Research Article

Uniform Nonlinear Constitutive Model and Parameters for Clay in Different Consolidation Conditions Based on Regression Method

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The nonlinear constitutive relations of clay are investigated considering different initial conditions. Highly compressible clay is selected as the test sample. Two groups of tri-axial compression tests are performed, respectively, after K_0 consolidation and isotropic consolidation. On the basis of the framework of $E \sim \nu$ model, a uniform nonlinear constitutive model is proposed by fitting the test data. With the average slope of the unloading-reloading curve selected as the unloading modulus, the unloading function is constructed as the loading-unloading criterion. Moreover, a comparison between the experimental stress-strain curves and the results predicted by the constitutive model is made. It is shown that the prediction is reasonable, which can reflect the stress-strain behavior of the soil under the K_0 consolidation and isotropic consolidation conditions. The maximum relative error of the two series of curves is not remarkable, less than 6%.

1. Introduction

In geotechnical engineering, the nonlinearity nature of soil not only is influenced by a stress path but also has a close relationship with a load and a loading method. Therefore, it is a key to choose a suitable mechanical model of the medium material in geotechnical calculation.

Duncan [1, 2] presented an incremental nonlinear elastic model based on this relationship curve, which was generally called the Duncan-Chang model. A hyperbolic function was adopted to fit the tri-axial experiment stress-strain curve of soil [3]. The hyperbolic model has been a widely used nonlinear constitutive relationship for clay because of its simple model structure and accessible parameters [4]. This constitutive model was based on the data of tri-axial shear tests [5] under regular loading path. Thus, it had a simple framework and can raise the calculation efficiency.

Subsequently, some researchers attempted to improve the Duncan-Chang model and studied different soil constitutive models and their influence on results [6–8]. They concluded

that the modified Duncan-Chang model was a good compromise between prediction accuracy and availability of parameters from conventional tri-axial compression testing. It was developed for cohesionless soils that had two parts [9, 10]: an elastic nonlinear part and two mechanisms of plasticity. It was allowed to take into account the nonlinearity of the behavior at low stress level for overconsolidated materials [11]. The description of the model and its parameters are given [12]. To consider the effect of different stress paths, some scholars [13, 14] developed the Duncan-Chang model, derived a formula of tangent elastic modulus under different stress paths, and provided approximate elastic-plastic models for clay [15, 16]. Lode parameter was introduced into Duncan-Chang model to reflect the effect of intermediate principal stress [17]. To consider the soil structural damage during loading, the concept of damage rate was introduced to modify Duncan-Chang model [18, 19].

In these modified models, the initial stress state was assumed to be isotropic. However, the initial stress state of naturally deposited clay is usually anisotropic, known

FIGURE 1: Experiment testing machine and sample.

FIGURE 2: Stress path of compression experiment.

as K_0 consolidation [20]. The anisotropic initial stress and strain states result in different strength and deformation characteristics, and the constitutive relationship was different from that of isotropic stress states [21]. To overcome these differences, a unified nonlinear constitutive model was proposed for clay under different initial conditions. Two groups of tri-axial compression tests for highly compressible clay are performed, as K_0 consolidation and isotropic consolidation. By fitting the test data, a factor of anisotropic consolidation is introduced into the framework of $E \sim v$ model to construct a uniform nonlinear constitutive model. With the average slope of the unloading-reloading curve selected as the unloading modulus, the unloading function is constructed as the loading-unloading criterion. Furthermore, the rationality of this model is checked. A comparison is made between the experimental stress-strain curves and the computational results predicted by the constitutive model. It is shown that the model can reflect the stress-strain behavior of the soil under the K_0 consolidation and isotropic consolidation conditions.

2. Experiment Analysis

2.1. Sample Preparation. In this study, natural saturation clay sampled from Wuhan is studied. Considering the anisotropic initial stress and strain state, two parallel drain tri-axial tests were carried out. In these tests, the static lateral pressure

coefficient $K_0 = \sigma_3/\sigma_1$ was set to 0.562 [22]. Then, two parallel lateral confinement compression tests were carried out to acquire the natural compression coefficient a , and it equals 0.567 MPa^{-1} . Some other physical properties of the studied clay are shown in Table 1.

Considering the inevitable influence of disturbance and the difference characteristics in mechanics and physics of undisturbed soil sampling from different place, definite divergence was inevitable. Therefore, the moisture content is decided by field investigation. The remodeling soil sample of test passed by a series process: the undisturbed soil samples are made into uniform saturation samples, then tamping in 5 layers in tramper, and then extraction air before test. After the sample is installed, counter-pressure saturation is carried out until the saturation reaches larger than 99%. Then tri-axial shear test can be carried out under specified stress path.

2.2. Tri-Axial Test Paths. Consolidated drained tri-axial compression tests (CD) were carried out in two different stress paths, by using SJ-1AG conventional tri-axial testing machine (shown as Figure 1). One test path is drained tri-axial compression tests under K_0 consolidation conditions (CK_0D). As comparison, the other is chosen as isotropic consolidation (CD). The stress paths of the two group tests are shown in Figure 2.

In the CD test, prophase average consolidation pressures p_a were varied from 50 to 300 kPa, and the interval between

TABLE 1: Physical properties of the studied clay.

Physical property	w (%)	ρ (g/cm ³)	G_s	w_p (%)	w_L (%)	e	S_r (%)
Value	28.53	1.961	2.697	28.18	47.82	0.897	98.9

TABLE 2: Initial stresses of CK_0D tests.

p_a /kPa	σ_1 /kPa	σ_3 /kPa
50	70.6	39.7
100	141.3	79.4
150	211.9	119.1
200	282.4	158.7
250	353.1	198.4
300	423.9	238.2

every single test was 50 kPa. The tri-axial compression test cannot be carried out unless the pore pressure dissipation reaches 95%; otherwise consolidation will be sustained. Similarly, CK_0D tests also consist of 6 series of parallel tests with different consolidation pressures. In consolidation, the axial pressure and lateral pressure are added step by step until initial consolidation pressure p_a reaches 50~300 kPa. Unlike the CD test, the initial σ_3 is different from σ_1 in the CK_0D tests. As $p_a = (\sigma_1 + 2\sigma_3)/3$, the initial stresses of CK_0D tests are determined and shown in Table 2.

2.3. Result of Test. Figure 3 is the relationship between generalized shear stress q and generalized shear strain $\bar{\epsilon}$ as initial consolidation pressure is 200 kPa of two different stress paths. Similarly, Figure 4 shows the relationship between generalized shear stress q and generalized volumetric strain ϵ_v .

The two figures indicate that there are different stress-strain relationships for different stress paths. In the initial stage of load, the stress in isotropic pressure test (CD test) increases slower than that of K_0 consolidation (CK_0D test). In the later stage, both $\bar{\epsilon}$ and ϵ_v in CD test increase faster. Finally, the peak values in the CK_0D test are slightly above those of CD test. It is because the lateral deformation constraints in CK_0D test are less than CD . Thus, it is also shown that the influence of initial anisotropy on the constitutive relationship cannot be neglected.

3. Framework of Model

3.1. Tangent Elastic Modulus E_t . Assuming that the elastic relationship of $(\sigma_1 - \sigma_3) \sim \epsilon_1$ is hyperbolic based on the generalized Hooke law and test data, we choose the tangent elastic model ($E\sim\nu$ model) as the model framework. In view of initial anisotropy, we introduce the lateral pressure coefficient into this model. Thus, it can be expressed as

$$E_t = E_i \left[1 - \frac{R_f (1 - \sin \varphi) (\sigma_1 - \sigma_3)^{0.85/K_0}}{2c \cos \varphi + 2\sigma_3 \sin \varphi} \right]^2, \quad (1)$$

where K_0 is the lateral pressure coefficient; E_t is the tangent elastic modulus; R_f is the damage ratio, $R_f = (\sigma_1 - \sigma_3)_f / (\sigma_1 - \sigma_3)_{ult}$; $(\sigma_1 - \sigma_3)_{ult}$ is the asymptotic value of curve $(\sigma_1 - \sigma_3) \sim \epsilon_1$; $(\sigma_1 - \sigma_3)_f$ is the breakdown strength and it can be determined by the Mohr Coulomb failure criterion:

$$(\sigma_1 - \sigma_3)_f = \frac{(2c \cos \varphi + 2\sigma_3 \sin \varphi)}{(1 - \sin \varphi)}. \quad (2)$$

In the equation above, c denotes the cohesive strength of soil; φ denotes the internal friction angle of soil; σ_1 and σ_3 denote the axial and radial stresses, respectively, based on the nonlinear regression analysis method [23]; the initial modulus of the curve E_i can be determined by the following:

$$E_i = K p_a \left(\frac{\sigma_3}{p_a} \right)^n, \quad (3)$$

where p_a is the atmospheric pressure; K and n are experiment parameters. In log-log coordinates, there is linear relationship between E_i and σ_3 and the slope is n . K is the intercept of the line when $\sigma_3 = p_a$.

3.2. Tangent Poisson Ratio ν_t . The relationship between ϵ_1 and ϵ_3 is always hyperbolic. Equation (4) shows the expression of the Poisson ratio ν_t deduced as

$$\nu_t = \frac{\nu_i}{(1 - D\epsilon_1)^2}, \quad (4)$$

where D is a test parameter determined by the slope of the linear relationship between ϵ_r/ϵ_1 and ϵ_r , $\epsilon_r = (3\epsilon_v - \epsilon_1)/2$; ϵ_v, ϵ_1 represent the volumetric strain and the axial strain, respectively; ν_i is the Poisson ratio and can be determined by the following:

$$\nu_i = G - F \log \left(\frac{\sigma_3}{p_a} \right). \quad (5)$$

In the equation above, G and F are experiment parameters and can be determined according to the curve between ν_i and σ_3 in semilog coordinates. F is the slope of the $\nu_i - \log \sigma_3$ line and G is the intercept of the line when $\sigma_3 = p_a$. Axial strain ϵ_1 can be determined by the following:

$$\epsilon_1 = (\sigma_1 - \sigma_3) \times \left(E_i \left[1 - \frac{R_f (1 - \sin \varphi) (\sigma_1 - \sigma_3)^{0.85/K_0}}{2c \cos \varphi + 2\sigma_3 \sin \varphi} \right] \right)^{-1}. \quad (6)$$

3.3. Unloading Modulus E_{ur} . Based on experimental data, Duncan assumed that the unloading modulus E_{ur} does not change with the change of $\sigma_1 - \sigma_3$. It is only related to σ_3 .

FIGURE 3: $q \sim \bar{\varepsilon}$ curves of different compression experiments. FIGURE 4: $q \sim \varepsilon_v$ curves of different compression experiments.

According to experimental unloading-reloading curves, based on the nonlinear regression analysis method [23], the relationship between E_{ur} and σ_3 can be drawn. It is approximately linear in the logarithmic coordinates, as shown in the following:

$$E_{ur} = K_{ur} p_a \left(\frac{\sigma_3}{p_a} \right)^n, \quad (7)$$

where K_{ur} is an empirical constant and can be expressed as the intercept of the line between $\log(E_{ur}/p_a)$ and $\log(\sigma_3/p_a)$ when $\sigma_3 = p_a$.

4. Constitutive Model

4.1. Parameters for the Model. In K_0 consolidation, the clay appears to be anisotropic state. For p equals 100, 200, and 300 kPa, the lateral pressure σ_3 can be determined from K_0 . The corresponding values are 79.4, 158.7, and 238.1 kPa. By the relationship between $\varepsilon_1/(\sigma_1 - \sigma_3)$ and ε_1 (shown in Figure 5), the intercept $1/E_i$ can be determined. The Initial elastic modulus E_i can be calculated correspondingly. It is shown in Table 3.

TABLE 3: Initial tangent elastic modulus E_i .

$\sigma_3/100 \text{ kPa}$	$1/E_i$	$E_i/100 \text{ kPa}$	$\bar{E}_i/100 \text{ kPa}$
0.794	0.0045	222.2	
1.587	0.0040	250.0	242.8
2.381	0.0039	256.0	

In log-log coordinates, the relationship between E_i and σ_3 is almost linear, and it is shown in Figure 6.

Figure 6 indicates when $\sigma_3 = 100 \text{ kPa}$, the corresponding vertical coordinate $K = 230.1$, and the slope n of the line equals 0.139.

Based on the result of test, axial strain ε_1 and volumetric strain ε_v are determined; after conversion between strains, the relationship curve can be drawn between $\varepsilon_r/\varepsilon_1$ and ε_r . In view of that we can determine the parameter G in fitting line (Figure 8) only when $\sigma_3 = 100 \text{ kPa}$, and we choose some test data which can cover 100 kPa. That is, $\sigma_3 = 39.7 \text{ kPa}$, 79.4 kPa , and 158.7 kPa (as Table 1). Thus, the curve of $\varepsilon_r/\varepsilon_1 \sim \varepsilon_r$ is shown in Figure 7.

FIGURE 5: $\epsilon_1/(\sigma_1 - \sigma_3) \sim \epsilon_1$ curves of different compression experiments.

FIGURE 7: $\epsilon_r/\epsilon_1 \sim \epsilon_r$ curves.

FIGURE 6: $E_i \sim \sigma_3$ curve.

FIGURE 8: $\nu_i \sim \sigma_3$ curve.

Based on (5), the linear relationship between ν_i and σ_3 in semilog coordinates is shown in Figure 8. The figure indicates that corresponding straight slope and unit intercept are $F = 0.09$ and $G = 0.0642$.

Mohr's circle can be drawn based on parameters of limit states for different ambient pressures. It is shown in Figure 9 that c equals 21 kPa and φ equals 31.3° .

$(\sigma_1 - \sigma_3)_{\text{ult}}$ and R_f are determined by the curve relationship between $\sigma_1 - \sigma_3$ and ϵ_1 . The calculation result is shown in Table 4.

Similar to the solution to elastic modulus E_t , the relationship between E_{ur} and σ_3 can be drawn in log-log coordinates. It is shown in Figure 10. When σ_3 equals 100 kPa, the corresponding vertical coordinate K_{ur} equals 295.8, and the slope n equals 0.362.

TABLE 4: Limit state parameter.

$\sigma_3/100$ kPa	$(\sigma_1 - \sigma_3)_f$	$(\sigma_1 - \sigma_3)_{\text{ult}}$	R_f	$\bar{R}_f$
0.794	264.1	286.0	0.9140	
1.587	446.0	600.0	0.7433	0.820
2.381	630.7	786.0	0.8024	

4.2. Constitutive Model. Integrating the results of the above calculation, the $E \sim \nu$ model can be obtained for K_0 consolidation. The tangent elastic modulus is shown in the following:

$$E_t = 230.1 \left[1 - \frac{0.394(\sigma_1 - \sigma_3)^{0.85/K_0}}{0.403 + 1.039\sigma_3} \right]^2 p_a \left(\frac{\sigma_3}{p_a} \right)^{0.139}. \quad (8)$$

FIGURE 9: Mohr circle of shearing limit state.

FIGURE 10: $E_{ur} \sim \sigma_3$ curve.

The tangent Poisson ratio is shown in the following:

$$\nu_t = \left\{ 0.064 - 0.09 \log \left(\frac{\sigma_3}{p_a} \right) \right\} \times \left\{ 1 - \left(3.13 (\sigma_1 - \sigma_3) \right) \times \left(230.1 \left[1 - \frac{0.394 (\sigma_1 - \sigma_3)^{0.85/K_0}}{0.403 + 1.039 \sigma_3} \right]^2 \times p_a \left(\frac{\sigma_3}{p_a} \right)^{0.139} \right)^{-1} \right\}^{-2} \quad (9)$$

The coefficient K_0 is determined by stress values in (8) and (9). Correspondingly, the uniform relationship between the stress and strain increments is shown in the following:

$$\{d\sigma\} = [D] \{d\varepsilon\}, \quad (10)$$

FIGURE 11: Computation diagram of FEA.

where $\{d\sigma\} = \{d\sigma_{x(r)}, d\sigma_{y(\theta)}, d\sigma_z, d\tau_{xy(rz)}\}^T$; $\{d\varepsilon\} = \{d\varepsilon_{x(r)}, d\varepsilon_{y(\theta)}, d\varepsilon_z, d\varepsilon_{xy(rz)}\}^T$.

In (10), $[D]$ can be described as follows:

$$[D] = \frac{E}{(1 + \nu_t)(1 - 2\nu_t)} \begin{bmatrix} 1 - \nu_t & \nu_t & \nu_t & 0 \\ \nu_t & 1 - \nu_t & \nu_t & 0 \\ \nu_t & \nu_t & 1 - \nu_t & 0 \\ 0 & 0 & 0 & 1 - 2\nu_t \end{bmatrix}, \quad (11)$$

where E represents the elastic modulus of clay. In the equation above, loading and unloading states can be determined by (7) and (8). At loading stage, $E = E_t$; at unloading stage $E_{ur} = 295.8 p_a (\sigma_3 / p_a)^{0.362}$; at reloading stage, $E = E_{ur}$. This modulus is used to separate elastic strain from plastic one.

5. Numerical Analysis

To verify the validity of the model proposed in this paper, the authors have implemented the constitutive relationship into a finite element program and made a numerical simulation of tri-axial tests. The height and diameter of the clay sample are 8 cm and 3.91 cm, respectively. Taking the symmetry of the sample into consideration, a half of the sample is adopted in the numerical simulation. In view of symmetry, horizontal displacement of nodes on the axes of symmetry is zero. Similarly, because of rigid fastening of test base, vertical constraints are applied to all the nodes of the sample. Moreover, the other two boundaries are force boundaries. The upper boundary is applied with σ_1 and the right is σ_3 . The computation model is shown in Figure 11. There are 45 nodes and 64 3-node triangular elements in the simulation.

The relationship of $q \sim \varepsilon_1$ is obtained, in which, q is the shear stress of the sample and ε_1 is the average axial strain of the sample. Comparison between the numerical results and that of the corresponding test has been made. The part of comparison is shown in Figure 12.

Figure 12 indicates that the nonlinear model proposed in this paper can reflect the relationship of tri-axial shear test for K_0 consolidation (in CK_0D test, $K_0 = 0.562$; in CD test, $K_0 = 1$). Separation appears only at the later stage when the

FIGURE 12: Comparison between numerical simulation and test curves.

sample is close to failure. At this stage, the result from test is slightly larger than that of simulation, but the maximum error is only 6.0%.

6. Conclusion

The initial condition of natural clay is usually under K_0 stress states. To display the different mechanical characteristics, two series of initial consolidation condition tri-axial compress experiments are done. It is shown that there are some differences of a soil under isotropic conditions. In the initial stage of load, the stress in isotropic pressure test increases slower than that of K_0 consolidation. In the later stage, the shear strain in K_0 consolidation increases faster. It indicates that the influence of stress paths to the constitutive relationship cannot be neglected.

According to the tests, a factor of lateral pressure coefficient is introduced into the $E \sim \nu$ model. A uniform non-linear constitutive model is proposed by fitting of the stress-strain behavior of clay with different initial conditions. Based on nonlinear regression analysis, some model parameters are provided.

With the average slope of the unloading-reloading curve selected as the unloading modulus, the constitutive relations can be promoted to an elastoplastic model. The proposed model can reflect the stress-strain relationship under K_0 conditions. Some discrepancy between the model prediction and the test data occurs at the stage close to failure. The maximum error is just 6.0%.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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