Research Article

A Semianalytical Solution for Multifractured Horizontal Wells in Box-Shaped Reservoirs

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This paper presented a 3D point sink model through using Dirac function. Then, 3D point sink solution in boxed reservoirs was obtained through using Laplace transform and Fourier transform methods. Based on the flux and pressure equivalent conditions in Laplace space, a semianalytical solution for multifractured horizontal wells was also proposed for the first time. The production rate distribution was discussed in detail for multifractured horizontal wells. The calculative results show the outermost fractures had higher production ratio due to larger drainage area and the inner fractures were lower due to the strong interface between fractures. Type curves were established to analyze the flow characteristics, which would be divided into six stages, for example, bilinear flow region, the first linear flow region, the first radial flow region, the second linear flow region, the second radial flow region, and the boundary dominated flow region, respectively. Finally, effects of some sensitive parameters on type curves were also analyzed in detail.

1. Introduction

The development of tight reservoirs has been paid more and more attention in China. Stimulation for a horizontal well in tight reservoirs may further enhance well productivity. Unlike a vertical well, a horizontal well may be more than one point along the well length. During the last two decades, horizontal wells have become a common applied completion in the petroleum industry. With a large reservoir contact area, horizontal wells can greatly improve well productivity; however, it is most advantageous to drill horizontal well in thin and tight reservoirs with vertical fractures [1, 2].

The main difficulty of transient-pressure analysis for multifractured horizontal wells (MFHWs) is how to solve those problems of multiple-fracture interference, finite conductivity transverse fractures coupled with the formation flow. Some typical research papers are worth reviewing [2–9]. Since 1972, some attempts have been made to simulate the pressure transient behavior for either horizontal or vertical wells, with or without hydraulic fractures. Larsen and Hegre [3] rigorously presented a transient pressure solution of horizontal wells with circular finite-conductivity fractures

in the three-dimensional unbounded formation. The results just showed the early-stage and middle-stage features of fracture system because the effects of outer boundary were not considered into their model. Guo et al. [2] presented the pressure transient behavior for a horizontal well with multiple randomly distributed vertical fractures in the infinite reservoir and bounded reservoir. Wan and Aziz [6] described a new semianalytical solution for horizontal wells with multiple hydraulic fractures. The fractures can be rotated at any horizontal angle to the well and need not fully penetrate the formation in the vertical direction. Al-Kobaisi et al. [7] presented a hybrid numerical analytical model with a finite-conductivity vertical fracture intercepted by a horizontal well, which dynamically couples a numerical fracture model with an analytical reservoir model. Their approach allows us to include finer details of the fractures characteristics while keeping the computational work manageable. Valkó and Amini [8] proposed a DVS (distributed volumetric sources) method to predict gas production for a horizontal well with multiple transverse fractures in a bounded reservoir. But it was only an approximate approach and the fracture conductivity was not considered in their model. Al Rbeawi

FIGURE 1: Physical model for multiple fractured horizontal wells.

and Tiab [9] introduced a new technique for interpreting the pressure transient behavior for a horizontal well with multiple infinite conductivity fractures which could be longitudinal or transverse, vertical or inclined, or symmetrical or asymmetrical. Recently, some meshless methods have been presented [10–13] and were possibly applied to solve the fracture problems.

The main objective of this paper is to develop a computational model to investigate the transient-pressure behavior for multifracted horizontal wells (MFHWs) in box-shaped reservoirs. A semianalytical solution for MFHWs in boxed reservoirs is obtained through using Laplace transform and Fourier transform. The fluid flow in the fracture system and reservoir system is computed separately, and the flux and pressure equivalent conditions in Laplace space are applied in the fracture wall to couple the fluids flow in both systems. The result is validated accurately through comparing with previous results in the literature.

2. Physical Model

Figures 1 and 2 show the physical model assumed and the assumptions of this model are as follows.

- (1) A single-phase flow was assumed in the reservoir. There is a fractured horizontal well in the boxed, closed, and homogeneous formation whose length is x_e , width is y_e , and height is h (see Figure 1).
- (2) The well produces with constant viscosity and slightly compressible fluid at the total flow rate of Q , but the flow rates from each fracture may change over time.
- (3) The oil production is assumed to be an isothermal process. Darcy flow is assumed in the fracture. The linear flow occurs in the fracture (see Figure 2).
- (4) Two wings of each fracture may be of unequal length, and the tips of the fractures are assumed as no-flow boundaries (see Figure 2).
- (5) The horizontal wellbore is assumed to be infinitely conductive and the fractures are assumed to be finitely conductive.
- (6) The effect of conductivity can be described by the common vertically fractured wells model.
- (7) At the starting time of production, the pressure is uniformly distributed and is equal to the initial pressure (p_i).

FIGURE 2: Flow model within the fractures.

3. Mathematical Model

3.1. Reservoir Model. To obtain the fracture solution in the boxed reservoirs, we must firstly obtain the point sink solution. The partial differential equation which is closed in all directions in the Cartesian coordinate system is given by

$$\phi c_t \frac{\partial p}{\partial t} - \left(\frac{\partial ((k/\mu) (\partial p / \partial x))}{\partial x} + \frac{\partial ((k/\mu) (\partial p / \partial y))}{\partial y} + \frac{\partial ((k/\mu) (\partial p / \partial z))}{\partial z} \right) = q_f (\gamma, t) \delta(x - x_w) \delta(y - y_w) \delta(z - z_w), \quad (1)$$

where ϕ is porosity, c_t is total compressibility, t is the time, p is reservoir pressure, k is reservoir permeability, q_f is point flux, δ is Dirac function, (x_w, y_w, z_w) is the point position, and x_e and y_e are the boundary length and width. All the boundaries of the box-shaped drainage volume are closed; therefore, boundary conditions can be given by

$$\left. \frac{\partial p}{\partial x} \right|_{x=0, x_e} = 0 \quad \left. \frac{\partial p}{\partial y} \right|_{y=0, y_e} = 0 \quad \left. \frac{\partial p}{\partial z} \right|_{z=0, z_e} = 0. \quad (2)$$

And the initial condition can be given by

$$p(x, y, z, 0) = p_i. \quad (3)$$

In order to simplify (1)–(3), we take the following dimensionless transforms:

$$\begin{aligned} p_D &= \frac{2\pi kh(p_i - p)}{Q\mu B}; & t_D &= \frac{kt}{\phi\mu c_t L^2}; \\ y_D &= \frac{y}{L}; & y_{wD} &= \frac{y_w}{L}; & y_{eD} &= \frac{y_e}{L}; \\ x_D &= \frac{x}{L}; & x_{wD} &= \frac{x_w}{L}; & x_{eD} &= \frac{x_e}{L}; \\ z_D &= \frac{z}{L}; & z_{wD} &= \frac{z_w}{L}; & z_{eD} &= \frac{z_e}{L}, \end{aligned} \quad (4)$$

where L is the characteristic length, q_f is the point convergence intensity at the point sink (x_w, y_w, z_w) , and the total productivity of the well is Q ; there holds

$$q_{fD} = \frac{q_f}{BQ}. \quad (5)$$

Then, through using dimensionless transforms, (1) becomes

$$\begin{aligned} \frac{\partial p_D}{\partial t_D} - \left(\frac{\partial^2 p_D}{\partial x_D^2} + \frac{\partial^2 p_D}{\partial y_D^2} + \frac{\partial^2 p_D}{\partial z_D^2} \right) \\ = 2\pi q_{fD}(\gamma_D, t_D) \delta(x_D - x_{wD}) \\ \times \delta(y_D - y_{wD}) \delta(z_D - z_{wD}). \end{aligned} \quad (6)$$

Equation (2) becomes

$$\left. \frac{\partial p_D}{\partial x_D} \right|_{x_D=0, x_{eD}} = 0 \quad \left. \frac{\partial p_D}{\partial y_D} \right|_{y_D=0, y_{eD}} = 0 \quad \left. \frac{\partial p_D}{\partial z_D} \right|_{z_D=0, z_{eD}} = 0. \quad (7)$$

And initial condition becomes

$$p_D(x_D, y_D, z_D, 0) = 0. \quad (8)$$

The Laplace transform is based on t_D and functions as follows:

$$\tilde{p}_D(r_D, s) = \int_0^\infty p_D(r_D, s) e^{-st_D} dt_D. \quad (9)$$

Applying the Laplace transforms to (6)–(8), we have

$$\begin{aligned} s\tilde{p}_D - \left(\frac{\partial^2 \tilde{p}_D}{\partial x_D^2} + \frac{\partial^2 \tilde{p}_D}{\partial y_D^2} + \frac{\partial^2 \tilde{p}_D}{\partial z_D^2} \right) \\ = 2\pi \tilde{q}_{fD}(s) \delta(x_D - x_{wD}) \\ \times \delta(y_D - y_{wD}) \delta(z_D - z_{wD}). \end{aligned} \quad (10)$$

Outer boundary conditions in Laplace space are

$$\left. \frac{\partial \tilde{p}_D}{\partial x_D} \right|_{x_D=0, x_{eD}} = 0 \quad \left. \frac{\partial \tilde{p}_D}{\partial y_D} \right|_{y_D=0, y_{eD}} = 0 \quad \left. \frac{\partial \tilde{p}_D}{\partial z_D} \right|_{z_D=0, z_{eD}} = 0. \quad (11)$$

Fourier cosine transform based on x_D can be defined as

$$\bar{p}_D(u_n) = \int_0^{x_{eD}} p_D \cos(u_n x_D) dx_D. \quad (12)$$

The characteristic equation is

$$\sin(u_n x_{eD}) = 0. \quad (13)$$

Through solving (13), characteristic number is obtained by

$$u_m = \frac{m\pi}{x_{eD}}. \quad (14)$$

Imposing the Fourier cosine transform on the variable x_D in (10), we have

$$\begin{aligned} s\bar{\tilde{p}}_D - \left(-u_m^2 \bar{\tilde{p}}_D + \frac{\partial^2 \bar{\tilde{p}}_D}{\partial y_D^2} + \frac{\partial^2 \bar{\tilde{p}}_D}{\partial z_D^2} \right) \\ = 2\pi \tilde{q}_{fD}(s) \cos(u_m x_{wD}) \\ \times \delta(y_D - y_{wD}) \delta(z_D - z_{wD}). \end{aligned} \quad (15)$$

Similarly, imposing the Fourier cosine transform on the variable y_D in (15), we have

$$\begin{aligned} s\bar{\bar{\tilde{p}}}_D - \left(-u_m^2 \bar{\bar{\tilde{p}}}_D - v_n^2 \bar{\bar{\tilde{p}}}_D + \frac{\partial^2 \bar{\bar{\tilde{p}}}_D}{\partial z_D^2} \right) \\ = 2\pi \tilde{q}_{fD}(s) \cos(u_m x_{wD}) \cos(v_n y_{wD}) \delta(z_D - z_{wD}). \end{aligned} \quad (16)$$

Similarly, imposing the Fourier cosine transform on the variable z_D in (16), we have

$$\bar{\bar{\bar{\tilde{p}}}}_D = \frac{2\pi \tilde{q}_{fD}(s) \cos(u_m x_{wD}) \cos(v_n y_{wD}) \cos(w_p z_{wD})}{s + u_m^2 + v_n^2 + w_p^2}, \quad (17a)$$

where

$$u_m = \frac{m\pi}{x_{eD}} \quad v_n = \frac{n\pi}{y_{eD}} \quad w_p = \frac{p\pi}{z_{eD}}. \quad (17b)$$

Imposing the Fourier inverse transform on the variable z_D in (17a) and (17b), we obtain

$$\begin{aligned} \frac{\bar{\bar{\tilde{p}}}_D}{\tilde{q}_{fD}(s)} \\ = \frac{1}{z_{eD}} \frac{2\pi \cos(u_m x_{wD}) \cos(v_n y_{wD})}{s + u_m^2 + v_n^2} \\ + \frac{2}{z_{eD}} \sum_{p=1}^{\infty} \cos(w_p z_D) \\ \times \frac{2\pi \cos(u_m x_{wD}) \cos(v_n y_{wD}) \cos(w_p z_{wD})}{s + u_m^2 + v_n^2 + w_p^2}. \end{aligned} \quad (18)$$

Imposing the Fourier inverse transform on the variable y_D in (18), we have

$$\begin{aligned} \frac{\bar{\tilde{p}}_D}{\tilde{q}_{fD}(s)} \\ = \frac{1}{y_{eD}} \frac{1}{z_{eD}} \frac{2\pi \cos(u_m x_{wD})}{s + u_m^2} \\ + \frac{2}{y_{eD}} \sum_{n=1}^{\infty} \frac{1}{z_{eD}} \cos(v_n y_D) \\ \times \frac{2\pi \cos(u_m x_{wD}) \cos(v_n y_{wD})}{s + u_m^2 + v_n^2} \\ + \frac{1}{y_{eD}} \frac{2}{z_{eD}} \sum_{p=1}^{\infty} \cos(w_p z_D) \\ \times \frac{2\pi \cos(u_m x_{wD}) \cos(w_p z_{wD})}{s + u_m^2 + w_p^2} \end{aligned}$$

$$\begin{aligned}
& + \frac{2}{y_{eD}} \frac{2}{z_{eD}} \sum_{n=1}^{\infty} \sum_{p=1}^{\infty} \cos(v_n y_D) \cos(w_p z_D) \\
& \times \frac{2\pi \cos(u_m x_{wD}) \cos(v_n y_{wD}) \cos(w_p z_{wD})}{s + u_m^2 + v_n^2 + w_p^2}.
\end{aligned} \quad (19)$$

Imposing the Fourier inverse transform on the variable x_D in (19), we have

$$\begin{aligned}
& \frac{\tilde{p}_D}{\tilde{q}_{fD}(s)} \\
& = \frac{2\pi}{x_{eD} y_{eD} z_{eD} s} + \frac{2}{x_{eD} y_{eD} z_{eD}} \\
& \times \sum_{m=1}^{\infty} \frac{2\pi \cos(u_m x_{wD}) \cos(u_m x_D)}{s + u_m^2} \\
& + \frac{2}{x_{eD} y_{eD}} \frac{1}{z_{eD}} \sum_{n=1}^{\infty} \cos(v_n y_D) \\
& \times \frac{2\pi \cos(v_n y_{wD})}{s + v_n^2} + \frac{4}{x_{eD} y_{eD} z_{eD}} \\
& \times \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \cos(v_n y_D) \cos(u_m x_D) \\
& \times \frac{2\pi \cos(v_n y_{wD}) \cos(u_m x_{wD})}{s + u_m^2 + v_n^2} \\
& + \frac{1}{x_{eD} y_{eD}} \frac{2}{z_{eD}} \sum_{p=1}^{\infty} \cos(w_p z_D) \frac{2\pi \cos(w_p z_{wD})}{s + w_p^2} \\
& + \frac{2}{x_{eD} y_{eD}} \frac{2}{z_{eD}} \\
& \times \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} \cos(w_p z_D) \cos(u_m x_D) \\
& \times \frac{2\pi \cos(u_m x_{wD}) \cos(w_p z_{wD})}{s + u_m^2 + w_p^2} \\
& + \frac{4}{x_{eD} y_{eD} z_{eD}} \sum_{n=1}^{\infty} \sum_{p=1}^{\infty} \cos(v_n y_D) \cos(w_p z_D) \\
& \times \frac{2\pi \cos(v_n y_{wD}) \cos(w_p z_{wD})}{s + v_n^2 + w_p^2} \\
& + \frac{8}{x_{eD} y_{eD} z_{eD}} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{p=1}^{\infty} \cos(u_m x_D) \\
& \times \cos(v_n y_D) \cos(w_p z_D) \\
& \times \frac{2\pi \cos(u_m x_{wD}) \cos(v_n y_{wD}) \cos(w_p z_{wD})}{s + u_m^2 + v_n^2 + w_p^2},
\end{aligned} \quad (20)$$

where we use the following formulas:

$$\sum_{k=1}^{\infty} \frac{\cos kx}{k^2 + \alpha^2} = \frac{\pi}{2\alpha} \frac{\cosh \alpha(\pi - x)}{\sinh \alpha\pi} - \frac{1}{2\alpha^2}. \quad (21)$$

Therefore, we have

$$\begin{aligned}
& \sum_{n=1}^{\infty} \frac{\cos((n\pi/y_{eD})(y_D - y_{wD}))}{s + (n\pi/y_{eD})^2} \\
& = \frac{y_{eD}}{2\sqrt{s}} \frac{\cosh \sqrt{s}(y_{eD} - (y_D - y_{wD}))}{\sinh \sqrt{s}y_{eD}} - \frac{1}{2s}
\end{aligned} \quad (22)$$

and the cosine product formula can be given by

$$\begin{aligned}
& 2 \cos(v_n y_D) \cos(v_n y_{wD}) \\
& = \cos(v_n (y_D - y_{wD})) + \cos(v_n (y_D + y_{wD})) \\
& = \cos\left(\frac{n\pi}{y_{eD}}(y_D - y_{wD})\right) + \cos\left(\frac{n\pi}{y_{eD}}(y_D + y_{wD})\right).
\end{aligned} \quad (23)$$

Through using (22)-(23), the following formulas can be obtained:

$$\begin{aligned}
& \sum_{n=1}^{\infty} \frac{2 \cos(v_n y_D) \cos(v_n y_{wD})}{s + v_n^2} \\
& = \sum_{n=1}^{\infty} (\cos((n\pi/y_{eD})(y_D - y_{wD})) \\
& \quad + \cos((n\pi/y_{eD})(y_D + y_{wD}))) \\
& \quad \times (s + v_n^2)^{-1} \\
& = \frac{y_{eD}}{\sqrt{s}} \frac{\cosh \sqrt{s}(y_{eD} - (y_D - y_{wD}))}{\sinh \sqrt{s}y_{eD}} - \frac{1}{2s} \\
& \quad + \frac{y_{eD}}{\sqrt{s}} \frac{\cosh \sqrt{s}(y_{eD} - (y_D + y_{wD}))}{\sinh \sqrt{s}y_{eD}} - \frac{1}{2s} \\
& = \frac{y_{eD}}{\sqrt{s}} \\
& \quad \times (\cosh \sqrt{s}(y_{eD} - (y_D - y_{wD})) \\
& \quad + \cosh \sqrt{s}(y_{eD} - (y_D + y_{wD}))) \\
& \quad \times (\sinh \sqrt{s}y_{eD})^{-1} - \frac{1}{s}, \\
& \sum_{n=1}^{\infty} \frac{2 \cos(v_n y_D) \cos(v_n y_{wD})}{s + u_m^2 + v_n^2} \\
& = \frac{y_{eD}}{2\sqrt{s + u_m^2}} \\
& \quad \times (\cosh \sqrt{s + u_m^2}(y_{eD} - (y_D - y_{wD})))
\end{aligned}$$

$$\begin{aligned}
& + \cosh \sqrt{s + u_m^2} (y_{eD} - (y_D + y_{wD})) \Big) \\
& \times \left(\sinh \sqrt{s + u_m^2} y_{eD} \right)^{-1} - \frac{1}{s + u_m^2}, \\
& \sum_{p=1}^{\infty} \frac{2 \cos(v_n y_D) \cos(v_n y_{wD})}{s + v_n^2 + w_p^2} \\
& = \frac{y_{eD}}{2 \sqrt{s + w_p^2}} \\
& \times \left(\cosh \sqrt{s + w_p^2} (y_{eD} - (y_D - y_{wD})) \right. \\
& \quad \left. + \cosh \sqrt{s + w_p^2} (y_{eD} - (y_D + y_{wD})) \right) \\
& \times \left(\sinh \sqrt{s + w_p^2} y_{eD} \right)^{-1} \\
& - \frac{1}{s + w_p^2}, \\
& \sum_{p=1}^{\infty} \frac{2 \cos(v_n y_D) \cos(v_n y_{wD})}{s + u_m^2 + v_n^2 + w_p^2} \\
& = \frac{y_{eD}}{2 \sqrt{s + u_m^2 + w_p^2}} \\
& \times \left(\cosh \sqrt{s + u_m^2 + w_p^2} (y_{eD} - (y_D - y_{wD})) \right. \\
& \quad \left. + \cosh \sqrt{s + u_m^2 + w_p^2} (y_{eD} - (y_D + y_{wD})) \right) \\
& \times \left(\sinh \sqrt{s + u_m^2 + w_p^2} y_{eD} \right)^{-1} \\
& - \frac{1}{s + u_m^2 + w_p^2}.
\end{aligned} \tag{24}$$

Substituting (21)–(24) into (20), we can have

$$\begin{aligned}
& \frac{\tilde{p}_D}{\tilde{q}_{fD}(s)} \\
& = \frac{2\pi}{x_{eD} y_{eD} z_{eD} s} + \frac{2\pi}{x_{eD} y_{eD} z_{eD}} \\
& \times \sum_{m=1}^{\infty} \frac{2 \cos(u_m x_{wD}) \cos(u_m x_D)}{s + u_m^2} + \frac{2\pi}{x_{eD} y_{eD} z_{eD}} \\
& \times \left[\frac{y_{eD}}{2 \sqrt{s}} \right. \\
& \quad \times \left(\cosh \sqrt{s} (y_{eD} - (y_D - y_{wD})) \right. \\
& \quad \left. + \cosh \sqrt{s} (y_{eD} - (y_D + y_{wD})) \right) \\
& \quad \left. \times \left(\sinh \sqrt{s} y_{eD} \right)^{-1} - \frac{1}{s} \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{4\pi}{x_{eD} y_{eD} z_{eD}} \sum_{m=1}^{\infty} \cos(u_m x_{wD}) \cos(u_m x_D) \\
& \times \left[\frac{y_{eD}}{2 \sqrt{s + u_m^2}} \right. \\
& \quad \times \left(\cosh \sqrt{s + u_m^2} (y_{eD} - (y_D - y_{wD})) \right. \\
& \quad \left. + \cosh \sqrt{s + u_m^2} (y_{eD} - (y_D + y_{wD})) \right) \\
& \quad \left. \times \left(\sinh \sqrt{s + u_m^2} y_{eD} \right)^{-1} - \frac{1}{s + u_m^2} \right] \\
& + \frac{2}{x_{eD} y_{eD} z_{eD}} \\
& \times \sum_{p=1}^{\infty} \frac{2\pi \cos(w_p z_D) \cos(w_p z_{wD})}{s + w_p^2} \\
& + \frac{4}{x_{eD} y_{eD} z_{eD}} \\
& \times \sum_{p=1}^{\infty} \cos(w_p z_D) \cos(u_m x_D) \\
& \times \sum_{m=1}^{\infty} \frac{2\pi \cos(u_m x_{wD}) \cos(w_p z_{wD})}{s + u_m^2 + w_p^2} \\
& + \frac{4\pi}{x_{eD} y_{eD} z_{eD}} \sum_{p=1}^{\infty} \cos(w_p z_D) \cos(w_p z_{wD}) \\
& \times \left[\frac{y_{eD}}{2 \sqrt{s + w_p^2}} \right. \\
& \quad \times \left(\cosh \sqrt{s + w_p^2} (y_{eD} - (y_D - y_{wD})) \right. \\
& \quad \left. + \cosh \sqrt{s + w_p^2} (y_{eD} - (y_D + y_{wD})) \right) \\
& \quad \left. \times \left(\sinh \sqrt{s + w_p^2} y_{eD} \right)^{-1} - \frac{1}{s + w_p^2} \right] \\
& + \frac{8\pi}{x_{eD} y_{eD} z_{eD}} \sum_{m=1}^{\infty} \cos(u_m x_{wD}) \cos(u_m x_D) \\
& \times \sum_{p=1}^{\infty} \cos(w_p z_D) \cos(w_p z_{wD}) \\
& \times \left[\frac{y_{eD}}{2 \sqrt{s + u_m^2 + w_p^2}} \right.
\end{aligned}$$

$$\begin{aligned} & \times \left(\cosh \sqrt{s + u_m^2 + w_p^2} (y_{eD} - (y_D - y_{wD})) \right. \\ & \quad \left. + \cosh \sqrt{s + u_m^2 + w_p^2} (y_{eD} - (y_D + y_{wD})) \right) \\ & \times \left(\sinh \sqrt{s + u_m^2 + w_p^2} y_{eD} \right)^{-1} - \frac{1}{s + u_m^2 + w_p^2} \Bigg]. \end{aligned} \quad (25)$$

Further, (25) could be simplified as the following equation:

$$\begin{aligned} & \frac{\tilde{p}_D}{\tilde{q}_{fD}(s)} \\ &= \frac{2\pi}{x_{eD} y_{eD} z_{eD}} \\ & \times \left[\frac{y_{eD}}{2\sqrt{s}} \left(\cosh \sqrt{s} (y_{eD} - (y_D - y_{wD})) \right. \right. \\ & \quad \left. \left. + \cosh \sqrt{s} (y_{eD} - (y_D + y_{wD})) \right) \right. \\ & \quad \left. \times (\sinh \sqrt{s} y_{eD})^{-1} \right] \\ & + \frac{4\pi}{x_{eD} y_{eD} z_{eD}} \sum_{m=1}^{\infty} \cos(u_m x_{wD}) \cos(u_m x_D) \frac{y_{eD}}{2\sqrt{s + u_m^2}} \\ & \times \left(\cosh \sqrt{s + u_m^2} (y_{eD} - (y_D - y_{wD})) \right. \\ & \quad \left. + \cosh \sqrt{s + u_m^2} (y_{eD} - (y_D + y_{wD})) \right) \\ & \times \left(\sinh \sqrt{s + u_m^2} y_{eD} \right)^{-1} \\ & + \frac{4\pi}{x_{eD} y_{eD} z_{eD}} \sum_{p=1}^{\infty} \cos(w_p z_D) \cos(w_p z_{wD}) \frac{y_{eD}}{2\sqrt{s + w_p^2}} \\ & \times \left(\cosh \sqrt{s + w_p^2} (y_{eD} - (y_D - y_{wD})) \right. \\ & \quad \left. + \cosh \sqrt{s + w_p^2} (y_{eD} - (y_D + y_{wD})) \right) \\ & \times \left(\sinh \sqrt{s + w_p^2} y_{eD} \right)^{-1} \\ & + \frac{8\pi}{x_{eD} y_{eD} z_{eD}} \sum_{m=1}^{\infty} \cos(u_m x_{wD}) \cos(u_m x_D) \\ & \times \sum_{p=1}^{\infty} \cos(w_p z_D) \cos(w_p z_{wD}) \frac{y_{eD}}{2\sqrt{s + u_m^2 + w_p^2}} \\ & \times \left(\cosh \sqrt{s + u_m^2 + w_p^2} (y_{eD} - (y_D - y_{wD})) \right. \\ & \quad \left. + \cosh \sqrt{s + u_m^2 + w_p^2} (y_{eD} - (y_D + y_{wD})) \right) \\ & \times \left(\sinh \sqrt{s + u_m^2 + w_p^2} y_{eD} \right)^{-1}. \end{aligned} \quad (26)$$

Equation (26) is the point sink solution in box-shaped reservoirs.

3.2. *Fracture Model.* Fluid flow inside n th fracture in the system may be defined by the following set of equations:

$$\begin{aligned} & \frac{\partial((k_{fn}/\mu)(\partial p_{fn}/\partial z_n))}{\partial z_n} \\ & + \frac{q_{whfn}}{w_{fn} h} \delta(z_n - z_{whfn}) - \Omega_{fn}(z_n, t) = 0, \end{aligned} \quad (27)$$

where

$$\Omega_{fn}(z_n, t) = \int_0^{L_{fn}} \frac{q_{zfn}(z_n, t)}{w_{fn} h L_{fn}} \delta(z_n - \gamma_n) d\gamma_n, \quad (28)$$

where z_{whfn} is the well position in the fracture, k_{fn} is the fracture permeability, q_{whfn} is well rate in the fracture, L_{fn} is the fracture length, γ_n is the source position, h is the reservoir thick, and q_{zfn} is the fracture flux. In order to simplify (27), we take the following dimensionless transforms:

$$\begin{aligned} P_{fD} &= \frac{2\pi k h (p_i - p_{fn})}{BQ\mu}, \\ C_{fDn} &= \frac{k_{fn} w_{fn}}{k L_{fn}}, \quad q_{whfDn} = \frac{q_{whfn}}{Q}, \\ L_{fDn} &= \frac{L_{fn}}{L}, \\ \gamma_{Dn} &= \frac{\gamma_n}{L}, \quad z_{Dn} = \frac{z_n}{L}, \\ z_{whfDn} &= \frac{z_{whfn}}{L}, \quad q_{zfDn} = \frac{q_{zfn}}{Q}. \end{aligned} \quad (29)$$

Therefore, the following equations can be obtained by

$$\begin{aligned} & \frac{\partial^2 P_{fDn}}{\partial z_{Dn}^2} + \frac{2\pi q_{whfDn}}{L_{fDn} C_{fDn}} \delta(z_{Dn} - z_{whfDn}) \\ & - \int_0^{L_{fDn}} \frac{2\pi q_{zfDn}(z_{Dn}, t_D)}{L_{fDn}^2 C_{fDn}} \delta(z_{Dn} - \gamma_{Dn}) d\gamma_{Dn} = 0. \end{aligned} \quad (30)$$

The initial condition can be given by

$$P_{fDn}(z_{Dn}, 0) = 0. \quad (31)$$

The boundary conditions are given by

$$\begin{aligned} & \frac{\partial P_{fDn}(0, t_D)}{\partial z_{Dn}} = 0, \\ & \frac{\partial P_{fDn}(L_{fDn}, t_D)}{\partial z_{Dn}} = 0. \end{aligned} \quad (32)$$

Imposing Laplace transforms on t_D in (30)–(32), we have

$$\begin{aligned} \frac{\partial^2 \tilde{p}_{fDn}}{\partial z_{Dn}^2} + \frac{2\pi \tilde{q}_{whfDn}}{L_{fDn} C_{fDn}} \delta(z_{Dn} - z_{whfDn}) \\ - \int_0^{L_{fDn}} \frac{2\pi \tilde{q}_{zfDn}(z_{Dn}, s)}{L_{fDn}^2 C_{fDn}} \delta(z_{Dn} - \gamma_{Dn}) d\gamma_{Dn} = 0. \end{aligned} \quad (33)$$

The boundary conditions become

$$\begin{aligned} \frac{\partial \tilde{p}_{fDn}(0, s)}{\partial z_{Dn}} &= 0, \\ \frac{\partial \tilde{p}_{fDn}(L_{fDn}, s)}{\partial z_{Dn}} &= 0. \end{aligned} \quad (34)$$

3.3. Semianalytical Solution. The reservoir- and fracture-flow equations are described in Sections 3.1 and 3.2. To build the reservoir-flow model, Fourier transformation on space and Laplace transformation on time have been applied and point sink solution has been also obtained in Section 3.1. Thus, line sink solution can be derived by integrating the solution given in (26) with respect to z_{wD} over 0 to z_{eD} . (Note that only the integral of $\cos(w_p z_{wD})$ needs to be determined.) The result can be given by

$$\tilde{p}_D = \tilde{q}_{fD}(\gamma_D, s) F(x_D, \gamma_D, x_{wD}, \gamma_{wD}, x_{eD}, \gamma_{eD}, s), \quad (35)$$

where

$$\begin{aligned} F(x_D, \gamma_D, x_{wD}, \gamma_{wD}, x_{eD}, \gamma_{eD}, s) \\ = \frac{\pi}{x_{eD}} \\ \times (\cosh \sqrt{s} (y_{eD} - (y_D - \gamma_{wD}))) \\ + \cosh \sqrt{s} (y_{eD} - (y_D + \gamma_{wD}))) \\ \times (\sqrt{s} \sinh \sqrt{s} y_{eD})^{-1} \\ + \frac{2\pi}{x_{eD}} \sum_{m=1}^{\infty} \cos(u_m x_{wD}) \cos(u_m x_D) \\ \times \left(\cosh \sqrt{s + u_m^2} (y_{eD} - (y_D - \gamma_{wD})) \right. \\ \left. + \cosh \sqrt{s + u_m^2} (y_{eD} - (y_D + \gamma_{wD})) \right) \\ \times \left(\sqrt{s + u_m^2} \sinh \sqrt{s + u_m^2} y_{eD} \right)^{-1}. \end{aligned} \quad (36)$$

Fracture number for MFHW is N_f ; the pressure drop formulas at each fracture segment for MFHW can be obtained

through using superposition principle and point sink integral method,

$$\begin{aligned} \tilde{p}_D = \sum_{n=1}^{N_f} \int_0^{L_{fDn}} \tilde{q}_{fDn}(\gamma_D, s) \\ \times F(x_D, \gamma_D, x_{fDn} + \gamma_{Dn} \cos \theta_{fn}, \gamma_{fDn} \\ + \gamma_{Dn} \sin \theta_{fn}, x_{eD}, \gamma_{eD}, s) d\gamma_D, \end{aligned} \quad (37)$$

where θ_{fn} is the angle between n th fracture and x -axis, F is defined in (36), and γ_{Dn} is source position in the fracture. Now, we divide per fracture into N_{sn} segments; thus, pressure drop formulas for j th segment of n th fracture can be given by

$$\begin{aligned} \tilde{p}_{Dj} = \sum_{n=1}^{N_f} \sum_{i=1}^{N_{sn}} \tilde{q}_{fDni}(\gamma_D, s) \\ \times F(x_{Dj}, \gamma_{Dj}, x_{fDn} + \gamma_{Dn} \cos \theta_{fn}, \gamma_{fDn} \\ + \gamma_{Dn} \sin \theta_{fn}, x_{eD}, \gamma_{eD}, s) d\gamma_D. \end{aligned} \quad (38)$$

To obtain the solution for the fracture-flow models, Laplace transformations on both space and time have been carried out. The pressure-drop evaluation for a fracture is presented in the following equation. The pressure drop at k th segment on the n th fracture is given by

$$\begin{aligned} \tilde{p}_{fDnk}(z_{Dnk}, s) = \tilde{p}_{fDn}(0, s) \\ + \frac{1}{L_{fDn}^2 C_{fDn}} R_1 - \frac{2\pi}{L_{fDn} C_{fDn}} R_2, \end{aligned} \quad (39)$$

where

$$\begin{aligned} R_1 = \sum_{i=1}^{k-1} \tilde{q}_{fDni} \left(\frac{1}{2} + k - i \right) \Delta z_{Dn}^2 + \tilde{q}_{fDnk} \frac{\Delta z_{Dn}^2}{8}, \\ R_2 = \tilde{q}_{whfDn}(z_{whfDn}, s) [z_{fDnk} - z_{whfDn}] \\ \times H(z_{fDnk} - z_{whfDn}), \end{aligned} \quad (40)$$

where H is the Heaviside unit step function.

For each fracture, the total flux equation can be given by

$$\sum_{i=1}^{N_{sn}} \tilde{q}_{fDni} \Delta x_{Dn} = \tilde{q}_{whfDn} \quad (41)$$

and the pressure for each fracture is equal to the pressure of horizontal well; it must have

$$\begin{aligned} \tilde{p}_{fD1}(z_{whfD1}, s) \\ = \tilde{p}_{fD2}(z_{whfD2}, s) \cdots = \tilde{p}_{fDN_f-1}(z_{whfDN_f-1}, s) \\ = \tilde{p}_{fDN_f}(z_{whfDN_f}, s). \end{aligned} \quad (42)$$

TABLE 1: Basic data.

Parameters	Dimensionless value
Fracture number N_f	1
Fracture segment number N_{sn}	16
Fracture position x_{fDn}	2000
Fracture position y_{fDn}	2000
Fracture length L_{fDn}	1
Well position Z_{whfDn}	0.5
Region length x_{eD}	5000
Region width y_{eD}	5000
Fracture angle θ_{fn}	$\pi/2$

FIGURE 3: Validation of solutions presented in this paper.

In addition, total rate of horizontal wells should be equal to sum of flux for each fracture; thus,

$$\sum_{i=1}^{N_f} \tilde{q}_{whfDn} = \frac{1}{s}. \quad (43)$$

Coupling the fracture- and reservoir-flow models at the fracture faces, we obtain a system of $N_{ts} + N_f + N_f$ equations with $N_{ts} + N_f + N_f$ unknowns. N_{ts} is the total number of fracture segments, and N_f is the total number of fractures. This system of equations could be rearranged and written in the matrix format. Once the matrix is solved for unknowns, the rate at each fracture segment and the pressure drop at each fracture will be obtained. These rates and pressure drops are used to calculate the pressure drop at the wellbore for MFHW.

4. Results and Discussion

4.1. Validation of Solutions in This Paper. Riley et al. [14] presented an analytical solution for a single fracture case. To validate the solutions proposed in this paper, some data were utilized (see Table 1) for comparing the solutions with those from Riley et al. [14] (see Figure 3). For the convenience of the comparison, only a fracture is considered into the MFHWs. As shown in Figure 3, there is a good agreement between the solutions obtained in this work and the results presented by

FIGURE 4: The production rate distribution of MFHWs with five transversal fractures.

FIGURE 5: Type curves used to analyze the flow characteristics.

Riley. This might indicate that previous models are special cases of our MFHWs model and also implies that our model is validated accurately.

4.2. The Production Rate Distribution. The production rate distribution is presented in Figure 4 and the data is shown in Table 2. In this case, five transversal fractures have identical length and uniform distribution along a horizontal well located in a box-shaped reservoir. As shown in Figure 4, the outermost fractures have higher production ratio due to larger drainage area and the inner fractures are lower due to strong interference between fractures. For a specific fracture, the flux distribution along the discrete elements has the same property as the whole fracture system.

4.3. Flow Characteristics Analysis. The transient transport characteristics are graphically showed by type curves, which can be used to analyzing transient pressure and rate decline so as to recognize the flow characteristics of fluids in the reservoir. In addition, by type curves matching some reservoir property parameters, such as permeability, skin factor, gas in

TABLE 2: Basic data.

Parameters	Frac1	Frac2	Frac3	Frac4	Frac5
Fracture number N_f	5	5	5	5	5
Fracture segment number N_{sn}	20	20	20	20	20
Fracture position x_{fDn}	2000	2000	2000	2000	2000
Fracture position y_{fDn}	1900	1950	2000	2050	2100
Fracture length L_{fDn}	1	1	1	1	1
Well position Z_{whfDn}	0.5	0.5	0.5	0.5	0.5
Region length x_{eD}	4000	4000	4000	4000	4000
Region width y_{eD}	4000	4000	4000	4000	4000
Fracture angle θ_{fn}	$\pi/2$	$\pi/2$	$\pi/2$	$\pi/2$	$\pi/2$
Fracture conductivity C_{fDn}	6π	6π	6π	6π	6π

place, fracture half-length, and gas reservoir drainage area, can be obtained [15, 16].

The pressure and its derivative curves are presented in Figure 5, which shows flow characteristics for a fractured horizontal well with 5 identical lengths and uniform-distribution fractures, and the used data is shown in Table 2. As shown in Figure 5, the flow can be divided into six stages, which could be corresponding with Figure 6 and described as follows.

Stage 1. In this stage, the segment has a straight line with 1/4 slope, reflecting the bilinear flow region (see Figure 6(a)). In this region, fluids flow from fracture to wellbore and from reservoirs to fracture at the same time. This region only occurs when the fracture conductivity is relatively small.

Stage 2. The segment has a 1/2 slope straight line on both pressure and pressure derivative curves, namely, the first linear flow region. In this region, flow occurs linearly and directly from formation to individual fractures and each fracture behaves independently of the other fractures (see Figure 6(b)). The linear flow is the optimal flow mode because it can reduce the seepage resistance.

Stage 3. If the pressure derivative curve shows the $1/(2N)$ constant, the first radial flow region will be observed. In this region, radial flow occurs directly (see Figure 6(c)) from formation to individual fractures and the continuous time of radial flow for each fracture depends on the fracture length and fracture spacing. A big fracture length or big spacing between the fractures will lead to a short time for radial flow.

Stage 4. The segment has a 0.36 slope straight line on both pressure and pressure derivative curves, namely, the second linear flow region. The reservoir linear flow occurs in this region (Figure 6(d)). Fluids flow will start to occur when the pressure wave spreads farther. The linear flow starts from reservoir to the vertical plane which contains the horizontal wellbore.

Stage 5. The segment has a straight line with 0.5 constant, namely, the second radial flow region (Figure 6(e)). During

this flow period, flow across the outer most producing elements becomes dominant.

Stage 6. The segment has a unit slope straight line on both pressure and pressure derivative curves, namely, boundary dominated flow region. This flow period occurs when the reservoir boundary is reached (Figure 6(f)). Due to the fractured-horizontal wells centered in the reservoir, the fluids flow reached the six boundaries nearly at the same time. If the fractured-horizontal wells were not centered in the reservoir, the characteristics of boundary dominated flow may be not a unit slope straight line on both pressure and pressure derivative curves.

The proposed flow regions above may not exist in a single test. Depending on the specific properties of fracture and reservoir, some of the flow regions may be absent. For the moderate ratios of fracture length and spacing, the boundary dominated flow region may be nonexistent or replaced by a transitional flow region. It is also possible that radial flow region may not exist because of well interference or boundary effects. Admittedly, affected by fracture conductivity and relative position, some characteristics of flow pattern may not clearly emerge at times. In the real case, engineers should analyze the results of flow regions according to the concrete case.

4.4. Effects of Sensitive Parameters on Type Curves. Based on the presented model, the main factors affecting transient-pressure characteristics of fractured horizontal wells in coal reservoirs are presented in Figures 7–9. Those parameters are including fracture number, fracture conductivity C_{fDn} , and reservoir width y_{eD} . The basic used parameters are listed in Table 2.

The effect of fracture number is shown in Figure 7 and fracture number can be set to 3 and 7, respectively. It appears that the fracture number mainly affects the flow characteristics of bilinear flow region, the first linear flow region, and the first radial flow region, which could be seen on the pressure derivative curve. It can be seen from pressure derivative curve that the time of the first radial flow region

FIGURE 6: Six kinds of different flow patterns for MFHWs.

with big fracture number is longer than that with small fracture number and the time of the second linear flow region with small fracture number is longer than that with big fracture number. It is known that the production mainly benefits from the second linear flow region, and the longer the second linear flow region is, the better the production is. Therefore, through the analysis above, increasing the fracture number is helpful to production, but it is not the more

the fracture, the better the development. It can be analyzed from the pressure curve that pressure curve with big fracture number is always higher than the one with small fracture number, which means the increasing fractures number will cause small pressure depletion.

The effect of fracture conductivity is shown in Figure 8 and fracture conductivity can be set to 6 and 7, respectively. The derivative curve shows that the effect of the fracture

FIGURE 7: The effect of fracture number on type curve.

conductivity on flow characteristics occurs obviously in the bilinear flow region. As shown in the figure, the bilinear flow characteristic will gradually disappear as the conductivity increases; finally, linear flow will appear when $C_{fD} > 300$. It can be also found from pressure curve that the pressure with small conductivity is higher than the one with big conductivity, which means fracture with high conductivity will cause big pressure depletion at early time.

The effect of reservoir width is shown in Figure 9 and reservoir width can be set to 1000 and 4000, respectively. Reservoir width mainly affects the boundary dominated flow region. It can be found from the pressure curve that pressure with big reservoir width is higher than the one with small reservoir width, which means that a small reservoir width will cause big pressure depletion in the boundary dominated flow region. We also see from the pressure derivative curve that a small reservoir width value will shorten the time of the second radial flow and reach the boundary dominated flow region early.

5. Conclusions

Through using Fourier cosine transform and Laplace transform method, a semianalytical solution for multifractured horizontal wells in box-shaped reservoirs is presented to investigate the pressure transient behavior. The accuracy of the semianalytical solutions is verified by graphically comparing them with the published analytical solutions which are special cases of this paper. According to the presented solutions, the production rate distribution is also proposed for MFHWs, which show the outermost fractures have higher production ratio due to larger drainage area and the inner fractures are lower due to the strong interface between fractures. Type curves are established to analyze the flow characteristics, which can be divided into six stages. Effects of main sensitive parameters on type curves are analyzed in detail including fracture number, fracture conductivity, and reservoir width. The presented model could be applicable to

FIGURE 8: The effect of fracture conductivity on type curve.

FIGURE 9: The effect of reservoir width on type curve.

develop well test soft for MFHWs, which will be our future research topic.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

- [1] B. E. Roberts, H. van Engen, and C. P. J. W. van Kruysdijk, "Productivity of multiply fractured horizontal wells in tight gas reservoirs," in *Proceedings of the Offshore Europe Conference*, pp. 133–141, September 1991, SPE paper no. 23113.

- [2] G. Guo, R. D. Evans, and M. M. Chang, "Pressure-transient behavior for a horizontal well intersecting multiple random discrete fractures," in *Proceedings of the SPE Annual Technical Conference & Exhibition*, pp. 277–290, September 1994, SPE paper no. 28390.
- [3] L. Larsen and T. M. Hegre, "Pressure transient analysis of multi-fractured horizontal wells," in *Proceedings of the SPE Annual Technical Conference & Exhibition*, pp. 265–276, September 1994, SPE paper no. 28389.
- [4] R. N. Horne and K. O. Temeng, "Relative productivities and pressure transient modeling of horizontal wells with multiple fractures," in *Proceedings of the 9th Middle East Oil Show & Conference*, pp. 563–574, March 1995, SPE paper no. 29891.
- [5] C. C. Chen and R. Raghavan, "A multiply-fractured horizontal well in a rectangular drainage region," in *Proceedings of the 2nd International Conference on Horizontal Well Technology*, pp. 311–319, November 1996, SPE paper no. 37072.
- [6] J. Wan and K. Aziz, "Semi-analytical well model of horizontal wells with multiple hydraulic fractures," *SPE Journal*, vol. 7, no. 4, pp. 437–445, 2002.
- [7] M. Al-Kobaisi, E. Ozkan, and H. Kazemi, "A hybrid numerical-analytical model of finite-conductivity vertical fractures intercepted by a horizontal well," in *Proceedings of the SPE International Petroleum Conference in Mexico*, pp. 599–609, November 2004, SPE paper no. 92040.
- [8] P. P. Valkó and S. Amini, "The method of distributed volumetric sources for calculating the transient and pseudosteady-state productivity of complex well-fracture configurations," in *Proceedings of the SPE Hydraulic Fracturing Technology Conference*, pp. 426–439, January 2007, SPE paper no. 106279.
- [9] S. Al Rbeawi and D. Tiab, "Transient pressure analysis of a horizontal well with multiple inclined hydraulic fractures using type-curve matching," in *Proceedings of the SPE International Symposium and Exhibition on Formation Damage Control*, pp. 1–20, 2012, SPE paper no. 149902.
- [10] P. Zhu, L. W. Zhang, and K. M. Liew, "Geometrically nonlinear thermomechanical analysis of moderately thick functionally graded plates using a local Petrov-Galerkin approach with moving Kriging interpolation," *Composite Structures*, vol. 107, pp. 298–314, 2014.
- [11] L. W. Zhang, P. Zhu, and K. M. Liew, "Thermal buckling of functionally graded plates using a local Kriging meshless method," *Composite Structures*, vol. 108, pp. 472–492, 2014.
- [12] L. W. Zhang, Z. X. Lei, K. M. Liew, and J. L. Yu, "Static and dynamic of carbon nanotube reinforced functionally graded cylindrical panels," *Composite Structures*, vol. 111, pp. 205–212, 2014.
- [13] K. M. Liew, Z. X. Lei, J. L. Yu, and L. W. Zhang, "Postbuckling of carbon nanotube-reinforced functionally graded cylindrical panels under axial compression using a meshless approach," *Computer Methods in Applied Mechanics and Engineering*, vol. 268, pp. 1–17, 2014.
- [14] M. F. Riley, W. E. Brigham, and R. N. Horne, "Analytic solutions for elliptical finite-conductivity fractures," in *Proceedings of the SPE Annual Technical Conference and Exhibition*, pp. 35–48, October 1991, SPE paper no. 22656.
- [15] L. Wang, X. D. Wang, J. Q. Li, and J. Wang, "Simulation of pressure transient behavior for asymmetrically fractured wells in coal reservoirs," *Transport in Porous Media*, vol. 97, no. 3, pp. 352–372, 2013.
- [16] R.-S. Nie, Y.-F. Meng, J.-C. Guo, and Y.-L. Jia, "Modeling transient flow behavior of a horizontal well in a coal seam," *International Journal of Coal Geology*, vol. 92, pp. 54–68, 2012.