Research Article

Grey Accumulation Generation Relational Analysis Model for Nonequidistance Unequal-Length Sequences and Its Application

Xuemei Li,¹ Yaoguo Dang,¹ Song Ding,¹ and Juan Zhang²

¹ College of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing 211106, China

² College of Science, Nanjing University of Aeronautics and Astronautics, Nanjing 211106, China

Correspondence should be addressed to Xuemei Li; xuemeili85@163.com

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As research is required on nonequidistance unequal-length sequences, so grey accumulation generation relational analysis model based on grey exponential law (AGRA) for nonequidistance unequal-length (NDUL) sequences is put forward in this paper. The original data is accumulated generation firstly and the generation sequences are simulated. Then the generation rate is established as the ratio of the tangent slope and the mean of the simulation function. Furthermore, the dynamic similarity of change trend of the original time sequences is characterized by the proximity of generation rate sequences. Meanwhile, properties of AGRA model for nonequidistance unequal-length sequences are discussed. The new relational analysis model is available for equal interval sequences, nonequidistance sequences, sequences which have relationship before transformation and sequences which have relationship after accumulation; therefore, the AGRA model has expanded the scope of application of grey relational analysis. Lastly, factors which affect the amount of passenger cars in China are sorted using AGRA model for NDUL sequences. This application is presented to illustrate the effectiveness and practicality of the proposed model.

1. Introduction

Grey relational analysis (GRA) has emerged as an effective model to measure the degree of correlation between sequences by calculating their similarity or proximity. Some research on GRA and its application has been done, such as the improvement and development of the existing GRA [1–5], extension of objects of studying [6], parallelism and consistency [7], consistency for affine transformation [8], and similarity and proximity of GRA [9].

GRA has been employed in various fields where they have produced promising results due to not limited data distribution and quantity. On the theoretical side, GRA plays its advantage during combination of clustering [10], decision making [11–14], and evaluating [15]. There are also a range of practical or industrial applications employing GRA, for instance, in the period of Chinese regional economic discrepancy [16], affecting factors of service brand [17], company financial performance [18], software effort estimation, energy consumption [19], Chinese economic development and energy consumption [20], residential energy consumption

and renewable energy utilization [21], sustainability indicator of renewable energy system [22], maximum peak discharge at upper stream [23], and track association algorithm for distributed multitarget system [24]. As a summary, Liu et al. review the research approaches of GRA modeling systematically [25]. Yin conducts a bibliometric study on publication and citation patterns of GRA published from 1996 to 2010 through a systemic search using the ISI web-based databases; results demonstrate that there has been a substantial increase in the number of peer-reviewed papers on GRA indexed by the ISI Web of Knowledge [26].

From the view of grey systems, although the objective system looks complex, it always has the overall function and certain inherent law. There is a method called grey generation which can explore the real law of the original data. Accumulation generation is an effective kind of grey generation, which whitens grey process and inherent development tendency of accumulation process. Original data may have no regular raw seemingly but emerge integral characteristics by accumulation generation [27]. Because the economic system, ecological system, and agricultural system could be treated as

generalized energy system, and the accumulation of energy follows the exponential rule, so accumulation generation has extensive adaptability and has been applied into grey forecasting models. Grey forecasting models based on grey exponential law are employed in a range of applications [28–30].

The existing GRA model examines geometric similarity or proximity of data directly or after dimensionless without considering other kinds of law, and research on GRA for nonequidistance unequal-length (NDUL) sequences is required, so AGRA for NDUL sequences based on grey exponential law will be put forward in this paper. Generation rate for NDUL sequences will be established. Then the dynamic change trend similarity of the original time series will be characterized by the proximity of generation rate sequence. Properties of AGRA for NDUL sequences will also be studied. Lastly, an application will be researched using the model proposed in this paper.

The paper is structured as follows. In the next section, AGRA for NDUL sequences is provided. Properties of AGRA for NDUL sequences are researched in Section 3. An application using the model is furnished in Section 4. In Section 5, appropriate conclusions are drawn.

2. Establishment of Grey Generation Rate Relational Analysis Model for Nonequidistance Unequal-Length Sequences

2.1. Simulation of Accumulation Generation Sequence of Nonequidistance Unequal-Length Sequences

Definition 1. Let $X^{(0)} = (x^{(0)}(t_1), x^{(0)}(t_2), \dots, x^{(0)}(t_n))$ be a time sequence, where $\Delta t_k = t_k - t_{k-1} \neq \text{const}$; then $X^{(0)}$ is a nonequidistance sequence.

Definition 2. Let $X_1^{(0)} = (x_1^{(0)}(t_1), x_1^{(0)}(t_2), \dots, x_1^{(0)}(t_n))$ and $X_2^{(0)} = (x_2^{(0)}(t_1), x_2^{(0)}(t_2), \dots, x_2^{(0)}(t_m))$ be time sequences, and $m \neq n$; then $X_1^{(0)}$ and $X_2^{(0)}$ are unequal-length sequences.

Definition 3. Let $X^{(1)} = (x^{(1)}(t_1), x^{(1)}(t_2), \dots, x^{(1)}(t_n))$ be the sequence of 1-AGO (one-step accumulating generation operator), where

$$x^{(1)}(t_k) = \sum_{j=1}^k x^{(0)}(t_j) \Delta t_j, \quad k = 1, 2, \dots, n, \quad (1)$$

are generated from $X^{(0)}$.

The background value optimized nonequidistance GM (1,1) model in the literature [18] will be used to simulate accumulation generation sequences. The differential equation is

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b. \quad (2)$$

The discrete form is

$$x^{(0)}(t_k) \Delta t_k + az^{(1)}(t_k) = b \Delta t_k. \quad (3)$$

If $\hat{a} = (a, b)^T$ is a sequence of parameters and

$$Y = \begin{bmatrix} x^{(0)}(2) \Delta t_2 \\ x^{(0)}(3) \Delta t_3 \\ \vdots \\ x^{(0)}(n) \Delta t_n \end{bmatrix}, \quad B = \begin{bmatrix} -z^{(1)}(2) \Delta t_2 \\ -z^{(1)}(3) \Delta t_3 \\ \vdots \\ -z^{(1)}(n) \Delta t_n \end{bmatrix},$$

$$z^{(1)}(k) = \frac{(\Delta t_k)^2 x^{(0)}(t_k)}{\ln x^{(0)}(t_k) - \ln x^{(0)}(t_{k-1})} + x^{(0)}(t_1) \Delta t_k - \left([x^{(0)}(t_k)]^2 [x^{(0)}(t_k) / x^{(0)}(t_{k-1})]^{(t_1 - t_k) / \Delta t_k} \times (\Delta t_k)^2 \right) (x^{(0)}(t_k) - x^{(0)}(t_{k-1}))^{-1}. \quad (4)$$

Then the parameters of the model based on the least squares method are

$$(a, b)^T = (B^T B)^{-1} B^T Y. \quad (5)$$

The solution, also known as time response function, of the whitenization equation $dx^{(1)}(t)/dt + ax^{(1)}(t) = b$ is given by

$$\hat{x}^{(1)}(t) = ce^{-a(t-t_1)} + \frac{b}{a}, \quad (6)$$

where

$$c = x^{(1)}(t_1) - \frac{b}{a}. \quad (7)$$

Its discrete form is

$$\hat{x}^{(1)}(t_k) = \left(x^{(1)}(t_1) - \frac{b}{a} \right) e^{-a(t_k - t_1)} + \frac{b}{a}. \quad (8)$$

2.2. Generation of Equidistance Equal-Length Sequences from Nonequidistance Unequal-Length Sequences. Suppose that $X_0^{(0)} = (x_0^{(0)}(t_0^1), x_0^{(0)}(t_0^2), \dots, x_0^{(0)}(t_0^{q_0}))$ and $X_i^{(0)} = (x_i^{(0)}(t_i^1), x_i^{(0)}(t_i^2), \dots, x_i^{(0)}(t_i^{q_i}))$ ($i = 1, 2, \dots, m$) are non-negative sequences, and $n = \max\{q_i\}$ ($i = 0, 1, 2, \dots, m$); then equidistance equal-length sequences $X'_0 = (x'_0(1), x'_0(2), \dots, x'_0(n))$ and $X'_i = (x'_i(1), x'_i(2), \dots, x'_i(n))$ ($i = 0, 1, 2, \dots, m$) can be obtained by time response equation in Section 2.1 based on $X_0^{(0)}$ and $X_i^{(0)}$. The time intervals of new sequences are one and lengths are n .

2.3. Establishment of Grey Generation Rate Relational Analysis Model Based on Generated Sequences. The slope of the tangent of $X^{(1)}$ at time t can be got as (9) by the function of $X^{(1)}$ and the meaning of slope:

$$h(t) = \frac{d\hat{x}^{(1)}(t)}{dt} = -ace^{-a(t-t_1)}. \quad (9)$$

The mean value of $X^{(1)}$ between $[1, n]$ can be calculated as

$$\begin{aligned}\bar{X}^{(1)} &= \frac{1}{n-1} \int_1^n \hat{x}^{(1)}(t) dt \\ &= \frac{1}{n-1} \int_1^n \left(ce^{-a(t-t_1)} + \frac{b}{a} \right) dt \\ &= \frac{c}{(n-1)a} \left(e^{-a(1-t_1)} - e^{-a(n-t_1)} \right) + \frac{b}{a}.\end{aligned}\quad (10)$$

The slope is the measure of the tilt of the linear. The slope of the tangent of $X^{(1)}$ at time t is the derivative value of $X^{(1)}$ at time t , which is the change rate of $X^{(1)}$ at time t . Then the slope of the tangent is divided by the mean value to establish the generation rate which eliminates the order of magnitudes.

Definition 4. Generation rate of $X^{(0)}$ at time t is defined as (11), where $t \in [a, b]$:

$$s(t) = \frac{h(t)}{\bar{X}^{(1)}}. \quad (11)$$

Definition 5. Let $s_0(k) = h_0(k)/\bar{X}_0^{(1)}$ and $s_i(k) = (h_i(k)/\bar{X}_i^{(1)})$ ($i = 1, 2, \dots, m$) be generation rates of $X_0^{(0)}$ and $X_i^{(0)}$ ($i = 1, 2, \dots, m$) at time k ; then $S_0 = (s_0(1), s_0(2), \dots, s_0(n))$ is the generation rate sequence of system characteristic sequence X_0 and $S_i = (s_i(1), s_i(2), \dots, s_i(n))$ ($i = 1, 2, \dots, m$) are the generation rate sequences of system behavioral sequence X_i ($i = 1, 2, \dots, m$), where

$$\begin{aligned}h_0(k) &= -a_0 c_0 e^{-a_0(k-t_0^1)}, & h_i(k) &= -a_i c_i e^{-a_i(k-t_i^1)}, \\ & & k &= 1, 2, \dots, n, \\ \bar{X}_0^{(1)} &= \frac{c_0}{(n-1)a_0} \left(e^{-a_0(1-t_0^1)} - e^{-a_0(n-t_0^1)} \right) + \frac{b_0}{a_0}, \\ \bar{X}_i^{(1)} &= \frac{c_i}{(n-1)a_i} \left(e^{-a_i(1-t_i^1)} - e^{-a_i(n-t_i^1)} \right) + \frac{b_i}{a_i}, \\ & & i &= 1, 2, \dots, m.\end{aligned}\quad (12)$$

Definition 6. $X_0^{(0)} = (x_0^{(0)}(t_0^1), x_0^{(0)}(t_0^2), \dots, x_0^{(0)}(t_0^{q_0}))$ is the nonnegative system characteristic sequence and $X_i^{(0)} = (x_i^{(0)}(t_i^1), x_i^{(0)}(t_i^2), \dots, x_i^{(0)}(t_i^{q_i}))$ ($i = 1, 2, \dots, m$) are nonnegative system behavioral sequence sequences. Equidistance equal-length sequences $X'_0 = (x'_0(1), x'_0(2), \dots, x'_0(n))$ and $X'_i = (x'_i(1), x'_i(2), \dots, x'_i(n))$ ($i = 1, 2, \dots, m$) are generated from $X_0^{(0)}$ and $X_i^{(0)}$ according to Section 2.1. Then,

$$\xi_i(k) = \frac{|s_0(k)|}{|s_0(k)| + |s_0(k) - s_i(k)|}, \quad (i = 1, 2, \dots, m), \quad (13)$$

is named as the grey relational coefficient of AGRA between $X_0^{(0)}$ and $X_i^{(0)}$ at time k .

$|s_0(k) - s_i(k)|$ reflects the proximity of change trend at time k between $X_0^{(0)}$ and $X_i^{(0)}$, so the closer development

tendency at time k of $X_0^{(0)}$ and $X_i^{(0)}$ is, the smaller $|s_0(k) - s_i(k)|$ is, meanwhile, the greater the relational coefficient of $X_0^{(0)}$ and $X_i^{(0)}$ is, and vice versa. The constriction of function for the relational coefficient is consistent with the meaning of grey relational analysis.

Definition 7. Grey relational degree of AGRA between X_i and X_j is constructed as

$$\zeta_i = \frac{1}{n} \sum_{k=1}^n \xi_i(k), \quad (i = 1, 2, \dots, m). \quad (14)$$

The grey relational coefficient of AGRA between sequences reflects the degree of consensus of sequences with change trend at one moment, and the grey relational degree of AGRA between sequences is the mean of the grey relational coefficient, so the grey relational degree of AGRA between sequences is measurement of consensus of sequences with development tendency for the whole time.

2.4. The Step of AGRA Model for NDUL Sequences. The steps to obtain the order of sequences by AGRA model for NDUL sequences are concluded as follows.

- (1) Make qualitative analysis on system characteristics sequence X_0 to confirm system behavioral sequences X_i .
- (2) Get the accumulation generation sequences of X_0 and X_i by (1).
- (3) Obtain the fitting curve functions of generation sequences according to (4)–(8).
- (4) Construct the generation rate sequences using (9)–(11).
- (5) Calculate the grey relational coefficients of AGRA between $X_0^{(0)}$ and $X_i^{(0)}$ at time t ($t = 1, 2, \dots, n$) by (13).
- (6) Determine the grey relational degree between $X_0^{(0)}$ and $X_i^{(0)}$ ($i = 1, 2, \dots, m$) based on grey relational coefficients and (14). Confirm the order of behavioral sequences between characteristics sequence according to the grey relational degree of AGRA.

3. Properties of AGRA Model for NDUL Sequences

Theorem 8. AGRA model for NDUL sequences has the following properties.

- (1) $0 < \zeta_i \leq 1$. $\zeta_i = 1$, iff $s_0(k) = s_i(k)$ for $\forall k \in \{1, 2, \dots, n\}$.
- (2) *Uniqueness and Independence.* ζ_i is unique for fixed two sequences and is not influenced by other sequences.
- (3) *Proximity.* The closer the development tendency of sequences is, the greater ζ_i is.
- (4) *Transitivity.* If $\zeta_i \geq \zeta_j$ and $\zeta_j \geq \zeta_q$, then $\zeta_i \geq \zeta_q$.

Proof. (1) $\zeta_i = 0 \Leftrightarrow |s_0(k)| = 0, \forall k \in \{1, 2, \dots, n\} \Leftrightarrow x_0^{(1)}(k) = r, \forall k \in \{1, 2, \dots, n\}$ where $r = \text{const}$; then

$$x_0^{(0)}(k) = x_0^{(1)}(k) - x_0^{(1)}(k-1) = r - r = 0, \quad k = 2, 3, \dots, n. \quad (15)$$

There is no sense to research correlation between these sequences. So $\zeta_i \neq 0$.

As $|s_0(k)| > 0, |s_0(k) - s_i(k)| \geq 0$, so $0 < |s_0(k)|/|s_0(k)| + |s_0(k) - s_i(k)| \leq 1$; that is $0 < \xi_i(k) \leq 1$, and $0 < (1/n) \sum_{k=1}^n \xi_i(k) \leq 1$; therefore

$$0 < \zeta_i \leq 1. \quad (16)$$

If $s_0(k) = s_i(k)$, then $\xi_i(k) = 1$; if $s_0(k) \neq s_i(k), \forall k \in \{1, 2, \dots, n\}$, then $\zeta_i = 1$.

(2) According to the definition of the grey relational coefficient and grey relational degree of AGRA, ζ_i is unique for fixed $X_0^{(0)}$ and $X_i^{(0)}$ and is not influenced by other sequences.

(3) The closer change trends at time k of $X_0^{(0)}$ and $X_i^{(0)}$ is, the smaller $|s_0(k) - s_i(k)|$ is. meanwhile, the greater the relational coefficient of $X_0^{(0)}$ and $X_i^{(0)}$ is, and vice versa. The closer development tendency of whole $X_0^{(0)}$ and $X_i^{(0)}$ is, the greater ζ_i is.

(4) Transitivity can be known by Properties 2 and 3. $\square$

Theorem 9. Let $t \in [a, b]$, $X_0^{(0)} = (x_0^{(0)}(t_0^1), x_0^{(0)}(t_0^2), \dots, x_0^{(0)}(t_0^{q_0}))$ is the system characteristics sequence, $X_i^{(0)} = (x_i^{(0)}(t_i^1), x_i^{(0)}(t_i^2), \dots, x_i^{(0)}(t_i^{q_i}))$ ($i = 1, 2, \dots, m$) are the behavioral sequences, and equidistance equal-length sequences based on $X_0^{(0)}$ and $X_i^{(0)}$ are $X'_0 = (x'_0(1), x'_0(2), \dots, x'_0(n))$ and $X'_i = (x'_i(1), x'_i(2), \dots, x'_i(n))$, $n = \max\{q_i\}$, ($i = 0, 1, 2, \dots, m$). The grey relational degree of AGRA will remain the same after $X_0^{(0)}$ and $X_i^{(0)}$ are divided by the initial value or the mean value, or multiplied; in other words, the grey relational degree of AGRA will not change by dimensionless of sequences.

Proof. The simulation functions of $X_0^{(1)}$ and $X_i^{(1)}$ which are the accumulation generation sequences of $X_0^{(0)}$ and $X_i^{(0)}$, $i = 1, 2, \dots, m$, are

$$\begin{aligned} \hat{x}_0^{(1)}(t) &= \left(x_0^{(0)}(t_0^1) - \frac{b_0}{a_0} \right) e^{-a_0(t-t_0^1)} + \frac{b_0}{a_0}, \\ \hat{x}_i^{(1)}(t) &= \left(x_i^{(0)}(t_i^1) - \frac{b_i}{a_i} \right) e^{-a_i(t-t_i^1)} + \frac{b_i}{a_i}. \end{aligned} \quad (17)$$

Then the generation rates of $X_0^{(0)}$ and $X_i^{(0)}$ ($i = 1, 2, \dots, m$) at time k are

$$\begin{aligned} s_0(k) &= \left(-a_0^2 \left(x_0^{(0)}(t_0^1) - b_0/a_0 \right) (n-1) e^{-a_0(k-t_0^1)} \right. \\ &\quad \times \left(\left(x_0^{(0)}(t_0^1) - b_0/a_0 \right) \left(e^{-a_0(1-t_0^1)} - e^{-a_0(n-t_0^1)} \right) \right. \\ &\quad \left. \left. + (n-1)b_0 \right)^{-1}, \end{aligned}$$

$$\begin{aligned} s_i(k) &= \left(-a_i^2 \left(x_i^{(0)}(t_i^1) - b_i/a_i \right) (n-1) e^{-a_i(k-t_i^1)} \right) \\ &\quad \times \left(\left(x_i^{(0)}(t_i^1) - b_i/a_i \right) \left(e^{-a_i(1-t_i^1)} - e^{-a_i(n-t_i^1)} \right) \right. \\ &\quad \left. + (n-1)b_i \right)^{-1}, \end{aligned}$$

$$k = 1, 2, \dots, n, \quad i = 1, 2, \dots, m. \quad (18)$$

Now let $Y_0^{(0)} = \lambda X_0^{(0)}, Y_i^{(0)} = \beta X_i^{(0)}$ ($i = 1, 2, \dots, m$), and $Y_0^{(1)}, Y_i^{(1)}$ ($i = 1, 2, \dots, m$) are the accumulation generation sequences of $Y_0^{(0)}$ and $Y_i^{(0)}$. The simulation functions of $Y_0^{(1)}$ and $Y_i^{(1)}$ ($i = 1, 2, \dots, m$) are

$$\begin{aligned} \hat{y}_0^{(1)}(t) &= \left(y_0^{(0)}(t_0^1) - \frac{b'_0}{a'_0} \right) e^{-a'_0(t-t_0^1)} + \frac{b'_0}{a'_0}, \\ \hat{y}_i^{(1)}(t) &= \left(y_i^{(0)}(t_i^1) - \frac{b'_i}{a'_i} \right) e^{-a'_i(t-t_i^1)} + \frac{b'_i}{a'_i}, \end{aligned} \quad (19)$$

($i = 1, 2, \dots, m$).

Then the generation rates after the dimensionless of dates are

$$\begin{aligned} s'_0(k) &= \left(-a'^2_0 \left(y_0^{(0)}(t_0^1) - b'_0/a'_0 \right) (n-1) e^{-a'_0(k-t_0^1)} \right) \\ &\quad \times \left(\left(y_0^{(0)}(t_0^1) - b'_0/a'_0 \right) \left(e^{-a'_0(1-t_0^1)} - e^{-a'_0(n-t_0^1)} \right) \right. \\ &\quad \left. + (n-1)b'_0 \right)^{-1}, \\ s'_i(k) &= \left(-a'^2_i \left(y_i^{(0)}(t_i^1) - b'_i/a'_i \right) (n-1) e^{-a'_i(k-t_i^1)} \right) \\ &\quad \times \left(\left(y_i^{(0)}(t_i^1) - b'_i/a'_i \right) \left(e^{-a'_i(1-t_i^1)} - e^{-a'_i(n-t_i^1)} \right) \right. \\ &\quad \left. + (n-1)b'_i \right)^{-1}, \end{aligned}$$

$k = 1, 2, \dots, n, \quad i = 1, 2, \dots, m. \quad (20)$

Meanwhile, $a_0 = a'_0, a_i = a'_i, b'_0 = \lambda b_0, b'_i = \beta b_i$ ($i = 1, 2, \dots, m$) can be got by Li [31].

So, $s'_0(k) = s_0(k), s'_i(k) = s_i(k)$; then

$$\begin{aligned} \xi'_i(k) &= \frac{|s'_0(k)|}{|s'_0(k)| + |s'_i(k) - s'_i(k)|} \\ &= \frac{|s_0(k)|}{|s_0(k)| + |s_0(k) - s_i(k)|} \\ &= \xi_i(k). \end{aligned} \quad (21)$$

Therefore, the grey relational coefficients of generation rate between $X_0^{(0)}$ and $X_i^{(0)}$ will remain the same after $X_0^{(0)}$ and $X_i^{(0)}$ are divided by the initial value or the mean value, or multiplied; in other words, the grey relational coefficients

TABLE 1: The original data of factors which affect the number of commercial passenger cars.

	2005	2006	2007	2008	2009	2010	2011	2012
Commercial passenger cars (ten thousand) X_0	2132.46			3838.92	4845.09	6124.13	7478.37	8943.01
Road mileage (ten thousand of kilometers) X_1	334.52	345.70	358.37	373.02	386.08	400.82	410.64	423.75
Turnover passengers on highway (Million passenger-kilometer) X_2	9292.08	10130.85	11506.77	12476.11	13511.44	15020.81	16760.25	18467.55
The number of civil aircrafts X_3	1386			1961	2181	2405	3191	3589

Date resource: 2013 China's Transportation Almanac.

of AGRA will not be affected by dimensionless so do the grey relational degree of AGRA. Another conclusion can be got; the AGRA model for NDUL sequences is suitable for both positive sequences and negative sequences. $\square$

Theorem 10. Let behavioral sequences be $X_i^{(0)} = (x_i^{(0)}(t_1), x_i^{(0)}(t_2), \dots, x_i^{(0)}(t_n))$, and $x_i^{(0)}(t_k) = \theta x_0^{(0)}(t_k)$, $\theta = \text{const}$, $k = 1, 2, \dots, n$; then $\zeta_i = 1$.

Proof. The generation rates of $X_0^{(1)}$ and $X_i^{(1)}$ at time k are

$$\begin{aligned}
 s_0(k) &= \left(-a_0^2 (x_0^{(0)}(t_0^1) - b_0/a_0) (n-1) e^{-a_0(k-t_0^1)} \right) \\
 &\quad \times \left((x_0^{(0)}(t_0^1) - b_0/a_0) (e^{-a_0(1-t_0^1)} - e^{-a_0(n-t_0^1)}) \right. \\
 &\quad \left. + (n-1)b_0 \right)^{-1}, \\
 s_i(k) &= \left(-a_i^2 (x_i^{(0)}(t_i^1) - b_i/a_i) (n-1) e^{-a_i(k-t_i^1)} \right) \\
 &\quad \times \left((x_i^{(0)}(t_i^1) - b_i/a_i) (e^{-a_i(1-t_i^1)} - e^{-a_i(n-t_i^1)}) \right. \\
 &\quad \left. + (n-1)b_i \right)^{-1}, \\
 k &= 1, 2, \dots, n, \quad i = 1, 2, \dots, m.
 \end{aligned} \tag{22}$$

As $x_i^{(0)}(t_k) = \theta x_0^{(0)}(t_k)$, $t_0^1 = t_i^1$, so $a_0 = a_i$, $b_i = \theta b_0$ ($i = 1, 2, \dots, m$), then $s_i(k) = s_0(k)$; therefore

$$\begin{aligned}
 \xi_i(k) &= \frac{|s_0(k)|}{|s_0(k)| + |s_0(k) - s_i(k)|} \\
 &= \frac{|s_0(k)|}{|s_0(k)|} \\
 &= 1.
 \end{aligned} \tag{23}$$

So

$$\zeta_i = \frac{1}{n} \sum_{k=1}^n \xi_i(k) = 1. \tag{24}$$

$\square$

Theorem 11. The AGRA model for NDUL sequences has noise reduction effect and is stable for noise.

Proof. As initial sequences in AGRA model for NDUL sequences are accumulation generated before taken into the model, and accumulation generation has noise reduction effect, so the model reduces the impact of noise. Stability and validity for noise interference in this model will be demonstrated as follows.

Let $x = x_0 + \varepsilon$ be the sequence which has noise, x_0 the sequence without noise, and ε the noise; then

$$\begin{aligned}
 s(t) &= \frac{h(t)}{\bar{X}^{(1)}} = \frac{d(x_0 + \varepsilon)/dt}{\bar{X}_0^{(1)} + \varepsilon} \\
 &= \frac{dx_0/dt + d\varepsilon/dt}{\bar{X}_0^{(1)} + \varepsilon}.
 \end{aligned} \tag{25}$$

As ε can be ignored when compared with x_0 and $\bar{X}_0^{(1)}$, and the derivative of ε is infinitesimal, so

$$s(t) = \frac{dx_0/dt + d\varepsilon/dt}{\bar{X}_0^{(1)} + \varepsilon} \approx \frac{dx_0/dt}{\bar{X}_0^{(1)}} = s_0(t). \tag{26}$$

Therefore, the AGRA model for NDUL sequences is stable for noise. $\square$

4. Case Study

Traffic jam is becoming one of the most difficult problems in which cities are facing as the urban space is expanding and the population is accumulating. Traffic accidents, air pollution, noise pollution, resource shortages, and other issues which are brought by traffic jam are hindering the global economy and even threatening human survival; that is why traffic jam is one of the most focused on problems for academic organizations and governments. It is significant to sort factors which affect the amount of commercial passenger cars for traffic management and congestion controlling. The road mileage, turnover passengers on highway, and the number of civil aircrafts are selected as affecting factors according to the existing literatures and expert advices. The original data of affecting factors are in Table 1.

Due to the limitation of data source, the numbers of commercial passenger cars and civil aircrafts in 2006 and 2007 are lacking. Common GRA models fail to handle this situation. In order to make full use of existing data, the AGRA model for NDUL sequences is required to rank factors

TABLE 2: The parameters during the mathematical calculation.

	i	0	1	2	3
The parameters in simulation functions	a_i	-0.14017	-0.03390	-0.09820	-0.09141
	b_i	3403.67530	330.23450	8799.98560	1793.96982
The generation rates of sequences	$s_i(1)$	0.179630	0.209493	0.184575	0.195901
	$s_i(2)$	0.206659	0.216716	0.203620	0.214652
	$s_i(3)$	0.237754	0.224189	0.224630	0.235198
	$s_i(4)$	0.273529	0.231919	0.247808	0.257710
	$s_i(5)$	0.314686	0.239916	0.273378	0.282377
	$s_i(6)$	0.362037	0.248189	0.301586	0.309405
	$s_i(7)$	0.416512	0.256747	0.332705	0.339020
	$s_i(8)$	0.479184	0.265600	0.367034	0.371470
The grey relational coefficients of AGRA	$\xi_i(1)$	0.85745	0.97321	0.91694	0.85745
	$\xi_i(2)$	0.95359	0.98551	0.96276	0.95359
	$\xi_i(3)$	0.94602	0.94769	0.98936	0.94602
	$\xi_i(4)$	0.86796	0.91405	0.94533	0.86796
	$\xi_i(5)$	0.80801	0.88396	0.90689	0.80801
	$\xi_i(6)$	0.76077	0.85692	0.87308	0.76077
	$\xi_i(7)$	0.72276	0.83249	0.84313	0.72276
	$\xi_i(8)$	0.69169	0.81034	0.81647	0.69169

TABLE 3: The grey relational degree of AGRA and the ranking of factors which affect the number of commercial passenger cars.

	Road mileage	Turnover passengers on highway	Civil aircrafts
The grey relational degrees of AGRA	0.82603	0.90052	0.90674
The racking	3	2	1

which affect the number of commercial passenger cars. The parameters during the mathematical calculation are provided in Table 2. The grey generation rate degrees of incidence and the ranking list are in Table 3.

As shown in Table 3, the number of civil aircrafts is more affected by requirement of society than social capacity. The driving action of requirement of society is greater than the restriction of social capacity in front of the increasing of cars.

5. Conclusions and Future Studies

AGRA model for NDUL sequences is established in this paper. The original data is accumulated firstly and the curve is fitted by optimized GM (1,1) model for nonequidistance sequences; then the fitting equation is obtained. Generation rate is established as the ratio of the tangent slope and the mean of the fitting equation. The similarity of dynamic change trend of the original time series is characterized by the proximity of generation sequences. The AGRA model for NDUL sequences is available for both equal interval sequences and nonequidistance sequences. Meanwhile, the new model can handle sequences which have relationship before transformation and sequences which have relationship after accumulation. Accumulation generation reflects development tendency of data and expands the scope of application of grey relational analysis. Meanwhile, properties of AGRA model for NDUL sequences are researched. It is

shown that AGRA model for NDUL sequences is not limited to positive sequences. In the last section, factors which affect the number of passenger cars in China are ordered using the model proposed in this paper. It is illustrated that the proposed model is effective and practical.

Because the economic system, ecological system, and agricultural system could be treated as generalized energy system, the accumulation of energy follows the exponential rule which is used to establish AGRA model. Meanwhile, data missing is common in practical problem so that NDUL sequences are general. In conclusion, AGRA model for NDUL sequences can be applied but not limited to economic system, ecological system, and agricultural system problems with NDUL sequences.

There are some potential studies that can be done in the future according to the ideas presented in this paper, including conducting a new AGRA model based on other kinds of GRA models; in this way, more applications of other kinds of AGRA models can be determined. The AGRA model for NDUL sequences can also be combined with clustering methods and decision making methods in the presence of NDUL sequences.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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