Distributed Wireless Networked $H_{\infty}$ Control for a Class of Lurie-Type Nonlinear Systems

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A new approach to solving the distributed control problem for a class of discrete-time nonlinear systems via a wireless neural control network (WNCN) is presented in this paper. A unified Lurie-type model termed delayed standard neural network model (DSNNM) is used to describe these nonlinear systems. We assume that all neuron nodes in WNCN which have limited energy, storage space, and computing ability can be regarded as a subcontroller, then the whole WNCN is characterized by a mesh-like structure with partially connected neurons distributed over a wide geographical area, which can be considered as a fully distributed nonlinear output feedback dynamic controller. The unreliable wireless communication links within WNCN are modeled by fading channels. Based on the Lyapunov functional and the S-procedure, the WNCN is solved and configured for the DSNNM to absolutely stabilize the whole closed-loop system in the sense of mean square with a $H_{\infty}$ disturbance attenuation index using LMI approach. A numerical example shows the effectiveness of the proposed design approaches.

1. Introduction

Artificial neural networks (ANNs) are one of the effective technologies in modeling and controlling complex nonlinear systems due to the universal nonlinear function approximation property of ANNs. Because all biological neural networks (BNNs) have the recursive properties and most industrial processes are nonlinear dynamic system, the recurrent neural networks (RNNs) which have internal feedback loops and are suitable for dynamic mapping have attracted increasing attention in the control field [1]. Many researchers have extensively investigated RNNs-based design methods for nonlinear control systems. For example, in [2], diagonal recurrent neural networks (DRNNs) are constructed to identify and control, respectively, for both BIBO and non-BIBO nonlinear plants. Lin et al. [3] studied an FPGA-based computed force control system based on the Elman neural network (ENN) considered as a particular class of RNN to achieve the high-accuracy position control of linear ultrasonic motor. A neural controller using recurrent learning (RTRL) network updating algorithm for nonlinear plants with unknown dynamics is presented in [4]. Guaranteed cost control for exponential synchronization of cellular neural networks (CNNs) with various activation functions and mixed time-varying delays is investigated in [5]. As a dynamic system, the stability analysis of RNNs and stabilization synthesis of RNNs-based control systems are a primary consideration. One of the main characteristics of RNNs is that the nonlinear activation functions in RNNs are of the sigmoidal type. Since the various sigmoidal functions in RNNs belong to a subset of nonlinear functions of Lurie-type system [6], during the past two decades, there have been a large number of research contributions concerning the absolute stability of RNNs such as [7–14]. It is worth noting that a new neural network model termed by the standard neural network model (SNNM) is proposed in [12]. Most nonlinear control systems based on delayed (or nondelayed) RNNs can be converted into the SNNMs, the absolute stability of which can be analyzed using a unified approach in the sense of Lurie [6, 13, 14]. However, the traditional static and dynamic control methods [12–14] for SNNMs are centralized and do not apply to distributed networked control systems.

Boosted by advances in computing, communications, and sensing technologies, cyber-physical systems (CPSs) in
which computational and physical components are closely
conjoined and coordinated are becoming increasingly ubiq-
utous [15, 16]. A large number of embedded devices (such
as sensors, actuators, and controllers) distributed over a vast
geographical area in CPSs will depend more and more on
communications networks to achieve information inter-
action and manipulate physical entities; therefore, wireless net-
worked control systems (WNCs) represent a new research
frontier of CPSs and have recently received a great deal of
attention [17]. Employing wireless networks for CPSs will
enhance the flexibility and expandability of system (e.g.,
network nodes are easy to move or be deployed in scenes
which have difficulties in wiring) whilst reducing installation,
maintenance, debugging, and labour costs. However, the
unreliable communication channels, resource constraints,
and limited bandwidth that characterize the wireless tech-
nology require special care and raise new challenges to
communication, signal processing, closed-loop control, and
so forth. Recently, many researchers have investigated these
issues and some significant results were obtained and many
are in progress. Shi and Zhang [18] investigate the remote
state estimation and optimal schedule for two sensors under
bandwidth constraint. Guo et al. [19] consider the control
and actuators/sensors scheduling problem for linear system
and then propose a novel stability criterion based on the
modes of Markov chains and the transmission delays. The
problem of joint design of an output feedback controller and
the medium access scheduling policy are investigated for
networked control systems in [20, 21]. The analysis and design
of state feedback controllers for linear systems where there
are limitations on the number of active actuators and trans-
mission delays are studied in [22]. A decentralized event-
triggered control method over wireless sensor and actuator
network (WSAN) of centralized controllers is discussed in
[23]. Furthermore, with network scale unceasingly expand-
ing, any of the sensing/actuating nodes cannot access/act to
the full state of the physical plant, so development and design
of distributed control methods for a large-scale WNCs are
still hotspot issues in both engineering and academic fields
[24]. At present, the wireless network (WN) is considered
primarily as a communication medium in most of the
research results [17, 23–26] for WNCs. It means that the
nodes in WN will only achieve the data communication and
transmission tasks among sensing/actuating nodes and one
or more dedicated controllers. However, these works have
potential drawbacks such as that the WNCs is susceptible
to the failures of those dedicated controllers and the packet
losses and delays over unreliable wireless communication
links among nodes. Pajic et al. [27] propose the basic concept
of wireless control network (WCN), a new fully distributed
control method for WNCs, in which the control function is
achieved over a multihop WN. For WCN, the entire multihop
network fulfills itself as a distributed controller where every
node can be regarded as a local (small) linear dynamical
controller for linear physical plants [28].

In this paper, we focus on the distributed networked $H_{\infty}$
control and absolute stability analysis of delayed standard
neutral network model (DSNNM) based on a wireless neuron
control network (WNCN) introduced in [29], which is an
improved nonlinear WCN. In summary, the aim for intro-
ducing WNCN stems from the need of a distributed control
approach for WNCs. There are many practical application
requirements that also motivate this study. Typical examples
include industrial humidity, ventilation, air conditioning
(HVAC) control systems in [24], the networked process
control for the distillation column in [30], the drip irrigation
control for agriculture using wireless sensor and actuator
network (WSAN) [31], and so forth.

Compared with normal RNN being the fully connected
among neurons and having a layered architecture as shown
in Figure I(a), the WCN, as a special kind of control-
oriented RNN, is characterized by a mesh-like structure
with partially connected neurons distributed over a wide
geographical area. Consider a scenario where several neu-
ral nodes forming with limited computation and wireless
communication capabilities are deployed around an indus-
trial plant and can exchange information with immediate
neighbor neuron nodes to form a wireless mesh network,
some of which can also receive state values of the plant
from neighbor sensors or send control signals to neighbor
actuators, respectively, as shown in Figure I(b). Compared
with WCN behaving as a linear dynamical system, WNCN
is essentially a nonlinear wireless mesh RNN system. To
the best of our knowledge, the problem formulation is
novel.

The remainder of this paper is organized as follows. In
Section 2, we first briefly cover the delayed standard neutral
network model (DSNNM) and then describe the nonlinear
dynamics of WNCN. Section 3 investigates the absolute stability and the $H_{\infty}$
performance of the closed-loop system. The criteria to synthesis of the optimal $H_{\infty}$
controller based on WNCN without stochastic packet dropping are
first presented in Section 4, and then the result is extended
to study the robust case based on stochastic WNCN with
fading communication channels in Section 5. In Section 6, a
numerical example is given to demonstrate the effectiveness
of the derived results. And finally, conclusions are drawn in
Section 7.

Notation. $\mathbb{R}^n$ is the n-dimensional Euclidean space. $\mathbb{R}^{n\times m}$ is
the set of real $n \times m$ matrices. $A^T$ denotes the transpose of
matrix $A$. $\text{Tr}(A)$ denotes the trace of a square matrix $A$. $S^n$
denotes the set of symmetric $n \times n$ matrices. $S^n_+$ denotes
the set of positive definite $n \times n$ matrices. $S^n_{++}$ denotes
the set of positive definite $n \times n$ matrices. The curved inequality
symbol $\succ (\preceq, \succeq, \prec)$ is used to denote generalized inequality:
$A, B \in S^n$, the matrix inequality $A \succeq (\succeq, \preceq, \prec) 0$ means
that $A \in S^n_+(S^n_+ - S^n_+ - S^n_+), \text{and } A \succeq (\preceq, \succeq, \prec) 0 \Rightarrow
A - B \succeq (\succeq, \preceq, \prec) 0$. $k(I)$ denotes the cardinality of set $I$, $I$
denotes an identity matrix of appropriate order. $C_e$ denotes
the $r$th vector of the standard basis of $\mathbb{R}^n$. $E()$ denotes
the estimation operator. $\text{diag}()$ denotes a diagonal matrix. $\ast$
is used as an ellipsis for terms induced by symmetry. If
$X \in \mathbb{R}^p$ and $Y \in \mathbb{R}^q$, $C(X;Y)$ denotes the space of all
continuous functions mapping $\mathbb{R}^p \rightarrow \mathbb{R}^q$, $L_2 [0; \infty)$ is
the space of square integrable vectors. $\|\cdot\|$ denotes the Euclidean
norm for vectors or the spectral norm of matrices.
2. Problem Formulation

2.1. Delayed Standard Neural Network Model. Consider the following discrete-time DSNNM with input-output:

\[
\begin{align*}
\mathcal{P} & : \begin{cases} 
    x(k+1) = Ax(k) + A_dx(k-d) + B_d\phi(e(k)) + B_w\omega(k) + B_uu(k), \\
    e(k) = C_x x(k) + C_d x(k-d) + D_g\phi(e(k)) + D_w\omega(k) + D_uu(k), \\
    y(k) = C_y x(k),
\end{cases}
\end{align*}
\]

(1)

with the initial condition function \(x(k) = \phi(k), \forall k \in [-d, 0]\), where \(x(k) \in \mathbb{R}^n\) is the state vector, \(u(k) \in \mathbb{R}^m\) is the control input vector, \(\epsilon(k) \in \mathbb{R}^l\) is the measured output vector, \(\omega(k) \in \mathbb{R}^d\) is the disturbance that belongs to \(l_2[0, \infty)\), \(\phi(e(k)) \in C(\mathbb{R}^l; \mathbb{R}^l)\) is the activation function with the input vector \(e(k) \in \mathbb{R}^l\), \(L \in \mathbb{R}\) is the number of nonlinear activation functions, \(d > 0\) is the time delay, \(A \in \mathbb{R}^{n \times n}, A_d \in \mathbb{R}^{n \times n}\), \(B_d \in \mathbb{R}^{n \times m}, B_u \in \mathbb{R}^{n \times m}\), \(C_x \in \mathbb{R}^{l \times n}, C_d \in \mathbb{R}^{l \times n}\), \(D_g \in \mathbb{R}^{l \times l}, D_w \in \mathbb{R}^{l \times l}, D_u \in \mathbb{R}^{l \times l}\). Assume that the activation function \(\phi\) satisfies \(\phi(0) = 0\) and belongs to a type of set \(\Omega(K)\) as follows:

\[
\Omega(K) \triangleq \left\{ \phi \mid 0 \leq \frac{\phi(e_i(k))}{e_i(k)} \leq k_i, i = 1, \ldots, L \right\},
\]

(2)

which means that \(\phi\) is sector restricted to the interval \([0, K]\), where \(K = \text{diag}(k_1, \ldots, k_L) > 0\).

2.2. Wireless Neural Control Network. The traditional design approaches of dynamic controllers based on SNNMs are centralized and the dimension of controller and plant must remain consistent [12–14]. However, without losing system stability, the WNCN can be structured as a distributed recurrent neurocontroller (RNC) with arbitrary dimension which will be in favor of controlling the complex nonlinear systems with multiple geographically distributed sensors (multi-output) and actuators (multi-input). So, the motivation for introducing WNCN stems from the need for distributed control approaches for WNCNs.

Assume that we use a WNCN consisting of \(N\) neuron nodes to control the aforementioned DSNNM \(\mathcal{P}\). The wireless network in the whole system can be described by a directed graph as follows:

\[
\mathcal{G} \triangleq \left\{ \mathcal{V} \cup \mathcal{E}, \mathcal{E}^{\infty} \right\}.
\]

(3)

where \(\mathcal{V} = \{v_1, \ldots, v_N\}\) is the set of \(N\) neuron nodes, \(\mathcal{A} = \{a_1, \ldots, a_m\}\) is the set of \(m\) actuators which can execute the input vector \(u(k) = [u_1(k), \ldots, u_m(k)]^T\), \(\mathcal{S} = \{s_1, \ldots, s_l\}\) is the set of \(l\) sensors used to measure the output vector \(y(k) = [y_1(k), \ldots, y_l(k)]^T\), and edge sets \(\mathcal{E}^{\mathcal{F}} = \{(v_i, a_p) \mid v_i \in \mathcal{V}, a_p \in \mathcal{A}\}, \mathcal{E}^{\mathcal{C}} = \{(v_i, s_q) \mid v_i \in \mathcal{V}, s_q \in \mathcal{S}\}, \mathcal{E}^{\mathcal{G}} = \{(v_i, v_j) \mid v_i, v_j \in \mathcal{V}\}\) correspond to the physical radio communication links in the wireless network. Define the following three sets:

- the neighbor sensors of \(v_i, \forall i \in \{1, \ldots, N\}\)

\[
\mathcal{S}^{\mathcal{C}} \triangleq \left\{ s_q \mid s_q \in \mathcal{S}, \exists (s_q, v_i) \in \mathcal{E}^{\mathcal{C}} \right\} = \left\{ s_q \mid w_{iq}^c \neq 0 \right\}.
\]

(4)

- the neighbor neurons of \(a_p, \forall p \in \{1, \ldots, m\}\)

\[
\mathcal{A}^{\mathcal{F}} \triangleq \left\{ v_j \mid v_j \in \mathcal{V}, \exists (v_j, a_p) \in \mathcal{E}^{\mathcal{F}} \right\} = \left\{ v_j \mid w_{jp}^f \neq 0 \right\}.
\]

(5)

- the neighbor neurons of \(v_i, \forall i \in \{1, \ldots, N\}\)

\[
\mathcal{G}^{\mathcal{E}} \triangleq \left\{ v_j \mid v_j \in \mathcal{V}, \exists (v_j, v_i) \in \mathcal{E}^{\mathcal{G}} \right\} = \left\{ v_j \mid w_{ij}^g \neq 0 \right\}.
\]

(6)
where \( w_{ij}^x, w_{ij}^y, w_{ij}^o \) are the weights of edge \((v_i, a_p), (v_j, v_i), \) and \((s_q, v_i)\), respectively. This implies that \( s_q \in \delta^o \) if \( v_i \) can receive data directly from \( s_q, v_j \in \gamma^o \) if \( a_p \) can receive data directly from \( v_j, v_j \in \gamma^r \) if \( v_i \) can receive data directly from \( v_j \).

The dynamic behavior of the neuron nodes \( v_j \) may be represented by the following pair of nonlinear equations:

\[
\begin{align*}
    z_i(k+1) &= \psi_i(\xi_i(k)), \\
    \xi_i(k) &= w_i^x z_i(k) + \sum_{j \in \gamma^o} w_{ij}^x z_j(k) + \sum_{s \in \delta^s} w_{is}^o y_q(k),
\end{align*}
\]

where \( z_i(k) \) is the state of neuron node \( v_i, z_j(k) \) is the state of neuron node \( v_j, v_j \in \gamma^o, y_q(k) \) is the measurement value of sensor \( s_q, \xi_i(k) \) is the weighted linear combination of \( v_i \)’s present state and exogenous input signals (from neuron nodes in \( \gamma^o \) or sensors in \( \delta^s \)), and \( \psi_i(\cdot) \) is the activation function of neuron node \( v_i \), where \( K = \text{diag}(k_{1,1}, \ldots, k_{L,N}) > 0 \). Each plant input \( u_p(k), p \in \{1, \ldots, m\} \) is a weighted linear combiner output due to neighbor neuron nodes of the activator \( a_p \) as follows:

\[
u_p(k) = \sum_{v_j \in \gamma^o} w_{pj}^x z_i(k).
\]

If each neuron node is regarded as a nonlinear dynamical subcontroller, the whole WNCN consisting of \( N \) neuron nodes may act as a fully distributed RNC whose dynamic behavior may be described as

\[
\begin{align*}
    x(k+1) &= \psi(\xi(k)), \\
    \xi(k) &= W^x z(k) + W^o y(k), \\
    u(k) &= W^r z(k),
\end{align*}
\]

where \( z \in \mathbb{R}^N \) is the state vector of WNCN, \( \psi \in C(\mathbb{R}^N; \mathbb{R}^N), \xi \in \mathbb{R}^{N}, W^x \in \mathbb{R}^{N \times N}, W^o \in \mathbb{R}^{N \times 1}, \) and \( W^r \in \mathbb{R}^{m \times N}. \) In the above-mentioned equations, \( \forall i \in \{1, \ldots, N\}, \psi_i(\cdot) = 0 \) if \( v_j \notin \gamma^o \cup \{v_i\}, w_{ij}^o = 0 \) if \( s_q \notin \delta^s \), and \( w_{pi}^o = 0 \) if \( v_i \notin \gamma^o \). Therefore, the weight matrices \( W^x, W^o, \) and \( W^r \) have the sparsity constraints. This means that the WNCN has considerably fewer weights (accounting for little computational overhead) than the fully connected neural network, which is conducive to industrial real-time control.

In this paper, a MAC synchronized network protocol based on time division multiple access (TDM) architecture is used to schedule neuron nodes in WNCN to accomplish the cooperative control for the system (1). Under the scheme, every neuron node \( v_i, i \in \{1, \ldots, N\} \) transmits its state information once per time frame. In the beginning, \( v_i \) has an arbitrary initial state value and then successively receives information from its neighbors in \( \gamma^i \) and \( \delta^s \) in each time slot of frame. After \( v_i \) has received all the information from its neighbors, \( v_i \) will update its state by (7). Furthermore, in a similar way, every actuator \( a_p, p \in \{1, \ldots, m\} \) can receive the combination of control signals from neighbor neuron nodes in \( \delta^s \) and then act to system (1) by (8).

Define vectors \( \bar{x} = [x^T, z^T]^T \in \mathbb{R}^{m+N}, \bar{e} = [e^T, \xi^T]^T \in \mathbb{R}^{L+N}, \) and \( \bar{y} = [\phi^T, \psi^T]^T \in \Omega(\bar{K}), \bar{K} = \text{diag}(k_{1,1}, \ldots, k_{L,N}) > 0 \). Then the overall closed-loop system \( \mathcal{G} \) of the DSNNM \( \mathcal{P} \) and the WNCN \( \mathcal{H} \) is described as

\[
\begin{align*}
    \bar{x}(k+1) &= \begin{bmatrix} A & B_\bar{e} W_r^x \\
    0 & 0 \end{bmatrix} \bar{x}(k) + \begin{bmatrix} \tilde{A}_d & 0 \\
    \tilde{A}_d & 0 \end{bmatrix} \bar{y}(k-d) + \begin{bmatrix} B_{\bar{e}} \phi(\bar{e}(k)) \\
    B_{\bar{e}} \omega(\bar{e}(k)) \end{bmatrix} \\
    \bar{e}(k) &= \begin{bmatrix} C_x W_r^x C_y & D_x W_r^y \\
    0 & 0 \end{bmatrix} \bar{x}(k) + \begin{bmatrix} \tilde{C}_d & 0 \\
    \tilde{C}_d & 0 \end{bmatrix} \bar{y}(k-d) + \begin{bmatrix} D_{\bar{e}} \phi(\bar{x}(k)) \\
    D_{\bar{e}} \psi(\bar{x}(k)) \end{bmatrix} + \begin{bmatrix} B_{\omega} \omega(\bar{e}(k)) \\
    B_{\omega} \omega(\bar{e}(k)) \end{bmatrix},
\end{align*}
\]

Consider that the performance output of the closed-loop system \( \mathcal{G} \) is described by \( \bar{y}(k) = \bar{C}_y \bar{x}(k) \), where \( \bar{C}_y = [I \ 0] \), then the following definition is introduced.

**Definition 1** (see [6, 32, 33]). Given a scalar \( \gamma > 0 \), the closed-loop system \( \mathcal{G} \) is said to be absolutely stable with a \( H_\infty \)-norm bound \( \gamma \) if there exists a distributed dynamic neural controller WNCN \( \mathcal{H} \) such that the following conditions are satisfied (Algorithm 1).

1. With zero disturbance, that is, \( \omega(k) = 0 \), the zero solution of the closed-loop system \( \mathcal{G} \) is globally asymptotically stable, \( \forall \bar{x}(0), \forall \bar{\phi} \in \Omega(\bar{K}). \)

2. Under the zero-initial condition, the performance output \( \bar{y}(k) \) satisfies

\[
\sum_{k=0}^{\infty} \| \bar{y}(k) \|^2 \leq \gamma^2 \sum_{k=0}^{\infty} \| \omega(k) \|^2, \quad \forall \text{nonzero } \omega(k).
\]

Then the WNCN \( \mathcal{H} \) is said to be a \( H_\infty \) controller for the DSNNM \( \mathcal{P} \). Furthermore, if we can find a minimal \( \gamma^* \) to satisfy the above conditions, the WNCN \( \mathcal{H} \) is an optimal \( H_\infty \) controller.

Our aim is to design the WNCN \( \mathcal{H} \) for DSNNM \( \mathcal{P} \) such that the closed-loop system \( \mathcal{G} \) satisfies the requirements (1) and (2) in Definition 1.
3. $H_\infty$ Performance Analysis of the Closed-Loop System

In this section, we will investigate the absolute stability and $H_\infty$ performance of the closed-loop system $\mathcal{G}$. Before deducing the main results, we need to make use of the following two lemmas.

Lemma 2 (S-procedure [34]). Let $T_0, T_1, \ldots, T_p \in \mathbb{S}^n$. If there exists $\tau_i \geq 0$, $i = 1, \ldots, p$ such that

$$T_0 - \sum_{i=1}^{p} \tau_i T_i < 0$$

then $\zeta^T T_0 \zeta < 0$ for all $\zeta \neq 0$ such that $\zeta^T T_i \zeta \leq 0$, $i = 1, \ldots, p$.

Lemma 3 (Schur complement [35]). Consider a matrix $X \in \mathbb{S}^n$ partitioned as

$$X = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix},$$

where $A \in \mathbb{S}^k$. If $A$ is nonsingular, the matrix $S = C - B^T A^{-1} B$ is called the Schur complement of $A$ in $X$. Then, the following characterizations of positive definiteness or semidefiniteness of the block matrix $X$ hold:

$$\begin{array}{l}
(1) \ X > 0, \quad \text{iff} \quad A > 0, S > 0, \\
(2) \ \text{If} \ A > 0, \ \text{then} \ X \geq 0 \ \text{iff} \ S \geq 0.
\end{array}$$

Theorem 4. Given $\nu > 0$ and WNCN $\mathcal{H}$ with parameter set $\mathcal{H} = \{ W^\nu, W^\nu_\nu, W^\nu_T \}$, if there exist appropriate dimension matrices $P > 0, R > 0$, and $T \succeq 0$, such that the following matrix inequality holds:

$$\Xi_1 = \begin{bmatrix} A^T P A - P & A^T P B \phi + C T K & A^T P B \omega \\ R + C^T \phi^T & A^T d P A_d - R & A^T d P B \phi + C^T d T K \\ * & * & A^T d P B \omega \\ * & * & \bar{B}^T \phi^T + \bar{D}^T \phi^T \\ * & * & \bar{B}^T \omega \end{bmatrix} > 0,$$

where $K = \text{diag}(k_1, \ldots, k_{L+N})$, then the zero solution of closed-loop system $\mathcal{G}$ is absolutely stable and the $H_\infty$-norm constraint (11) is achieved for all nonzero $\omega(k)$.

Proof. From system $\mathcal{G}$ with $\omega(k) = 0$, one can obtain

$$\begin{bmatrix} \bar{x}(k + 1) = \bar{A} \bar{x}(k) + \bar{A}_d \bar{x}(k - d) + \bar{B}_\phi \phi(\bar{e}(k)) \\
\bar{e}(k) = \bar{C}_e \bar{x}(k) + \bar{C}_d \bar{x}(k - d) + \bar{D}_\phi \phi(\bar{e}(k)) \end{bmatrix}.$$\hspace{1cm} (16)

Assume that the $\bar{x}(k) = 0$ is the only equilibrium of $\mathcal{G}$. Consider the following Lyapunov-Krasovskii functional for systems $\mathcal{G}$ as

$$V(\bar{x}(k)) = \bar{x}^T(k) P \bar{x}(k) + \sum_{i=1}^{d} \bar{x}^T(k - i) R \bar{x}(k - i).$$\hspace{1cm} (17)

According to the sector bound set $\Omega(\bar{K})$ of $\phi$, we have

$$\phi_1(\bar{e}_i(k)) \cdot \tau_i \cdot \left[ \phi_1(\bar{e}_i(k)) - k_i \bar{e}_i(k) \right] = \tau_i \phi_1^2(\bar{e}_i(k)) - \tau_i k_i \bar{e}_i(k) \phi_1(\bar{e}_i(k)) \leq 0,$$

where $\bar{e}_i \geq 0$, $i = 1, \ldots, L + N$. Now, by defining the difference of $V(\bar{x}(k))$ along $\mathcal{G}$ as $\Delta V(\bar{x}(k)) = V(\bar{x}(k + 1)) - V(\bar{x}(k))$ and using Lemma 2 (S-procedure), we can obtain

\begin{align*}
\Delta V(\bar{x}(k)) \\
= \bar{x}^T(k + 1) P \bar{x}(k + 1) - \bar{x}^T(k) P \bar{x}(k) \\
+ \bar{x}(k)^T R \bar{x}(k) - \bar{x}^T(k - d) R \bar{x}(k - d) \\
\leq \left[ \bar{A} \bar{x}(k) + \bar{A}_d \bar{x}(k - d) + \bar{B}_\phi \phi(\bar{e}(k)) \right]^T \\
\times P \left[ \bar{A} \bar{x}(k) + \bar{A}_d \bar{x}(k - d) + \bar{B}_\phi \phi(\bar{e}(k)) \right] \\
- \bar{x}^T(k) P \bar{x}(k) + \bar{x}^T(k) R \bar{x}(k) \\
- \bar{x}^T(k - d) R \bar{x}(k - d) - 2 \sum_{i=1}^{L+N} \tau_i \phi_1^2(\bar{e}_i(k)) \\
+ 2 \sum_{i=1}^{L+N} \tau_i k_i \bar{e}_i(k) \phi_1(\bar{e}_i(k)) \\
= \begin{bmatrix} \bar{x}(k) \\
\bar{x}(k - d) \\
\phi(\bar{e}(k)) \end{bmatrix}^T \\
\begin{bmatrix} A & B \\
B^T & C \end{bmatrix} \begin{bmatrix} \bar{x}(k) \\
\bar{x}(k - d) \\
\phi(\bar{e}(k)) \end{bmatrix}.
\end{align*}
where $T = \text{diag}(\tau_1, \tau_2, \ldots, \tau_{\omega N}) \succeq 0$. By Lemma 3 (Schur complement), if $\Xi_1 < 0$ (15) holds, $\Xi_0 < 0$ also holds. So, if $\Xi_1 < 0$, system $G$ with $\omega(k) = 0$, that is, system $\overline{G}$, is globally asymptotically stable, $\forall \xi(k) \in \Omega(\bar{K})$.

Next, for all $\ell > 0$, define

$$J_\ell = \sum_{k=0}^{\ell} \| \bar{y}(k) \|^2 - \gamma \sum_{k=0}^{\ell} \| \omega(k) \|^2$$

and according to (19) and (20), we have

$$J_\ell = \sum_{k=0}^{\ell} \Delta V(\bar{x}(k)) > 0.$$ (21)

Therefore, for system $\overline{G}$, defining vector $\zeta(k) = [\bar{x}^T(k) (k-d) \bar{\phi}^T(\bar{e}(k)) \omega^T(k)]^T$ and according to (19) and (20), we have

$$J_\ell = \sum_{k=0}^{\ell} \left[ \bar{y}^T(k) \bar{y}(k) - \gamma^2 \omega^T(k) \omega(k) + \Delta V(\bar{x}(k)) \right]$$

$$- V(\bar{x}(\ell + 1))$$ (22)

If $\Xi_1 < 0$ (15) holds, $\lim_{\ell \to \infty} J_\ell = \lim_{\ell \to \infty} \sum_{k=0}^{\ell} \zeta^T(k) \Xi \zeta(k) < 0$. Thus, for all nonzero $\omega(k) \in L_2 [0, \infty)$ and the $H_{\infty}$-norm constraint (11) is achieved. This completes the proof.

4. $H_{\infty}$ Controller Design Based on WNCN

In the previous stage, the matrix inequality condition $\Xi_1 < 0$ (15) is not an LMI, which cannot be solved by LMI tools. In what follows, we first convert the matrix inequality condition $\Xi_1 < 0$ (15) into a cone complementarity problem (CCP) and then use the B-type linearization algorithm introduced in [36] to formulate a convex optimization problem with LMI constraints to obtain the appropriate parameters of WNCN (i.e., interconnection weight matrices set $\mathcal{H} = \{W^O, W^W, W^-\}$).

**Lemma 5** (see [27]). There exist matrices $P, Q \in S^n_{++}$ satisfying the constraint $Q = P^{-1}$ if and only if they are optimal points for the problem

$$\min \ \text{Tr}\{QP\}$$

subject to $\begin{bmatrix} Q & I \\ I & P \end{bmatrix} \succeq 0, \ P, Q \in S^n_{++}$, (23)

and the optimal cost of the problem is $n$.

**Theorem 6.** Given a scalar $\gamma > 0$, the closed-loop system $\overline{G}$ is said to be absolutely stabilizable by using a WNCN $\mathcal{H}$ and the $H_{\infty}$-norm constraint (11) is achieved for all nonzero $\omega(k)$ if there exist appropriate dimension matrices $P > 0$, $Q > 0$, $R > 0$, $\Sigma \succeq 0$, and $W^O, W^W, W^- \in \mathcal{H}$ such that the following optimization problem:

$$\min \ \text{Tr}\{QP\}$$

subject to

$$\begin{bmatrix} -P + R & \bar{C}^T \Sigma & \bar{C}^T \gamma \\ 0 & -R & \bar{C}^T \\ \bar{C}^T \gamma & \bar{C}^T \gamma & -A^T + \bar{A}^T \end{bmatrix} < 0,$$ (24)

$$\begin{bmatrix} 0 & \bar{C}^T \gamma \\ -R & \bar{C}^T \gamma & 0 \\ \bar{C}^T \gamma & \bar{C}^T \gamma & \bar{C}^T \gamma \end{bmatrix} < 0.\]$$ (25)

$$\begin{bmatrix} Q & I & P \\ I & P \end{bmatrix} \succeq 0, \ Q, P \in S^n_{+ + N},$$ (26)
where
\[ \tilde{A} = \begin{bmatrix} A & B \omega^T \\ 0 & 0 \end{bmatrix}, \quad \tilde{C}_\epsilon = \begin{bmatrix} C_{\epsilon} & D_{\epsilon} \omega^T \\ W_{\epsilon} & W_{\epsilon} \end{bmatrix}, \] (27)
is feasible with optimal cost \( n + N \).

Proof. Inequality \( \Xi_1 < 0 \) in Theorem 4 can be rewritten as
\begin{align*}
\left[ \begin{array}{cccc}
-P + R & 0 & \tilde{C}_\epsilon^T \tilde{T} & 0 \\
* & -R & \tilde{C}_d^T \tilde{T} & 0 \\
* & * & -\tilde{D}_\phi^T \tilde{T}K + \tilde{T}K \tilde{D}_\phi - 2T \tilde{T}K \tilde{D}_\phi & 0 \\
* & * & * & -\gamma^2 I
\end{array} \right] < 0.
\end{align*}
(28)
Then, using Lemma 3 (Schur complement), the inequality (28) is equivalent to
\begin{align*}
\left[ \begin{array}{cccc}
\tilde{A}_T^T & 0 & 0 & 0 \\
\tilde{A}_d^T & \tilde{C}_d^T \tilde{T}K & 0 & 0 \\
\tilde{D}_\phi^T \tilde{T}K + \tilde{T}K \tilde{D}_\phi - 2T \tilde{T}K \tilde{D}_\phi & 0 & -\gamma^2 I & 0 \\
\tilde{C}_y & 0 & 0 & 0
\end{array} \right] \prec 0.
\end{align*}
(29)
and pre- and postmultiplying the left-hand side matrix of (29) by \( \text{diag}(I, I, I, I, I) \), respectively, the inequality (29) is equivalent to
\begin{align*}
\left[ \begin{array}{cccc}
-P + R & 0 & \tilde{C}_\epsilon^T & 0 \\
* & -R & \tilde{C}_d^T & 0 \\
* & * & \tilde{S}_\phi^T + \tilde{D}_\phi S - 2\Sigma \tilde{D}_\phi & 0 \\
* & * & * & -\gamma^2 I & 0 \\
* & * & * & * & -Q & 0 \\
* & * & * & * & * & -I
\end{array} \right] < 0.
\end{align*}
(31)
According to (30), the following equations hold:
\[ S^{-1} = T\tilde{K}, \quad T = S^{-1}\Sigma S^{-1}. \] (32)
Form (32), we know
\[ S = \Sigma\tilde{K}. \] (33)
Substituting (33) into (31), one can obtain \( \Xi_2 < 0 \) (25).

So far the WNCN has been designed to guarantee the absolute stability with a given \( H_\infty \)-norm bound \( \gamma \) of the closed-loop system. In what follows, we give the Algorithm 2 based on the bisection method to design WNCN for the optimal \( H_\infty \) control problem: \( \min \gamma \) s.t. (15), \( P > 0, R > 0, T \succeq 0 \).

5. Robust \( H_\infty \) Controller Design Based on Fading WNCN

Due to the large geographical nature of the closed-loop system \( \mathcal{G} \) over a WN, a realistic distributed control design approach for WNCN should take the communication packet losses into account.

According to [37], we adopt the fading channel models to simulate the unreliable wireless communication links in WNCN as shown in Figure 2(a). First, define a bijective mapping \( \Omega : \{(a,b)\} \to \{\tau\}, (a,b) \in \mathcal{E}_0 \cup \mathcal{E}_c \cup \mathcal{E}_i \), where \( \tau = \{1, \ldots, \rho\} \) and \( \rho = k(\mathcal{E}_0) + k(\mathcal{E}_c) + k(\mathcal{E}_i) \) is the total number of wireless links in WNCN, to concisely enumerate all links in the network. Therefore, the weights of links \( (a,b) \) can be mapped to \( \omega_0^\tau, \omega_c^\tau, \omega_i^\tau, \forall \tau = \Omega(a,b) \) and then compacted into the following weight vector as
\begin{align*}
\omega = \left[ (\omega_0^\tau)^T, (\omega_c^\tau)^T, (\omega_i^\tau)^T \right]^T.
\end{align*}
(34)
Step 1. Set $k = 0$. If there exists an initial feasible solution set $\mathcal{Y}_0 = \{ P, Q, R, \Sigma, W^\sigma, W^\varrho, W^r \}$ satisfying the constraints (25)-(26), let $\mathcal{X}_0 = P$, $\mathfrak{y}_0 = Q$. Otherwise, exit.

Step 2. If $k \leq \kappa$, go to Step 3 where $\kappa$ is the assumed maximum number of iteration. Otherwise, exit.

Step 3. At $k \geq 0$, obtain the feasible solution set $\mathcal{Y}_{k+1} = \{ P, Q, R, \Sigma, W^\sigma, W^\varrho, W^r \}$ by solving the following LMI problem: 

\[
\min \text{Tr} \{ \mathcal{X}_k Q + \mathfrak{y}_k P \} \quad \text{s.t.} \quad (25)-(26).
\]

Step 4. Substitute $\mathcal{Y}_{k+1}$ into (15). If (15) holds, stop the algorithm. Otherwise, set $k = k + 1$, $\mathcal{X}_k = P$, $\mathfrak{y}_k = Q$ and go to Step 2.

**Algorithm 1**: Given a scalar $\gamma > 0$, solving the $H_\infty$ WNCN for closed-loop system $\mathfrak{X}$.

\[
t(k) = \begin{bmatrix}
W^\sigma \gamma & 0 \\
0 & W^\varrho \gamma \\
\mathcal{C} \gamma & 0 \\
0 & \mathcal{I}_N
\end{bmatrix}
\begin{bmatrix}
x(k) \\
z(k)
\end{bmatrix} = \mathcal{W}^\infty x(k),
\]

(35)

where $W^\sigma = \text{diag}(w^\sigma) \in \mathbb{R}^{\rho \times \rho}$, $W^\varrho = \text{diag}(w^\varrho) \in \mathbb{R}^{\rho \times \rho}$, and $w \in \mathbb{R}^\rho$. Let $t_{rs}$ denote the data packet transmitted over the $r$th communication link at time $k$. Then, aggregating all of $t_{rs}$ in a vector $t_k \in \mathbb{R}^\rho$, we obtain

Remark 7. $\mathcal{W} = \mathbb{R}^{\rho \times (N+1)}$ is a row selection matrix whose each row contains a single nonzero element which equals to a corresponding weight $w^\sigma$, $w^\varrho$ or $w^r$.

Next, let $r(k)$ denote the received date from $t(k)$ via the unreliable wireless communication links. $y_t(k)$, $r \in \{1, \ldots, \rho\}$ is independent and identically distributed (I.I.D) Bernoulli random variable with mean $\mu_r = \mathbb{E}[y_t(k)]$ and variance $\sigma_r^2 = \mathbb{E}(y_t(k) - \mu_r)^2$. $y_t(k)$ indicates whether packet $t_{rs}$ is successfully received by $r_s(k)$; that is, $y_t(k) = 1$ if packet arrives and $y_t(k) = 0$ otherwise. If $\Delta_s(k)$, $\tau \in \{1, \ldots, \rho\}$ denotes an I.I.D Bernoulli random variable with zero-mean and variance $\sigma_s^2$, $y_t(k)$ can be transformed into a robust form such that $y_t(k) = \mu_s + \Delta_s(k)$, where $\mu_s$ is nominal value and $\Delta_s(k)$ is random perturbation value. Thus, the fading channel model is described by the following bijective mapping:

\[
\Gamma : t(k) \rightarrow r(k) = \Gamma(k) t(k) = (M + \Delta(k)) t(k),
\]

where $\Gamma(k) = \text{diag}(y_1(k), \ldots, y_p(k))$, $M = \text{diag}(\mu_1, \ldots, \mu_p)$, $\Delta(k) = \text{diag}(\Delta_1(k), \ldots, \Delta_p(k))$, $\mathbb{E}[\Delta(k) \Delta(k)^T] = \text{diag}(\sigma_1^2, \ldots, \sigma_p^2)$, and $\mathbb{E}[\Delta(k)] = 0$.

Thus, the dynamic behavior of the fading WNCN with stochastic packet losses can be described as follows:

\[
\begin{align*}
\hat{z}(k+1) &= \psi(\hat{z}(k)), \\
\hat{x}(k+1) &= \hat{x}(k) + \mathcal{W}_\gamma \mathcal{X}_k (k) t(k) + u(k),
\end{align*}
\]

(37)

where $\mathcal{W}_\gamma \mathcal{X}_k = \mathcal{W}_\gamma \mathcal{X}_k + \mathcal{W}_\gamma$ and

\[
W^\sigma = \left[ \begin{array}{l}
\mu_r w^\tau, \quad \exists \nu \in \mathcal{Y}, \nu = \nu, \\
0,
\end{array} \right]_{\mathbb{R}^{N \times 1}},
\]

where $W^\sigma = \left[ \begin{array}{l}
\mu_r w^\tau, \quad \exists \nu \in \mathcal{Y}, \nu = \nu, \\
0,
\end{array} \right]_{\mathbb{R}^{N \times 1}}$, and

\[
W^\tau = \left[ \begin{array}{l}
\mu_r w^\tau, \quad \exists \nu \in \mathcal{Y}, \nu = \nu, \\
0,
\end{array} \right]_{\mathbb{R}^{N \times 1}}.
\]
\[ W_\Delta^o = [(w_\Delta^o)_{i\tau}]_{N \times \rho}, \]

where \( \begin{cases} 1, \exists v_i \in V, s_i \in \delta, \\
\Omega(s_i, v_i) = \tau \end{cases} \) for \( i, \rho \).

\[ W_\Delta^e = [(w_\Delta^e)_{i\tau}]_{N \times \rho}, \]

where \( \begin{cases} 1, \exists v_i \in V, v_i \in \varphi, \\
\Omega(v_i, v_i) = \tau \end{cases} \) for \( i, \rho \).

\[ W_\Delta^f = [(w_\Delta^f)_{p\tau}]_{m \times \rho}, \]

where \( \begin{cases} 1, \exists v_i \in V, a_i \in \delta, \\
\Omega(v_i, a_i) = \tau \end{cases} \) for \( i, \rho \).

\[ (\omega_\Delta)_{i\tau} = \begin{cases} 1, \exists V_i \in V, s_i \in \delta, \\
\Omega(s_i, v_i) = \tau \end{cases} \]

\[ \forall i, \rho. \]

**Remark 8.** Similar to \( W^{or} \), matrices \( W^{eo} \) and \( W^{ef} \) are used to select which elements of \( \Delta(k) \) are added to \( \xi(k) \) and \( u(k) \), respectively.

As shown in Figure 2(b), DSNNM (1) is controlled by a fading network composed by the mean WNCN (MWNcn) and the stochastic perturbation \( \Delta \). Consider the following stochastic closed-loop system:

\[
\begin{align*}
\dot{\bar{x}}(k+1) &= \bar{A}_\mu \bar{x}(k) + \bar{A}_d \bar{x}(k-1) + \bar{B}_d \bar{\epsilon}(k) + \bar{B}_\omega \omega(k) + \bar{W}_{dat}(k) t(k), \\
\bar{\epsilon}(k) &= \bar{C}_d \bar{x}(k) + \bar{C}_\omega \bar{x}(k-1) + \bar{D}_d \bar{\epsilon}(k) + \bar{D}_\omega \omega(k) + \bar{W}_{dat}(k) t(k),
\end{align*}
\]

(39)

\[
\begin{align*}
\min \ 	ext{Tr} \{QP\}
\begin{bmatrix}
-P + R + \Pi & 0 & \bar{C}_d^T & 0 & \bar{A}_d^T & \bar{C}_\omega^T \\
* & -R & \bar{C}_d^T & \bar{A}_d^T & 0 & 0 \\
* & * & -\bar{D}_d^T \Sigma K + \Sigma K \bar{D}_\omega^T - 2\Sigma \bar{D}_\omega \Sigma K \bar{B}_\phi^T & 0 \\
* & * & * & 0 & -\gamma^2 I & 0 \\
* & * & * & -Q & 0 & 0 \\
* & * & * & 0 & 0 & -I
\end{bmatrix} < 0,
\end{align*}
\]

(41)

\[
\begin{align*}
\Pi = \begin{bmatrix} \bar{W}_{dat}^d & \bar{W}_{dat}^e & \bar{W}_{dat}^f \end{bmatrix}^T, \\
\Theta = \text{diag}(\theta_1, \ldots, \theta_\rho), \text{ and } \theta_i = \sigma_i^2 (\bar{W}_{dat}^d)^T P (\bar{W}_{dat}^d), \text{ denote the } i\text{th column of the matrix } \bar{W}_{dat}^d, \text{ is feasible with optimal cost } n + N.
\end{align*}
\]

**Theorem 10.** Given a scalar \( \gamma > 0 \), the stochastic closed-loop system \( \bar{\Xi} \) is said to be absolutely stabilizable in mean-square by using a fading WNCN \( \bar{\Xi} \) and the \( H_{\infty} \)-norm constraint (40) is achieved for all nonzero \( \omega(k) \) if there exist appropriate dimension matrices \( P > Q > R > 0, \Sigma > 0, \bar{W}^e, \bar{W}^e, \bar{W}^e, \bar{W}^f, \bar{W}^f, \) and scalar \( \theta_i, i = 1, \ldots, \rho, \) such that the following optimization problem:

\[
\begin{align*}
\min \ 	ext{Tr} \{[QP]\}
\begin{bmatrix}
-P + R + \Pi & 0 & \bar{C}_d^T & 0 & \bar{A}_d^T & \bar{C}_\omega^T \\
* & -R & \bar{C}_d^T & \bar{A}_d^T & 0 & 0 \\
* & * & -\bar{D}_d^T \Sigma K + \Sigma K \bar{D}_\omega^T - 2\Sigma \bar{D}_\omega \Sigma K \bar{B}_\phi^T & 0 \\
* & * & * & 0 & -\gamma^2 I & 0 \\
* & * & * & -Q & 0 & 0 \\
* & * & * & 0 & 0 & -I
\end{bmatrix} < 0,
\end{align*}
\]

(41)

\[
\begin{align*}
\begin{bmatrix} Q & I \\
I & P \end{bmatrix} \geq 0, \quad Q, P \in S^{n+\rho},
\end{align*}
\]

(44)

\[
\begin{align*}
\text{Proof. Consider a Lyapunov candidate as follows:}
\end{align*}
\]

\[
V[M(k)] = \text{Tr} \{M(k)P\}.
\]

(45)
The difference of $V[M(k)]$ along the trajectory of stochastic closed-loop system $\overline{\mathcal{G}}$ with $\omega(k) = 0$ is given by

$$
\Delta V[M(k)] = V[M(k+1)] - V[M(k)] = \operatorname{Tr} [M(k+1)P] - \operatorname{Tr} [M(k)P] = \operatorname{Tr} \left[ E \left[ (x^T(k+1)P\bar{x}(k+1) - \bar{x}^T(k+1)P\bar{x}(k) \right] \right] = \operatorname{Tr} \left[ E \left[ x^T(k+1)P\bar{x}(k+1) - \bar{x}^T(k+1)P\bar{x}(k) \right] \right].
$$

By considering $\Delta(k)$ with $E[\Delta(k)] = 0$ and $E[\Delta(k)\Delta^T(k)] = \operatorname{diag}(\sigma^2_1, \ldots, \sigma^2_\ell)$ is independent from $\bar{x}(k)$ and $\phi(\bar{x}(k))$ and using Lemma 2 (S-procedure) one can obtain

$$
\Delta V[M(k)] = \begin{bmatrix} \bar{x}(k) \\ \bar{x}(k-d) \\ \phi(\bar{x}(k)) \end{bmatrix}^T \begin{bmatrix} \bar{A}^T_P \bar{A}_P \bar{P} + P + R + \Pi \\ \bar{A}^T_P \bar{A}_P \bar{P} - R \\ \bar{A}^T_P \bar{P}B_\varphi + \bar{C}^T_\varphi \bar{K} \\ \bar{A}^T_P \bar{P}B_\varphi + \bar{C}^T_\varphi \bar{K} \\ \bar{A}^T_P \bar{P}B_\varphi + \bar{C}^T_\varphi \bar{K} \\ \bar{A}^T_P \bar{P}B_\varphi + \bar{C}^T_\varphi \bar{K} \end{bmatrix} \begin{bmatrix} \bar{x}(k) \\ \bar{x}(k-d) \\ \phi(\bar{x}(k)) \end{bmatrix},
$$

where $T = \operatorname{diag}(\tau_1, \tau_2, \ldots, \tau_{\ell+N})$, $\bar{K} = \operatorname{diag}(k_1, \ldots, k_{\ell+N})$, $\Pi = (\bar{W}^m)\bar{W}^m$, $\Theta = \operatorname{diag}(\theta_1, \ldots, \theta_\ell)$, $\theta_i = \sigma^2_{\ell}(\bar{W}^m_{i,\ell})$, $\bar{W}^m_{i,\ell}$ denote the $i$th column of the matrix $\bar{W}^m$.

According to (20)–(22), $\forall \ell > 0$, we have

$$
E \{J_\ell\} = \sum_{k=0}^\ell E \left\{ \left\| y^T(k) \right\|^2 \right\} - \gamma^2 \sum_{k=0}^\ell E \left\{ \left\| \omega(k) \right\|^2 \right\} = \sum_{k=0}^\ell E \left\{ y^T(k) \bar{y}(k) - \gamma^2 \omega^T(k) \omega(k) \right\} \leq \sum_{k=0}^\ell \eta^2(k) \bar{E}_\varphi \zeta(k),
$$

where

$$
\bar{E}_\varphi = \begin{bmatrix} \bar{A}^T_P \bar{A}_P \bar{P} + P + R + \Pi \\ \bar{A}^T_P \bar{A}_P \bar{P} - R \\ \bar{A}^T_P \bar{P}B_\varphi + \bar{C}^T_\varphi \bar{K} \\ \bar{A}^T_P \bar{P}B_\varphi + \bar{C}^T_\varphi \bar{K} \\ \bar{A}^T_P \bar{P}B_\varphi + \bar{C}^T_\varphi \bar{K} \\ \bar{A}^T_P \bar{P}B_\varphi + \bar{C}^T_\varphi \bar{K} \end{bmatrix}.
$$

As in the previous section, we present Algorithms 3 and 4.

6. Numerical Simulation

Consider the following nonlinear system [38]:

$$
x_1(k+1) = -x_1^3(k) + 0.3x_2(k) + 0.1x_1^3(k-2) - 0.2x_1(k-2)x_2(k-2) + u_1(k),
$$
Step 1. Set \(k = 0\). If there exists an initial feasible solution set \( \Upsilon_0 = \{P, Q, R, \Sigma, W_0, \mu_0\} \) satisfying the constraints (42)–(44), let \( \mathcal{X}_0 = P, \mathcal{Y}_0 = Q \). Otherwise, exit.

Step 2. If \(k \leq k\) go to Step 3 where \(k\) is the assumed maximum number of iteration. Otherwise, exit.

Step 3. At \(k \geq 0\), obtain the feasible solution set \( \Upsilon_{k+1} = \{P, Q, R, \Sigma, W_k, \mu_k\} \) by solving the following LMI problem:

\[
\min \text{Tr} [XQ + YP] \quad \text{s.t.} \quad (42)–(44).
\]

Step 4. Substitute \( \Upsilon_{k+1} \) into matrix \( \tilde{\Xi} \). If inequality \( \tilde{\Xi} \leq 0 \) holds, stop the algorithm. Otherwise, set \( k = k + 1 \), \( \mathcal{X}_k = P, \mathcal{Y}_k = Q \) and go to Step 2.

Algorithm 3: Given a scalar \( \gamma > 0 \), solving the \( H_{\infty} \) fading WNCN with unreliable communication links for closed-loop system \( \tilde{G} \).

Algorithm 4: Minimizing \( \gamma \) to solve the optimal \( H_{\infty} \) fading WNCN with unreliable communication links for closed-loop system \( \tilde{G} \).

\[
x_2(k+1) = 0.1x_1(k) + x_2(k) + 0.5u_2(k),
\]
\[
y(k) = 0.6x_1(k).
\]

(50)

According to [39, 40], when we consider the disturbance \( w(k) \), the nonlinear system (50), where \( A = [\begin{bmatrix} -0.5 & 0.3 \\ 0.1 & -0.2 \end{bmatrix}], A_2 = [\begin{bmatrix} 0.05 & -0.1 \\ 0.1 & 0.2 \end{bmatrix}], B_f = [0], B_h = [\begin{bmatrix} 1 \\ 0.5 \end{bmatrix}], C_r = [1 \ 0], C_d = [\begin{bmatrix} 0.9 \ 0.1 \end{bmatrix}], D_h = 0, D_s = 0.1,\) and \( K = I \).

\[
P =
\begin{bmatrix}
3.6031 & -1.0546 & -0.0825 & -0.0825 & 0.0047 & 0.0047 & -0.0617 & -0.0617 \\
-1.0546 & 11.3398 & 1.2546 & 1.2546 & -1.1665 & -1.1665 & -1.1256 & -1.1256 \\
-0.0825 & 1.2546 & 1.1610 & 0.0840 & -0.0821 & -0.0821 & -0.0785 & -0.0785 \\
-0.0825 & 1.2546 & 0.0840 & 1.1610 & -0.0821 & -0.0821 & -0.0785 & -0.0785 \\
0.0047 & -1.1665 & -0.0821 & -0.0821 & 1.1945 & 0.0725 & 0.0708 & 0.0708 \\
0.0047 & -1.1665 & -0.0821 & -0.0821 & 0.0725 & 1.1945 & 0.0708 & 0.0708 \\
-0.0617 & -1.1256 & -0.0785 & -0.0785 & 0.0708 & 0.0708 & 1.1454 & 0.0684 \\
-0.0617 & -1.1256 & -0.0785 & -0.0785 & 0.0708 & 0.0708 & 0.0684 & 1.1454
\end{bmatrix}
\]

\[
Q =
\begin{bmatrix}
0.2931 & 0.0501 & -0.0192 & -0.0192 & 0.0378 & 0.0378 & 0.0545 & 0.0545 \\
0.0501 & 0.1789 & -0.1421 & -0.1421 & 0.1359 & 0.1359 & 0.1342 & 0.1342 \\
-0.0192 & -0.1421 & 0.9929 & 0.9929 & -0.0579 & -0.0579 & -0.0576 & -0.0576 \\
-0.0192 & -0.1421 & 0.0644 & 0.9929 & -0.0579 & -0.0579 & -0.0576 & -0.0576 \\
0.0378 & 0.1359 & -0.0579 & -0.0579 & 0.9884 & 0.0599 & 0.0594 & 0.0594 \\
0.0378 & 0.1359 & -0.0579 & -0.0579 & 0.9884 & 0.0599 & 0.0594 & 0.0594 \\
0.0545 & 0.1342 & -0.0576 & -0.0576 & 0.0594 & 0.0594 & 0.0605 & 0.9890 \\
0.0545 & 0.1342 & -0.0576 & -0.0576 & 0.0594 & 0.0594 & 0.0605 & 0.9890
\end{bmatrix}
\]

Consider that the double-input-single-output (DISO) discrete-time DSNNM described above is synthesized by a WNCN which consists of 6 wireless neuron nodes shown in Figure 3. In WNCN, each wireless communication link is modeled as a fading channel with same packet arrival rate (mean) \( \delta \) and variance \( \sigma^2 = \delta(1 - \delta) \). For \( \delta = 0.95\% \), Algorithm 4 can be solved by CVX, a package for specifying and solving convex programs [41]. Then, we obtain the minimum optimal \( H_{\infty} \) performance index \( \gamma^* = 0.7921 \), the solutions of (41)–(44), and the interconnection weight matrix parameters of WNCN as follows:
\[
x(k + 1) = Ax(k) + A_d x(k - d) + B_p \phi(e(k)) + B_o \omega(k)
\]
\[
e(k) = C_x x(k) + C_d x(k - d) + D_o \omega(k)
\]
\[
y(k) = C_y x(k)
\]

Figure 3: A discrete-time DSNNM synthesized by a WNCN with 6 wireless neuron nodes.

\[
R = \begin{bmatrix}
1.1713 & -0.1182 & 0.0122 & 0.0122 & -0.0342 & -0.0342 & -0.1895 & -0.1895 \\
0 & 0.5427 & -0.0197 & -0.0197 & -0.0580 & -0.0580 & 0.0405 & 0.0405 \\
0 & 0 & 0.7390 & -0.7183 & 0.0078 & 0.0078 & -0.0614 & -0.0614 \\
0 & 0 & 0 & 0.1738 & 0.0657 & 0.0657 & -0.5153 & -0.5153 \\
0 & 0 & 0 & 0 & 0.7494 & -0.6879 & 0.0713 & 0.0713 \\
0 & 0 & 0 & 0 & 0 & 0.2973 & 0.3447 & 0.3447 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.7422 & -0.7089 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2198
\end{bmatrix},
\]
\[
\Sigma = \text{diag}(0.5313, 1.2461, 0.1357, 0.4233, 1.4452, 0.7652, 1.2891),
\]
\[
W^\mu_x = \begin{bmatrix}
0.0484 & 0.0484 & 0.0003 & 0.0003 & 0 & 0 \\
0.0484 & 0.0484 & 0.0003 & 0.0003 & 0 & 0 \\
-0.4925 & -0.4925 & 0.0110 & 0.0110 & -0.1193 & -0.1193 \\
-0.4925 & -0.4925 & 0.0110 & 0.0110 & -0.1193 & -0.1193 \\
0 & 0 & 0.4870 & 0.4870 & -0.0879 & -0.0879 \\
0 & 0 & 0.4870 & 0.4870 & -0.0879 & -0.0879
\end{bmatrix},
\]
\[
W^{\mu^c}_x = \begin{bmatrix}
-1.3588 \\
-1.3588 \\
0 \\
0 \\
0 \\
0
\end{bmatrix},
\]
\[
W^{\mu^f}_x = \begin{bmatrix}
0 & 0 & 0 & -0.0646 & -0.0646 \\
0 & 0 & 0 & -0.2989 & -0.2989
\end{bmatrix}.
\]

Figure 4: State response of the closed-loop system under $H_\infty$ controller WNCN with $y^* = 0.7921$. (51)
Figure 4 shows the simulation results of the state trajectories of controlled discrete-time DSNNM, where state $x(k)$ is initialized arbitrarily in interval $[-0.5,0.5]$ at $k = 0$ and $k = 40$, respectively, and the disturbance input is nonlinear load $1/\kappa^2$. It is easily seen that the WNSN with a distributed architecture solved by Algorithm 4 using CVX toolbox can ensure the absolute stability of the closed-loop system in the mean-square sense with optimal $H_{\infty}$ performance.

7. Conclusions
A novel wireless networked $H_{\infty}$ control approach based on WNCN has been considered for a class of Lurie-type nonlinear systems named DSNNM. The WNCN which can absolutely stabilize the closed-loop system in mean-square with a desired $H_{\infty}$ disturbance rejection level can be obtained by solving LMIs using a CVX toolbox (release 2.0 (beta)). Simulation results have illustrated the feasibility of the distributed control methods presented in this paper.

Conflict of Interests
The authors declare that there is no conflict of interests regarding the publication of this paper.

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