Research Article

An Improved Interacting Multiple Model Algorithm Used in Aircraft Tracking

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There are some problems in traditional interacting multiple model algorithms (IMM) when used in target tracking systems. For instance, the mode transition matrix is inaccurate and cannot be determined when the sojourn times are not known. To solve these problems, an optimal mode transition matrix IMM (OMTM-IMM) algorithm is proposed in this paper. The linear minimum variance theory is used to calculate the mode transition matrix which depends on the continuous system state rather than the sojourn times in this algorithm. Moreover, the correlation of the subfilter is considered; hence the covariance matrices are utilized to compute mode transition matrix. In this algorithm, the model probability is defined as a diagonal matrix which is combined with the filters outputs; thus the effects produced by each state can be distinguished. Finally, to verify the superiority of the new algorithm, the theoretical proof and simulation results are given. They show that the OMTM-IMM algorithm can improve the tracking accuracy and can be utilized in the complex environment.

1. Introduction

For an aircraft target tracking system, related technologies include target tracking, control algorithms, and fault detection. For control algorithm and fault detection, it can be referred to [1–4] for the latest research results. Our paper focuses on the target tracking problem which is currently popular in researches on tracking system. Interacting multiple model algorithm (IMM) is originally proposed by Blom in 1984 [5]. Because of the excellent compromise between performance and complexity, it is widely applied in maneuvering target tracking [6–8]. To further enhance the capability of maneuvering target tracking, many improved IMM algorithms are generated, such as interacting multiple bias model (IMBM) algorithm and variable structure interacting multiple model (VSIMM) algorithm [9–14]. In these papers, the mode transition matrix only depends on the sojourn times, rather than continuous state. This is not conforming to the hybrid systems [13, 15–18].

In [19], the authors pointed out that, in IMM algorithm, if the mode transition matrix only depends on the sojourn times, the mode transition matrix is inaccurate and cannot be determined when the sojourn times are not known. To

improve its performance, Li and Bar-Shalom proposed a variable structure IMM algorithm in [12]. The set of modes are chosen based on the continuous state estimation in the algorithm. However, the mode transition matrices are constant in this algorithm. It means that the mode transition matrices are influenced by the sojourn times. The literature [15–17] also has been devoted to solving the problem. They estimate the mode transition probabilities based on complex guard conditions. However the computational costs would be very high if the dimension of the continuous state space is large. Thus, the algorithm causes the maneuvering target tracking system to lose the real-time performance. To achieve better real-time performance, in [18], the authors proposed a hybrid estimation algorithm based on the IMM approach with continuous-state-dependent transitions. They derived two models to express the guard conditions. The computational cost is dependent only on the dimension of the guard conditions and is independent of the dimension of continuous state space. However their method to obtain parameters of mode transition probabilities only gives a solution set rather than a definite explicit solution.

To solve the problem where the mode transition matrix is only dependent on the sojourn times, a new IMM algorithm

with the optimal mode transition matrix is proposed, named as optimal mode transition matrix IMM (OMTM-IMM) algorithm. Our work differs from those of the above papers [9–18] in that we calculate the mode transition matrix without the guard conditions and the sojourn times. In this paper, we derive the optimal mode transition matrix based on the linear minimum variance theory. To further improve the estimation accuracy of this algorithm, we consider the effects produced by each state and the correlations between subfilters. Thereby, we define the mode probability as a diagonal matrix. As a result, the effects produced by each state can be distinguished. Then, the covariance of estimation is used to calculate the optimal mode transition matrix. Ultimately, the mode transition matrix of OMTM-IMM algorithm depends on the continuous state of the systems and the correlations between subfilters. Then, the switch of maneuvering target motion state will be more accurate. Therefore, the estimation accuracy of OMTM-IMM algorithm is improved and the algorithm is more suitable for maneuvering target tracking.

This rest of the paper is organized as follows. Section 2 discusses the jump linear target tracking problem. Section 3 shows the proposed algorithm. Section 4 provides the analysis of proposed algorithms. Simulation results are shown in Section 5. Finally, Section 6 gives the conclusions of our work.

2. The Jump Linear Target Tracking Problem

The jump linear maneuvering tracking system can be modeled by the following equations:

$$\begin{aligned} \mathbf{x}(k) &= \mathbf{F}(k | k-1, m_k) \mathbf{x}(k-1) \\ &\quad + \mathbf{\Gamma}(k | k-1, m_k) \mathbf{w}(k-1, m_k), \end{aligned} \quad (1)$$

$$\mathbf{z}(k) = \mathbf{H}(k, m_k) \mathbf{x}(k) + \mathbf{v}(k, m_k). \quad (2)$$

Formula (1) is state equation, and formula (2) is measurement equation. The state $\mathbf{x}(k)$ is an a -dimensional vector; the observation process $\mathbf{z}(k)$ is a b -dimensional vector; m_k is model state (system mode index) which denotes the mode or model (the models are linear-Gaussian models) in effect during the sampling period starting at k , $m_k \in \mathbb{N}$, $\mathbb{N} = \{1, 2, \dots, n\}$; $\mathbf{w}(k-1, m_k)$ and $\mathbf{v}(k, m_k)$ are mutually uncorrelated white Gaussian noise vector with variance $\mathbf{Q}(k-1, m_k)$ and $\mathbf{R}(k, m_k)$, respectively, and they are uncorrelated with the Gaussian initial state $\mathbf{x}(0)$; $\mathbf{F}(k | k-1, m_k)$, $\mathbf{\Gamma}(k | k-1, m_k)$, and $\mathbf{H}(k, m_k)$ are the state matrices corresponding to the mode m_k at time k . Let $\mathbf{F}_j$, $\mathbf{\Gamma}_j$, $\mathbf{H}_j$, $\mathbf{Q}_j$, and $\mathbf{R}_j$ denote $\mathbf{F}(k | k-1, m_k)$, $\mathbf{\Gamma}(k | k-1, m_k)$, $\mathbf{H}(k, m_k)$, $\mathbf{Q}(k-1, m_k)$, and $\mathbf{R}(k, m_k)$ when $m_k = j$ ($\mathbf{F}_j$, $\mathbf{\Gamma}_j$, $\mathbf{H}_j$, $\mathbf{Q}_j$, and $\mathbf{R}_j$ are known).

In traditional IMM algorithm, to deal with jump linear maneuvering tracking problem, it is assumed that mode (model) switching process is a Markov process (Markov chain) with known mode transition probabilities [11]. In other words, these mode transition probabilities will be assumed to be time-invariant and independent of the base state. However, this assumption is inappropriate when the sojourn times are unknown. Hence, we can assume that mode switching process is a Markov process with time-variant and dependent on the base state mode transition probabilities. Based on this assumption, we propose an OMTM-IMM algorithm with

the optimal mode transition probabilities. The optimal mode transition probabilities depend on the continuous state of the systems and the correlations between subfilters and can describe the switching process of target motions accurately.

3. OMTM-IMM Algorithm

3.1. The Optimal Mode Transition Probability. In IMM algorithm, mode transition probabilities are used to estimate mixing probability; accordingly the initial state is calculated by the mixing probability and state estimation of last time interval. We find that the accuracy of initial state and state estimation are tightly correlated with the accuracy of mode transition probabilities (the proofs are given in Section 4). Therefore, if the mode transition probabilities can make the initial state have the least error, then the mode transition probabilities are the optimal ones. Based on this result, we can utilize the linear minimum variance theory to minimize the error of initial state; the purpose is to derive the optimal mode transition probabilities. Besides, in order to distinguish the effects produced by each state, we define the mode probability as a diagonal matrix.

The derivation steps of the optimal mode transition probabilities are as follows.

Define the mode transition probability as $\pi_{ij}(k-1)$, mixing probability as $\mu_{ilj}(k-1)$, and mode probability as $\mu_j(k-1)$, $i, j \in \mathbb{N}$. They are all $n \times n$ diagonal matrices and satisfy the following equations:

$$\begin{aligned} \pi_{1j}(k-1) + \pi_{2j}(k-1) + \dots + \pi_{nj}(k-1) &= \mathbf{I}, \\ \mu_{1lj}(k-1) + \mu_{2lj}(k-1) + \dots + \mu_{nlj}(k-1) &= \mathbf{I}, \\ \mu_1(k-1) + \mu_2(k-1) + \dots + \mu_j(k-1) &= \mathbf{I}, \end{aligned} \quad (3)$$

where $\mathbf{I}$ is the identity matrix.

Let $\hat{\mathbf{x}}_j(k-1)$ ($j \in \mathbb{N}$) be the estimation of state vector $\mathbf{x}_j(k-1)$ of j th model, $\hat{\mathbf{x}}_{0j}(k-1)$ the mixed initial estimation, and $\mathbf{x}(k-1)$ the true state (i.e., true mixed initial state based on j th model). The optimal information fusion (i.e., linear minimum variance) estimation is given by

$$\begin{aligned} \hat{\mathbf{x}}_{0j}(k-1) &= \sum_{i=1}^n \mu_{ilj}(k-1) \hat{\mathbf{x}}_i(k-1), \\ \mathbf{x}(k-1) &= \sum_{i=1}^n \mu_{ilj}(k-1) \mathbf{x}_i(k-1). \end{aligned} \quad (4)$$

Step 1. Derive the variance of the initial state.

Define mixed initial error $\tilde{\mathbf{x}}_{0j}(k-1)$ and estimation error $\tilde{\mathbf{x}}_j(k-1)$ of j th model as follows:

$$\tilde{\mathbf{x}}_{0j}(k-1) = \mathbf{x}(k-1) - \hat{\mathbf{x}}_{0j}(k-1), \quad (5)$$

$$\tilde{\mathbf{x}}_j(k-1) = \mathbf{x}_j(k-1) - \hat{\mathbf{x}}_j(k-1). \quad (6)$$

Assuming that $\tilde{\mathbf{x}}_{0i}(k-1)$ and $\tilde{\mathbf{x}}_{0j}(k-1)$ ($i \neq j, i, j \in \mathbb{N}$) are correlated, accordingly, the variance and cross covariance are denoted by $\tilde{\mathbf{P}}_{ii}(k-1)$, $\tilde{\mathbf{P}}_{jj}(k-1)$, and $\tilde{\mathbf{P}}_{ij}(k-1)$, respectively. According to (4), and (6), (5) can be written as

$$\tilde{\mathbf{x}}_{0j}(k-1) = \sum_{i=1}^n \mu_{ilj}(k-1) \tilde{\mathbf{x}}_i(k-1). \quad (7)$$

Define

$$\begin{aligned} \mathbf{D}_j(k-1) &= [\boldsymbol{\mu}_{1j}(k-1) \quad \boldsymbol{\mu}_{2j}(k-1) \quad \cdots \quad \boldsymbol{\mu}_{nj}(k-1)]^T, \\ \tilde{\mathbf{X}}(k-1) &= [\tilde{\mathbf{x}}_1(k-1) \quad \tilde{\mathbf{x}}_2(k-1) \quad \cdots \quad \tilde{\mathbf{x}}_n(k-1)]^T. \end{aligned} \quad (8)$$

Equation (7) can be simplified as

$$\tilde{\mathbf{x}}_{0j}(k-1) = \mathbf{D}_j^T(k-1) \tilde{\mathbf{X}}(k-1). \quad (9)$$

Because of (3), we find

$$\mathbf{D}_j^T(k-1) \mathbf{U} = \mathbf{I}, \quad (10)$$

where $\mathbf{U} = [\mathbf{I} \quad \mathbf{I} \quad \cdots \quad \mathbf{I}]^T$.

The variance of initial state is

$$\begin{aligned} \tilde{\mathbf{P}}_{0j}(k) &= E[\tilde{\mathbf{x}}_{0j}(k) \tilde{\mathbf{x}}_{0j}^T(k)] \\ &= \mathbf{D}_j^T(k-1) E[\tilde{\mathbf{X}}(k-1) \tilde{\mathbf{X}}^T(k-1)] \mathbf{D}_j(k-1) \quad (11) \\ &= \mathbf{D}_j^T(k-1) \tilde{\mathbf{P}}(k-1) \mathbf{D}_j(k-1), \end{aligned}$$

where $E[\cdot]$ is the expectation,

$$\begin{aligned} \tilde{\mathbf{P}}(k-1) &= \begin{bmatrix} \tilde{\mathbf{P}}_{11}(k-1) & \tilde{\mathbf{P}}_{12}(k-1) & \cdots & \tilde{\mathbf{P}}_{1n}(k-1) \\ \tilde{\mathbf{P}}_{21}(k-1) & \tilde{\mathbf{P}}_{22}(k-1) & \cdots & \tilde{\mathbf{P}}_{2n}(k-1) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{P}}_{n1}(k-1) & \tilde{\mathbf{P}}_{n2}(k-1) & \cdots & \tilde{\mathbf{P}}_{nn}(k-1) \end{bmatrix}, \\ \tilde{\mathbf{P}}_{ij}(k-1) &= E[\tilde{\mathbf{x}}_i(k-1) \tilde{\mathbf{x}}_j^T(k-1)]. \end{aligned} \quad (12)$$

From (11), we can find that the $\tilde{\mathbf{P}}_{0j}(k)$ use all states covariance information between different models.

Step 2. Minimize the variance of the initial state.

In this part, we apply the Lagrange multiplier method to minimize the variance of initial state. The performance index is defined as

$$\mathbf{J} = \text{tr}(\tilde{\mathbf{P}}_{0j}(k)), \quad (13)$$

where $\text{tr}(\cdot)$ denotes the trace of matrix. The problem is to find the optimal solution of $\mathbf{D}_j(k-1)$ under restriction (10). Thus, we should minimize the performance index $\mathbf{J}$. Introducing the Lagrange multiplier method, we have

$$\mathbf{G} = \mathbf{J} + 2 \text{tr}[\mathbf{Y}(\mathbf{D}_j^T(k-1) \mathbf{U} - \mathbf{I})], \quad (14)$$

where $\mathbf{G}$ is an auxiliary function and $\mathbf{Y}$ is Lagrangian with $n \times n$ dimensions. Set

$$\left. \frac{\partial \mathbf{G}}{\partial \mathbf{D}_j(k-1)} \right|_{\mathbf{D}_j(k-1)=\mathbf{D}_j^{\text{opt}}(k-1)} = \mathbf{0}, \quad (15)$$

where $\mathbf{D}_j^{\text{opt}}(k-1)$ is the optimal value of $\mathbf{D}_j(k-1)$ when the $\mathbf{J}$ is the least. We have

$$\tilde{\mathbf{P}}(k-1) \mathbf{D}_j^{\text{opt}}(k-1) + \mathbf{U} \mathbf{Y} = \mathbf{0}. \quad (16)$$

Combining (10) with (16) yields the matrix equation as

$$\begin{pmatrix} \tilde{\mathbf{P}}(k-1) & \mathbf{U} \\ \mathbf{U}^T & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{D}_j^{\text{opt}}(k-1) \\ \mathbf{Y} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{I} \end{pmatrix}. \quad (17)$$

It is clearly that $\tilde{\mathbf{P}}(k-1)$ is a symmetric positive matrix. Hence, $\mathbf{U}^T \tilde{\mathbf{P}}^{-1}(k-1) \mathbf{U}$ is nonsingular; then we have the optimal solution to minimize the performance index $\mathbf{J}$

$$\mathbf{D}_j^{\text{opt}}(k-1) = \tilde{\mathbf{P}}^{-1}(k-1) \mathbf{U} (\mathbf{U}^T \tilde{\mathbf{P}}^{-1}(k-1) \mathbf{U})^{-1}. \quad (18)$$

Then, substituting (18) into (11), we can obtain the minimum variance matrix of the optimal initial state of j th model

$$\tilde{\mathbf{P}}_{0j}^{\text{opt}}(k-1) = (\mathbf{U}^T \tilde{\mathbf{P}}^{-1}(k-1) \mathbf{U})^{-1}. \quad (19)$$

Step 3. Obtain the optimal mode transition probabilities.

Because of

$$\begin{aligned} \boldsymbol{\mu}_{ij}(k-1) &= \boldsymbol{\pi}_{ij} \boldsymbol{\mu}_i(k-1) \bar{\mathbf{c}}_j^{-1}, \\ \bar{\mathbf{c}}_j &= \sum_{i=1}^n \boldsymbol{\pi}_{ij} \boldsymbol{\mu}_i(k-1). \end{aligned} \quad (20)$$

Using (8), (18), and (20), we can obtain the optimal solution of $\boldsymbol{\pi}_{ij}(k-1)$

$$\begin{aligned} \boldsymbol{\pi}_{ij}^{\text{opt}}(k-1) &= \prod_{b \neq j}^n \boldsymbol{\mu}_b(k-1) \sum_{a=1}^n \boldsymbol{\alpha}_{ja}(k-1) \\ &\times \left[\sum_{d=1}^n \left(\prod_{b \neq d}^n \boldsymbol{\mu}_b(k-1) \sum_{a=1}^n \boldsymbol{\alpha}_{da}(k-1) \right) \right]^{-1} \\ &\quad (i, j, a, b, d \in \mathbb{N}), \end{aligned} \quad (21)$$

where

$$\begin{aligned} \tilde{\mathbf{P}}^{-1}(k-1) &= \boldsymbol{\alpha}(k-1), \\ \boldsymbol{\alpha}(k-1) &= \begin{bmatrix} \boldsymbol{\alpha}_{11}(k-1) & \boldsymbol{\alpha}_{12}(k-1) & \cdots & \boldsymbol{\alpha}_{1n}(k-1) \\ \boldsymbol{\alpha}_{21}(k-1) & \boldsymbol{\alpha}_{22}(k-1) & \cdots & \boldsymbol{\alpha}_{2n}(k-1) \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\alpha}_{n1}(k-1) & \boldsymbol{\alpha}_{n2}(k-1) & \cdots & \boldsymbol{\alpha}_{nn}(k-1) \end{bmatrix}. \end{aligned} \quad (22)$$

Since $\partial^2 \mathbf{G} / \partial [\mathbf{D}_j(k-1)]^2 = \tilde{\mathbf{P}}(k-1) > \mathbf{0}$, $\mathbf{D}_j^{\text{opt}}(k-1)$ is the optimal solution to minimize the performance index $\mathbf{J}$; we also have the relationship

$$\text{tr}(\tilde{\mathbf{P}}_{0j}^{\text{opt}}(k-1)) \leq \text{tr}(\tilde{\mathbf{P}}_{0j}(k-1)). \quad (23)$$

3.2. The OMTM-IMM Algorithm

Step 1. Calculate the optimal mode transition probabilities. Consider

$$\begin{aligned} \boldsymbol{\pi}_{ij}^{\text{opt}}(k-1) &= \prod_{b \neq j}^n \boldsymbol{\mu}_b(k-1) \sum_{a=1}^n \boldsymbol{\alpha}_{ja}(k-1) \\ &\times \left[\sum_{d=1}^n \left(\prod_{b \neq d}^n \boldsymbol{\mu}_b(k-1) \sum_{a=1}^n \boldsymbol{\alpha}_{da}(k-1) \right) \right]^{-1} \\ &\quad (i, j, a, b, d \in \mathbb{N}). \end{aligned} \quad (24)$$

Step 2. Calculate the mixed initial probabilities for the j th model ($i, j \in \mathbb{N}$). Consider

$$\boldsymbol{\mu}_{ij}(k-1) = \boldsymbol{\pi}_{ij}(k-1) \boldsymbol{\mu}_i(k-1) \bar{\mathbf{c}}_j^{-1}. \quad (25)$$

Step 3. Calculate the mixed initial state and corresponding variance for the j th ($i, j \in \mathbb{N}$) model.

$$\begin{aligned} \hat{\mathbf{x}}_{0j}(k-1) &= \sum_{i=1}^n \boldsymbol{\mu}_{ij}(k-1) \hat{\mathbf{x}}_i(k-1), \\ \tilde{\mathbf{P}}_{0j}^{\text{opt}}(k-1) &= (\mathbf{U}^T \tilde{\mathbf{P}}^{-1}(k-1) \mathbf{U})^{-1}. \end{aligned} \quad (26)$$

Step 4. Consider mode-matched filtering ($i, j \in \mathbb{N}$). The estimation $\hat{\mathbf{x}}_{0j}(k-1)$ and variance $\tilde{\mathbf{P}}_{0j}^{\text{opt}}(k-1)$ are used as input to the j th ($i, j \in \mathbb{N}$) model, which uses $\mathbf{z}(k)$ to yield $\tilde{\mathbf{x}}_j(k)$ and $\tilde{\mathbf{P}}_{jj}(k)$. The likelihood functions corresponding to the j th filter are represented as

$$\Lambda_j(k) = \begin{bmatrix} \Lambda_j^1(k) & 0 & \cdots & 0 \\ 0 & \Lambda_j^2(k) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Lambda_j^n(k) \end{bmatrix},$$

$$\begin{aligned} \Lambda_j^i(k) &= p[z^i(k) | m_k = j, \hat{\mathbf{x}}_{0j}^i(k-1), \tilde{\mathbf{P}}_{0j}^i(k-1)] \\ &= N[z^i(k); \hat{\mathbf{z}}_j^i[k | k-1, \hat{\mathbf{x}}_{0j}^i(k-1)], S_j^i[k; \tilde{\mathbf{P}}_{0j}^i(k-1)]], \end{aligned} \quad (27)$$

where subscript denotes the model index and superscript denotes the state index. $\Lambda_j^i(k)$ is the likelihood function corresponding to the i th state of the j th filter and $\tilde{\mathbf{P}}_{0j}^i(k-1)$ is the variance of $\hat{\mathbf{x}}_{0j}^i(k-1)$.

Step 5. Update mode probability and combine the state estimates and corresponding variance ($i, j \in \mathbb{N}$). Consider

$$\boldsymbol{\mu}_j(k) = \Lambda_j(k) \bar{\mathbf{c}}_j^{-1}, \quad (28)$$

where $\mathbf{c}$ is the normalization constant matrix,

$$\mathbf{c} = \sum_{i=1}^n \Lambda_j(k) \bar{\mathbf{c}}_j,$$

$$\hat{\mathbf{x}}(k) = \sum_{j=1}^n \boldsymbol{\mu}_j(k) \hat{\mathbf{x}}_j(k),$$

$$\mathbf{P}(k) = \sum_{j=1}^n \boldsymbol{\mu}_j(k) \left\{ \tilde{\mathbf{P}}_{jj}(k) + [\hat{\mathbf{x}}_j(k) - \hat{\mathbf{x}}(k)] [\hat{\mathbf{x}}_j(k) - \hat{\mathbf{x}}(k)]^T \right\}. \quad (29)$$

4. Accuracy Analyses

4.1. Accuracy Analysis of Parallel Filters of OMTM-IMM. Under the linear minimum variance criterions, the optimal mode transition probability $\boldsymbol{\pi}_{ij}(k-1)$ is derived, and the optimal mixed initial states have the minimum variance

matrix. Thereof, the following proof is for the influence of optimal mixed initial states on parallel filters.

Let $\tilde{\mathbf{x}}_j(k | k-1) \triangleq \mathbf{x}_j(k) - \hat{\mathbf{x}}_j(k | k-1)$, $\tilde{\mathbf{x}}_j(k) \triangleq \mathbf{x}_j(k) - \hat{\mathbf{x}}_j(k)$, and $\tilde{\mathbf{x}}_{0j}(k) \triangleq \mathbf{x}_{0j}(k) - \hat{\mathbf{x}}_{0j}(k)$.

Thereby,

$$\begin{aligned} \tilde{\mathbf{x}}_j(k) &= \mathbf{x}_j(k) - \hat{\mathbf{x}}_j(k | k-1) \\ &\quad - \mathbf{K}_j(k) [\mathbf{z}_j(k) - \mathbf{H}_j \hat{\mathbf{x}}_j(k | k-1)] \\ &= [\mathbf{I} - \mathbf{K}_j(k) \mathbf{H}_j] \tilde{\mathbf{x}}_j(k | k-1) - \mathbf{K}_j(k) \mathbf{v}_j(k), \end{aligned} \quad (30)$$

where $\mathbf{K}_j$ is the gain matrix. Assuming that $\tilde{\mathbf{x}}_j(k)$ and $\mathbf{v}_j(k)$ are uncorrelated, we can obtain the states variance

$$\begin{aligned} \tilde{\mathbf{P}}_{jj}(k) &= E[\tilde{\mathbf{x}}_j(k) \tilde{\mathbf{x}}_j^T(k)] \\ &= [\mathbf{I} - \mathbf{K}_j(k) \mathbf{H}_j] E[\tilde{\mathbf{x}}_j(k | k-1) \tilde{\mathbf{x}}_j^T(k | k-1)] \\ &\quad \times [\mathbf{I} - \mathbf{K}_j(k) \mathbf{H}_j]^T + \mathbf{K}_j(k) \mathbf{R}_j \mathbf{K}_j^T(k). \end{aligned} \quad (31)$$

According to [20], the above formula can be written as

$$\tilde{\mathbf{P}}_{jj}^{-1}(k) = E[\tilde{\mathbf{x}}_j(k | k-1) \tilde{\mathbf{x}}_j^T(k | k-1)]^{-1} + \mathbf{H}_j^T \mathbf{R}_j^{-1} \mathbf{H}_j. \quad (32)$$

Meanwhile,

$$\begin{aligned} \tilde{\mathbf{x}}_j(k | k-1) &= \mathbf{x}_j(k) - \hat{\mathbf{x}}_j(k | k-1) \\ &= \mathbf{F}_j \tilde{\mathbf{x}}_{0j}(k-1) + \boldsymbol{\Gamma}_j \mathbf{w}(k-1). \end{aligned} \quad (33)$$

Assuming that $\tilde{\mathbf{x}}_{0j}(k)$ and $\mathbf{w}_j(k)$ are uncorrelated, thereby,

$$\begin{aligned} E[\tilde{\mathbf{x}}_j(k | k-1) \tilde{\mathbf{x}}_j^T(k | k-1)] \\ = \mathbf{F}_j \tilde{\mathbf{P}}_{0j}(k-1) \mathbf{F}_j^T + \boldsymbol{\Gamma}_j \mathbf{Q}_j \boldsymbol{\Gamma}_j^T. \end{aligned} \quad (34)$$

By using (32) and (34), we can obtain

$$\begin{aligned} \text{tr}(\tilde{\mathbf{P}}_{jj}^{-1}(k)) \\ = \text{tr}([\mathbf{F}_j \tilde{\mathbf{P}}_{0j}(k-1) \mathbf{F}_j^T + \boldsymbol{\Gamma}_j \mathbf{Q}_j \boldsymbol{\Gamma}_j^T]^{-1} + \mathbf{H}_j^T \mathbf{R}_j^{-1} \mathbf{H}_j). \end{aligned} \quad (35)$$

Using the linear minimum variance criterions, we have

$$\begin{aligned} \text{tr}(\tilde{\mathbf{P}}_{jj}^{\text{opt}-1}(k)) \\ = \text{tr}([\mathbf{F}_j \tilde{\mathbf{P}}_{0j}^{\text{opt}}(k-1) \mathbf{F}_j^T + \boldsymbol{\Gamma}_j \mathbf{Q}_j \boldsymbol{\Gamma}_j^T]^{-1} + \mathbf{H}_j^T \mathbf{R}_j^{-1} \mathbf{H}_j). \end{aligned} \quad (36)$$

Because of (23), comparing (35) with (36), we have the relationship that

$$\text{tr}(\tilde{\mathbf{P}}_{jj}^{\text{opt}}(k)) \leq \text{tr}(\tilde{\mathbf{P}}_{jj}(k)), \quad (37)$$

which shows that the accuracy of all parallel filters in OMTM-IMM are better than the ones in traditional IMM.

4.2. Accuracy Analysis of OMTM-IMM. The state $\mathbf{x}$ and observation $\mathbf{z}$ are given in Section 2. We define the linear space spanned by $\mathbf{z}$ as $L(\mathbf{z})$. Let $\hat{\mathbf{x}}_{\text{OMTM-IMM}}$ be the estimation of $\mathbf{x}$ based on $\hat{\mathbf{x}}_{0j}^{\text{opt}}$ and $L_{\text{OMTM-IMM}}(\hat{\mathbf{x}}_1, \dots, \hat{\mathbf{x}}_n)$ and let $\hat{\mathbf{x}}_{\text{IMM}}$ be the estimation of $\mathbf{x}$ based on $L_{\text{IMM}}(\hat{\mathbf{x}}_1, \dots, \hat{\mathbf{x}}_n)$, where $L_{\theta}(\hat{\mathbf{x}}_1, \dots, \hat{\mathbf{x}}_n)$ ($\theta = \text{OMTM-IMM, IMM}$) are the set $\{\mu_1 \hat{\mathbf{x}}_1 + \dots + \mu_n \hat{\mathbf{x}}_n\}$. Define the variance of $\hat{\mathbf{x}}_{\text{OMTM-IMM}}$ and $\hat{\mathbf{x}}_{\text{IMM}}$ as $\mathbf{P}_{\text{OMTM-IMM}}$ and $\mathbf{P}_{\text{IMM}}$. Thereof, the following is the influence of optimal mode transition probabilities on OMTM-IMM.

In traditional IMM, $\pi_{ij}(k-1)$ is a constant, $\mu_j(k-1)$ is a scalar, and $\tilde{\mathbf{x}}_{0i}(k-1)$ and $\tilde{\mathbf{x}}_{0j}(k-1)$ ($i \neq j$) are uncorrelated; in OMTM-IMM, $\pi_{ij}(k-1)$ is a variation, $\mu_j(k-1)$ is a diagonal matrix, and $\tilde{\mathbf{x}}_{0i}(k-1)$ and $\tilde{\mathbf{x}}_{0j}(k-1)$ ($i \neq j$) are correlated. That means that IMM is a special case of OMTM-IMM; thus we have the relationship

$$L_{\text{IMM}}(\hat{\mathbf{x}}_1, \dots, \hat{\mathbf{x}}_n) \subseteq L_{\text{OMTM-IMM}}(\hat{\mathbf{x}}_1, \dots, \hat{\mathbf{x}}_n). \quad (38)$$

From (38), we can get

$$\text{tr}(\mathbf{P}_{\text{OMTM-IMM}}) \leq \text{tr}(\mathbf{P}_{\text{IMM}}). \quad (39)$$

Equation (39) shows that the accuracy of OMTM-IMM is better than traditional IMM.

5. Simulations

In this section, we consider two aircraft tracking examples. In the first one, the sojourn time τ_i is known. In the other one, τ_i is unknown.

5.1. The Known Sojourn Time

Simulation Scenario. As we just consider the horizontal motion, the change of aircraft's height can be ignored. Figure 1 shows the path of aircraft in X-Y plan. The initial position is (10 m, 20 m), and the initial velocity is (20 m/s, 20 m/s). For the tracking of the aircraft, three models are employed: constant velocity (CV) motion, left constant turn (LCT) motion, and right constant turn (RCT) motion. And it executes a 3-motion sequence (CV-LCT-RCT): (1) CV motion in 100 s, (2) LCT model in 100 s, and (3) RCT model in 100 s; State $\mathbf{x} = [P_x \ V_x \ P_y \ V_y]^T$ consists of the x -axis position and velocity and y -axis position and velocity.

CV model is

$$F_1 = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad Q_1 = 9 \begin{bmatrix} \frac{t^4}{4} & \frac{t^4}{4} & 0 & 0 \\ \frac{t^3}{2} & t^2 & 0 & 0 \\ 0 & 0 & \frac{t^4}{4} & \frac{t^4}{4} \\ 0 & 0 & \frac{t^3}{2} & t^2 \end{bmatrix}; \quad (40)$$

FIGURE 1: x-y trajectory of target.

LCT model is

$$F_2 = \begin{bmatrix} 1 & \frac{\sin(\omega_l T)}{\omega_l} & 0 & -\frac{(1 - \cos(\omega_l T))}{\omega_l} \\ 0 & \cos(\omega_l T) & 0 & -\sin(\omega_l T) \\ 0 & \frac{(1 - \cos(\omega_l T))}{\omega_l} & 1 & \frac{\sin(\omega_l T)}{\omega_l} \\ 0 & \sin(\omega_l T) & 0 & \cos(\omega_l T) \end{bmatrix}, \quad (41)$$

$$Q_2 = 4 \begin{bmatrix} \frac{t^4}{4} & \frac{t^4}{4} & 0 & 0 \\ \frac{t^3}{2} & t^2 & 0 & 0 \\ 0 & 0 & \frac{t^4}{4} & \frac{t^4}{4} \\ 0 & 0 & \frac{t^3}{2} & t^2 \end{bmatrix};$$

RCT model is

$$F_3 = \begin{bmatrix} 1 & \frac{\sin(\omega_r T)}{\omega_r} & 0 & -\frac{(1 - \cos(\omega_r T))}{\omega_r} \\ 0 & \cos(\omega_r T) & 0 & -\sin(\omega_r T) \\ 0 & \frac{(1 - \cos(\omega_r T))}{\omega_r} & 1 & \frac{\sin(\omega_r T)}{\omega_r} \\ 0 & \sin(\omega_r T) & 0 & \cos(\omega_r T) \end{bmatrix}, \quad (42)$$

$$Q_3 = Q_2,$$

where $T = 1$, $t = 0.1$, $\omega_l = 2\pi/180$, and $\omega_r = -2\pi/180$.

In traditional IMM, π can be calculated by τ_i . Because of no uniqueness requirement for the transition matrix if its

FIGURE 2: States estimation error of Section 5.1.

dimension is no less than three, without loss of generality, we assume $\pi_{ij} = \pi_{iq}$ ($i \neq j \neq q, i, j, q \in \mathbb{N}$); we have

$$\pi = \begin{bmatrix} 0.99 & 0.005 & 0.005 \\ 0.005 & 0.99 & 0.005 \\ 0.005 & 0.005 & 0.99 \end{bmatrix}. \quad (43)$$

In OMTM-IMM, π can be calculated by (21). The simulation results are shown in Figures 2 and 3 after 100 Monte Carlo runs.

Figures 2 and 3 show the contrast of the tracking accuracy and mode estimation accuracy between the traditional IMM algorithm and the proposed OMTM-IMM algorithm. The mode transitions occur at $t = 100$ s and $t = 200$ s, respectively. In Figure 2, the total error consists of the error of x -axis and the error of y -axis, and the total tracking error of IMM algorithm is larger than the new algorithm. To simplify Figure 3, we only give the mode probabilities of x -axis velocity in OMTM-IMM; the mode probabilities of other states are roughly analogous. It is clearly from Figure 3 that the mode probabilities of traditional IMM algorithm are not good and have large delay in detecting mode transitions. The OMTM-IMM has consistently better tracking accuracy and mode estimation accuracy than the IMM, and the mode probabilities change with little delay. This is achieved with the use of the optimal mode transition matrix which is derived based on linear minimum variance criterions and more dynamics information.

5.2. The Unknown Sojourn Time

Simulation Scenario. The models are the same as Section 5.1. The difference is that the τ_i is unknown. In IMM, π cannot be calculated; therefore we assumed that it is the same as the one in example 1. The simulation results are shown in Figures 4 and 5.

FIGURE 3: Mode probabilities of Section 5.1.

FIGURE 4: States estimation error of Section 5.2.

Figure 4 shows that the states estimation accuracy of OMTM-IMM algorithms is higher than the IMM algorithm. In particular, in Figure 5, the mode probabilities of IMM are worse than Section 5.1; however, the new algorithm is satisfactory as the shown result in Section 5.1. The optimal mode transition matrix of OMTM-IMM changes with aircraft dynamics. It is suitable for the stochastic motion. How-

FIGURE 5: Mode probabilities of Section 5.2.

TABLE 1: Run-time statistics.

	Simulation A	Simulation B
IMM	10.6306 s	5.1381 s
OMTM-IMM	12.1974 s	6.2586 s

The above simulations' results show that the OMTM-IMM has an obvious advantage in maneuvering target tracking. Nevertheless, the computational effort is slightly increased.

ever the classical IMM algorithm departs from the practical situation and causes the degradation of performance.

The computation complexity of the algorithms can be shown through the run-time statistics in Table 1. The algorithms are implemented in MATLAB R2011b on 3.4 GHz Intel(R) Core(TM) i3-2130 CPU computer operating under Windows 7. In simulation A, the running time is for 100 times' Monte Carlo simulations, each time consisting of 300 steps, and, in simulation B, the running time is for 100 times' Monte Carlo simulations, each time consisting of 150 steps.

6. Conclusions

To solve the problem that the mode transition matrix is inaccurate and cannot be determined when the sojourn times are not known, the OMTM-IMM algorithm is proposed based on the linear minimum variance theory. Therefore, the optimal mode transition matrix depends on continuous state estimation rather than sojourn time, and the relevance of the subfilter and the effects produced by each state is considered. The simulation results show that the tracking accuracy of OMTM-IMM algorithm is better than IMM algorithm. And we can conclude that OMTM-IMM algorithm is more suitable for the maneuvering target tracking problem. Although the computation complexity of OMTM-IMM algorithm is

slightly larger than IMM algorithm, it can be remedied by more advanced computer technology in the future.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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