Research Article

Robust $D$-Stability Controller Design for a Ducted Fan Unmanned Aerial Vehicle

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This paper deals with the aerodynamic modeling of a small ducted fan UAV and the problem of attitude stabilization when the parameter of the vehicle is varied. The main aerodynamic model of the hovering flight UAV is first presented. Then, an attitude control is designed from a linearization of the dynamic model around the hovering flight, which is based on the $H_{\infty}$ output feedback control theory with $D$-stability. Simulation results show that such method has good robustness to the attitude system. They can meet the requirements of attitude control and verify further the feasibility of such a control strategy.

1. Introduction

The ducted fan unmanned aerial vehicle is a new popular UAV in the recent years. Almost all system forces and moments come from the ducted fan. This UAV with VTOL (vertical takeoff and landings) and hovering flight capability is shown in Figure 1.

The ducted fan UAV can be adapted to a variety of complex environments and complete difficult flight missions [1, 2]. It is small with a compact and flexible layout. It is capable of maneuvering in any angle in addition to the normal hover and vertical takeoff and landing capabilities; therefore it can complete some special tasks [3, 4]. The peripheral duct not only enables the fan to be effectively protected, but also provides some aerodynamic lift for the vehicle’s flight. The experiments show that the ducted unique geometry can provide some aerodynamic lift for the vehicle [5].

Compared with ordinary fixed-wing aircraft, ducted fan UAV has such advantages.

1. Flexibility. Fixed-wing aircraft needs ground support in takeoff and landing. So it cannot provide timely information assurance to combat team in the city and mountain area. The ducted fan UAV can do vertical takeoff and landing and hover to adapt to the complex environment of the city or mountain.

2. Compact Structure and High Propulsive Force. Compared with unmanned helicopter, in the same power consumption, the ducted fan will produce greater tension than isolated propeller with the same diameter. The ducted fan UAV has more compact structure, less drag. And its flight attitude is closer to fixed-wing aircraft. So it can fly faster than the equivalent helicopter.

3. Low Noise, More Concealment, and Better Security. The propeller is placed in the ducted fan, which can obstruct the transmission of aerodynamic noise. So the noise intensity and propagation distance can be reduced to some extent. Meanwhile the engine is surrounded by the ducted fan, which can obstruct the engine’s thermal radiation, and then the vehicle’s thermal radiation characteristic can be reduced. So the ducted fan UAV has better concealment, which can provide better survivability to the vehicle in the battle.

Compared with traditional fixed-wing aircraft, the flow field distribution of the ducted fan UAV is more complex, which makes the vehicle’s modeling difficult. The attitude control is the key problem in the ducted fan UAV control, and it will affect the vehicle’s other actions. In the flight, because the vehicle’s flow field is complex [8–13], the nonlinear effect is obvious [14–17], and these will be big problems in the vehicle’s
control. In references [18, 19], an evolutionary pinning control is put toward which is applied in unmanned aerial vehicle coordination. Reference [20] used a PID control method. The PID method has a simple control structure and a low calculation amount, but the parameter adjustment is too cumbersome, and it has a poor adaptability for the coupling between the axes. Reference [21] used a robust $H_\infty$ method. The $H_\infty$ control has good inhibitory effect on the outside interference, but it is conservative to the uncertainty of the system. Reference [22] used a feedback linearization method, which has a good antijamming capability, but complex calculations.

This paper uses $H_\infty$ output feedback control with $D$-stability for ducted fan UAV attitude control system with parameter uncertainty. The status of the system may be difficult to observe in the practical engineering. So the output variable which can be observed is used as feedback variable in this paper. Therefore, the output feedback control system has good physical realizability. Using $H_\infty$ control method can resolve the uncertainties of the system but cannot provide location information of the closed-loop poles. $H_\infty$ control method used in this paper which is combined with regional pole assignment not only ensures the robustness of control system, but also places the poles of the closed-loop system in a specified region. Dynamic performance of the UAV can be satisfied. This controller is designed by solving a set of linear matrix inequalities. Simulation results show this controller not only guarantees the closed-loop system to be $D$-stable, but also achieves the given $H_\infty$ disturbance index. The vehicle can meet the requirements of attitude control and has good practicality, which illustrates the effectiveness of the proposed method.

2. UAV Attitude Modeling

2.1. Rotor Aerodynamics. Compared with the fixed-wing aircraft, the ducted fan vehicle has a more complicated air environment. The modeling of the fan is the main difficulty to the modeling of the whole vehicle.

There are two theories to study the fan, the momentum theory, and the blade element theory. This paper uses the two theories above to present the fan’s aerodynamics equations.

The thrust can be expressed as [23]

$$ T_{\text{fan}} = (\nu_b - \nu_i) \cdot \frac{\rho \cdot \omega \cdot a \cdot n \cdot d \cdot R^2}{4}, $$

(1)

where $\omega$ represents angular velocity of rotor, $\rho$ is free stream density, $R$ is the radius of the rotor, $a$ is rotor blade lift curve slope, $n$ represents the number of rotor blades, $d$ is the rotor blade chord, $\nu_b$ represents the flow through the rotor, and $\nu_i$ is rotor induced velocity. The far field velocity can be expressed as

$$ |\nu| = \sqrt{\nu_x^2 + \nu_y^2 + (\nu_z - \nu_i)^2}. $$

(2)

The induced velocity can be now expressed as follows:

$$ \nu_i = \frac{T}{2\pi n R^2 |\nu|}. $$

(3)

The airflow along the $z$-axis can be expressed as

$$ \nu_b = \nu_z + \frac{1}{2} \omega_t K_{\text{twist}}, $$

(4)

where $K_{\text{twist}}$ is the twist of the blades.

2.2. Duct. The airflow tends to follow the direction of the duct contour due to the Coandă effect. As a result, the wake will have a larger area than that of a helicopter. Comparing with a traditional helicopter, the duct provides additional thrust which can be attributed to the pressure distributions above and below the propeller. As shown in [24], the total thrust is the sum of the propeller or fan thrust and the force experienced by the duct:

$$ T = T_{\text{fan}} + T_{\text{duct}}, $$

(5)

where

$$ \frac{T_{\text{fan}}}{T} = \frac{1}{2a_d}, $$

(6)

where $a_d$ represents the ratio between the area of the wake and the propeller disk area.

The force on the vehicle due to the ducted fan can now be expressed as

$$ F_{\text{rotor}} = \begin{bmatrix} 0 \\ 0 \\ -T \end{bmatrix}. $$

(7)

The rotor twisting moment can be expressed as

$$ M_{\text{rotor}} = \begin{bmatrix} 0 \\ 0 \\ -P \omega \end{bmatrix}, $$

where $P$ is rotor output power.
2.3. Control Vane. The rudder control torque is

\[
L = \frac{1}{2} \rho V^2 S_L C_{Lr} (\delta),
\]
\[
D = \frac{1}{2} \rho V^2 S_D C_{Dr} (\delta),
\]

where \( S_L \) is rudder surface area and \( C_{Lr} \), and \( C_{Dr} \) are dimensionless lift coefficient and resistance coefficient, which are related to the deflection angle of the rudder.

The rudder surface distribution is shown in Figure 2; the control volume is \( \delta_1 = \delta_p + \delta_r, \delta_2 = \delta_q - \delta_r, \delta_3 = \delta_p - \delta_r, \) and \( \delta_4 = \delta_q + \delta_r, \) where \( \delta_p, \delta_q, \) and \( \delta_r \) are roll, pitch, and yaw control volume and \( \delta_1, \delta_2, \delta_3, \) and \( \delta_4 \) are the deflections angles of the rudder.

2.4. Gyroscopic Moment. The expression for gyroscopic moment vector is given as

\[
M_{\text{gyro}} = n I_s \omega_r \begin{bmatrix} -\omega_y \\ \omega_z \\ 0 \end{bmatrix},
\]

where \( I_s \) is the rotational inertia of the blade.

3. Modeling

The force and moment vectors can be expressed as

\[
F = F_{\text{aero}} + F_{\text{rotor}} + F_{\text{vane}},
\]
\[
M = M_{\text{aero}} + M_{\text{rotor}} + M_{\text{vane}} + M_{\text{gyro}}.
\]

The UAV dynamics and kinematics equations are given as

\[
\dot{\mathbf{x}} = (A + \Delta A) \mathbf{x}(t) + B_1 \mathbf{\alpha}(t) + (B_2 + \Delta B) \mathbf{u}(t),
\]
\[
\mathbf{z} = C_1 \mathbf{x}(t) + D_1 \mathbf{u}(t),
\]
\[
\mathbf{y} = C_2 \mathbf{x}(t),
\]

where \( \mathbf{x}(t) \in \mathbb{R}^n \) is the state vector of generalized object, \( \mathbf{u}(t) \in \mathbb{R}^m \) is the control input, \( \mathbf{w}(t) \in \mathbb{R}^p \) is the interference signal, \( \mathbf{z}(t) \in \mathbb{R}^q \) is the output to adjust, \( \mathbf{y}(t) \in \mathbb{R}^r \) is the output to measure, \( A \) is \( n \times n \) real constant matrix, \( B_1 \) is \( n \times p \) real constant matrix, \( B_2 \) is \( n \times m \) real constant matrix, \( C_1 \) is \( q \times n \) real constant matrix, \( D \) is \( q \times m \) real constant matrix, \( C_2 \) is \( f \times n \) real constant matrix, \( \Delta A \), \( \Delta B \) represent the uncertainty of the real parameters in system state matrix and input matrix, \( \Delta A \Delta B = H F (E_1 \ E_2) \), \( H \), \( E_1 \), and \( E_2 \) represent the constant matrix which have the corresponding dimension, and \( F \) represents unknown matrices and meets \( FF^T \leq I \).
Definition 1. Given performance index \( \gamma > 0 \), region \( D(-q, r) \), as shown in Figure 3, the feedback controller (14) is said to be \( H_\infty \) output feedback controller with \( D \)-stability, if the following conditions are established.

(1) If the uncertainties of the control system meet \((\Delta A \ \Delta B) = HF (E_1 \ E_2) \) and \( FF^T \leq I \), then the close-loop system is internal stability.

(2) The poles of the augmented closed-loop system are placed in region \( D \) shown in Figure 3.

(3) \( \|T_{zw}\|_\infty \) is minimized, where \( T_{zw}(s) \) is the closed-loop transfer function from \( w \) to \( z \); meanwhile, given \( \gamma > 0 \), \( \|T_{zw}\|_\infty < \gamma \),

\[
\dot{x}_c = A_c x_c + B_c y, \\
u = C_c x_c + D_c y. 
\] (14)

Augmented closed-loop system represents as

\[
\dot{x}_{cl} = A_{cl} x_{cl} + B_{cl} \omega, \\
z = C_{cl} x_{cl}, 
\] (15)

where \( x_{cl} = [x \ x_c]^T \) is the state vector of the augmented closed-loop system and system matrixes are represented, respectively, as

\[
A_{cl} = \begin{bmatrix} A + \Delta A + (B_2 + \Delta B) D_c C_2 - (B_2 + \Delta B) C_c \\ B_c C_2 \end{bmatrix}, \\
B_{cl} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \\
C_{cl} = \begin{bmatrix} C_1 + DD_c C_2 \\ DC_c \end{bmatrix}. 
\] (16)

Lemma 2. Let \( U, W, \) and \( V \) be any given real matrixes which have the corresponding dimension. For a given symmetric matrix \( M \), the following statements are equivalent.

(1) For any given matrix \( V \) which meets \( V^T V \leq I, U, W, \) and \( V \) satisfy

\[
M + UVW + W^T V^T U^T < 0. 
\] (17)

(2) There exists a real number \( \epsilon > 0 \) which makes

\[
M + \epsilon^{-1} U U^T + \epsilon W^T W < 0. 
\] (18)

Theorem 3. Given upper bound \( \gamma > 0 \) and region \( D(-q, r) \), for uncertainty system (13), if there exists symmetric positive matrix \( P = P^T > 0 \) and real number \( \epsilon > 0 \) for which

\[
N^* B_{cl} P C_{cl}^T (\overline{A}_{cl} + \alpha I) P P F^T \epsilon \overline{H} < 0. 
\] (19)

Then (1) the poles of the augmented closed-loop system are placed in region \( D(-q, r) \); (2) the close-loop system is stability internal and \( \|T_{zw}\|_\infty < \gamma \), where

\[
\alpha = q - r, \\
N^* = (\overline{A}_{cl} + \alpha I) P + P (\overline{A}_{cl} + \alpha I)^T, \\
\overline{A}_{cl} = \begin{bmatrix} A + B_2 D_c C_2 & B_2 C_c \\ B_c C_2 & A_c \end{bmatrix}, \\
\overline{H} = \begin{bmatrix} H & H \\ 0 & 0 \end{bmatrix}, \\
F = \begin{bmatrix} F & 0 \\ 0 & F \end{bmatrix}, \\
E = \begin{bmatrix} E_1 & 0 \\ E_2 D_c C_2 & E_2 C_c \end{bmatrix}. 
\] (20)
Proof. Consider
\[
A_{cl} = \overline{A}_{cl} + \Delta A_{cl},
\]
(21)
\[
\Delta A_{cl} = \begin{bmatrix}
\Delta A + \Delta BD_c C_2 & \Delta BC_c \\
0 & 0
\end{bmatrix}
\]
(22)
\[
= \begin{bmatrix}
HFE_1 + HFE_2 D_c C_2 & HFE_2 C_c \\
0 & 0
\end{bmatrix} = \overline{HFE} E_e,
\]
where
\[
egin{bmatrix}
N^* & B_{cl} & PC_{cl}^T (\overline{A}_{cl} + \alpha I) P \\
B_{cl}^T & -\gamma^2 & 0 & 0 \\
C_{cl} P & 0 & -I & 0 \\
P (\overline{A}_{cl} + \alpha I)^T & 0 & 0 & -r P
\end{bmatrix}
\]
(23)
\[
\begin{bmatrix}
\Delta A_{cl} P + P \Delta A_{cl}^T & 0 & 0 & \Delta A_{cl} P \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
P \Delta A_{cl}^T & 0 & 0 & 0
\end{bmatrix}
< 0,
\]
where (23) is equivalent to
\[
\begin{bmatrix}
\overline{HFE} E_e P + P (\overline{HFE})^T & 0 & 0 & \overline{HFE} E_e P \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
P (\overline{HFE})^T & 0 & 0 & 0
\end{bmatrix}
< 0.
\]
(25)
Then
\[
\begin{bmatrix}
\overline{H} E_e P & 0 & 0 & \overline{E} P \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
< 0.
\]
(26)
Let
\[
\overline{M} = \begin{bmatrix}
H \\
0 \\
0 \\
0
\end{bmatrix},
\]
(27)
\[
\overline{N} = \begin{bmatrix}
\overline{E} P & 0 & 0 & \overline{E} P
\end{bmatrix},
\]
where (26) is equivalent to
\[
\overline{Z} + \overline{M} \overline{F} \overline{N} + \overline{N}^T \overline{F} \overline{M}^T < 0.
\]
(28)
According to Lemma 2, (25) is founded when all \( F \) meet \( FF^T \leq I \), only if there exists a positive constant \( \epsilon \), let
\[
\overline{Z} + \epsilon \overline{M} \overline{M}^T + \epsilon^{-1} \overline{N}^T \overline{N} < 0.
\]
(29)
Which is equivalent, by the Schur complement, to
\[
\begin{bmatrix}
Z & N^T & M \\
N & -\epsilon I & 0 \\
M^T & 0 & -\epsilon^{-1} I
\end{bmatrix} < 0.
\]
(30)
Let inequality (30) premultiply and postmultiply \( \text{diag}(I, I, \epsilon I) \).
The proof is completed.

Theorem 5. Given \( H_{in} \), performance index \( \gamma > 0 \), positive real number \( \epsilon \), and region \( D(-q, r) \), if there exist corresponding dimension real matrices \( R = R^T \) and \( S = S^T \), matrices \( \overline{A}_c, \overline{B}_c, \overline{C}_c, \overline{D}_c \) satisfy inequalities (33) and (34):
\[
\begin{bmatrix}
R & M \\
M^T & * \\
S & N \\
I & I
\end{bmatrix} > 0.
\]
(31)
(32)
(33)

Lemma 4. There exists symmetry positive definite matrix \( X \); the necessary and sufficient condition that meets both (31) and (32) is
\[
X = \begin{bmatrix}
R & M \\
M^T & * \\
S & N \\
I & I
\end{bmatrix},
\]
\[
X^{-1} = \begin{bmatrix}
R & I \\
I & S
\end{bmatrix} > 0.
\]
(34)
\[ \Phi_{16} = A + B_2 \bar{D}_c C_2 + \alpha I, \]
\[ \Phi_{25} = \bar{A}_c + \alpha I, \]
\[ \Phi_{26} = S A + \bar{B}_c C_2 + \alpha S, \]
\[ \Phi_{18} = C_2^T D_c^T E_2^T, \]
\[ \Phi_{28} = C_2^T D_c^T E_2^T. \]

(35)

Then all poles of the closed-loop system are placed within the circular region \( \mathcal{D}(\gamma_1, r) \), and the system is stable and \( \|T_{z_{\infty}}(s)\|_{\infty} < \gamma \).

**Proof.** By Lemma 4, there exists a symmetric positive definite matrix \( P = \begin{bmatrix} R & M \\ M^T & U \end{bmatrix} \) and inverse matrix \( P^{-1} = \begin{bmatrix} S & N \\ N^T & V \end{bmatrix} \), in which \( R \), and \( S \) are symmetric positive definite matrices, as \( PP^{-1} = I \), and can get \( MN^T = I - RS \); because \( \begin{bmatrix} R & I \\ M^T & 0 \end{bmatrix} > 0 \), then \( I - RS \) is invertible and then \( M \), and \( N \) are full-ranked; let

\[ \Gamma_1 = \begin{bmatrix} R & I \\ M^T & 0 \end{bmatrix}, \]
\[ \Gamma_2 = \begin{bmatrix} I & S \\ 0 & N^T \end{bmatrix}. \]

(36)

Obviously

\[ P \Gamma_1 = \Gamma_1, \]
\[ \Gamma_1^T P \Gamma_2 = \begin{bmatrix} R & I \\ I & S \end{bmatrix}. \]

(37)

Then matrix \( P = \Gamma_1 \Gamma_2^{-1} \); furthermore \( \begin{bmatrix} R & I \\ M^T & 0 \end{bmatrix} > 0 \).

Let inequality (34) premultiply diag(\( \begin{bmatrix} I & I \end{bmatrix} \)) and postmultiply diag(\( \begin{bmatrix} I & I \end{bmatrix} \)) and

we can have the following inequality:

\[
\begin{bmatrix}
\Theta_{11} & \Theta_{12} & B_1 & \Theta_{14} & \Theta_{15} & \Theta_{16} & \Theta_{18} & eH & eH \\
\Theta_{22} & \Theta_{23} & \Theta_{24} & \Theta_{25} & E_1^T & \Theta_{28} & eSH & eSH \\
B_1^T & B_1^T S & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 & 0 \\
\Theta_{14} & \Theta_{15} & 0 & -I & 0 & 0 & 0 & 0 & 0 \\
\Theta_{22} & \Theta_{23} & 0 & 0 & -rR & -rI & R E_1^T & \Theta_{18} & 0 & 0 \\
\Theta_{24} & \Theta_{25} & 0 & 0 & -rI & -rS & E_1^T & \Theta_{28} & 0 & 0 \\
E_1 R & E_1 & 0 & 0 & E_1 R & E_1 & -\epsilon I & 0 & 0 & 0 \\
\Theta_{18} & \Theta_{28} & 0 & 0 & 0 & 0 & -\epsilon I & 0 & 0 \\
\epsilon H^T & \epsilon H^T S & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & 0 \\
\end{bmatrix} < 0,
\]

(38)

where

\[
\begin{align*}
\Theta_{11} & = AR + RA^T + B_1 D_c C_2 R + B_2 C_c M^T + RC_2^T D_c^T B_2^T + MC_c^T B_2^T + 2aR, \\
\Theta_{12} & = A + B_2 D_c C_2 + RA^T S + RC_2^T D_c^T B_2^T S + RC_2^T B_2^T N^T + MC_c^T B_2^T S + MA_c^T N^T + 2aI, \\
\Theta_{22} & = SA + A^T S + SB_2 D_c C_2 + NB_c C_2 + C_2^T B_2^T N^T + C_2^T D_c^T B_2^T S + 2aS, \\
\Theta_{14} & = RC_2^T + RC_2^T D_c^T D_c^T + MC_c^T D_c^T, \\
\Theta_{24} & = C_2^T + C_2^T D_c^T D_c^T, \\
\Theta_{15} & = AR + B_2 D_c C_2 R + B_2 C_c M^T + \alpha R, \\
\Theta_{16} & = A + B_2 D_c C_2 + \alpha I, \\
\Theta_{25} & = SAR + SB_2 D_c C_2 R + NB_c C_2 R + SB_2 C_c M^T + NA_c M^T + \alpha I, \\
\end{align*}
\]

Since the inequality contains the nonlinear terms, the variable substitution should be performed.

Define variable substitution as

\[ \bar{A}_c = SAR + NA_c M^T + SB_2 C_c M^T + SB_2 D_c C_2 R + NB_c C_2 R, \]
\[ \bar{B}_c = SB_2 D_c + NB_c, \]
\[ \bar{C}_c = C_c M^T + D_c C_2 R, \]
\[ \bar{D}_c = D_c. \]

(40)

Substituting inequality (40) into inequality (38) results in inequality (34). The proof is completed. \( \square \)
3.2. Robust D-Stability Dynamic Output Feedback Controller Design. Based on the above analysis, the nonlinear inequalities are converted to linear inequalities by variable substitution. From Theorem 5, for the system with parameter uncertainty, given $H_\infty$ performance index $\gamma > 0$, positive real number $\varepsilon > 0$, and circular region $D(-q, r)$, then the output feedback controller with $H_\infty$-$D$ stability is designed as follows.

(1) Get a feasible solution of linear matrix inequalities $R$, $S$, $\bar{A}_c$, $\bar{B}_c$, $\bar{C}_c$, and $\bar{D}_c$.

(2) Verify that the matrix $I - RS$ is reversible. If reversible, make full rank decomposition where $MN^T = I - RS$.

(3) According to variable substitution equation (40), we can get

$$D_c = \bar{D}_c,$$

$$C_c = (\bar{C}_c - D_c CR)M^{-T},$$

$$B_c = N^{-1}(\bar{B}_c - SB_c \bar{D}_c),$$

$$A_c = N^{-1}\left[ \bar{A}_c - S (A + B_c \bar{D}_c) R \right] M^T - B_c CR^{-T} - B_c CR^{-T} SB_c C_c,$$

(41)

where $(A_c, B_c, C_c, D_c)$ is the output feedback controller.

3.3. Simulation of Ducted Fan UAV Attitude Control. Based on the above model, the dynamic output feedback controller of the closed-loop system can be built. According to Theorem 5, the reconstruction of control law of systems is designed to place the closed-loop system poles into the circle region $D(-5, 4.5)$; let $\gamma = 0.92$, $\varepsilon = 1$.

Perturbation parameters $H = [0.5 \ 0.5 \ 0.5 \ 0.5 \ 0.5 \ 0.5 \ 0.5 \ 0.5 \ 0.5]^T$ and $(E_1 \ E_2) = [0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1]$. Then satisfy $F^T(t)F(t) \leq I$, according to previous methods, the matrices in control law reconfigurable systems are

$$A_c = \begin{bmatrix}
-22.64 & -0.2732 & -0.09441 & -1.568 & -0.4542 & -0.2776 & 0.5601 & 0.601 & 0.1774 \\
0.06462 & -8.671 & 0.6483 & 16.53 & 15.78 & 3.741 & 0.5845 & 1.741 & 0.7923 \\
-0.8520 & -2.610 & -0.8109 & 5.448 & -0.1541 & 0.8185 & -0.9721 & 0.3199 & 0.3014 \\
1.983 & 5.107 & -11.26 & 6.820 & -0.9133 & 0.8767 & 0.6171 & -0.5441 \\
-2.984 & 0.04552 & -3.048 & -1.52 & -23.44 & -2.365 & -1.378 & -0.9767 & 0.06624 \\
-7.701 & 2.591 & -5.136 & -10.90 & -23.26 & -3.992 & -0.2918 & -0.5453 & -0.4525 \\
-6.050 & 476.2 & 28.74 & 93.85 & 171.2 & 38.17 & 1.440 & 10.77 & 8.601 \\
114.2 & -590.1 & -39.47 & -118.6 & -239.3 & -50.20 & -0.7837 & -14.37 & -10.601 \\
\end{bmatrix},$$

$$B_c = \begin{bmatrix}
-0.02361 & -0.01612 & -0.2993 & -0.1272 & -0.1637 & -0.9465 & 0.04412 & 0.3329 & -0.3412 \\
0.1168 & -0.3121 & 0.1372 & 0.9954 & 0.4566 & 0.8167 & 0.8109 & -0.4011 & -0.2643 \\
-0.04551 & -0.04553 & -0.6308 & 0.4096 & -0.7956 & 0.4148 & -0.01628 & 0.4089 & 0.9448 \\
-0.03100 & 0.1068 & -0.1281 & -0.7726 & 0.3786 & -0.3129 & 0.3200 & -0.1454 & -0.2687 \\
-0.03587 & -0.09831 & -0.3434 & -0.9012 & -0.5590 & -0.1777 & -0.7323 & 0.7785 & -0.1931 \\
0.5540 & -0.2112 & 0.4984 & 0.3361 & 0.8961 & -0.5204 & -0.7233 & 0.3424 & 0.5452 \\
1.985 & -110.9 & 3.859 & 16.64 & 4.160 & 15.07 & -29.60 & -3.188 & -2.672 \\
1.338 & 0.9083 & -2.200 & -0.2673 & 1.703 & -0.3880 & 2.000 & 0.7796 & 1.650 \\
\end{bmatrix},$$

$$C_c = \begin{bmatrix}
301.8 & -4.545 & 10.80 & 51.19 & 95.25 & 15.79 & -6.855 & -6.062 & -1.385 \\
0.4401 & 3.211 & 0.2304 & -7.527 & -3.683 & -1.338 & -0.1323 & -0.5338 & -0.3353 \\
\end{bmatrix},$$

$$D_c = \begin{bmatrix}
1.320 & 0.7305 & 1.317 & 0.6735 & 6.697 & 0.4660 & -0.3071 & -0.8931 & 0.3908 \\
-0.4978 & 0.4364 & 1.101 & 0.9777 & 5.969 & 0.3108 & 0.6912 & 0.3042 & 0.05631 \\
-0.3687 & 0.9844 & -0.2012 & -4.332 & -0.8031 & -3.081 & 0.9728 & 0.04252 & 0.09001 \\
\end{bmatrix}.$$
does not change significantly. The above results show that the dynamic output feedback controller effectively solves the system’s uncertainty problem.

From the simulation results in Figures 5–10, three-axis attitude angle and angular velocity can quickly maintain stability with strong robustness. The ducted fan UAV has system uncertainty during the flight. As can be seen from the response curve before and after the load change, in the presence of uncertainty, system poles can be maintained within the designated region and has good stability and dynamic performance.

4. Conclusion

This paper has established the ducted fan UAV’s mathematical model and proposed a $H_\infty-D$ stabilization control method for linear uncertainty. This method gives the condition that meets both the interference suppression index constraints and regional pole constraints, in the form of linear matrix inequality (LMI).
This control method is applied in the ducted fan UAV’s attitude control system. The simulation results demonstrate the effectiveness of this method. The experimental results show that this method has good robustness for both model linear uncertainty and controller uncertainty. This method can effectively suppress the crosswind interference to the control system. The regional pole constraints also give the system a good dynamic performance.

Conflict of Interests

There is no conflict of interests regarding the publication of this paper.

References


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