Research Article

Slope Stability Analysis Using Slice-Wise Factor of Safety

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The concept of slice-wise factor of safety is introduced to investigate the state of both the whole slope and each slice. The assumption that the interslice force ratio is the same between any two slices is made and the eccentric moment of slice weight is also taken into account. Then four variables equations are formulated based on the equilibrium of forces and moment and the assumption of interslice forces, and then the slice-wise factor of safety along the slip surface can be obtained. The active and passive sections of the slope can be determined based on the distribution of factor of safety. The factor of safety of the whole slope is also defined as the ratio of the sum of antisliding force to the sum of sliding force on the slip surface. Two examples with different slip surface shapes are analysed to demonstrate the usage of the proposed method. The slice-wise factor of safety enables us to determine the sliding mechanism and pattern of a slope. The reliability is verified by comparing the overall factor of safety with that calculated by conventional methods.

1. Introduction

Among various methods currently available for slope stability analysis, conventional methods based on the concept of limit equilibrium have been most widely used in engineering practice. Though finite element analysis is becoming an attractive alternative [1–3], the limit equilibrium technique will probably continue to play an important role in the further slope engineering due to its simplicity and its ease of use.

The key procedure and main purpose of slope stability analysis using the limit equilibrium technique are the calculation of the factor of safety. Given a predefined slip surface, the factor of safety is determined with these methods from the equilibrium of force and/or momentum of the mass contained between the slip surface and the free ground surface. Some of the proposed methods are only for circular slip surfaces [4, 5], while more recent ones are for any shape of slip surfaces [6–9]. The list is not exhaustive. In addition, these methods are also different in the equilibrium conditions that they satisfy. The ordinary methods of slices [4] satisfy only the moment equilibrium; Bishop's modified method

[5] satisfies the moment equilibrium and vertical force equilibrium. Morgenstern and Price's method [6], Janbu's generalized procedure of slice [10], Sarma's method [11], and slope stability charts [9] satisfy all conditions of equilibrium and differ from each other in the assumptions about interslice forces. Recently, researches focus on methods to find the critical slip surfaces instead of the limit equilibrium technique itself. Sarma and Tan [12] used the stress acceptability criterion to locate the critical slip surface; Li et al. [13] employed a real-coded genetic algorithm to develop a search approach for locating the noncircular critical slip surface.

However, we realized that the limit equilibrium technique for slope analyses still has some aspects to improve. The equation in each method mentioned above is an implicit equation of the factor of safety and there is a connotative assumption that the factor of safety of each slice is equal to the factor of safety of the slope. The assumption is just a simple treatment of the factor of safety in order to solve the implicit equation more conveniently. Wright et al. [14], Tavenas et al. [15], and others noted that the factor of safety varies from place to place along the slip surface.

FIGURE 1: Sketch of a slope section and forces acting on the i th slice.

Furthermore, the slice-wise factor of safety is helpful for engineers to determine the active or passive section of a slope and the countermeasures for slope stabilization. Misleading measures based on incorrectly determined passive sections resulted in disastrous consequences in many cases [16, 17].

In this paper, the slice-wise factor of safety is first proposed (the definition of factor of safety is based on the conception of shear strength reduction), and the influence of its distribution along the slip surface on the stability of the whole slope is discussed. For each slice, four variables, including the horizontal and vertical interslice forces, the normal force on the slip surface, and the slice-wise factor of safety, are used to establish the slice-wise equations, and then the group of equations are solved iteratively using the boundary conditions at the first and the last slice.

2. Basic Assumption

2.1. Definition of Slice-Wise Factor of Safety. For the limit equilibrium method, the mass of a slope between the slip surface and the free ground surface is evenly divided into n slices as shown in (Figure 1(a)). The main forces acted on the i th slice (Figure 1(b)) are the weight (W_i), the horizontal earthquake force (Q_{ci}), the horizontal and vertical load (Q_{xi}, Q_{yi}), the water pressure (U_i), the interslice forces ($E_i, X_i, E_{i+1}, X_{i+1}$), the normal force (N_i), and the sliding force (T_i) of the slip surface.

The slice-wise factor of safety is defined as the ratio of the antisliding force to the sliding force, which is the same as the conception of shear strength reduction [1],

$$F_i = \frac{N_i \tan \phi_i + c_i b_i \sec \alpha_i}{T_i}, \quad (1)$$

where c_i is the shear strength parameters of the slip surface and b_i the width of the slice.

In the conventional strength reduction method, the factor of safety (F_s) means that the shear strength of all the

slices' bottom surface should be reduced by F_s times, which is obviously not the realistic state of the slope. By defining the slice-wise factor of safety, an individual reduction coefficient is given to each slice and all the slip surfaces can reach the limit equilibrium state simultaneously.

2.2. Discussion on the Height of Thrust Line. The rigorous Janbu's method [10] suggests that the height of the thrust line is between 1/3 and 1/2 of the height of the slice's profile. Actually the height of the thrust line is dependent on the property of the slope mass. For a loose deposit slope, the distribution of horizontal soil pressure would be triangular and thus the height of the thrust line is one-third of the height of the slice's profile. For a hard soil or block rock slope, the distribution of horizontal soil would be a parallelogram, and the height of the thrust line is half of the height of the slice's profile. Therefore, the determination of the thrust line height should be based on the geological investigation of the slope and differ from slope to slope.

2.3. Equilibrium of Forces and Boundary Condition. From the horizontal and vertical equilibrium of the forces

$$E_{i+1} + T_i \cos \alpha_i - N_i \sin \alpha_i = E_i + (Q_{ci} + U_i \sin \alpha_i - Q_{xi}), \quad (2a)$$

$$X_{i+1} + N_i \cos \alpha_i + T_i \sin \alpha_i = X_i + (W_i + Q_{yi} - U_i \cos \alpha_i). \quad (2b)$$

According to (1), the sliding force T_i in Figure 1 can be replaced by $(N_i \tan \phi_i + c_i b_i \sec \alpha_i)/F_i$. Substituting T_i into (2a) and (2b) yields

$$E_{i+1} + \left(\frac{\tan \phi_i \cdot \cos \alpha_i}{F_i} - \sin \alpha_i \right) N_i \quad (3a)$$

$$= E_i - \frac{c_i b_i}{F_i} + (Q_{ci} + U_i \sin \alpha_i - Q_{xi}),$$

$$X_{i+1} + \left(\frac{\tan \phi_i \cdot \sin \alpha_i}{F_i} + \cos \alpha_i \right) N_i \quad (3b)$$

$$= X_i - \frac{c_i b_i}{F_i} \tan \alpha_i + (W_i + Q_{yi} - U_i \cos \alpha_i).$$

At the boundaries, the horizontal force acting on the left side of the first slice E_1 and the force on the right side of the last slice E_{n+1} are usually assigned certain values, 0 in this case.

2.4. Assumption of Interslice Forces. The major difference among all the limit equilibrium methods is the assumption about the interslice force. No matter when a slope is sliding or stable, the interaction pattern between slices is generally similar to one another and seldom influenced by the states of the slices. Therefore, we assume the interslice force ratio

(the horizontal force to the vertical force) equal to the same value λ :

$$\lambda = \begin{cases} \frac{X_{i+1}}{c_i H_{i+1} + E_{i+1} \tan \phi_i} & \text{if } E_{i+1} \neq 0 \\ 0 & \text{if } E_{i+1} = 0, \end{cases} \quad (4)$$

where H_{i+1} is the height of the profile of the slice. The zero horizontal force indicates there is no interaction between the two slices, and thus λ must be zero too.

As a sign of global mobilization of the slope, the value of the interslice force ratio λ must be between 0 and 1. The larger the λ is, the better the antisliding forces are exerted. When the factor of safety of a slope is less than 1 and the slope is unstable, the antisliding force between the slices is exerted to the maximum at that time and the value of λ should be larger. In contrast, when the slope is stable, the antisliding force between the slices will not be exerted sufficiently and the value of λ should be much smaller.

2.5. Discussion on Balance of Moment. From the equilibrium of the moment at the midpoint of slice bottom, we can obtain

$$E_{i+1} \left(z_{i+1} - \frac{b_i \tan \alpha_i}{2} \right) = E_i \left(z_i + \frac{b_i \tan \alpha_i}{2} \right) - \frac{b_i}{2} (X_{i+1} + X_i) + Q_{c_i} \frac{h_i}{2} - Q_{x_i} h_i. \quad (5)$$

An assumption connoted in (5) is that the weight W_i , the vertical load Q_{y_i} , and the normal force N_i all pass through the midpoint of slice bottom. According to (5), the boundary condition at the first slice will be $E_1 = X_1 = 0$ when the horizontal load Q_{x_i} is zero regardless of the earthquake force Q_{c_i} . Consequently, a simple calculation using (4) suggests that all the interslice forces are zero, which is not consistent with actual stress state of the slope.

In order to avoid this erroneous situation, we recommended that the eccentric moment M_{w_i} of slice weight is to be taken into account. For a parallelogram-shape slice, the mass center passes through the midpoint of slice bottom, whereas the eccentric moment is no longer zero in the case of a trapezoidal slice. A straightforward way is to divide a trapezoid into a triangle and a parallelogram, and the eccentric moment M_{w_i} is only caused by the weight of triangle part. Hence, we can define the eccentric moment M_{w_i} as

$$M_{w_i} = \begin{cases} \frac{H_{i+1} - H_i}{H_{i+1} + H_i} \frac{W_i b_i}{6} & \text{if } H_{i+1} \geq H_i \\ 0 & \text{if } H_{i+1} < H_i, \end{cases} \quad (6)$$

where H_i is the height of the upper profile of the slice and H_{i+1} is the height of the lower profile of the slice. We neglect the negative eccentric moment M_{w_i} and assign zero to it when H_{i+1} is lower than H_i .

After introducing the eccentric moment M_{w_i} , the balance of moment is expressed as follows:

$$E_{i+1} \left(z_{i+1} - \frac{b_i \tan \alpha_i}{2} \right) = E_i \left(z_i + \frac{b_i \tan \alpha_i}{2} \right) - \frac{b_i}{2} (X_{i+1} + X_i) + Q_{c_i} \frac{h_i}{2} - Q_{x_i} h_i + M_{w_i}. \quad (7)$$

3. Solution of the Slice-Wise and the Overall Factor of Safety

3.1. Solution of the Slice-Wise Factor of Safety. From (3a), (3b), (4), and (6) of the four variables E_{i+1} , X_{i+1} , N_i , F_i , we can express E_{i+1} as the following formula:

$$E_{i+1}(\lambda) \Phi(i, \lambda) = E_i(\lambda) \Psi(i, \lambda) + \Omega(i, \lambda), \quad (8)$$

where

$$\Phi(i, \lambda) = 1 + \frac{\lambda b_i \tan \phi_i^j}{2(z_{i+1} - b_i \tan \alpha_i/2)}, \quad (9a)$$

$$\Psi(i, \lambda) = \frac{z_i + b_i \tan \alpha_i/2}{z_{i+1} - b_i \tan \alpha_i/2} - \frac{\lambda b_i \tan \phi_i^j}{2(z_{i+1} - b_i \tan \alpha_i/2)}, \quad (9b)$$

$$\Omega(i, \lambda) = \frac{Q_{c_i} h_i/2 - Q_{x_i} h_i + M_{w_i}}{z_{i+1} - b_i \tan \alpha_i/2} - \frac{\lambda b_i (c_{i+1}^j H_{i+1} + c_i^j H_i)}{2(z_{i+1} - b_i \tan \alpha_i/2)}. \quad (9c)$$

We can attain $E_2(\lambda)$ by substituting the boundary condition at the first slice $E_1 = 0$ into (8). The expression $E_{n+1}(\lambda)$ can be obtained successively, and then the value of λ can be solved with the boundary condition $E_{n+1}(\lambda) = 0$. The interslice forces E_{i+1} and X_{i+1} can be obtained using the value of λ and (4) and (8). After substituting E_{i+1} into (3a) and (3b), we can solve the simultaneous equations for the normal force N_i and the slice-wise factor of safety F_i .

After obtaining all the slice-wise factors of safety, the sliding mechanism of the slope also can be discussed accordingly.

3.2. Solution of the Overall Factor of Safety. The factor of safety of the whole slope is also defined by the ratio of the sum of antisliding force to the sum of sliding force on the slip surface:

$$F = \frac{\sum_{i=1}^n R_i}{\sum_{i=1}^n T_i}, \quad (10)$$

where

$$R_i = N_i \tan \phi_i + \frac{c_i b_i}{\cos \alpha_i}. \quad (11)$$

The method for slope stability analysis proposed in this paper can satisfy all the equilibrium conditions of forces and moment. Compared with the conventional limit equilibrium methods, the method can obtain not only the factor of the

TABLE 1: Factor of safety of the slope with a circular slip surface calculated by different methods.

Calculation method	Factor of slope safety
Donald (introduced)	1.000
Ordinary method	0.967
Bishop's method	0.992
Janbu's method	0.963
Morgenstern and Price's method	0.991
Proposed method	0.996

whole slope, but also the slice-wise factor of safety which can enable us to determine the mechanism and sliding pattern of a slope. In addition, the interslice force ratio λ also has the special physical meaning that reflects the exertion degree of the antisliding forces of slope mass.

4. Two Examples

In this section, we select two classical examples from the tests published by ACADS [18] to verify the method proposed in this paper. One slope has circular slip surface and the other has polylines slip surface.

4.1. Slope with Circular Slip Surface. The calculation example of circular slip surface is the first one of the ten tests published by ACADS EX1(a). Figure 2 shows the profile of the slope and the parameter of the soil mass.

According to the method proposed, firstly we should solve the value of λ by the equation $E_{n+1}(\lambda) = 0$.

The slope is composed of soil mass, and thus the height of thrust line of every slice is equal to 1/3 of the height of the slice's profile, $z_{i+1} = h_i/3$. There are two solutions for equation $E_{n+1}(\lambda) = 0$: (1) $\lambda_1 = 0.17392$ and (2) $\lambda_2 = 0.82783$. For the first one, the factor of safety of the whole slope is 0.968; for the second, the corresponding factor of safety is 0.996. The two factors of safety that are less than 1.0 indicate the slope is unstable. Hence, the antisliding forces between the slices should be exerted to the maximum and we must select the bigger $\lambda_2 = 0.82783$ and the corresponding factor of safety of the slope as 0.996.

Figure 3 plots the distribution of slice-wise factor of safety along the slip surface when $\lambda = 0.82783$.

Figure 3 shows that the slice-wise factors of safety of the slices from 22 to 30 at the lower region of the slope mass are all less than 1.0, which indicates that the region is the active section and will slide before the other slices. Hence, the sliding mechanism of the slope can be judged as traction sliding.

Table 1 shows the factor of safety calculated by a number of methods. The difference of the factor of safety between the proposed method and the standard answer introduced by Donald is only 0.4%, which verifies the reliability of the method.

4.2. Slope with Polyline Slip Surface. The calculation example of polyline slip surface is the first one of the ten tests

FIGURE 2: Calculation diagram of the slope with a circular slip surface.

FIGURE 3: Slice-wise factor of safety along the circular slip surface.

TABLE 2: Soil parameters.

Soil	c (kPa)	ϕ ($^\circ$)	γ (kN/m ^{3})
Soil-1	28.5	20	18.84
Soil-2	0	10	18.84

published by ACADS EX3(b). Figure 4 and Table 2 show the profile of the slope and the parameters of the soil mass, respectively.

The slope is composed of soil mass, and thus the height of thrust line of every slice is 1/3 of the height of the slice's profile, $z_{i+1} = h_i/3$. There is only one solution for equation $E_{n+1}(\lambda) = 0$, $\lambda = 0.02662$, and the factor of safety of the whole slope 1.247. The near-zero λ indicates that the slope mass is integrated tightly, and the vertical forces are small enough to be ignored. In this case, the result is very close to the result given by Bishop's method which ignores the influence of vertical forces.

Figure 5 shows that the slice-wise factors of safety of slices from 11 to 29 in the middle of the slope are smaller than 1 and distribute reposefully. Hence, this region is probably the active section, and the region from slices 1 to 10 and the region from slices 30 to 32 are the passive sections of the slope.

Table 3 shows the calculation results of different methods. From Table 3 we can conclude that the error of the factors of slope safety between the proposed method and Morgenstern

FIGURE 4: Calculation diagram of the slope with a circular slip surface.

FIGURE 5: Distribution of slice-stability coefficients along the poly-lines slip surface.

TABLE 3: Factor of safety of the slope with polylines slip surface calculated by different methods.

Calculation method	Factor of slope safety
Bishop's method	1.258
Janbu's method	1.199
Spencer's method	1.275
Morgenstern and Price's method	1.261
Proposed method	1.247

and Price's methods is only 1.4%. With an accuracy of about $\pm 6\%$, the factor of safety calculated using methods that satisfy all conditions of equilibrium can be considered to be the correct answer [19]. Because the error is small enough to be ignored, the proposed method is quite reliable.

4.3. Influence of the Height of Thrust Line. In order to evaluate the influence of thrust line height on the factor of safety, we use $1/2$ of the height of the slice's profile as the thrust line height and calculate the two examples again.

For the example of circular slip surface, the single solution of equation $E_{n+1}(\lambda) = 0$ is $\lambda = 0.40789$, and the factor of safety F is 0.972. For the example of polylines slip surface, the single solution of equation $E_{n+1}(\lambda) = 0$ is $\lambda = 0.09262$, and the corresponding factor of safety is 1.263. Though the

height of thrust line has little effect on the factor of safety of the slope, it has great effect on the interslice forces ratio λ . Hence the height of thrust line should be determined based on the property of the slope mass. Usually, we use $h_i/3$ for soft soil and $h_i/2$ for rigid rock.

5. Conclusion

Considering the limitation of the definition of factor of safety in traditional slice methods, the authors introduce the concept of slice-wise factor of safety and propose a new limit equilibrium method based on it. In the method, an assumption that the interslice force ratio λ is the same between any two slices is first made to ease the solution process. The ratio reflects the exertion degree of the antisliding forces of slip mass. In addition, the eccentric moment is considered in the analysis. A four-variable implicit equation is established based on the equilibrium of forces and moment and the assumption of interslice forces. And then the interslice force ratio λ can be solved from the equation $E_{n+1}(\lambda) = 0$ using the boundary condition. With the ratio, the slice factors of safety along the slip surface can be obtained straightforwardly. The slice-wise factor of safety is useful for engineers to determine the sliding mechanism and passive section of a slope, which can help the engineers with the design of stabilizing piles. The factor of safety of a slope is also defined by the ratio of the sum of antisliding force to the sum of sliding force on the slip surface. The results of the two calculation examples verified the reliability of the proposed method. The height of thrust line has little effect on the factor of slope safety but has great effect on the state of interslice forces.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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