$$\begin{aligned} \mathcal{L}V(x(t), t) &= \lim_{\Delta \rightarrow 0^+} \frac{1}{\Delta} [E(V(x(t+\Delta), t+\Delta) - V(x(t), t))] . \end{aligned} \quad (5)$$

Definition 4 (see [20, 21]). A stochastic system is p th moment exponential stability. If for any fixed initial condition, there exist $\alpha > 0$ and $\beta \geq 1$, then

$$E\|x(t)\|^p \leq \beta \exp(-\alpha t) \sup_{-d \leq \theta \leq 0} E\|\xi(\theta)\|^p, \quad \forall t \geq 0. \quad (6)$$

Especially, when $p = 2$, we called the system mean square exponential stability.

Definition 5 (see [20, 21]). A stochastic system is p th moment exponential asymptotic stability. If for any fixed initial condition, then

$$\lim_{t \rightarrow \infty} E\|x(t)\|^p = 0. \quad (7)$$

Especially, when $p = 2$, we called the system mean square exponential asymptotic stability.

Lemma 6 (see [20, 21]). A stochastic system $u(t) = Fy(t)$ is mean square stability if there exists a Lyapunov-Krasovskii functional $V(x(t), t) > 0$ and its infinitesimal generation operator satisfies $\mathcal{L}V(x(t), t) < 0$.

Lemma 7 (see [14]). For any constant matrix $M \in R^{m \times m}$, $M = M^T > 0$, scalar $\gamma > 0$, and vector function $\omega : [0, \gamma] \rightarrow R^m$, the integrations in the following are well defined; then

$$\gamma \int_0^\gamma \omega^T(\beta) M \omega(\beta) d\beta \geq \left(\int_0^\gamma \omega(\beta) d\beta \right)^T M \left(\int_0^\gamma \omega(\beta) d\beta \right). \quad (8)$$

Lemma 8(a) (see [22] (Schur complement 1)). Given the Hermite matrix $S = \begin{pmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{pmatrix}$, then $S < 0$ is equivalent to any one of the following conditions:

- (1) $S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$,
- (2) $S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 8(b) (see [23, 24] (Schur complement 2)). Assuming that $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{12}^T & A_{22} \end{pmatrix}$ is a Hermite matrix, then

- (1) $A > 0 \Leftrightarrow A_{11} > 0, A_{22} - A_{12}^T A_{11}^{-1} A_{12} > 0$,
- (2) If $A_{11} > 0$, then $A \geq 0 \Leftrightarrow A_{22} - A_{12}^T A_{11}^{-1} A_{12} \geq 0$.

Lemma 9 (see [25] (Gronwall inequality)). If $x(t) \leq h(t) + \int_{t_0}^t k(s)x(s)ds$, $t \in [t_0, T)$, where all the functions involved are continuous on $[t_0, T)$, $T \leq +\infty$, and $k(t) \geq 0$, then $x(t)$ satisfies

$$\begin{aligned} x(t) &\leq h(t) + \int_{t_0}^t h(s) k(s) \exp \left[\int_s^t k(u) du \right] ds \\ &\quad t \in [t_0, T). \end{aligned} \quad (9)$$

If, in addition, $h(t)$ is nondecreasing, then

$$x(t) \leq h(t) \exp \left(\int_{t_0}^t k(s) ds \right), \quad t \in [t_0, T). \quad (10)$$

3. Stability Analysis

The block diagram of the PWM feedback system (3) is shown in Figure 1.

Now, consider the following stochastic feedback system with time-varying delays:

$$\begin{aligned} dx(t) = & Ax(t)dt + A_d x(t-d)dt + Bp(t)dt \\ & + g(t, x(t), x(t-d))dW_t, \end{aligned} \quad (11)$$

where $x \in R^n$, $p(t)$ is a function of $x(t), x(t-d)$, g is the nonlinear stochastic perturbations, A, A_d, B are matrices of appropriate dimensions, $0 < d \leq T_k$, and W_t is a scalar wiener process which is defined on the probability space (Ω, F, P) .

For system (11), we let

$$g(t) \triangleq g(t, x(t), x(t-d)), \quad (12)$$

which satisfies that

$$\text{tr} [g^T(t)g(t)] \leq \|G_1(x(t))\|^2 + \|G_2(x(t-d))\|^2, \quad (13)$$

where G_1, G_2 are constant matrices of appropriate dimensions.

We construct the Lyapunov-Krasovskii functional as follows:

$$V(x(t), t) = \sum_{i=1}^5 V_i(x(t), t), \quad (14)$$

with

$$\begin{aligned} V_1(x(t), t) &= x^T(t)Px(t), \\ V_2(x(t), t) &= \int_{t-d}^t x^T(s)R_1x(s)ds, \\ V_3(x(t), t) &= \int_{t-d}^t \int_s^t x^T(\gamma)R_2x(\gamma)d\gamma ds, \\ V_4(x(t), t) &= 2x^T(t)Z \int_{t-d}^t x(s)ds, \\ V_5(x(t), t) &= \int_{-d}^0 \int_{-d}^0 x^T(t)Qx(t+\zeta)d\zeta ds. \end{aligned} \quad (15)$$

Denoting the parameter set as $\Phi := \{P, R_1, R_2, Z, Q\}$, P, R_1, R_2, Z, Q are Hermite matrices of appropriate dimensions.

Proposition 10 (see [13]). *Lyapunov-Krasovskii functional (14) is positive definite if the parameter set Φ satisfies*

$$R_1 > 0, \quad R_2 > 0, \quad \begin{bmatrix} P & Z \\ Z^T & d^{-1}R_1 + Q \end{bmatrix} > 0. \quad (16)$$

Theorem 11 (see [13]). *For a fixed scalar $d > 0$, the stochastic system (11) with time-varying delays is mean square exponential stability if there exist scalars $\rho > 0$, $\varepsilon > 0$, and*

FIGURE 1: Block diagram of PWM feedback systems subjected to multiplicative disturbances.

the parameter set Φ satisfies Proposition 10, then there exists a linear matrix inequality

$$\begin{aligned} \Theta = & \begin{bmatrix} \Theta_{11} & PA_d - Z + \varepsilon E_1^T E_2 & dA^T Z + dQ & PB \\ * & -R_1 + \rho G_2^T G_2 + \varepsilon E_2^T E_2 & dA_d^T Z - dQ & 0 \\ * & * & -h^{-1}R_2 & dZ^T B \\ * & * & * & -\varepsilon I \end{bmatrix} \\ < 0, \end{aligned} \quad (17)$$

where $P \leq \rho I$:

$$\begin{aligned} \Theta_{11} = & PA + A^T P + R_1 + dR_2 + Z + Z^T \\ & + \varepsilon E_1^T E_1 + \rho G_1^T G_1. \end{aligned} \quad (18)$$

From [13], one obtains that, for $t > 0$,

$$E\|x(t)\|^2 \leq \frac{\lambda_{\max}(P) + \Delta}{\lambda_{\min}(P)} \sup_{-d \leq \theta \leq 0} E\|\xi(\theta)\|^2 \exp\left(-\frac{\mu t}{\lambda_{\min}(P)}\right), \quad (19)$$

where $\Delta = \lambda_{\max}(Z^T Z) + d + \lambda_{\max}(R_1) + d\lambda_{\max}(R_2) + d\lambda_{\max}(Q)$.

Now, let us consider the stochastic PWM feedback system (3) with time-varying delays (Figure 2). After variable substitution, system (3) can be represented as

$$\begin{aligned} dx(t) &= Ax(t)dt + A_d x(t-d)dt + u'(t)dt + g'(t)dW_t, \\ u'(t) &= Hu(t), \\ g'(t) &= G\vartheta(t, x(t), x(t-d)), \\ z(t) &= C_1 x(t) + C_2 x(t). \end{aligned} \quad (20)$$

Note that in system (20),

$$\begin{aligned} \|u'(t)\| &\leq \left\| MH \sum_{k=1}^n \text{sgn}(e(kt)) \right\| \leq M \| \lambda H \|, \\ -n &\leq \lambda \leq n, \end{aligned} \quad (21)$$

where n is the amount of the PWM period.

By choosing appropriate PWM, we can get $0 \leq \lambda \leq \varepsilon$. Then

$$\|u'(t)\| \leq \lambda M \|H\|. \quad (22)$$

FIGURE 2: Sample response of system (3) with time-varying delays in Example 1 when $M = 0.3$.

Associating with (13), we also assume that the stochastic perturbations $g'(t)$ satisfy

$$\text{tr} [g'^T(t) g'(t)] \leq \|DG_1(x(t))\|^2 + \|DG_2(x(t-d))\|^2. \quad (23)$$

where $P \leq \rho I$, $\Psi_{11} = PA + A^T P + R + Z + Z^T + \lambda^2 \varepsilon M^2 H^T H + \rho D^T G_1^T G_1 D$.

Proof. Using the Itô isometry, we get infinitesimal generation operators of functional (20). Consider

$$\begin{aligned} \mathcal{L}V_1(y(t), t) &= \lim_{\Delta \rightarrow 0^+} \frac{1}{\Delta} [E(V_1(y(t+\Delta), t+\Delta) - V_1(y(t), t))] \\ &= 2y^T(t) P [Ay(t) + A_d y(t-d) + u'(t)] \\ &\quad + \text{Trace} [G^T W^T(t) P W(t) G] \\ &\leq 2y^T(t) P [Ay(t) + A_d y(t-d) + u'(t)] \\ &\quad + \|GW_1(y(t))\|^2 + \|GW_2(y(t))\|^2, \\ \mathcal{L}V_2(y(t), t) &= y^T(t) Ry(t) - y^T(t-d) Ry(t-d), \end{aligned}$$

By simplifying functional (14), we get a new Lyapunov-Krasovskii functional

$$V(y(t), t) = \sum_{i=1}^4 V_i(y(t), t), \quad (24)$$

with

$$\begin{aligned} V_1(y(t), t) &= y^T(t) P y(t), \\ V_2(y(t), t) &= \int_{t-d}^t y^T(s) R y(s) ds, \\ V_3(y(t), t) &= 2y^T(t) Z \int_{t-d}^t y(s) ds, \\ V_4(y(t), t) &= \int_{-d}^0 \int_{-d}^0 y^T(t) Q y(t+\zeta) d\zeta ds. \end{aligned} \quad (25)$$

Denoting the parameter set as $\Phi_1 := \{P, R, Z, Q\}$, $R > 0$, $y(t) = x^{p/2}(t)$, $p \in N_+$, P, R, Z, Q are Hermite matrices of appropriate dimensions.

Let functional (24) satisfy Proposition 10, then it is positive definite.

Theorem 12. For time-varying delay $d > 0$, the stochastic PWM feedback system (3) with time-varying delays is p th moment exponential stability if there exist scalars $\rho > 0$, $\varepsilon > 0$, and the parameter set Φ_1 and if the following linear matrix inequality holds:

$$\Psi = \begin{bmatrix} \Psi_{11} & PA_d - Z & dA^T Z + dQ & P \\ * & -R + \rho D^T G_2^T G_2 D + \lambda^2 \varepsilon M^2 H^T H & dA_d^T Z - dQ & 0 \\ * & * & 0 & dZ^T \\ * & * & * & -\varepsilon I \end{bmatrix} < 0, \quad (26)$$

$$\begin{aligned} \mathcal{L}V_3(y(t), t) &= 2[Ay(t) + A_d y(t-d) + u'(t)]^T \\ &\quad \times Z \int_{t-d}^t y(s) ds \\ &\quad + 2y^T(t) Z [y(t) - y(t-d)], \end{aligned}$$

$$\begin{aligned} \mathcal{L}V_4(y(t), t) &= 2y^T(t) Q \int_{t-d}^t y(s) ds \\ &\quad + 2y^T(t-d) Q \int_{t-d}^t y(s) ds. \end{aligned} \quad (27)$$

Based on (22), we can get $\lambda^2 \varepsilon M^2 H^T H - \varepsilon u'^T u' \geq 0$.

Then

$$\begin{aligned} \mathcal{L}V(y(t), t) &= \sum_{i=1}^4 \mathcal{L}V_i(y(t), t) \\ &\leq \frac{1}{d} \int_{t-d}^t \eta^T(t, s) \Psi \eta(t, s) ds, \end{aligned} \quad (28)$$

where $\eta^T(t, s) = [y^T(t) \ y^T(t-d) \ y^T(s) \ u^T(t)]$. Consider

$$\mathcal{L}V(y(t), t) < 0 \quad \text{if } \Psi \text{ satisfies } \Psi < 0. \quad (29)$$

By Schur complement 1, there exists a scalar $\mu > 0$, which makes

$$\mathcal{L}V(y(t), t) \leq -\mu \|y(t)\|^2. \quad (30)$$

Then, by Dynkin formula, one observes that

$$\begin{aligned} EV(y(t), t) &= EV(y(0), 0) + E \int_0^t \mathcal{L}V(y(s), s) ds, \\ V_2(y(t), t) &\leq \lambda_{\max}(R) \int_{-d}^0 \|y(t+\theta)\|^2 d\theta, \\ V_3(y(t), t) &\leq \lambda_{\max}(Z^T Z) \|y(t)\|^2 + d \int_{-d}^0 \|y(t+\theta)\|^2 d\theta, \\ V_4(y(t), t) &\leq d \lambda_{\max}(Q) \int_{-d}^0 \|y(t+\theta)\|^2 d\theta. \end{aligned} \quad (31)$$

Then

$$\begin{aligned} EV(y(0), 0) &\leq (\lambda_{\max}(P) + \nabla) \sup_{-d \leq \theta \leq 0} E \|\xi(\theta)\|^2, \\ EV(y(t), t) &\geq \lambda_{\min}(P) E \|y(t)\|^2, \\ \nabla &= \lambda_{\max}(Z^T Z) + d + \lambda_{\max}(R) + d \lambda_{\max}(Q). \end{aligned} \quad (32)$$

Thus, by Lemma 9 (Gronwall inequality), we have that

$$\begin{aligned} E \|y(t)\|^2 &= E \|x(t)^{p/2}\|^2 \\ &= E \|x(t)\|^p \leq \frac{\lambda_{\max}(P) + \nabla}{\lambda_{\min}(P)} \\ &\quad \times \sup_{-d \leq \theta \leq 0} E \|\xi(\theta)\|^2 \exp\left(-\frac{\mu t}{\lambda_{\min}(P)}\right), \\ E \|x(t)\|^p &\leq \frac{\lambda_{\max}(P) + \nabla}{\lambda_{\min}(P)} \\ &\quad \times \sup_{-d \leq \theta \leq 0} E \|\zeta(\theta)\|^p \exp\left(-\frac{\mu t}{\lambda_{\min}(P)}\right), \\ \zeta(\theta) &= (\xi(\theta))^{p/2}. \end{aligned} \quad (33)$$

Associating with Definition 4, we know that the system (3) is p th moment exponential stability. $\square$

Corollary 13. *A stochastic PWM feedback system with time-varying delays which satisfies the conditions of Theorem 12 is p th moment exponential asymptotic stability.*

Proof. From the above proof, we obtain

$$E \|x(t)\|^p \leq \frac{\lambda_{\max}(P) + \nabla}{\lambda_{\min}(P)} \sup_{-d \leq \theta \leq 0} E \|\zeta(\theta)\|^p \exp\left(-\frac{\mu t}{\lambda_{\min}(P)}\right), \quad (34)$$

where $\lambda_{\max}(p), \lambda_{\min}(p), \nabla, \sup E \|\zeta(\theta)\|^p, \mu$ are all constants.

Similarly,

$$\begin{aligned} E \|x(t)\|^p &\leq K_1 K_2 \exp(-K_3 t), \\ \text{where } K_1, K_2, K_3 &\text{ are constants.} \end{aligned} \quad (35)$$

Then, we have

$$\lim_{t \rightarrow \infty} E \|x(t)\|^p \leq \lim_{t \rightarrow \infty} K_1 K_2 \exp(-K_3 t) = 0. \quad (36)$$

By Definition 5, the system (3) is p th moment exponential asymptotic stability.

Then, we discuss the functional (24) based on its parameter set.

As the parameter set satisfies $R > 0, \begin{bmatrix} P & Z \\ Z^T & d^{-1}R + Q \end{bmatrix} > 0$, by Schur complement 2, we observe that

$$P > 0, \quad d^{-1}R + Q - Z^T P^{-1} Z > 0. \quad (37)$$

Then we can obtain an estimate about matrix P , such that

$$P > 0 \cap P < (Z^T)^{-1} (d^{-1}R + Q) Z^{-1}. \quad (38)$$

$\square$

Remark 14. For system (3), only H can be controlled easily in reality. So we may let $P = H$ and H satisfies formula (38). Then we get another corollary which can be applied in reality.

Corollary 15. *For time-varying delay $d > 0$, the system (3) is p th moment exponential stability and p th moment exponential asymptotic stability if there exist scalars $\rho > 0, \varepsilon > 0$, and the parameter set Φ_1 such that the following linear matrix inequality holds:*

$$\Psi = \begin{bmatrix} \Psi_{11} & HA_d - Z & dA^T Z + dQ & H \\ * & -R + \rho D^T G_2^T G_2 D + \lambda^2 \varepsilon M^2 H^T H & dA_d^T Z - dQ & 0 \\ * & * & 0 & dZ^T \\ * & * & * & -\varepsilon I \end{bmatrix} < 0, \quad (39)$$

FIGURE 3: Sample response of system (3) with time-varying delays in Example 1 when $M = 0.4$.

where $H \leq \rho I$, $\Psi_{11} = HA + A^T H + R + Z + Z^T + \lambda^2 \varepsilon M^2 H^T H + \rho D^T G_1^T G_1 D$ (Figure 3).

4. Example

Example 1. Consider the system (3). Letting $A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}$, $A_d = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $H = I$, the amplitude of the PWM $M = 0.3$, the sampling period $T = 1$, and the constant $\beta = 1$, the perturbations satisfy $g'(t) = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.01 \end{bmatrix}$.

We assume the time accuracy is 0.01 seconds and the system has a tenfold lag; that is, $d = 0.1$. And also at the system initial, there is no 0.1 second lag, while it has a 0.1 second lag at system shutdown. The perturbations function on the whole process. For initial-value $x(0) = (2, 1)^T$, we observe that the system (3) is stable.

By modulating the pulse-width control matrix H , we set $M = 0.4$. Then, we can get better result of the system (3).

5. Conclusions

In this paper, we studied the stochastic PWM feedback systems with time-varying delays and stochastic perturbations. Then, several criteria are established by employing the linear matrix inequality (LMI). Based on the result, an example is presented, which illustrates the effectiveness of the new criteria.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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