We investigate the effects of handoff on system performance in two-tier cellular networks. Two of the main performance metrics are new call blocking probability and handoff drop rate. We develop analytical models to evaluate the performance of two different handoff schemes. One scheme considers only femto-to-macrocell handoff while the other is bidirectional including macro-to-femtocell handoff. Our model is more elaborate than existing ones which have not considered the mobility of mobile stations. Numerical results show that the bidirectional scheme performs better than the femto-to-macrocell handoff as it achieves lower blocking probability and drop rate.

1. Introduction

Two-tier wireless networks [1–3] have been proposed to accommodate the ever-increasing data traffic in various outdoor and indoor environments by supporting high throughput and improving the energy efficiency of wireless communications [4–6]. The two-tier networks are configured by including femtocells within the coverage of a macrocell. To this end, network operators need to choose cell access scheme. They can have mobile stations try femtocells first or macrocells when accessing network resources. We believe it is worth developing mathematical model of different access schemes to evaluate performance and understand further the characteristics of the two-tier networks.

Various research efforts regarding the performance of the two-tier network have appeared in [7–9]. Spectrum-sharing schemes between macrocell and femtocell, as well as among femtocells, have been proposed to improve spatial reuse gain in [7]. The schemes limit the maximum intensity of simultaneously transmitting femtocells that satisfy a given per-tier outage constraint. Stochastic geometry is used to model and analyze performance of two-tier networks in a multichannel environment [8]. The proposed model quantifies the performance gain in outage probability and shows the existence of an optimal spectrum sensing threshold. A semidistributed interference management scheme is proposed in [9]. The management of both the cross-tier and cotier interferences is a critical performance obstacle for two-tier cellular networks.

Different studies on the cell access scheme in two-tier networks have appeared in the literature [1,10,11]. An incentive mechanism to motive the femtocell operators to open portion of the access opportunities to macrousers was proposed in [1]. The users in two-tier networks choose service cells dynamically according to the performance satisfaction level, pricing, and spectrum sharing schemes between macrocells and femtocells. Comprehensive investigation is given on the key aspects and challenges in two-tier networks [10]. Particularly, cell identification schemes, access control schemes, cell search, and handover schemes were discussed and three representative algorithms were described. Poisson point process was proposed to model the coverage of macrocells and femtocells in [11]. It proposes an approximation to the coverage probability for the open access and closed access policies, respectively.

An analytical model for two-tier networks has been proposed in [12]. Its model analyzes the performance of different cell access schemes from the traffic perspective. However, it has not considered in its analysis the movement of the mobile
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stations; thus, the handoff is not taken into account. A model with soft handoff was proposed in CDMA cellular networks [13]. The application of the model to the two-tier networks is difficult due to the different characteristics of femtocells overlaying macrocells.

In this paper, we develop a comparative performance evaluation study of two different types of the handoffs that are particular in two-tier wireless networks. One is the handoff from femtocells to a macrocell, called femto-to-macrocell handoff. The other is the opposite directional handoff, that is, from a macrocell to femtocells, macro-to-femtocell handoff. They are different from each other when handoff attempts fail. The femto-to-macrocell handoff drops the failed calls while the macro-to-femtocell handoff has the calls to continue to be served through the macrocell. The femto-to-macrocell handoff should obviously drop the call when a mobile station moves beyond the coverage of femtocells, but a macrocell is not able to provide available channels. On the contrary, the macro-to-femtocell handoff does not necessarily drop the call even though a target femtocell does not have available channels because the call is still able to occupy the macrocell channel.

Considering the two different types of the handoffs, we propose two handoff schemes: femto-to-macrocell handoff and bidirectional handoff scheme. The former supports only the femto-to-macrocell handoff. The latter supports both directions of handoff. This work is different from [12] in that we consider handoffs in performance analysis.

We note that, for new call arrivals, both of the proposed schemes work in the same way as the femtocell priority scheme of [12]. In the scheme, if a new call arrives at the system, the femtocells try to admit the call if the mobile station is within the coverage of the femtocells and there are available channels before the macrocell attempts. Thus, the two proposed schemes are similar in that they deal with new call admission by giving preference to the femtocells.

The rest of the paper is organized as follows. Section 2 describes the system model of two-tier cellular networks. In Sections 3 and 4, two continuous time Markov chain models are proposed, which takes into account handoff activities. In Section 5, we define performance metric, the blocking probability, and the handoff drop rate and present the analysis results of the different handoff schemes. Section 6 concludes the paper.

2. System Model

Figure 1 shows a scenario of a two-tier wireless network. It consists of a single macrocell and several femtocells. There are three different types of functional units involved: the macrocell base station (mBS), femtocell base stations (fBSs), and mobile stations (MSs). We assume that the mBS provides $C_m$ channels to the MSs and its coverage area is $S_m$, while there are $N$ femtocells randomly distributed within the macrocell coverage area. The coverage of femtocells does not overlap with each other. We denote by $S_i$ the coverage area of the fBS $i$ and by $C_i$ the number of channels: $1 \leq i \leq N$. As for the workload, it is assumed that new call admissions are attempted following Poisson process of rate $\lambda$ and call durations are exponentially distributed with rate $\mu$. An ongoing call is supposed to occupy one channel at any moment. Since our model considers mobility of MSs, the handoff interval of accepted calls follows an exponential distribution with mean $\beta$.

However, for simplicity of our analysis, we assume that the MSs do not move beyond the coverage area of the mBS. Finally, an MS is able to access both mBS and any fBS as long as they are within its communication range.

3. Modeling of Handoff-Considered Schemes

We model the two proposed schemes as a birth-and-death process [14]. For brevity of explanation, we consider a system with a single macrocell and a single femtocell. We note that larger systems can be modeled based on this simple version without difficulties. As in [12], let vector $(x_m, x_i)$ represent one of the states of the system. We denote by $x_m$, $0 \leq x_m \leq C_m$, the number of ongoing calls at the mBS and by $x_i$, $0 \leq x_i \leq C_i$, the number of ongoing calls at the fBS $i$. We note that the set of the state vectors and the transition relationship among them define an irreducible and ergodic continuous time Markov chain.

Before describing our model that considers handoff, we present the model without handoff. Figure 2 shows the state transition diagram of the femtocell priority scheme [12]. For new call arrivals, both of our proposed schemes behave in the same way as in Figure 2. The transitions are as follows:

(1) $(x_m, x_i) \rightarrow (x_m, x_i + 1)$: a new call arrives within the coverage area of the fBS $i$ and it is accepted by the fBS $i$.
(2) $(x_m, C_i) \rightarrow (x_m + 1, C_i)$: a new call arrives within the coverage area of the fBS $i$ but it is accepted by the mBS because all the channels of the fBS $i$ are occupied;

(3) $(x_m, x_i) \rightarrow (x_m + 1, x_i)$: a new call arrives outside the coverage area of the fBS $i$ and it is accepted by the mBS;

(4) $(x_m, x_i) \rightarrow (x_m, x_i - 1)$: a call of the fBS $i$ terminates;

(5) $(x_m, x_i) \rightarrow (x_m - 1, x_i)$: a mBS call terminates.

Besides the set of the transitions of Figure 2, we add to the proposed model the transitions related to the femto-to-macrocell handoff. Figure 3 shows the transition diagram of the handoff. A mobile station which has an ongoing call moves out of the femtocell and makes handoff to a macrocell. The transitions are as follows:

$$(x_m, x_i) \rightarrow (x_m + 1, x_i - 1), \quad (1)$$

where $0 \leq x_m \leq C_m - 1$ and $1 \leq x_i \leq C_f$. A mobile station makes a handoff to the macrocell from the fBS $i$.

The macro-to-femtocell handoff adds other transitions to the model as shown in Figure 4. A mobile station which has an ongoing call in the macrocell moves to a femtocell and makes handoff. The transitions are as follows:

$$(x_m, x_i) \rightarrow (x_m - 1, x_i + 1), \quad (2)$$

where $1 \leq x_m \leq C_m$ and $0 \leq x_i \leq C_f - 1$. A mobile station makes a handoff to the fBS $i$ from the macrocell.
From the above transition diagrams, we now model the transition diagram of the femto-to-macrocell scheme by overlaying the transition diagram of Figure 3 on top of the diagram of Figure 2. In a similar way, the transition diagram of the bidirectional scheme is obtained by overlaying all the three transition diagrams.

In the rest of the paper, we identify the states not only by \((x_m, x_f)\) but also by assigning sequential numbers to them starting from 1. We denote by \(S\) the set of all the states of the system \(S = \{1, 2, \ldots, N\}\) where \(N\) is the number of the states.

We need \(\Pi\), the stationary state distribution of all the states, to analyze the performance. Let \(\pi(\mathbf{x}) = \pi(x_m, x_f)\) be the steady state probability of finding the system in state \(i = (x_m, x_f)\). Let \(P(x_t = \mathbf{x})\) be the probability of finding the system in state \(i\) at time \(t\). Then, we can define \(q_{ij}\) as the probability that the system makes a transition from state \(i\) to \(j\) as follows:

\[
q_{ij} = \lim_{\Delta t \to 0} \frac{P(x_{t+\Delta t} = j \mid x_t = i)}{\Delta t}, \quad i \neq j
\]

which implies the transition rate or the intensity. We denote by \(Q\) an \(N \times N\) matrix that has \(q_{ij}\), \(1 \leq i, j \leq N\), as its \((i, j)\)th element. \(Q\) is known to have the following properties [14]:

1. \(q_{ii} \leq 0\) for all \(i \in S\);
2. \(q_{ij} \geq 0\) for all \(i, j \in S\);
3. \(\sum_{j \in S} q_{ij} = 0\) for all \(i \in S\).

From the above, it is obvious that the total transition rate out of state \(i\), \(q_i\), is

\[
q_i = \sum_{j \neq i} q_{ij}
\]

and, for the equilibrium purpose, \(q_{ii}\) is defined as \(q_{ii} = -q_i\).

We can obtain \(\Pi\) by solving the following global balance condition equations:

\[
\begin{align*}
\Pi \times Q &= 0, \\
\Pi \times e^t &= 1,
\end{align*}
\]

where \(e = (1, 1, \ldots, 1) \in \mathbb{R}^N\).

Let \(E\) be \(N \times N\) matrix that every element is 1. Then,

\[
\Pi \times E = e.
\]

We add (5) and (7) to obtain

\[
\Pi \times (Q + E) = e.
\]

From (8), \(\Pi\) can be obtained by multiplying the inverse of \((Q + E)\)

\[
\Pi = e \times (Q + E)^{-1}.
\]

4. Performance Metrics

We consider two performance metrics to evaluate the proposed schemes. One is a new call blocking probability and the other is a handoff drop rate. To understand how these metrics are analyzed, we consider a system with one mBS and \(N\) fBSs. For simplicity, we assume that all the fBSs have the same number of channels; that is, \(C_i = C_f\) \(\forall i\). Also, all of them have the same areas of the coverage; that is, \(S_i = S_f\) \(\forall i\). We denote by \(p\) the probability of finding an MS in the coverage area of fBS \(i\), \(p = S_f/S_m\). We assume that the total coverage areas of the fBSs do not cover the whole macrocell by putting a constraint \(Np < 1\).

Recall that \(\pi(x_m, x_{i1}, x_{i2}, \ldots, x_{IN})\) is the steady state probability of finding \(x_m\) ongoing calls in the mBS and \(x_i\) ongoing

\[
\begin{align*}
(0, C_f) &\quad 2p\beta \\
(0, C_f-1) &\quad 2p\beta \\
(0, 1) &\quad p\beta \\
(0, 0) &\quad p\beta
\end{align*}
\]

\[
\begin{align*}
(1, C_f) &\quad 3p\beta \\
(1, C_f-1) &\quad 3p\beta \\
(1, 1) &\quad 3p\beta \\
(1, 0) &\quad 3p\beta
\end{align*}
\]

\[
\begin{align*}
(2, C_f) &\quad \cdots \\
(2, C_f-1) &\quad \cdots \\
(2, 1) &\quad \cdots \\
(2, 0) &\quad \cdots
\end{align*}
\]

\[
\begin{align*}
(C_m, C_f) &\quad \cdots \\
(C_m-1, C_f) &\quad \cdots \\
(C_m-1, 1) &\quad \cdots \\
(C_m-1, 0) &\quad \cdots
\end{align*}
\]

\[
\begin{align*}
(C_m, C_f-1) &\quad \cdots \\
(C_m-1, C_f-1) &\quad \cdots \\
(C_m-1, 0) &\quad \cdots \\
(C_m-1, -1) &\quad \cdots
\end{align*}
\]
calls in the fBS: \(1 \leq i \leq N\). Then, a new call blocking probability is given by

\[
\text{NBP} = (1 - Np) \pi(C_{m}, x_1, x_2, \ldots, x_N)
\]

\[+ p \pi(C_{m}, C_1, x_2, \ldots, x_N)
\]

\[+ p \pi(C_{m}, x_1, C_2, \ldots, x_N)
\]

\[+ \cdots + p \pi(C_{m}, x_1, x_2, \ldots, C_N).
\]

Thus, (9) is further simplified as

\[
\text{NBP} \approx (1 - Np) \cdot \pi(C_{m}, x_1, x_2, \ldots, x_N)
\]

\[+ N \cdot p \cdot \pi(C_{m}, C_1, x_2, \ldots, x_N).
\]

The other performance metric, the handoff drop rate, is given by

\[
\text{HDR} = x_1 \beta \pi(C_{m}, x_1, x_2, \ldots, x_N)
\]

\[+ x_2 \beta \pi(C_{m}, x_1, x_2, \ldots, x_N)
\]

\[+ \cdots + x_N \beta \pi(C_{m}, x_1, x_2, \ldots, x_N)
\]

\[= \beta \sum_{i=1}^{C_f} \pi(C_{m}, i, x_2, \ldots, x_N)
\]

\[+ \beta \sum_{i=1}^{C_f} \pi(C_{m}, x_1, i, \ldots, x_N)
\]

\[+ \cdots + \beta \sum_{i=1}^{C_f} \pi(C_{m}, x_1, x_2, \ldots, i)
\]

\[= \beta \sum_{i=1}^{C_f} \sum_{j=1}^{C_i} \pi(C_{m}, i, j),
\]

where \(\pi(C_{m}, i, j)\) is the sum of the steady state probabilities of finding \(C_{m}\) macrocell calls and \(i\) femtocell calls at \(j\)th fBS.

A handoff call is dropped when an MS associated with a fBS moves beyond the coverage of the fBS, but the mBS has all channels occupied. Note that this metric is measured as rate not probability. We assume that the handoff occurs with rate \(\beta\). Thus, the handoff dropping is inevitable when all mBS channels are occupied. It is more appropriate to measure how often the handoff occurs rather than whether it happens.

\[
\pi(C_{m}, \ldots, C_i, \ldots) = \pi(C_{m}, \ldots, C_j, \ldots)
\]

for \(1 \leq i, j \leq N\). Thus, (10) is further simplified as

\[
\text{NBP} \approx (1 - Np) \cdot \pi(C_{m}, x_1, x_2, \ldots, x_N)
\]

\[+ N \cdot p \cdot \pi(C_{m}, C_1, x_2, \ldots, x_N).
\]

\[
\text{HDR} \approx \beta \sum_{i=1}^{C_f} \sum_{j=1}^{C_i} \pi(C_{m}, i, j),
\]

where \(\pi(C_{m}, i, j)\) is the sum of the steady state probabilities of finding \(C_{m}\) macrocell calls and \(i\) femtocell calls at \(j\)th fBS.

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\]

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\text{HDR} \approx \beta \sum_{i=1}^{C_f} \sum_{j=1}^{C_i} \pi(C_{m}, i, j),
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\]

\[+ N \cdot p \cdot \pi(C_{m}, C_1, x_2, \ldots, x_N).
\]

\[
\text{HDR} \approx \beta \sum_{i=1}^{C_f} \sum_{j=1}^{C_i} \pi(C_{m}, i, j),
\]

where \(\pi(C_{m}, i, j)\) is the sum of the steady state probabilities of finding \(C_{m}\) macrocell calls and \(i\) femtocell calls at \(j\)th fBS.

A handoff call is dropped when an MS associated with a fBS moves beyond the coverage of the fBS, but the mBS has all channels occupied. Note that this metric is measured as rate not probability. We assume that the handoff occurs with rate \(\beta\). Thus, the handoff dropping is inevitable when all mBS channels are occupied. It is more appropriate to measure how often the handoff occurs rather than whether it happens.
6. Conclusions

We have modeled the two handoff schemes in a two-tier wireless network. One is the femto-to-macro handoff scheme where only handoff from femtocells to macrocells is permitted. Handoff drop occurs when a macrocell does not have available channel. The other scheme is the bidirectional scheme where the opposite direction handoff, that is, macrocell to femtocell, as well as femtocell to macrocell, is allowed. A similar work has modeled two cell access schemes. However, its model has not considered handoff for simplicity of the performance analysis. In this paper, we have modified the model to consider handoff.

We have modeled both schemes by continuous time Markov chain and defined two performance metrics: the new call blocking probability and the handoff drop rate. The numerical results showed that the modified model produced different results from the model without handoff, particularly in terms of the new call blocking probabilities. Comparing the two proposed schemes, as expected, the bidirectional scheme showed improved blocking probability than the femto-to-macro scheme.

The handoff drop rate is interesting in the sense that it is affected by the femtocell area. The drop rate of both schemes increases as the coverage area grows. The bidirectional scheme outperforms the femto-to-macro scheme as the femtocell coverage expands. Our results show that the model produces different outcomes depending on whether handoff is considered or not. Also, we note that the bidirectional scheme is superior to the femto-to-macro scheme because it is able to utilize the resources of femtocells more efficiently.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

This work was supported by the University of Incheon International Cooperative Research grant in 2012.

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