Research Article

A MOLP Method for Solving Fully Fuzzy Linear Programming with *LR* Fuzzy Parameters

Xiao-Peng Yang,^{1,2} Xue-Gang Zhou,^{1,3} Bing-Yuan Cao,¹ and S. H. Nasser⁴

¹ School of Mathematics and Information Science, Key Laboratory of Mathematics and Interdisciplinary Sciences of Guangdong, Higher Education Institutes, Guangzhou University, Guangzhou 510006, China

² Department of Mathematics and Statistics, Hanshan Normal University, Chaozhou 521041, China

³ Department of Applied Mathematics, Guangdong University of Finance, Guangzhou 510521, China

⁴ Department of Mathematics, Mazandaran University, Babolsar 47416-95447, Iran

Correspondence should be addressed to Bing-Yuan Cao; caobingy@163.com

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Kaur and Kumar, 2013, use Mehara's method to solve a kind of fully fuzzy linear programming (FFLP) problems with *LR* fuzzy parameters. In this paper, a new kind of FFLP problems is introduced with a solution method proposed. The FFLP is converted into a multiobjective linear programming (MOLP) according to the order relation for comparing the *LR* flat fuzzy numbers. Besides, the classical fuzzy programming method is modified and then used to solve the MOLP problem. Based on the compromised optimal solution to the MOLP problem, the compromised optimal solution to the FFLP problem is obtained. At last, a numerical example is given to illustrate the feasibility of the proposed method.

1. Introduction

The research on fuzzy linear programming (FLP) has risen highly since Bellman and Zadeh [1] proposed the concept of decision making in fuzzy environment. The FLP problem is said to be a fully fuzzy linear programming (FFLP) problem if all the parameters and variables are considered as fuzzy numbers. In recent years, some researchers such as Lofti and Kumar were interested in the FFLP problems, and some solution methods have been obtained to the fully fuzzy systems [2–4] and the FFLP problems [5–13]. FFLP problems can be divided in two categories: (1) FFLP problems with inequality constraints; (2) FFLP problems with equality constraints. If the FFLP problems are classified by the types of the fuzzy numbers, they will include the next three classes: (1) FFLP problems with all the parameters and variables represented by triangular fuzzy numbers; (2) FFLP problems with all the parameters and variables represented by trapezoidal fuzzy numbers; (3) FFLP problems with all

the parameters and variables expressed by *LR* fuzzy numbers (or *LR* flat fuzzy numbers).

Fuzzy programming method is a classical method to solve multiobjective linear programming (MOLP) [14, 15]. In this paper, the fuzzy programming method is modified and then used to obtain a compromised optimal solution of the MOLP. The modified fuzzy programming method is shown in Steps 4–10 of the proposed method in Section 3.

Dehghan et al. [2–4] employed several methods to find solutions of the fully fuzzy linear systems. Hosseinzadeh Lotfi et al. [6] used the lexicography method to obtain the fuzzy approximate solutions of the FFLP problems. Allahviranloo et al. [7] and Kumar et al. [5, 8] solved the FFLP problem by use of a ranking function.

Fan et al. [12] adopted the α -cut level to deal with a generalized fuzzy linear programming (GFLP) problem. The feasibility of fuzzy solutions to the GFLP was investigated and a stepwise interactive algorithm based on the idea of design of experiment was advanced to solve the GFLP problem.

Kaur and Kumar [9] introduced Mehar's method to the FFLP problems with LR fuzzy parameters. They consider the following model:

Maximize (or Minimize)

$$\begin{aligned} & \sum_{j=1}^n ((p_j, q_j, \alpha'_j, \beta'_j)_{LR} \odot (x_j, y_j, \alpha''_j, \beta''_j)_{LR}) \\ & \text{subject to} \\ & \sum_{j=1}^n ((a_{ij}, b_{ij}, \alpha_{ij}, \beta_{ij})_{LR} \odot (x_j, y_j, \alpha''_j, \beta''_j)_{LR}) \preceq, \approx, \succeq (b_i, g_i, \gamma_i, \delta_i)_{LR}, \\ & i = 1, 2, \dots, m, \end{aligned} \quad (1)$$

where the parameters and variables are LR flat fuzzy numbers and the order relation for comparing the numbers is defined as follows.

- (i) $\tilde{u} \preceq \tilde{v}$ if and only if $\mathfrak{R}(\tilde{u}) \leq \mathfrak{R}(\tilde{v})$,
- (ii) $\tilde{u} \succeq \tilde{v}$ if and only if $\mathfrak{R}(\tilde{u}) \geq \mathfrak{R}(\tilde{v})$,
- (iii) $\tilde{u} \approx \tilde{v}$ if and only if $\mathfrak{R}(\tilde{u}) = \mathfrak{R}(\tilde{v})$.

Here $\tilde{u}$ and $\tilde{v}$ are two arbitrary LR flat fuzzy numbers.

In our study, we consider a new kind of FFLP problems with LR flat fuzzy parameters as follows:

$$\begin{aligned} & \min \text{ (or max)} \\ & z(\tilde{x}) = \tilde{c}_1 \otimes \tilde{x}_1 \oplus \tilde{c}_2 \otimes \tilde{x}_2 \oplus \dots \oplus \tilde{c}_n \otimes \tilde{x}_n \\ & \text{subject to} \\ & \tilde{a}_{i1} \otimes \tilde{x}_1 \oplus \tilde{a}_{i2} \otimes \tilde{x}_2 \oplus \dots \oplus \tilde{a}_{in} \otimes \tilde{x}_n \leq \tilde{b}_i, \\ & i = 1, 2, \dots, m, \\ & \tilde{x}_j \geq \tilde{0}, \quad j = 1, 2, \dots, n, \end{aligned} \quad (2)$$

where the parameters and variables are LR flat fuzzy numbers and the order relation shown in Definition 4 is different from the one above.

In this paper, we modify the classical fuzzy programming method. The FFLP is changed into a MOLP problem solved by the modified fuzzy programming method. We get the compromised optimal solution to the MOLP and generate the corresponding compromised optimal solution to the FFLP.

The rest of the paper is organized as follows. In Section 2, the basic definitions and the FFLP model are introduced. In Section 3, we propose a MOLP method to solve the FFLP problems. Some results are discussed from the solutions obtained by the proposed method. In Section 4, a numerical example is given to illustrate the feasibility of the proposed method. In Section 5, we show some short concluding remarks.

2. Preliminaries

2.1. Basic Notations

Definition 1 (LR fuzzy number, see [2]). A fuzzy number $\tilde{u}$ is said to be an LR fuzzy number if

$$\tilde{u}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), & x \leq m, \quad \alpha > 0, \\ R\left(\frac{x-m}{\beta}\right), & x \geq m, \quad \beta > 0, \end{cases} \quad (3)$$

where m is the mean value of $\tilde{u}$ and α and β are left and right spreads, respectively, and function $L(\cdot)$ means the left shape function satisfying

- (1) $L(x) = L(-x)$;
- (2) $L(0) = 1$ and $L(1) = 0$;
- (3) $L(x)$ is nonincreasing on $[0, \infty)$.

Naturally, a right shape function $R(\cdot)$ is similarly defined as $L(\cdot)$.

Definition 2 (LR flat fuzzy number, see [9, 16]). A fuzzy number $\tilde{u}$, denoted as $(m, n, \alpha, \beta)_{LR}$, is said to be an LR flat fuzzy number if its membership function $\tilde{u}(x)$ is given by

$$\tilde{u}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), & x \leq m, \quad \alpha > 0, \\ R\left(\frac{x-n}{\beta}\right), & x \geq n, \quad \beta > 0, \\ 1, & m \leq x \leq n. \end{cases} \quad (4)$$

Definition 3 (see [5, 9]). An LR flat fuzzy number $\tilde{u} = (m, n, \alpha, \beta)_{LR}$ is said to be nonnegative LR flat fuzzy number if $m - \alpha \geq 0$ and is said to be nonpositive LR flat number if $n + \beta \leq 0$.

We define $\tilde{u} = (m, n, 0, 0)_{LR}$ as an LR fuzzy number with membership function

$$\tilde{u}(x) = \begin{cases} 1, & m \leq x \leq n, \\ 0, & \text{otherwise,} \end{cases} \quad (5)$$

and denote $(0, 0, 0, 0)_{LR}$ as $\tilde{0}$.

2.2. Arithmetic Operations. Let $\tilde{u} = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\tilde{v} = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ be two LR flat fuzzy numbers, $k \in R$. Then the arithmetic operations are given as follows [9, 16]:

$$\begin{aligned}\tilde{u} \oplus \tilde{v} &= (m_1 + m_2, n_1 + n_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)_{LR}, \\ \tilde{u} \ominus \tilde{v} &= (m_1 - m_2, n_1 - n_2, \alpha_1 - \alpha_2, \beta_1 - \beta_2)_{LR}, \\ k\tilde{u} &= \begin{cases} (km_1, kn_1, k\alpha_1, k\beta_1)_{LR}, & k \geq 0, \\ (kn_1, km_1, -k\beta_1, -k\alpha_1)_{RL}, & k < 0, \end{cases} \\ \tilde{u} \otimes \tilde{v} &= \begin{cases} (m_1 m_2, n_1 n_2, m_1 \alpha_2 + \alpha_1 m_2, n_1 \beta_2 + \beta_1 n_2)_{LR}, & \tilde{u} \geq \tilde{0}, \tilde{v} \geq \tilde{0}, \\ (m_1 n_2, n_1 m_2, \alpha_1 n_2 - m_1 \beta_2, \beta_1 m_2 - n_1 \alpha_2)_{LR}, & \tilde{u} \leq \tilde{0}, \tilde{v} \geq \tilde{0}, \\ (n_1 m_2, m_1 n_2, n_1 \alpha_2 - \beta_1 m_2, m_1 \beta_2 - \alpha_1 n_2)_{LR}, & \tilde{u} \geq \tilde{0}, \tilde{v} \leq \tilde{0}, \\ (n_1 n_2, m_1 m_2, -n_1 \beta_2 - \beta_1 n_2, -m_1 \alpha_2 - \alpha_1 m_2)_{LR}, & \tilde{u} \leq \tilde{0}, \tilde{v} \leq \tilde{0}. \end{cases}\end{aligned}\quad (6)$$

It is easy to verify that the operator $\oplus$ satisfies associative law. Hence, the formula $\sum_{j=1}^n \tilde{u}_j = \tilde{u}_1 \oplus \tilde{u}_2 \oplus \dots \oplus \tilde{u}_n$ is reasonable, where $\tilde{u}_1, \tilde{u}_2, \dots, \tilde{u}_n$ are LR flat fuzzy numbers.

2.3. Order Relation for Comparing the LR Flat Fuzzy Numbers. For comparing the LR flat fuzzy numbers, we introduce the order relation as follows.

Definition 4. Let $\tilde{u} = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\tilde{v} = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ be any LR flat fuzzy numbers. Then

- (i) $\tilde{u} = \tilde{v}$ if and only if $m_1 = m_2, n_1 = n_2, \alpha_1 = \alpha_2, \beta_1 = \beta_2$;
- (ii) $\tilde{u} \leq \tilde{v}$ if and only if $m_1 \leq m_2, n_1 \leq n_2, m_1 - \alpha_1 \leq m_2 - \alpha_2, n_1 + \beta_1 \leq n_2 + \beta_2$;
- (iii) $\tilde{u} \geq \tilde{v}$ if and only if $m_1 \geq m_2, n_1 \geq n_2, m_1 - \alpha_1 \geq m_2 - \alpha_2, n_1 + \beta_1 \geq n_2 + \beta_2$.

Based on the definition of order $\leq$, we may obtain that (i) $\tilde{u}$ is nonnegative if and only if $\tilde{u} \geq \tilde{0}$; (ii) $\tilde{u}$ is nonpositive if and only if $\tilde{u} \leq \tilde{0}$.

The following propositions are given to show the properties of the order relation defined above.

Proposition 5. Let $\tilde{u}, \tilde{v}, \tilde{w}, \tilde{\sigma}$ be four arbitrary LR flat fuzzy numbers and k an arbitrary real number. Then

$$\begin{aligned}(1) \quad & \tilde{u} \leq \tilde{v}, \quad \tilde{w} \leq \tilde{\sigma} \implies \tilde{u} \oplus \tilde{w} \leq \tilde{v} \oplus \tilde{\sigma}; \\ (2) \quad & \tilde{u} \leq \tilde{v} \implies \begin{cases} k\tilde{u} \leq k\tilde{v}, & k \geq 0, \\ k\tilde{u} \geq k\tilde{v}, & k \leq 0. \end{cases}\end{aligned}\quad (7)$$

Proof. Suppose $\tilde{u} = (m_1, n_1, \alpha_1, \beta_1)_{LR}$, $\tilde{v} = (m_2, n_2, \alpha_2, \beta_2)_{LR}$, $\tilde{w} = (m_3, n_3, \alpha_3, \beta_3)_{LR}$, and $\tilde{\sigma} = (m_4, n_4, \alpha_4, \beta_4)_{LR}$.

(1) It is obvious that $\tilde{u} \oplus \tilde{w} = (m_1 + m_3, n_1 + n_3, \alpha_1 + \alpha_3, \beta_1 + \beta_3)_{LR}$, $\tilde{v} \oplus \tilde{\sigma} = (m_2 + m_4, n_2 + n_4, \alpha_2 + \alpha_4, \beta_2 + \beta_4)_{LR}$. Since $\tilde{u} \leq \tilde{v}, \tilde{w} \leq \tilde{\sigma}$, we get

$$\begin{aligned}m_1 &\leq m_2, & n_1 &\leq n_2, & m_1 - \alpha_1 &\leq m_2 - \alpha_2, \\ n_1 + \beta_1 &\leq n_2 + \beta_2, \\ m_3 &\leq m_4, & n_3 &\leq n_4, & m_3 - \alpha_3 &\leq m_4 - \alpha_4, \\ n_3 + \beta_3 &\leq n_4 + \beta_4.\end{aligned}\quad (8)$$

So

$$\begin{aligned}m_1 + m_3 &\leq m_2 + m_4, \\ n_1 + n_3 &\leq n_2 + n_4, \\ (m_1 + m_3) - (\alpha_1 + \alpha_3) &\leq (m_2 + m_4) - (\alpha_2 + \alpha_4), \\ (n_1 + n_3) + (\beta_1 + \beta_3) &\leq (n_2 + n_4) + (\beta_2 + \beta_4).\end{aligned}\quad (9)$$

This indicates that $\tilde{u} \oplus \tilde{w} \leq \tilde{v} \oplus \tilde{\sigma}$.

(2) It is clear that $k\tilde{u} = (km_1, kn_1, k\alpha_1, k\beta_1)_{LR}$, $k\tilde{v} = (km_2, kn_2, k\alpha_2, k\beta_2)_{LR}$. From $\tilde{u} \leq \tilde{v}$, we get

$$\begin{aligned}m_1 &\leq m_2, & n_1 &\leq n_2, & m_1 - \alpha_1 &\leq m_2 - \alpha_2, \\ n_1 + \beta_1 &\leq n_2 + \beta_2.\end{aligned}\quad (10)$$

Therefore,

$$\begin{aligned}km_1 &\leq km_2, & kn_1 &\leq kn_2, & km_1 - k\alpha_1 &\leq km_2 - k\alpha_2, \\ kn_1 + k\beta_1 &\leq kn_2 + k\beta_2,\end{aligned}\quad (11)$$

for $k \geq 0$, and

$$\begin{aligned}km_1 &\geq km_2, & kn_1 &\geq kn_2, & km_1 - k\alpha_1 &\geq km_2 - k\alpha_2, \\ kn_1 + k\beta_1 &\geq kn_2 + k\beta_2,\end{aligned}\quad (12)$$

for $k \leq 0$. This indicates that

$$\begin{aligned}k\tilde{u} &\leq k\tilde{v}, & k &\geq 0, \\ k\tilde{u} &\geq k\tilde{v}, & k &\leq 0.\end{aligned}\quad (13)$$

□

Proposition 6. Let $\tilde{u}, \tilde{v}, \tilde{w}$ be three arbitrary LR flat fuzzy numbers. Then

- (1) $\tilde{u} \leq \tilde{u}$;
- (2) $\tilde{u} \leq \tilde{v}, \tilde{v} \leq \tilde{u} \implies \tilde{u} = \tilde{v}$;
- (3) $\tilde{u} \leq \tilde{v}, \tilde{v} \leq \tilde{w} \implies \tilde{u} \leq \tilde{w}$.

Proof. Suppose $\tilde{u} = (m_1, n_1, \alpha_1, \beta_1)_{LR}$, $\tilde{v} = (m_2, n_2, \alpha_2, \beta_2)_{LR}$, and $\tilde{w} = (m_3, n_3, \alpha_3, \beta_3)_{LR}$.

(1) Obviously, $\tilde{u} = \tilde{u}$; hence, we have $\tilde{u} \leq \tilde{u}$.

(2) Since $\bar{u} \leq \bar{v}, \bar{v} \leq \bar{u}$, we get

$$\begin{aligned} m_1 \leq m_2, \quad n_1 \leq n_2, \quad m_1 - \alpha_1 \leq m_2 - \alpha_2, \\ n_1 + \beta_1 \leq n_2 + \beta_2, \\ m_2 \leq m_1, \quad n_2 \leq n_1, \quad m_2 - \alpha_2 \leq m_1 - \alpha_1, \\ n_2 + \beta_2 \leq n_1 + \beta_1. \end{aligned} \quad (14)$$

This means

$$\begin{aligned} m_1 = m_2, \quad n_1 = n_2, \quad m_1 - \alpha_1 = m_2 - \alpha_2, \\ n_1 + \beta_1 = n_2 + \beta_2. \end{aligned} \quad (15)$$

That is

$$m_1 = m_2, \quad n_1 = n_2, \quad \alpha_1 = \alpha_2, \quad \beta_1 = \beta_2. \quad (16)$$

Therefore, we have $\bar{u} = \bar{v}$.

(3) From $\bar{u} \leq \bar{v}, \bar{v} \leq \bar{w}$, we get

$$\begin{aligned} m_1 \leq m_2, \quad n_1 \leq n_2, \quad m_1 - \alpha_1 \leq m_2 - \alpha_2, \\ n_1 + \beta_1 \leq n_2 + \beta_2, \\ m_2 \leq m_3, \quad n_2 \leq n_3, \quad m_2 - \alpha_2 \leq m_3 - \alpha_3, \\ n_2 + \beta_2 \leq n_3 + \beta_3. \end{aligned} \quad (17)$$

This indicates

$$\begin{aligned} m_1 \leq m_3, \quad n_1 \leq n_3, \quad m_1 - \alpha_1 \leq m_3 - \alpha_3, \\ n_1 + \beta_1 \leq n_3 + \beta_3. \end{aligned} \quad (18)$$

Therefore, we have $\bar{u} \leq \bar{w}$. $\square$

From Proposition 6, we know that the order relation $\leq$ is a partial order on the set of all LR fuzzy numbers.

2.4. Fully Fuzzy Linear Programming with LR Fuzzy Parameters. In this paper, we will consider the following model; that is,

$$\begin{aligned} \min \quad & z(\tilde{x}) = \tilde{c} \otimes \tilde{x} \\ \text{s.t.} \quad & \tilde{A} \otimes \tilde{x} \leq \tilde{b} \\ & \tilde{x} \geq \tilde{0}, \end{aligned} \quad (19)$$

or

$$\begin{aligned} \min \quad & z(\tilde{x}) = \tilde{c}_1 \otimes \tilde{x}_1 \oplus \tilde{c}_2 \otimes \tilde{x}_2 \oplus \cdots \oplus \tilde{c}_n \otimes \tilde{x}_n \\ \text{s.t.} \quad & \tilde{a}_{i1} \otimes \tilde{x}_1 \oplus \tilde{a}_{i2} \otimes \tilde{x}_2 \oplus \cdots \oplus \tilde{a}_{in} \otimes \tilde{x}_n \leq \tilde{b}_i, \\ & i = 1, 2, \dots, m, \end{aligned} \quad (20)$$

$$\tilde{x}_j \geq \tilde{0}, \quad j = 1, 2, \dots, n,$$

where $\tilde{c} = [\tilde{c}_j]_{1 \times n}$, $\tilde{b} = [\tilde{b}_i]_{m \times 1}$, $\tilde{A} = [\tilde{a}_{ij}]_{m \times n}$, and $\tilde{x} = [\tilde{x}_j]_{n \times 1}$ represent LR fuzzy matrices and vectors and $\tilde{c}_j$, $\tilde{b}_i$, $\tilde{a}_{ij}$, and $\tilde{x}_j$ are LR flat fuzzy numbers. The order relations for comparing

the LR flat fuzzy numbers both in the objective function and the constraint inequalities are as shown in Definition 4.

3. Proposed Method

Steps of the proposed method are given to solve problem (20) as follows. This method is applicable to minimization of FFLP problems, and the solution method of maximization problems is similar to that of minimization ones.

Step 1. If all the parameters $\tilde{c}_j, \tilde{b}_i, \tilde{a}_{ij}, \tilde{x}_j$ are represented by LR flat fuzzy numbers $(c_{j1}, c_{j2}, \alpha_{c_j}, \beta_{c_j})_{LR}$, $(b_{i1}, b_{i2}, \alpha_{b_i}, \beta_{b_i})_{LR}$, $(a_{ij1}, a_{ij2}, \alpha_{a_{ij}}, \beta_{a_{ij}})_{LR}$, and $(x_{j1}, x_{j2}, \alpha_{x_j}, \beta_{x_j})_{LR}$, then the FFLP (20) can be written as

$$\begin{aligned} \min \quad & z(\tilde{x}) = \sum_{j=1}^n \left((c_{j1}, c_{j2}, \alpha_{c_j}, \beta_{c_j})_{LR} \right. \\ & \left. \otimes (x_{j1}, x_{j2}, \alpha_{x_j}, \beta_{x_j})_{LR} \right) \\ \text{s.t.} \quad & \sum_{j=1}^n \left((a_{ij1}, a_{ij2}, \alpha_{a_{ij}}, \beta_{a_{ij}})_{LR} \otimes (x_{j1}, x_{j2}, \alpha_{x_j}, \beta_{x_j})_{LR} \right) \\ & \leq (b_{i1}, b_{i2}, \alpha_{b_i}, \beta_{b_i})_{LR}, \quad i = 1, 2, \dots, m, \\ & (x_{j1}, x_{j2}, \alpha_{x_j}, \beta_{x_j})_{LR} \geq (0, 0, 0, 0)_{LR}, \\ & j = 1, 2, \dots, n. \end{aligned} \quad (21)$$

Step 2. Calculate $(c_{j1}, c_{j2}, \alpha_{c_j}, \beta_{c_j})_{LR} \otimes (x_{j1}, x_{j2}, \alpha_{x_j}, \beta_{x_j})_{LR}$ and $(a_{ij1}, a_{ij2}, \alpha_{a_{ij}}, \beta_{a_{ij}})_{LR} \otimes (x_{j1}, x_{j2}, \alpha_{x_j}, \beta_{x_j})_{LR}$, respectively, and suppose that $(c_{j1}, c_{j2}, \alpha_{c_j}, \beta_{c_j})_{LR} \otimes (x_{j1}, x_{j2}, \alpha_{x_j}, \beta_{x_j})_{LR} = (x'_{j1}, x'_{j2}, \alpha'_{x_j}, \beta'_{x_j})_{LR}$ and $(a_{ij1}, a_{ij2}, \alpha_{a_{ij}}, \beta_{a_{ij}})_{LR} \otimes (x_{j1}, x_{j2}, \alpha_{x_j}, \beta_{x_j})_{LR} = (p_{ij}, q_{ij}, \gamma_{ij}, \delta_{ij})_{LR}$; then the FFLP problem obtained in Step 1 can be written as

$$\begin{aligned} \min \quad & z(\tilde{x}) = \left(\sum_{j=1}^n x'_{j1}, \sum_{j=1}^n x'_{j2}, \sum_{j=1}^n \alpha'_{x_j}, \sum_{j=1}^n \beta'_{x_j} \right)_{LR} \\ \text{s.t.} \quad & \left(\sum_{j=1}^n p_{ij}, \sum_{j=1}^n q_{ij}, \sum_{j=1}^n \gamma_{ij}, \sum_{j=1}^n \delta_{ij} \right)_{LR} \leq (b_{i1}, b_{i2}, \alpha_{b_i}, \beta_{b_i})_{LR}, \\ & i = 1, 2, \dots, m, \\ & (x_{j1}, x_{j2}, \alpha_{x_j}, \beta_{x_j})_{LR} \geq (0, 0, 0, 0)_{LR}, \\ & j = 1, 2, \dots, n. \end{aligned} \quad (22)$$

Step 3. According to the order relation defined above, the problem obtained in Step 2 is equivalent to

$$\begin{aligned}
 \min \quad & \sum_{j=1}^n x'_{j1} \\
 & \sum_{j=1}^n x'_{j2} \\
 & \sum_{j=1}^n (x'_{j1} - \alpha'_{x_j}) \\
 & \sum_{j=1}^n (x'_{j2} + \beta'_{x_j}) \\
 \text{s.t.} \quad & \sum_{j=1}^n p_{ij} \leq b_{i1}, \quad i = 1, 2, \dots, m, \\
 & \sum_{j=1}^n q_{ij} \leq b_{i2}, \quad i = 1, 2, \dots, m, \\
 & \sum_{j=1}^n (p_{ij} - \gamma_{ij}) \leq (b_{i1} - \alpha_{b_i}), \quad i = 1, 2, \dots, m, \\
 & \sum_{j=1}^n (q_{ij} + \delta_{ij}) \leq (b_{i2} + \beta_{b_i}), \quad i = 1, 2, \dots, m, \\
 & x_{j1} \leq x_{j2}, \quad \alpha_{x_j} \geq 0, \quad \beta_{x_j} \geq 0, \\
 & x_{j1} - \alpha_{x_j} \geq 0, \quad j = 1, 2, \dots, n.
 \end{aligned} \tag{23}$$

We denote $X = (x_{11}, x_{12}, \alpha_{x_1}, \beta_{x_1}, x_{21}, x_{22}, \alpha_{x_2}, \beta_{x_2}, \dots, x_{n1}, x_{n2}, \alpha_{x_n}, \beta_{x_n})^T$, $z_1(X) = \sum_{j=1}^n x'_{j1}$, $z_2(X) = \sum_{j=1}^n x'_{j2}$, $z_3(X) = \sum_{j=1}^n (x'_{j1} - \alpha'_{x_j})$, $z_4(X) = \sum_{j=1}^n (x'_{j2} + \beta'_{x_j})$ and $D = \{X \mid X \text{ satisfies the constraints of programming (23)}\}$. Programming (23) may be written as the programming (24), below for short, as follows:

$$\begin{aligned}
 \min \quad & z_1(X) \\
 & z_2(X) \\
 & z_3(X) \\
 & z_4(X) \\
 \text{s.t.} \quad & X \in D.
 \end{aligned} \tag{24}$$

Obviously, programming (24) is a crisp multiobjective linear programming problem. In fact we have $z(\tilde{x}) = (z_1(X), z_2(X), z_1(X) - z_3(X), z_4(X) - z_2(X))$.

Step 4. Solve the subproblems

$$\begin{aligned}
 \min \quad & z_t(X) \\
 \text{s.t.} \quad & X \in D,
 \end{aligned} \tag{25}$$

where $t = 1, 2, 3, 4$. We find optimal solutions X_1, X_2, X_3 , and X_4 , respectively. And the corresponding optimal values will be $z_1^{\min} = z_1(X_1)$, $z_2^{\min} = z_2(X_2)$, $z_3^{\min} = z_3(X_3)$, and $z_4^{\min} = z_4(X_4)$.

Step 5. Let $z_t^{\max} = \max\{z_t(X_1), z_t(X_2), z_t(X_3), z_t(X_4)\}$, $t = 1, 2, 3, 4$, and the membership function of $z_t(X)$ is given by

$$\mu_{z_t}(z_t(X)) = \begin{cases} 1, & z_t(X) < z_t^{\min}, \\ \frac{z_t^{\max} - z_t(X)}{z_t^{\max} - z_t^{\min}}, & z_t^{\min} \leq z_t(X) \leq z_t^{\max}, \\ 0, & z_t(X) > z_t^{\max}, \end{cases} \tag{26}$$

where $t = 1, 2, 3, 4$.

Step 6. Let $I_0 = \{1, 2, 3, 4\}$; the MOLP problem obtained in Step 3 can be equivalently written as

$$\begin{aligned}
 \max \quad & \lambda \\
 \text{s.t.} \quad & \mu_{z_t}(z_t(X)) \geq \lambda, \quad t \in I_0, \\
 & X \in D.
 \end{aligned} \tag{27}$$

Suppose X^1 is one of the optimal solutions (if there exists only one optimal solution, X^1 is the unique one), and λ^{1*} is the optimal objective value (in fact, the optimal solution should be written as (X^1, λ^{1*}) . Since λ is an auxiliary variable, we denote (X^1, λ^{1*}) as X^1 for simplicity). Then $\mu_{z_{s_1}}(z_{s_1}(X^1)) = \lambda^{1*}$ for at least one s_1 in I_0 . (s_1 is an arbitrary element in the set $J = \{j \mid \mu_{z_j}(z_j(X^1)) = \lambda^{1*}\}$).

Step 7. Let $I_1 = I_0 - \{s_1\}$, and solve the following crisp programming:

$$\begin{aligned}
 \max \quad & \lambda \\
 \text{s.t.} \quad & \mu_{z_t}(z_t(X)) \geq \lambda, \quad t \in I_1, \\
 & \mu_{z_{s_1}}(z_{s_1}(X)) = \lambda^{1*}, \\
 & X \in D.
 \end{aligned} \tag{28}$$

If X^2 is one of the optimal solutions and λ^{2*} is the optimal objective value, then $\mu_{z_{s_2}}(z_{s_2}(X^2)) = \lambda^{2*}$ for at least one s_2 in I_1 .

Step 8. Let $I_2 = I_0 - \{s_1, s_2\}$, and solve the following crisp programming:

$$\begin{aligned}
 \max \quad & \lambda \\
 \text{s.t.} \quad & \mu_{z_t}(z_t(X)) \geq \lambda, \quad t \in I_2, \\
 & \mu_{z_{s_1}}(z_{s_1}(X)) = \lambda^{1*}, \\
 & \mu_{z_{s_2}}(z_{s_2}(X)) = \lambda^{2*}, \\
 & X \in D.
 \end{aligned} \tag{29}$$

Suppose X^3 is one of the optimal solutions and λ^{3*} is the optimal objective value. Then $\mu_{z_{s_3}}(z_{s_3}(X^3)) = \lambda^{3*}$ for at least one s_3 in I_2 .

Step 9. Let $I_3 = I_0 - \{s_1, s_2, s_3\}$, and solve the following crisp programming:

$$\begin{aligned} \max \quad & \lambda \\ \text{s.t.} \quad & \mu_{z_t}(z_t(X)) \geq \lambda, \quad t \in I_3, \\ & \mu_{z_{s_1}}(z_{s_1}(X)) = \lambda^{1*}, \\ & \mu_{z_{s_2}}(z_{s_2}(X)) = \lambda^{2*}, \\ & \mu_{z_{s_3}}(z_{s_3}(X)) = \lambda^{3*}, \\ & X \in D. \end{aligned} \quad (30)$$

Suppose X^4 is one of the optimal solutions, and λ^{4*} is the optimal objective value. Then $\mu_{z_{s_4}}(z_{s_4}(X^4)) = \lambda^{4*}$ with s_4 in I_3 .

Step 10. Take $X^* = X^4$ as the compromised optimal solution to programming (23) and generate the compromised optimal solution $\tilde{x}^*$ to programming (21) by X^* . Assuming

$$\begin{aligned} X^* = & (x_{11}^*, x_{12}^*, \alpha_{x_1}^*, \beta_{x_1}^*, x_{21}, x_{22}^*, \alpha_{x_2}^*, \\ & \beta_{x_2}^*, \dots, x_{n1}, x_{n2}^*, \alpha_{x_n}^*, \beta_{x_n}^*)^T, \end{aligned} \quad (31)$$

we may obtain

$$\begin{aligned} \tilde{x}^* = & (\tilde{x}_1^*, \tilde{x}_2^*, \dots, \tilde{x}_n^*)^T \\ = & ((x_{11}^*, x_{12}^*, \alpha_{x_1}^*, \beta_{x_1}^*)_{LR}, (x_{21}, x_{22}^*, \alpha_{x_2}^*, \beta_{x_2}^*)_{LR}, \\ & \dots, (x_{n1}, x_{n2}^*, \alpha_{x_n}^*, \beta_{x_n}^*)_{LR})^T, \end{aligned} \quad (32)$$

and the corresponding objective value $z^* = z(\tilde{x}^*)$.

Remark 7 ($\{s_1, s_2, s_3, s_4\} = I = \{1, 2, 3, 4\}$). Some properties of the solutions obtained in Steps 6–10 are shown in the following proposition.

Proposition 8. Suppose $\mu_{z_{s_j}}(z_{s_j}(X^*)), X^j, \lambda^{j*}$ ($j = 1, 2, 3, 4$), and X^* are the notations described in Steps 1–10; then

$$\begin{aligned} (1) \quad & \mu_{z_{s_1}}(z_{s_1}(X^*)) = \lambda^{1*} = \mu_{z_{s_1}}(z_{s_1}(X^1)), \\ & \mu_{z_{s_2}}(z_{s_2}(X^*)) = \lambda^{2*} = \mu_{z_{s_2}}(z_{s_2}(X^2)), \\ & \mu_{z_{s_3}}(z_{s_3}(X^*)) = \lambda^{3*} = \mu_{z_{s_3}}(z_{s_3}(X^3)), \\ & \mu_{z_{s_4}}(z_{s_4}(X^*)) = \lambda^{4*} = \mu_{z_{s_4}}(z_{s_4}(X^4)), \\ (2) \quad & \lambda^{1*} \leq \lambda^{2*} \leq \lambda^{3*} \leq \lambda^{4*}. \end{aligned} \quad (33)$$

Proof. (1) From the results of Steps 6–9, it is obviously clear that

$$\begin{aligned} \lambda^{1*} &= \mu_{z_{s_1}}(z_{s_1}(X^1)), \\ \lambda^{2*} &= \mu_{z_{s_2}}(z_{s_2}(X^2)), \\ \lambda^{3*} &= \mu_{z_{s_3}}(z_{s_3}(X^3)), \\ \lambda^{4*} &= \mu_{z_{s_4}}(z_{s_4}(X^4)), \end{aligned} \quad (34)$$

and $\mu_{z_{s_4}}(z_{s_4}(X^*)) = \lambda^{4*}$ with $X^* = X^4$. Since $X^* = X^4$ is an optimal solution to programming (30), we know that X^* satisfies the constraints of programming (30), and so $\mu_{z_{s_1}}(z_{s_1}(X^*)) = \lambda^{1*}$, $\mu_{z_{s_2}}(z_{s_2}(X^*)) = \lambda^{2*}$, and $\mu_{z_{s_3}}(z_{s_3}(X^*)) = \lambda^{3*}$.

(2) In fact, (X^1, λ^{1*}) is an optimal solution to programming (27); therefore, it is a feasible solution. We have

$$\begin{aligned} \mu_{z_t}(z_t(X^1)) &\geq \lambda^{1*}, \quad t \in I_1 \subseteq I_0, \\ X^1 &\in D, \end{aligned} \quad (35)$$

and it is obvious that $\mu_{z_{s_1}}(z_{s_1}(X^*)) = \lambda^{1*}$ from the result of Step 6. Hence, (X^1, λ^{1*}) is a feasible solution to programming (28). The objective value of (X^1, λ^{1*}) is λ^{1*} , and the optimal objective value of programming (28) is λ^{2*} ; so we get $\lambda^{1*} \leq \lambda^{2*}$. It is similar to prove $\lambda^{2*} \leq \lambda^{3*}$ and $\lambda^{3*} \leq \lambda^{4*}$. $\square$

4. Numerical Example

In this section, we present a numerical example to illustrate the feasibility of the solution method proposed in Section 3.

We aim to find the compromised optimal solution and corresponding objective value of the following fully fuzzy linear programming problem:

$$\begin{aligned} \max \quad & z(\tilde{x}) = z(\tilde{x}_1, \tilde{x}_2) \\ & = (6, 7, 1, 2)_{LR} \otimes \tilde{x}_1 \oplus (7, 9, 1, 2)_{LR} \otimes \tilde{x}_2 \\ \text{s.t.} \quad & (9, 10, 2, 1)_{LR} \otimes \tilde{x}_1 \oplus (1, 1, 1, 1)_{LR} \otimes \tilde{x}_2 \\ & \leq (50, 55, 4, 3)_{LR}, \\ & (2, 3, 1, 1)_{LR} \otimes \tilde{x}_1 \oplus (4, 5, 1, 2)_{LR} \otimes \tilde{x}_2 \\ & \leq (66, 70, 3, 5)_{LR}, \\ & \tilde{x}_1 \geq \bar{0}, \quad \tilde{x}_2 \geq \bar{0}, \end{aligned} \quad (36)$$

where $\tilde{x}_1 = (x_{11}, x_{12}, \alpha_1, \beta_1)_{LR}$ and $\tilde{x}_2 = (x_{21}, x_{22}, \alpha_2, \beta_2)_{LR}$.

TABLE 1: The optimal values and solutions of the four subproblems.

	The optimal objective value	The optimal solution
(a)	$z_1^{\max} = 70$	$X_1 = (0, 0.3223, 0, 0.2887, 10, 10.1163, 7.0903, 0.1898)^T$
(b)	$z_2^{\max} = 103.3188$	$X_2 = (1.6015, 3.7101, 1.6039, 0, 4.2500, 8.5942, 3.1084, 0)^T$
(c)	$z_3^{\max} = 50$	$X_3 = (0, 0.2544, 0, 0.2973, 10, 10.0893, 0, 0.2179)^T$
(d)	$z_4^{\max} = 142.5882$	$X_4 = (1.4272, 4.2157, 1.2516, 0, 0, 0, 0, 11.6275)^T$

According to Steps 1 and 2 in the proposed method, we obtain the following programming:

$$\begin{aligned}
 \max \quad & z = (6x_{11} + 7x_{21}, 7x_{12} + 9x_{22}, 6\alpha_1 + 7\alpha_2 \\
 & + x_{11} + 2x_{21}, 7\beta_1 + 9\beta_2 + 2x_{12} + x_{22})_{LR} \\
 \text{s.t.} \quad & (9x_{11} + x_{21}, 10x_{12} + x_{22}, 9\alpha_1 + \alpha_2 + 2x_{11} \\
 & + x_{21}, 10\beta_1 + \beta_2 + x_{12} + x_{22})_{LR} \\
 & \leq (50, 55, 4, 3)_{LR}, \\
 & (2x_{11} + 4x_{21}, 3x_{12} + 5x_{22}, 2\alpha_1 + 4\alpha_2 + x_{11} \\
 & + x_{21}, 3\beta_1 + 5\beta_2 + x_{12} + 2x_{22})_{LR} \\
 & \leq (66, 70, 3, 5)_{LR}, \\
 & (x_{11}, x_{12}, \alpha_1, \beta_1)_{LR} \geq (0, 0, 0, 0)_{LR}, \\
 & (x_{21}, x_{22}, \alpha_2, \beta_2)_{LR} \geq (0, 0, 0, 0)_{LR}.
 \end{aligned} \tag{37}$$

By Step 3, the programming above is transformed into the following programming:

$$\begin{aligned}
 \max \quad & z_1 = 6x_{11} + 7x_{21} \\
 & z_2 = 7x_{12} + 9x_{22} \\
 & z_3 = 5x_{11} + 5x_{21} - 6\alpha_1 - 7\alpha_2 \\
 & z_4 = 9x_{12} + 10x_{22} + 7\beta_1 + 9\beta_2 \\
 \text{s.t.} \quad & 9x_{11} + x_{21} \leq 50, \\
 & 10x_{12} + x_{22} \leq 55, \\
 & 7x_{11} - 9\alpha_1 - \alpha_2 \leq 46, \\
 & 11x_{12} + 2x_{22} + 10\beta_1 + \beta_2 \leq 58, \\
 & 2x_{11} + 4x_{21} \leq 66, \\
 & 3x_{12} + 5x_{22} \leq 70, \\
 & x_{11} + 3x_{21} - 2\alpha_1 - 4\alpha_2 \leq 63, \\
 & 4x_{12} + 7x_{22} + 3\beta_1 + 5\beta_2 \leq 75, \\
 & x_{11} - \alpha_1 \geq 0, \quad x_{21} - \alpha_2 \geq 0, \\
 & x_{11} \leq x_{12}, \quad x_{21} \leq x_{22}, \\
 & \alpha_1, \alpha_2, \beta_1, \beta_2 \geq 0.
 \end{aligned} \tag{38}$$

Programming (38) can be abbreviated to the following programming:

$$\begin{aligned}
 \max \quad & z_1(X) \\
 & z_2(X) \\
 & z_3(X) \\
 & z_4(X) \\
 \text{s.t.} \quad & X \in D,
 \end{aligned} \tag{39}$$

where $X = (x_{11}, x_{12}, \alpha_1, \beta_1, x_{21}, x_{22}, \alpha_2, \beta_2)^T$.
Solve the following subproblems:

$$\begin{aligned}
 \text{(a) } \max \quad & z_1(X) \\
 \text{s.t.} \quad & X \in D, \\
 \text{(b) } \max \quad & z_2(X) \\
 \text{s.t.} \quad & X \in D, \\
 \text{(c) } \max \quad & z_3(X) \\
 \text{s.t.} \quad & X \in D, \\
 \text{(d) } \max \quad & z_4(X) \\
 \text{s.t.} \quad & X \in D,
 \end{aligned} \tag{40}$$

respectively, and we obtain the optimal objective value and one of the optimal solutions as shown in Table 1.

According to $z_i^{\min} = \min\{z_i(X_1), z_i(X_2), z_i(X_3), z_i(X_4)\}$, we acquire the lower objective values $z_1^{\min} = 8.5631$, $z_2^{\min} = 29.5098$, $z_3^{\min} = 0.3681$, and $z_4^{\min} = 107.2245$, with corresponding membership functions given below. Consider

$$\mu_{z_1}(z_1(X)) = \begin{cases} 1, & z_1(X) > 70; \\ \frac{z_1(X) - 8.5631}{70 - 8.5631}, & 8.5631 \leq z_1(X) \leq 70; \\ 0, & z_1(X) < 8.5631, \end{cases}$$

$$\mu_{z_2}(z_2(X)) = \begin{cases} 1, & z_2(X) > 103.3188; \\ \frac{z_2(X) - 29.5098}{103.3188 - 29.5098}, & 29.5098 \leq z_2(X) \leq 103.3188; \\ 0, & z_2(X) < 29.5098, \end{cases}$$

$$\mu_{z_3}(z_3(X)) = \begin{cases} 1, & z_3(X) > 50; \\ \frac{z_3(X) - 0.3681}{50 - 0.3681}, & 0.3681 \leq z_3(X) \leq 50; \\ 0, & z_3(X) < 0.3681, \end{cases}$$

$$\mu_{z_4}(z_4(X)) = \begin{cases} 1, & z_4(X) > 142.5882; \\ \frac{z_4(X) - 107.2245}{142.5882 - 107.2245}, & 107.2245 \leq z_4(X) \leq 142.5882; \\ 0, & z_4(X) < 107.2245. \end{cases} \quad (42)$$

By Steps 4–6, we get

$$\begin{aligned} \max \quad & \lambda \\ \text{s.t.} \quad & \mu_{z_1}(z_1(X)) = \frac{6x_{11} + 7x_{21} - 8.5631}{70 - 8.5631} \geq \lambda, \\ & \mu_{z_2}(z_2(X)) = \frac{7x_{12} + 9x_{22} - 29.5098}{103.3188 - 29.5098} \geq \lambda, \\ & \mu_{z_3}(z_3(X)) = \frac{5x_{11} + 5x_{21} - 6\alpha_1 - 7\alpha_2 - 0.3681}{50 - 0.3681} \geq \lambda, \\ & \mu_{z_4}(z_4(X)) = \frac{9x_{12} + 10x_{22} + 7\beta_1 + 9\beta_2 - 107.2245}{142.5882 - 107.2245} \geq \lambda, \\ & X \in D. \end{aligned} \quad (43)$$

The optimal objective value is $\lambda^{1*} = 0.6033$, and one of the optimal solutions is $X^1 = (2.5901, 3.9107, 0.4927, 0, 4.7997, 5.1848, 0.3109, 4.6127)^T$.

Calculate the value of the membership function of $z_t(X)$ ($t = 1, 2, 3, 4$) at $X = X^1$, and we get $\mu_{z_1}(z_1(X^1)) = 0.6604$, $\mu_{z_2}(z_2(X^1)) = 0.6033$, $\mu_{z_3}(z_3(X^1)) = 0.6336$, and $\mu_{z_4}(z_4(X^1)) = 0.6033$.

Solve the following problem:

$$\begin{aligned} \max \quad & \lambda \\ \text{s.t.} \quad & \mu_{z_1}(z_1(X)) = \frac{6x_{11} + 7x_{21} - 8.5631}{70 - 8.5631} \geq \lambda, \\ & \mu_{z_2}(z_2(X)) = \frac{7x_{12} + 9x_{22} - 29.5098}{103.3188 - 29.5098} \geq \lambda, \\ & \mu_{z_3}(z_3(X)) = \frac{5x_{11} + 5x_{21} - 6\alpha_1 - 7\alpha_2 - 0.3681}{50 - 0.3681} \geq \lambda, \\ & \mu_{z_4}(z_4(X)) = \frac{9x_{12} + 10x_{22} + 7\beta_1 + 9\beta_2 - 107.2245}{142.5882 - 107.2245} = 0.6033, \\ & X \in D. \end{aligned} \quad (44)$$

The optimal objective value is $\lambda^{2*} = 0.6033$, and one of the optimal solutions is $X^2 = (2.8, 3.9107, 0.3135, 0, 4.7161, 5.1850, 0.4310, 4.6124)^T$.

Calculate the value of the membership function of $z_t(X)$ ($t = 1, 2, 3, 4$) at $X = X^2$, and we get $\mu_{z_1}(z_1(X^2)) = 0.6714$, $\mu_{z_2}(z_2(X^2)) = 0.6033$, $\mu_{z_3}(z_3(X^2)) = 0.6511$, and $\mu_{z_4}(z_4(X^2)) = 0.6033$.

Solve the following problem:

$$\begin{aligned} \max \quad & \lambda \\ \text{s.t.} \quad & \mu_{z_1}(z_1(X)) = \frac{6x_{11} + 7x_{21} - 8.5631}{70 - 8.5631} \geq \lambda, \\ & \mu_{z_3}(z_3(X)) = \frac{5x_{11} + 5x_{21} - 6\alpha_1 - 7\alpha_2 - 0.3681}{50 - 0.3681} \geq \lambda, \\ & \mu_{z_2}(z_2(X)) = \frac{7x_{12} + 9x_{22} - 29.5098}{103.3188 - 29.5098} = 0.6033, \\ & \mu_{z_4}(z_4(X)) = \frac{9x_{12} + 10x_{22} + 7\beta_1 + 9\beta_2 - 107.2245}{142.5882 - 107.2245} = 0.6033, \\ & X \in D. \end{aligned} \quad (45)$$

The optimal objective value is $\lambda^{3*} = 0.7126$, and one of the optimal solutions is $X^3 = (2.6750, 3.9107, 0.1158, 0.0005, 5.1850, 5.1850, 0.2608, 4.6125)^T$.

Calculate the value of the membership function of $z_t(X)$ ($t = 1, 2, 3, 4$) at $X = X^3$, and we get $\mu_{z_1}(z_1(X^3)) = 0.7126$, $\mu_{z_2}(z_2(X^3)) = 0.6033$, $\mu_{z_3}(z_3(X^3)) = 0.7274$, and $\mu_{z_4}(z_4(X^3)) = 0.6033$.

TABLE 2: Values of the four membership functions at X^j .

	$\mu_{z_1}(z_1(X))$	$\mu_{z_2}(z_2(X))$	$\mu_{z_3}(z_3(X))$	$\mu_{z_4}(z_4(X))$
$X = X^1$	0.6604	0.6033	0.6336	0.6033
$X = X^2$	0.6714	0.6033	0.6511	0.6033
$X = X^3$	0.7126	0.6033	0.7274	0.6033
$X = X^4$	0.7126	0.6033	0.7844	0.6033

TABLE 3: Values of the objective function $z(\tilde{x})$ at $\tilde{x}^j$.

	$z(\tilde{x})$
$\tilde{x} = \tilde{x}^1$	$(49.13, 74.03, 17.32, 38.96)_{LR}$
$\tilde{x} = \tilde{x}^2$	$(49.81, 74.03, 17.13, 38.96)_{LR}$
$\tilde{x} = \tilde{x}^3$	$(52.34, 74.03, 15.56, 38.96)_{LR}$
$\tilde{x} = \tilde{x}^4$	$(52.34, 74.03, 13.04, 38.96)_{LR}$

Solve the following problem:

$$\begin{aligned}
& \max \quad \lambda \\
& \text{s.t.} \quad \mu_{z_3}(z_3(X)) = \frac{5x_{11} + 5x_{21} - 6\alpha_1 - 7\alpha_2 - 0.3681}{50 - 0.3681} \geq \lambda, \\
& \quad \mu_{z_1}(z_1(X)) = \frac{6x_{11} + 7x_{21} - 8.5631}{70 - 8.5631} = 0.7126, \\
& \quad \mu_{z_2}(z_2(X)) = \frac{7x_{12} + 9x_{22} - 29.5098}{103.3188 - 29.5098} = 0.6033, \\
& \quad \mu_{z_4}(z_4(X)) \\
& \quad = \frac{9x_{12} + 10x_{22} + 7\beta_1 + 9\beta_2 - 107.2245}{142.5882 - 107.2245} = 0.6033, \\
& \quad X \in D.
\end{aligned} \tag{46}$$

The optimal objective value is $\lambda^{4*} = 0.7844$, and one of the optimal solutions is $X^4 = (2.6754, 3.9106, 0, 0, 5.1843, 5.1848, 0, 4.6128)^T$.

Calculate the value of the membership function of $z_t(X)$ ($t = 1, 2, 3, 4$) at $X = X^4$, and we get $\mu_{z_1}(z_1(X^4)) = 0.7126$, $\mu_{z_2}(z_2(X^4)) = 0.6033$, $\mu_{z_3}(z_3(X^4)) = 0.7844$, and $\mu_{z_4}(z_4(X^4)) = 0.6033$.

Generate $\tilde{x}^j$ by X^j ($j = 1, 2, 3, 4$), following Step 10, and calculate the value of $z(\tilde{x}^j)$. As shown in Tables 2 and 3, the solution $\tilde{x}^{j+1}$ (or X^{j+1}) is better than $\tilde{x}^j$ (or X^j), $j = 1, 2, 3$.

Following Step 10, we find

$$X^* = X^4 = (2.6754, 3.9106, 0, 0, 5.1843, 5.1848, 0, 4.6128)^T. \tag{47}$$

Therefore,

$$\begin{aligned}
\tilde{x}^* &= \tilde{x}^4 \\
&= ((2.6754, 3.9106, 0, 0)_{LR}, \\
&\quad (5.1843, 5.1848, 0, 4.6128)_{LR})^T,
\end{aligned} \tag{48}$$

serves as the compromised optimal solution with corresponding objective value

$$z^* = z(\tilde{x}^*) = (52.34, 74.03, 13.04, 38.96)_{LR}. \tag{49}$$

5. Concluding Remarks

To the end, we show the following concluding remarks.

- (1) In this paper, we proposed a new method to find the compromised optimal solution to the fully fuzzy linear programming problems with all the parameters and variables represented by *LR* flat fuzzy numbers. The solution is also an *LR* flat fuzzy number. In this sense, we get an exact solution, which may give more help to the decision makers.
- (2) The order relation in the objective function is the same as that in the constraint inequalities. Based on the definition of the order relation, the FFLP can be equivalently transformed into a MOLP, which is a crisp programming that is easy to be solved.
- (3) Considering the MOLP problem, classical fuzzy programming method is modified for obtaining the compromised optimal solution.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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