Research Article

Effect of Rotation on Wave Propagation in Hollow Poroelastic Circular Cylinder

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The objective of this paper is to study the effect of rotation on the wave propagation in an infinite poroelastic hollow circular cylinder. The frequency equation for poroelastic hollow circular cylinder is obtained when the boundaries are stress free and is examined numerically. The frequency, phase velocity, and attenuation coefficient are calculated for a pervious surface for various values of rotation, wave number, and thickness of the cylinder which are presented for nonaxial symmetric vibrations for a pervious surface. The dispersion curves are plotted for the poroelastic elastic behavior of the poroelastic material. Results are discussed for poroelastic material. The results indicate that the effect of rotation, wave number, and thickness on the wave propagation in the hollow poroelastic circular cylinder is very pronounced.

1. Introduction

The study of wave propagation over a continuous medium is of practical importance in the fields of engineering, medicine, and bioengineering. Application of the poroelastic materials in medical fields such as orthopedic, dental, and cardiovascular is well known. In orthopedics, wave propagation over bone is used in monitoring the rate of fracture healing. There are two types of osseous tissue such as cancellous or trabecular and compact or cortical bone, which are of different materials, with respect to their mechanical behavior. In macroscopic terms, the percentage of porosity in the cortical bone is 3–5%, whereas, in the trabecular or cancellous, the percentage of porosity is up to 90% [1]. In fact, in a recent article, Ahmed and Abd-Alla [1] investigated the electromechanical wave propagation in a cylindrical poroelastic bone with cavity. Analytical solution of electromechanical wave propagation in long bones has been obtained by El-Naggar et al. [2]. Abd-Alla et al. [3] studied the wave propagation modeling in cylindrical human long wet bones with cavity. Hart [4] investigated the theoretical study of the influence of bone maturation rate on surface remodeling predictions. Qin et al. [5] studied

the thermoelectroelastic solutions for surface bone remodeling under axial and transverse loads. Martínez et al. [6] discussed the external bone remodeling through boundary elements and damage mechanics. Computational simulation of simultaneous cortical and trabecular bone change in human proximal femur during bone remodeling has been investigated by Jang and Kim [7]. Tsili [8] studied the theoretical solutions for internal bone remodeling of diaphyseal shafts using adaptive elasticity theory. Cowin and Firoozbakhsh [9] investigated the bone remodeling of diaphyseal surfaces under constant load: theoretical predictions, a contribution to the mechanics and thermodynamics of surface growth. Application to bone external remodeling has been studied by Ganghoffer [10]. Sims and Gooi [11] investigated bone remodeling: multiple cellular interactions required for coupling of bone formation and resorption. Zumsande et al. [12] discussed the general analysis of mathematical models for bone remodeling. Malachanne et al. [13] studied the numerical model of bone remodeling sensitive to loading frequency through a poroelastic behavior and internal fluid movements. A model for mechanical adaptation of trabecular bone incorporating cellular accommodation and effects of microdamage

and disuse has been studied by Vahdati and Rouhi [14]. Hazelwood et al. [15] investigated the mechanistic model for internal bone remodeling exhibiting different dynamic responses in disuse and overload. Qu et al. [16] studied the hypothetical mechanism of bone remodeling and modeling under electromagnetic loads. Papathanasopoulou et al. [17] investigated the poroelastic bone model for internal remodeling. Isotropic continuum damage/repair model for alveolar bone remodeling has been studied by Mengoni and Ponthot [18]. Boyle and Kim [19] investigated the three-dimensional microlevel computational study of Wolff's law via trabecular bone remodeling in the human proximal femur using design space topology optimization. Wang et al. [20] studied the theoretical analysis of alendronate and risedronate effects on canine vertebral remodeling and microdamage. A physiologically based mathematical model of integrated calcium homeostasis and bone remodeling has been discussed by Peterson and Riggs [21]. Qin and Ye [22] studied the thermoelectroelastic solutions for internal bone remodeling investigated under axial and transverse loads. Boyle and Kim [23] studied the comparison of different hip prosthesis shapes considering microlevel bone remodeling and stress-shielding criteria using three-dimensional design space topology optimization. Cowin and Van Buskirk [24] investigated surface bone remodeling induced by a medullary pin. Biot [25] explained the general theory of three-dimensional consolidation. Hegedus and Cowin [26] studied bone remodeling II, small strain adaptive elasticity. The extensive literature on the topic is now available and we can only mention a few recent interesting investigations in [27–33]. Recently, Abd-Alla and Abo-Dahab [34] investigated magnetic field effect on poroelastic bone model for internal remodeling.

In the present analysis, the free vibrations of an infinite hollow poroelastic circular cylinder are studied employing general displacement components in cylindrical polar coordinates, following Biot's [35] theory. The general frequency hollow cylinder is homogeneous and isotropic. Degenerate cases of the general frequency equation of pervious surfaces, when the longitudinal wave number k is considered. The numerical result displayed by figures and the physical meaning are explained. The results and discussions presented in this study may be helpful to further understand wave propagation in hollow poroelastic circular cylinder.

2. Governing Equations

Let us consider that the equations of motion of a homogeneous, isotropic poroelastic solid in the presence of dissipation b are

(A + N)

$$\times \left[\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} + \frac{1}{r} \frac{\partial^2 u_\theta}{\partial r \partial \theta} - \frac{1}{r^2} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_z}{\partial r \partial z} - \frac{1}{r} u_r \right]$$

$$\begin{aligned} & + N \left[\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_r}{\partial z^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} - \frac{2}{r} u_r \right] \\ & + Q \left[\frac{\partial^2 v_r}{\partial r^2} - \frac{v_r}{r^2} + \frac{1}{r} \frac{\partial^2 v_\theta}{\partial r \partial \theta} + \frac{\partial^2 v_z}{\partial r \partial z} - \frac{1}{r^2} \frac{\partial v_\theta}{\partial \theta} + \frac{1}{r} \frac{\partial v_r}{\partial r} \right] \\ & = \frac{\partial^2}{\partial t^2} (\rho_{11} u_r + \rho_{12} v_r) - \rho_{11} \Omega^2 u_r + b \frac{\partial}{\partial t} (u_r - v_r) \\ & (A + N) \\ & \times \left[\frac{1}{r} \frac{\partial^2 u_r}{\partial r \partial \theta} + \frac{1}{r} \frac{\partial^2 u_z}{\partial \theta \partial z} + \frac{1}{r^2} \frac{\partial u_r}{\partial \theta} + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial r^2} \right] \\ & + N \left[\frac{2}{r^2} \frac{\partial u_r}{\partial \theta} + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial u_\theta}{\partial r} - \frac{1}{r} u_\theta + \frac{\partial^2 u_\theta}{\partial z^2} \right] \\ & + \frac{Q}{r} \left[\frac{\partial^2 v_\theta}{\partial r \partial \theta} + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{1}{r} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_z}{\partial \theta \partial z} \right] \\ & = \frac{\partial^2}{\partial t^2} (\rho_{11} u_\theta + \rho_{12} v_\theta) + b \frac{\partial}{\partial t} (u_\theta - v_\theta) \\ & (A + N) \left[\frac{\partial^2 u_r}{\partial r \partial z} + \frac{1}{r} \frac{\partial^2 u_\theta}{\partial \theta \partial z} + \frac{1}{r} \frac{\partial u_r}{\partial z} + \frac{\partial^2 u_z}{\partial z^2} \right] \\ & + N \left[\frac{\partial^2 u_z}{\partial z^2} + \frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{1}{r} \frac{\partial u_z}{\partial z} \right] \\ & + Q \left[\frac{\partial^2 v_r}{\partial r \partial z} + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta \partial z} + \frac{1}{r} \frac{\partial v_r}{\partial z} + \frac{\partial^2 v_z}{\partial z^2} \right] \\ & = \frac{\partial^2}{\partial t^2} (\rho_{11} u_z + \rho_{12} v_z) - \rho_{11} \Omega^2 u_z + b \frac{\partial}{\partial t} (u_z - v_z) \\ & Q \left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} + \frac{1}{r} \frac{\partial^2 u_\theta}{\partial r \partial \theta} - \frac{1}{r^2} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_z}{\partial r \partial z} - \frac{1}{r} u_r \right) \\ & + R \left[\frac{\partial^2 v_r}{\partial r^2} - \frac{v_r}{r^2} + \frac{1}{r} \frac{\partial^2 v_\theta}{\partial r \partial \theta} + \frac{\partial^2 v_z}{\partial r \partial z} - \frac{1}{r^2} \frac{\partial v_\theta}{\partial \theta} + \frac{1}{r} \frac{\partial v_r}{\partial r} \right] \end{aligned}$$

$$\begin{aligned}
&= \frac{\partial^2}{\partial t^2} (\rho_{12} u_r + \rho_{22} v_r) - b \frac{\partial}{\partial t} (u_r - v_r), \\
&Q \left[\frac{1}{r} \frac{\partial^2 u_r}{\partial r \partial \theta} + \frac{1}{r} \frac{\partial^2 u_z}{\partial \theta \partial z} + \frac{1}{r^2} \frac{\partial u_r}{\partial \theta} + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial r^2} \right] \\
&\quad + \frac{R}{r} \left[\frac{\partial^2 v_\theta}{\partial r \partial \theta} + \frac{1}{r} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{1}{r} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_z}{\partial \theta \partial z} \right] \\
&= \frac{\partial^2}{\partial t^2} (\rho_{12} u_\theta + \rho_{22} v_\theta) - b \frac{\partial}{\partial t} (u_\theta - v_\theta), \\
&Q \left[\frac{\partial^2 u_r}{\partial r \partial z} + \frac{1}{r} \frac{\partial^2 u_\theta}{\partial \theta \partial z} + \frac{1}{r} \frac{\partial u_r}{\partial z} + \frac{\partial^2 u_z}{\partial z^2} \right] \\
&\quad + R \left[\frac{\partial^2 v_r}{\partial r \partial z} + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta \partial z} + \frac{1}{r} \frac{\partial v_r}{\partial z} + \frac{\partial^2 v_z}{\partial z^2} \right] \\
&= \frac{\partial^2}{\partial t^2} (\rho_{12} u_z + \rho_{22} v_z) - b \frac{\partial}{\partial t} (u_z - v_z),
\end{aligned} \tag{1}$$

where ∇^2 is the Laplace operator, $\vec{u} = (u_r, u_\theta, u_z)$, $\vec{v} = (v_r, v_\theta, v_z)$ are displacements of solid and liquid, respectively, e and ε are the dilatations of solid and liquid, A , N , Q , and R are all poroelastic constants, and ρ_{ij} ($i, j = 1, 2$) are the mass coefficients following Biot [35].

The stress-strain equations for an isotropic poroelastic material of solid and liquid are given as

$$\begin{aligned}
\sigma_{rr} &= 2NE_{rr} + AE + Q\varepsilon, \\
\sigma_{\theta\theta} &= 2NE_{\theta\theta} + AE + Q\varepsilon, \\
\sigma_{zz} &= 2NE_{zz} + AE + Q\varepsilon, \\
\tau_{\theta z} &= NE_{\theta z}, \\
\tau_{zr} &= NE_{zr}, \\
\tau_{r\theta} &= NE_{r\theta}, \\
s &= QE + R\varepsilon.
\end{aligned} \tag{2}$$

3. Solution of the Problem

Let us consider (r, θ, z) to be the cylindrical polar coordinates. Consider a homogeneous, isotropic, poroelastic cylinder with inner and outer radii being r_1 and r_2 , respectively, so that the thickness of poroelastic cylinder is $h = (r_2 - r_1) > 0$, whose axis is in the direction of z -axis; let

$$\begin{aligned}
\vec{v} &= \nabla \phi_2 + \nabla \wedge \vec{\psi}_2, \\
\vec{u} &= \nabla \phi_1 + \nabla \wedge \vec{\psi}_1,
\end{aligned} \tag{3}$$

where $\vec{u}$, $\vec{v}$ are displacements of solid and liquid, respectively, ϕ_1 , ϕ_2 are scalar potentials, and $\vec{\psi}_1$, $\vec{\psi}_2$ are vector potentials.

Assuming that

$$\begin{aligned}
\vec{\Psi}_1 &= (h_r, h_\theta, h_z), \\
\vec{\Psi}_2 &= (H_r, H_\theta, H_z),
\end{aligned} \tag{4}$$

from (1) and (3), we get

$$\begin{aligned}
P\nabla^2 \theta_1 + Q\nabla^2 \theta_2 &= (\rho_{11} \ddot{\theta}_1 (1 + \Omega^2) + \rho_{12} \ddot{\theta}_2) + b (\dot{\theta}_1 - \dot{\theta}_2) \\
Q\nabla^2 \theta_1 + R\nabla^2 \theta_2 &= (\rho_{12} \ddot{\theta}_1 + \rho_{22} \ddot{\theta}_2) - b (\dot{\theta}_1 - \dot{\theta}_2) \\
0 &= (\rho_{12} \ddot{\Psi}_1 + \rho_{22} \ddot{\Psi}_2) - b (\dot{\Psi}_1 - \dot{\Psi}_2) \\
N\nabla^2 \Psi_1 &= (\rho_{11} \ddot{\Psi}_1 (1 + \Omega^2) + \rho_{12} \ddot{\Psi}_2) + b (\dot{\Psi}_1 - \dot{\Psi}_2),
\end{aligned} \tag{5}$$

where $P = A + 2N$.

Assuming that

$$\begin{aligned}
\theta_1 &= f_1(r) \cos(n\theta) e^{i(kz + \omega t)}, \\
\theta_2 &= f_2(r) \cos(n\theta) e^{i(kz + \omega t)}, \\
h_r &= g_r(r) \sin(n\theta) e^{i(kz + \omega t)}, \\
h_\theta &= g_\theta(r) \cos(n\theta) e^{i(kz + \omega t)}, \\
h_z &= g_z(r) \sin(n\theta) e^{i(kz + \omega t)}, \\
H_r &= G_r(r) \sin(n\theta) e^{i(kz + \omega t)}, \\
H_\theta &= G_\theta(r) \cos(n\theta) e^{i(kz + \omega t)}, \\
H_z &= G_z(r) \sin(n\theta) e^{i(kz + \omega t)},
\end{aligned} \tag{6}$$

where k is the axial wave number, n is an integer number of waves around the circumference or also known as angular wave number, and ω is circular frequency. From the second and third equations of (5) with (4), when the first two equations of (5) remain the same, they are reduced to

$$\begin{aligned}
P\nabla^2 \theta_1 + Q\nabla^2 \theta_2 &= (\rho_{11} \ddot{\theta}_1 (1 + \Omega^2) + \rho_{12} \ddot{\theta}_2) + b (\dot{\theta}_1 - \dot{\theta}_2), \\
Q\nabla^2 \theta_1 + R\nabla^2 \theta_2 &= (\rho_{12} \ddot{\theta}_1 + \rho_{22} \ddot{\theta}_2) - b (\dot{\theta}_1 - \dot{\theta}_2),
\end{aligned}$$

$$\begin{aligned}
& N \left(\nabla^2 h_r - \frac{h_r}{r^2} - \frac{2}{r^2} \frac{\partial h_\theta}{\partial \theta} \right) \\
& = \left((\rho_{11} + \Omega^2) \ddot{h}_r + \rho_{12} \ddot{H}_r \right) + b (\dot{h}_r - \dot{H}_r), \\
& N \left(\nabla^2 h_\theta - \frac{h_\theta}{r^2} + \frac{2}{r^2} \frac{\partial h_r}{\partial \theta} \right) \\
& = \left((\rho_{11} + \Omega^2) \ddot{h}_\theta + \rho_{12} \ddot{H}_\theta \right) + b (\dot{h}_\theta - \dot{H}_\theta), \\
& N (\nabla^2 h_z) \\
& = \left((\rho_{11} + \Omega^2) \ddot{h}_z + \rho_{12} \ddot{H}_z \right) + b (\dot{h}_z - \dot{H}_z), \\
& 0 = (\rho_{12} \ddot{h}_r + \rho_{22} \ddot{H}_r) - b (\dot{h}_r - \dot{H}_r), \\
& 0 = (\rho_{12} \ddot{h}_\theta + \rho_{22} \ddot{H}_\theta) - b (\dot{h}_\theta - \dot{H}_\theta), \\
& 0 = (\rho_{12} \ddot{h}_z + \rho_{22} \ddot{H}_z) - b (\dot{h}_z - \dot{H}_z).
\end{aligned} \tag{7}$$

From (7) and (6), we obtain

$$\begin{aligned}
& p \Delta f_1 + Q \Delta f_2 = -w^2 [k_{11} f_1 + k_{12} f_2], \\
& Q \Delta f_1 + R \Delta f_2 = -w^2 [k_{12} f_1 + k_{22} f_2], \\
& N \left[\Delta g_r - \frac{g_r}{r^2} - \frac{2n}{r^2} g_\theta \right] = -w^2 [k'_{11} g_r + k_{12} G_r], \\
& N \left[\Delta g_\theta - \frac{g_\theta}{r^2} - \frac{2n}{r^2} g_r \right] = -w^2 [k'_{11} g_\theta + k_{12} G_\theta], \\
& N \Delta g_3 = -w^2 [k'_{11} g_z + k_{12} G_z], \\
& 0 = -w^2 [k_{12} g_r + k_{22} G_r], \\
& 0 = -w^2 [k_{12} g_\theta + k_{22} G_\theta], \\
& 0 = -w^2 [k_{12} g_z + k_{22} G_z],
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
\Delta &= \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{n^2}{r^2} - k^2, \\
k_{11} &= \rho_{11} (1 - \Omega^2) - \frac{ib}{w}, \\
k_{12} &= \rho_{12} + \frac{ib}{w}, \quad k_{22} = \rho_{22} - \frac{ib}{w}, \\
k'_{11} &= \rho_{11} (1 + \Omega^2) - \frac{ib}{w}.
\end{aligned} \tag{9}$$

The general solution of (8) can be obtained in terms of the Bessel function of the first and second kind J and Y depending on its arguments $\alpha_1 r, \alpha_2 r, \alpha_3 r$.

From (8), we obtain

$$\begin{aligned}
f_1 &= c_1 J_n(\alpha_1 r) + c_2 Y_n(\alpha_1 r) + c_3 J_n(\alpha_2 r) + c_4 Y_n(\alpha_2 r), \\
g_3(r) &= A_3 J_n(\alpha_3 r) + B_3 Y_n(\alpha_3 r), \\
2g_1 &= g_r - g_\theta = 2A_1 J_{n+1}(\alpha_3 r) + 2B_1 Y_{n+1}(\alpha_3 r), \\
2g_2 &= g_r + g_\theta = 2A_2 J_{n-1}(r\alpha_3) + 2B_2 Y_{n-1}(r\alpha_3),
\end{aligned} \tag{10}$$

where J_n is Bessel function of the first kind and order n and Y_n is the Bessel function of the second kind and order n . Consider that

$$\begin{aligned}
\alpha_1^2 &= \zeta_1^2 - k^2 + \Omega^2, \\
\alpha_2^2 &= \zeta_2^2 - k^2 + \Omega^2, \\
\alpha_3^2 &= \zeta_3^2 - k^2 - \Omega^2
\end{aligned} \tag{11}$$

are positive or negative and

$$\omega^2 = \zeta_i^2 v_i^2 \quad (i = 1, 2, 3), \tag{12}$$

where v_r, v_θ are the dilatational wave velocities of first and second kind, respectively, and v_z is shear wave velocity.

The gauge invariance property, following the analysis of [36], is used to eliminate two integration constants from (10). Any one of the potential functions g_1, g_2 , or g_3 can be set equal to zero, without loss of generality of the solution. Setting $g_2 = 0$, we can obtain

$$g_r = -g_\theta = g_1. \tag{13}$$

The displacement vector of solid $\vec{u} = (u_r, u_\theta, u_z)$ with the help of (3) and (4) is given by

$$\begin{aligned}
u_r &= \frac{\partial \theta_1}{\partial r} + \frac{1}{r} \frac{\partial h_z}{\partial \theta} - \frac{\partial h_\theta}{\partial z}, \\
u_\theta &= \frac{1}{r} \frac{\partial \theta_1}{\partial \theta} + \frac{\partial h_r}{\partial z} - \frac{\partial h_z}{\partial r}, \\
u_z &= \frac{\partial \theta_1}{\partial z} + \frac{\partial h_\theta}{\partial r} + \frac{h_\theta}{r} - \frac{1}{r} \frac{\partial h_r}{\partial \theta}.
\end{aligned} \tag{14}$$

Substituting from (6) into (14), the displacement components of solid are

$$\begin{aligned}
u_r &= \left[f'_1 + \frac{n}{r} g_z - ik g_\theta \right] \cos(n\theta) e^{i(kz + \omega t)}, \\
u_\theta &= \left[-\frac{n}{r} f_1 + ik g_r - g'_z \right] \sin(n\theta) e^{i(kz + \omega t)}, \\
u_z &= \left[ik f_1 + g'_\theta + g_\theta - \frac{n}{r} g_r \right] \cos(n\theta) e^{i(kz + \omega t)}.
\end{aligned} \tag{15}$$

Substituting from (13) into (15), the solid displacement is

$$u_r = \left[f'_1 + \frac{n}{r} g_3 - ik g_1 \right] \cos(n\theta) e^{i(kz + \omega t)},$$

$$\begin{aligned}
u_\theta &= \left[-\frac{n}{r} f_1 + ik g_1 - g'_3 \right] \sin(n\theta) e^{i(kz+\omega t)}, \\
u_z &= \left[ik f_1 + g'_1 - \frac{(n+1)}{r} g_1 \right] \cos(n\theta) e^{i(kz+\omega t)},
\end{aligned} \tag{16}$$

where “prime” over a quantity denotes differentiation with respect to r , $\beta_1, \beta_2, \beta_3$ being the absolute values of $\alpha_1, \alpha_2, \alpha_3$, respectively.

The dilatations of solid and liquid media are

$$\begin{aligned}
e &= \Delta f_1(r) \cos(n\theta) e^{i(kz+\omega t)}, \\
\epsilon &= \Delta f_2(r) \cos(n\theta) e^{i(kz+\omega t)}.
\end{aligned} \tag{17}$$

Substituting from (16) into strain displacement relations and then using (2), the stresses σ_{ij} and the liquid pressures are

$$\begin{aligned}
\sigma_{rr} + s &= [c_1 M_{11}(r) + c_2 M_{12}(r) \\
&+ c_3 M_{13}(r) + c_4 M_{14}(r) + A_3 M_{15}(r) \\
&+ B_3 M_{16}(r) + A_1 M_{17}(r) + B_1 M_{18}(r)] \\
&\times \cos(n\theta) e^{i(kz+\omega t)},
\end{aligned} \tag{18}$$

$$\begin{aligned}
\sigma_{r\theta} &= [c_1 M_{21}(r) + c_2 M_{22}(r) \\
&+ c_3 M_{23}(r) + c_4 M_{24}(r) + A_3 M_{25}(r) \\
&+ B_3 M_{26}(r) + A_1 M_{27}(r) + B_1 M_{28}(r)] \\
&\times \sin(n\theta) e^{i(kz+\omega t)},
\end{aligned} \tag{19}$$

$$\begin{aligned}
\sigma_{rz} &= [c_1 M_{31}(r) + c_2 M_{32}(r) \\
&+ c_3 M_{33}(r) + c_4 M_{34}(r) + A_3 M_{35}(r) \\
&+ B_3 M_{36}(r) + A_1 M_{37}(r) + B_1 M_{38}(r)] \\
&\times \cos(n\theta) e^{i(kz+\omega t)},
\end{aligned} \tag{20}$$

$$\begin{aligned}
s &= [c_1 M_{41}(r) + c_2 M_{42}(r) \\
&+ c_3 M_{43}(r) + c_4 M_{44}(r)] \\
&\times \cos(n\theta) e^{i(kz+\omega t)},
\end{aligned} \tag{21}$$

$$\begin{aligned}
\frac{\partial s}{\partial r} &= [c_1 M_{41}(r) + c_2 M_{42}(r) \\
&+ c_3 M_{43}(r) + c_4 M_{44}(r)] \\
&\times \cos(n\theta) e^{i(kz+\omega t)},
\end{aligned} \tag{22}$$

where the coefficients $M_{ij}(r)$ are given as

$$\begin{aligned}
M_{11}(r) &= \left\{ 2N \left[\frac{n(n-1)}{r^2} - \alpha_1^2 \right] \right. \\
&\quad \left. + [(Q+R)\delta_1^2 - (A+Q)](k^2 + \alpha_1^2) \right\} \\
&\quad \times J_n(\beta_1 r) + \frac{2N\beta_1}{r} J_{n+1}(\beta_1 r), \\
M_{12}(r) &= \left\{ 2N \left[\frac{n(n-1)}{r^2} - \alpha_1^2 \right] \right. \\
&\quad \left. + [(Q+R)\delta_1^2 - (A+Q)](k^2 + \alpha_1^2) \right\} \\
&\quad \times Y_n(\beta_1 r) + \frac{2N\beta_1}{r} Y_{n+1}(\beta_1 r), \\
M_{13}(r) &= \left\{ 2N \left[\frac{n(n-1)}{r^2} - \alpha_1^2 \right] \right. \\
&\quad \left. + [(Q+R)\delta_1^2 - (A+Q)](k^2 + \alpha_1^2) \right\} \\
&\quad \times J_n(\beta_1 r) + \frac{2N\beta_1}{r} J_{n+1}(\beta_1 r), \\
M_{14}(r) &= \left\{ 2N \left[\frac{n(n-1)}{r^2} - \alpha_1^2 \right] \right. \\
&\quad \left. + [(Q+R)\delta_1^2 - (A+Q)](k^2 + \alpha_1^2) \right\} \\
&\quad \times Y_n(\beta_1 r) + \frac{2N\beta_1}{r} Y_{n+1}(\beta_1 r), \\
M_{15}(r) &= \frac{2Nn(n-1)}{r^2} J_n(\beta_3 r) - \frac{2Nn\beta_3}{r} J_{n+1}(\beta_3 r), \\
M_{16}(r) &= \frac{2Nn(n-1)}{r^2} Y_n(\beta_3 r) - \frac{2Nn\beta_3}{r} Y_{n+1}(\beta_3 r), \\
M_{17}(r) &= 2Nik\beta_3 J_n(\beta_3 r) - \frac{2N(n+1)ik}{r} J_{n+1}(\beta_3 r), \\
M_{18}(r) &= 2Nik\beta_3 Y_n(\beta_3 r) - \frac{2N(n+1)ik}{r} Y_{n+1}(\beta_3 r), \\
M_{21}(r) &= \frac{2Nn(1-n)}{r^2} Y_n(\beta_1 r) + \frac{2Nn\beta_1}{r} J_{n+1}(\beta_1 r), \\
M_{22}(r) &= \frac{2Nn(1-n)}{r^2} Y_n(\beta_1 r) + \frac{2Nn\beta_1}{r} Y_{n+1}(\beta_1 r), \\
M_{23}(r) &= \frac{2Nn(1-n)}{r^2} J_n(\beta_2 r) + \frac{2Nn\beta_2}{r} J_{n+1}(\beta_2 r), \\
M_{24}(r) &= \frac{2Nn(1-n)}{r^2} Y_n(\beta_2 r) + \frac{2Nn\beta_2}{r} Y_{n+1}(\beta_2 r), \\
M_{25}(r) &= \left[\frac{2Nn(1-n)}{r^2} + N\alpha_3^2 \right] J_n(\beta_3 r) \\
&\quad - \frac{2N\beta_3}{r} J_{n+1}(\beta_3 r),
\end{aligned}$$

$$\begin{aligned}
M_{26}(r) &= \left[\frac{2Nn(1-n)}{r^2} + N\alpha_3^2 \right] Y_n(\beta_3 r) \\
&\quad - \frac{2N\beta_3}{r} Y_{n+1}(\beta_3 r), \\
M_{27}(r) &= ikN\beta_3 J_n(\beta_3 r) \\
&\quad - \frac{2N(n+1)ik}{r} J_{n+1}(\beta_3 r), \\
M_{28}(r) &= ikN\beta_3 Y_n(\beta_3 r) \\
&\quad - \frac{2N(n+1)ik}{r} Y_{n+1}(\beta_3 r), \\
M_{31}(r) &= \frac{2Nnik}{r} J_n(\beta_1 r) - 2Nik\beta_1 J_{n+1}(\beta_1 r), \\
M_{32}(r) &= \frac{2Nnik}{r} Y_n(\beta_1 r) - 2Nik\beta_1 Y_{n+1}(\beta_1 r), \\
M_{33}(r) &= \frac{2Nnik}{r} J_n(\beta_2 r) - 2Nik\beta_2 J_{n+1}(\beta_2 r), \\
M_{34}(r) &= \frac{2Nnik}{r} Y_n(\beta_2 r) - 2Nik\beta_2 Y_{n+1}(\beta_2 r), \\
M_{35}(r) &= \frac{Nnik}{r} J_n(\beta_3 r), \\
M_{36}(r) &= \frac{Nnik}{r} Y_n(\beta_3 r), \\
M_{37}(r) &= \frac{-Nn\beta_3}{r} J_n(\beta_3 r) - N(k^2 - \alpha_3^2) J_{n+1}(\beta_3 r), \\
M_{38}(r) &= \frac{-Nn\beta_3}{r} Y_n(\beta_3 r) - N(k^2 - \alpha_3^2) Y_{n+1}(\beta_3 r), \\
M_{41}(r) &= (R\delta_1^2 - Q)(k^2 + \alpha_1^2) J_n(\beta_1 r),
\end{aligned}$$

$$\begin{aligned}
M_{42}(r) &= (R\delta_1^2 - Q)(k^2 + \alpha_1^2) Y_n(\beta_1 r), \\
M_{43}(r) &= (R\delta_2^2 - Q)(k^2 + \alpha_1^2) J_n(\beta_2 r), \\
M_{44}(r) &= (R\delta_2^2 - Q)(k^2 + \alpha_2^2) Y_n(\beta_2 r), \\
M_{45}(r) &= 0, \quad M_{46}(r) = 0, \\
M_{47}(r) &= 0, \quad M_{48}(r) = 0.
\end{aligned} \tag{23}$$

In (23),

$$\begin{aligned}
\delta_i^2 &= \frac{1}{(Rk_{12} - Qk_{22})} \\
&\quad \times [(Rk_{11} - Qk_{12}) - V_i^{-2}(PR - Q^2)] \\
&\quad (i = 1, 2),
\end{aligned} \tag{24}$$

where we considered

$$|kv_1| < \omega. \tag{25}$$

4. Boundary Conditions and Frequency Equation

The boundary conditions for traction free inner and outer surfaces of the hollow poroelastic cylinder in case of a pervious surface are

$$\begin{aligned}
\sigma_{rr} + s &= 0, \quad \sigma_{r\theta} = 0, \quad \sigma_{rz} = 0, \\
s &= 0, \quad \text{at } r = r_1, \quad r = r_2.
\end{aligned} \tag{26}$$

Equations (18)–(21) together with (26) yield eight homogeneous equations for eight arbitrary constants $C_1, C_2, C_3, C_4, A_3, B_3, A_1$, and B_1 . A nontrivial solution can be obtained when the determinant of coefficients vanishes. Thus, the frequency equation for a pervious surface is

$$\begin{vmatrix}
M_{11}(r_1) & M_{12}(r_1) & M_{13}(r_1) & M_{14}(r_1) & M_{15}(r_1) & M_{16}(r_1) & M_{17}(r_1) & M_{18}(r_1) \\
M_{21}(r_1) & M_{22}(r_1) & M_{23}(r_1) & M_{24}(r_1) & M_{25}(r_1) & M_{26}(r_1) & M_{27}(r_1) & M_{28}(r_1) \\
M_{31}(r_1) & M_{32}(r_1) & M_{33}(r_1) & M_{34}(r_1) & M_{35}(r_1) & M_{36}(r_1) & M_{37}(r_1) & M_{38}(r_1) \\
M_{41}(r_1) & M_{42}(r_1) & M_{43}(r_1) & M_{44}(r_1) & 0 & 0 & 0 & 0 \\
M_{11}(r_2) & M_{12}(r_2) & M_{13}(r_2) & M_{14}(r_2) & M_{15}(r_2) & M_{16}(r_2) & M_{17}(r_2) & M_{18}(r_2) \\
M_{21}(r_2) & M_{22}(r_2) & M_{23}(r_2) & M_{24}(r_2) & M_{25}(r_2) & M_{26}(r_2) & M_{27}(r_2) & M_{28}(r_2) \\
M_{31}(r_2) & M_{32}(r_2) & M_{33}(r_2) & M_{34}(r_2) & M_{35}(r_2) & M_{36}(r_2) & M_{37}(r_2) & M_{38}(r_2) \\
M_{41}(r_2) & M_{42}(r_2) & M_{43}(r_2) & M_{44}(r_2) & 0 & 0 & 0 & 0
\end{vmatrix} = 0. \tag{27}$$

In (27), the element $M_{ij}(r)$ is defined in (23).

By ignoring the liquid effects in the frequency equation of a pervious surface (27), the results of purely elastic solid are

obtained as a special case considered by [36]. Now, we consider the particular cases of the general frequency equations (27) when the axial and angular wave numbers vanish.

4.1. *Motion Independent of z .* When the axial wave number k is taken to be equal to zero, that is, by considering zeroth azimuthal mode, the frequency equation of a pervious surface

(28) degenerates into the product of two determinants as follows:

$$D_1 D_2 = 0, \quad (28)$$

where

$$D_1 = \begin{vmatrix} M'_{11}(r_1) & M'_{12}(r_1) & M'_{13}(r_1) & M'_{14}(r_1) & M'_{15}(r_1) & M'_{16}(r_1) \\ M'_{21}(r_1) & M'_{22}(r_1) & M'_{23}(r_1) & M'_{24}(r_1) & M'_{25}(r_1) & M'_{26}(r_1) \\ M'_{41}(r_1) & M'_{42}(r_1) & M'_{43}(r_1) & M'_{44}(r_1) & 0 & 0 \\ M'_{11}(r_2) & M'_{12}(r_2) & M'_{13}(r_2) & M'_{14}(r_2) & M'_{15}(r_2) & M'_{16}(r_2) \\ M'_{21}(r_2) & M'_{22}(r_2) & M'_{23}(r_2) & M'_{24}(r_2) & M'_{25}(r_2) & M'_{26}(r_2) \\ M'_{41}(r_2) & M'_{42}(r_2) & M'_{43}(r_2) & M'_{44}(r_2) & 0 & 0 \end{vmatrix}, \quad (29)$$

$$D_2 = \begin{vmatrix} M'_{37}(r_1) & M'_{38}(r_1) \\ M'_{37}(r_2) & M'_{38}(r_2) \end{vmatrix}.$$

The terms $M_{ij}(r)$ appearing in D_1 and D_2 are given in Appendix A for $k = 0$. It clears that, for $k = 0$, $\alpha_1^2, \alpha_2^2, \alpha_3^2$ being all positive, Bessel functions of the first and second kind enter the solution. Equation (30) is satisfied, if either D_1 or D_2 is equal to zero. The case of $D_1 = 0$ corresponds to plane-strain vibrations of thick-walled hollow poroelastic cylinders for a pervious surface.

The case of $D_2 = 0$, corresponds to longitudinal vibrations which involve only longitudinal displacement w . Also, $D_2 = 0$ is the same for a pervious and an impervious surface; hence, it is clear that longitudinal shear vibrations are independent of the nature of the surface. From (27), it is clear that plane-strain vibrations and longitudinal shear vibrations are uncoupled for a pervious surface, when the motion is independent of longitudinal coordinated z , and these vibrations are coupled for nonzero longitudinal wave number k ; that is, $k \neq 0$. The frequency equation of

longitudinal shear vibrations $D_2 = 0$, when expanded, gives

$$J'_n(\alpha_3 r_1) Y'_n(\alpha_3 r_2) - J'_n(\alpha_3 r_2) Y'_n(\alpha_3 r_1) = 0. \quad (30)$$

And the amplitude ratio is given as

$$\frac{A_1}{B_1} = -\frac{Y'_n(\alpha_3 r_1)}{J'_n(\alpha_3 r_1)}, \quad (31)$$

the frequency equation of longitudinal shear vibrations of hollow poroelastic cylinders (31).

4.2. *Motion Independent of θ .* When the motion is independent of angular coordinated θ (i.e., $n = 0$), the frequency equation (27) for a pervious surface is reduced to the product of two determinants given by

$$D_3 D_4 = 0, \quad (32)$$

where

$$D_3 = \begin{vmatrix} M''_{11}(r_1) & M''_{12}(r_1) & M''_{13}(r_1) & M''_{14}(r_1) & M''_{17}(r_1) & M''_{18}(r_1) \\ M''_{31}(r_1) & M''_{32}(r_1) & M''_{33}(r_1) & M''_{34}(r_1) & M''_{37}(r_1) & M''_{38}(r_1) \\ M''_{41}(r_1) & M''_{42}(r_1) & M''_{43}(r_1) & M''_{44}(r_1) & 0 & 0 \\ M''_{11}(r_2) & M''_{12}(r_2) & M''_{13}(r_2) & M''_{14}(r_2) & M''_{17}(r_2) & M''_{18}(r_2) \\ M''_{31}(r_2) & M''_{32}(r_2) & M''_{33}(r_2) & M''_{34}(r_2) & M''_{37}(r_2) & M''_{38}(r_2) \\ M''_{41}(r_2) & M''_{42}(r_2) & M''_{43}(r_2) & M''_{44}(r_2) & 0 & 0 \end{vmatrix} \quad (33)$$

$$D_4 = \begin{vmatrix} M''_{25}(r_1) & M''_{26}(r_1) \\ M''_{25}(r_2) & M''_{26}(r_2) \end{vmatrix}.$$

The terms $M'_{ij}(r)$ in D_3 and D_4 are given in Appendix B for $n = 0$. From (33), now (32) is satisfied if $D_3 = 0$ or $D_4 = 0$. The case of $D_3 = 0$ gives the frequency equation of axial symmetric vibrations of an infinite hollow poroelastic cylinder for a pervious surface discussed by Ahmed Shah [37].

The case of $D_4 = 0$ when simplified yields the equation

$$J_2(\alpha_3 r_1) Y_2(\alpha_3 r_2) - J_2(\alpha_3 r_2) Y_2(\alpha_3 r_1) = 0, \quad (34)$$

which is the frequency of torsional vibrations of infinite hollow poroelastic cylinder studied by El-Naggar et al. [2] in the presence of dissipation. Moreover, the frequency equation (34) is independent of the nature of the surface, that is, pervious.

Also, it can be said that (34) is the same for pervious surfaces.

4.3. Equivoluminal Modes. For $n = 0$, the stress free boundary conditions on the inner and outer surfaces of the hollow poroelastic cylinder are satisfied if

$$\alpha_3^2 = k^2 > 0. \quad (35)$$

The dilatational and equivoluminal potentials f_1 and g_1 are coupled through the boundary conditions. In particular, to consider purely equivoluminal modes, we set

$$f_1 = f_2 = 0 \quad g_3 = 0. \quad (36)$$

Using (36) into (17), it is seen that the dilatations of solid and liquid media are zero. Hence, from (2), the liquid pressure is zero. Therefore, the equivoluminal modes are independent of the nature of surface, that is, pervious and impervious. Accordingly, no distinctions between pervious surfaces are seen. Therefore, from (35) and (36), we have

$$g'_1(\alpha_3 r_1) = g'_1(\alpha_3 r_2) = 0 \quad \sigma_{rr} + s = 0, \quad (37)$$

where

$$g_1(\alpha_3 r) = A_1 J_1(\alpha_3 r) + B_1 Y_1(\alpha_3 r). \quad (38)$$

Substituting from (38) into (37) and eliminating the constants A_1, B_1 , the frequency equation of purely equivoluminal modes is

$$J'_1(\alpha_3 r_1) Y'_1(\alpha_3 r_2) - J'_1(\alpha_3 r_2) Y'_1(\alpha_3 r_1) = 0, \quad (39)$$

with the amplitude ratio

$$\frac{A_1}{B_1} = -\frac{Y'_1(\alpha_3 r_1)}{J'_1(\alpha_3 r_1)}. \quad (40)$$

And the nonzero displacement and stress for equivoluminal modes are

$$\begin{aligned} U &= ik [A_1 J_1(\alpha_3 r_1) + B_1 Y_1(\alpha_3 r_2)] e^{i(kz+wt)} \\ W &= -\alpha_3 [A_1 J_0(\alpha_3 r) + B_1 Y_0(\alpha_3 r)] e^{i(kz+wt)} \\ \sigma_{rr} &= -2Nki [A_1 J'_1(\alpha_3 r) + B_1 Y'_1(\alpha_3 r)] e^{i(kz+wt)}. \end{aligned} \quad (41)$$

Equation (39) is the same as (34) of longitudinal shear vibrations for a case of $n = 1$.

4.3.1. For Thin Poroelastic Cylindrical Shell. When $h/r_1 \ll 1$, that is, for thin poroelastic cylindrical shell, the frequency equation (27) by using Hankel-Kirchhoff asymptotic approximations [38] is reduced to

$$\sin(\alpha_3 h) - \frac{7\alpha_3 h}{8\alpha_3 r_1 r_2} \cos(\alpha_3 h) + \frac{49}{64\alpha_3 r_1 r_2} \sin(\alpha_3 h) \approx 0. \quad (42)$$

As $\alpha_3 r_1 \rightarrow \infty$, $\alpha_3 r_2 \rightarrow \infty$, with the help of (41), (27) is simplified to

$$\omega \approx \sqrt{2} \frac{q\pi v_3}{h} \left[1 + \frac{7}{8q^2\pi^2} \left(\frac{h}{r_1} \right)^2 \right], \quad (q = 1, 2, 3, \dots). \quad (43)$$

Equation (43) determines the frequency of purely equivoluminal modes of a poroelastic plate of thickness h .

4.3.2. For Poroelastic Solid Cylinder. When $r_1/h \rightarrow 0$, that is, for the poroelastic solid cylinder, the frequency equation (39) is reduced asymptotically to

$$J'_1(\alpha_3 r) = 0, \quad (44)$$

which is the frequency equation of purely equivoluminal modes of a poroelastic solid cylinder of radius h .

5. Nondimensional Frequency Equation

For propagating modes in a nondissipative medium, the wave number k is real. To analyze the frequency equation of pervious and impervious surfaces, it is convenient to introduce the following nondimensional parameters:

$$\begin{aligned} P' &= \frac{P}{H}, & Q' &= \frac{Q}{H}, \\ R' &= \frac{R}{H}, & N' &= \frac{N}{H}, \\ \rho'_{11} &= \frac{\rho_{11}}{\rho}, & \rho'_{12} &= \frac{\rho_{12}}{\rho}, \\ \rho'_{22} &= \frac{\rho_{22}}{\rho}, & W &= \omega h C_0^{-1}, \\ \varepsilon_1 &= \frac{V_0^2}{V_1^2}, & \varepsilon_2 &= \frac{V_0^2}{V_2^2}, & \varepsilon_3 &= \frac{V_0^2}{V_3^2}, \\ \delta &= \frac{h}{L}, & \Omega' &= \frac{\Omega}{V_0^2 \rho}, \end{aligned} \quad (45)$$

where w is nondimensional frequency,

$$H = P + 2Q + R, \quad \rho = \rho_{11} + 2\rho_{12} + \rho_{22}, \quad (46)$$

where C_0 and V_0 are the reference velocities taking the following form

$$\left(C_0^2 = \frac{N}{\rho}, V_0^2 = \frac{H}{\rho} \right), \quad (47)$$

FIGURE 1: Effect of the frequency determinant Δ with respect to the rotation Ω for different values of k , h , and n .

where h is the thickness of the hollow poroelastic cylinder and L is the wavelength.

Let

$$g = \frac{r_2}{r_1} \quad \text{so that} \quad \frac{h}{r_1} = (g - 1). \quad (48)$$

6. Numerical Results and Discussion

The numerical results for frequency equations are computed for the poroelastic material. Since the frequency equation, phase velocity, and attenuation coefficient are transcendental in nature. The roots are obtained for $n = 0$, the axisymmetric mode, and for the flexural mode $n = 1$. The values of the elastic constants of poroelastic material, one being sandstone

saturated with kerosene [37] and the other one being sandstone with water [39], are as follows:

$$\begin{aligned} P' &= 0.843, & Q' &= 0.065, & R' &= 0.028, & N' &= 0.234 \\ \rho'_{11} &= 0.901, & \rho'_{12} &= 0.001, & \rho'_{22} &= 0.101, & \varepsilon_1 &= 0.999, \\ & & \varepsilon_2 &= 4.763, & \varepsilon_3 &= 3.851. \end{aligned} \quad (49)$$

The numerical technique outlined above was used to obtain frequency equation with respect to rotation Ω under the effect of wave number and thickness. For the sake of brevity, some computational results are being presented here. The variations are shown in Figures 1–5, respectively.

Figure 1 shows the variation of the frequency equation Δ , phase velocity $\text{Re}(\Delta)$, and attenuation coefficient $\text{Im}(\Delta)$

FIGURE 2: Effect of the frequency determinant D_1 with respect to the rotation Ω for different values of n and h .

of wave propagation in hollow poroelastic circular cylinders with respect to rotation Ω for different values of wave number k and thickness h . In both figures, it is clear that the frequency equation, phase velocity, and attenuation coefficient have a nonzero value only in a bounded region of space. It is observed that the frequency equation and phase velocity increase with increasing rotation, while it decreases with increasing wave number and thickness; as well, attenuation coefficient increases with increasing thickness, while it decreases with increasing of rotation.

Figure 2 shows the variation of the frequency equation D_1 , phase velocity $\text{Re}(D_1)$, and attenuation coefficient $\text{Im}(D_1)$ of wave propagation in hollow poroelastic circular cylinders with respect to rotation Ω subjected to motion independent of z for different values of longitudinal mode ($n = 0$), flexural mode ($n = 1$), and thickness h . In both figures, it is clear that the frequency equation, phase velocity, and attenuation coefficient have a nonzero value only in a bounded region of space. It is observed that the frequency equation and phase

velocity increase with increasing rotation, longitudinal mode, and flexural mode, while frequency equation increases with increasing rotation, while it decreases with increasing thickness; as well, attenuation coefficient increases with increasing thickness, while it decreases with increasing rotation.

Figure 3 shows the variation of the frequency equation D_2 of wave propagation in hollow poroelastic circular cylinders with respect to rotation Ω subjected to motion independent of z for different values of longitudinal mode ($n = 0$), flexural mode ($n = 1$), and thickness h . In both figures, it is clear that the frequency equation has a nonzero value only in a bounded region of space. It is observed that the frequency equation decreases with increasing rotation, while it increases with increasing longitudinal mode and flexural mode; as well, it decreases with increasing of thickness.

Figure 4 shows the variation of the frequency equation D_3 , phase velocity $\text{Re}(D_3)$, and attenuation coefficient $\text{Im}(D_3)$ of wave propagation in hollow poroelastic circular cylinders with respect to rotation Ω subjected to motion independent

FIGURE 3: Effect of the frequency determinant D_2 with respect to the rotation Ω for different values of h and n .

FIGURE 4: Effect of the frequency determinant D_3 with respect to the rotation Ω for different values of k and h .

FIGURE 5: Effect of the frequency determinant D_4 with respect to the rotation Ω for different values of k and h .

of θ for different values of wave number k and thickness h . In both figures, it is clear that the frequency equation, phase velocity, and attenuation coefficient have a nonzero value only in a bounded region of space. It is observed that the frequency equation and attenuation coefficient decrease with increasing wave number and thickness, respectively, while it increases with increasing rotation; as well, attenuation coefficient decreases with increasing thickness, while it increases with increasing rotation.

Figure 5 shows the variation of the frequency equation D_4 of wave propagation in hollow poroelastic circular cylinders with respect to rotation Ω subjected to motion independent of θ for different values of wave number k and thickness h . In both figures, it is clear that the frequency equation has a nonzero value only in a bounded region of space. It is observed that the frequency equation decreases with increasing thickness and, while it has oscillatory behavior in the whole range of Ω -axis, there is no effect on the frequency equation.

Comparing with previous studies, we find that our results (shown in Figures 1–5) without rotation for the wave propagation of poroelastic bone with the results obtained by Ahmed Shah [37]. Also, these results agree with those of Kumar [40] by ignoring the liquid and rotation effect and, after rearrangement of terms, the results obtained agree with the purely elastic solid. In case of gravity $\Omega = 0$, our results are in agreement with those of Biot [36]. The analytical results obtained by Sharma and Gogna [41] can be considered as a limiting case (by taking $\Omega = 0$), which are in agreement with earlier results obtained by Fotiadis et al. [42].

7. Conclusion

In this paper, the wave propagation of poroelastic bone with circular cylinder subjected to traction free surfaces is considered. We adopted the analysis of [3], and the solution

of the problem was expressed in terms of a Bessel function of the first and second kind, respectively.

The resulting frequency equation has been solved numerically. The contribution of the fluid term to wave propagation is a well-established possible mechanism of wave propagation connected to many biological phenomena observed in bone. Although the prediction of the model cannot be trusted quantitatively at this stage, its qualitative behavior complies with the predictions of other theoretical and experimental models referred to in the literature. A calibration of the model and its verification with experimental data is in progress. Important phenomena are observed in all these computations as follows:

- (i) the frequency equation of axially symmetric vibrations is independent of the nature of surface, rotation, and presence of fluid in the poroelastic cylindrical;
- (ii) by comparing Figures 1–5, it was found that the frequency equation, phase velocity, and attenuation coefficient have the same behavior in both media; but, with the passage of rotation, wave number, and thickness, numerical values of frequency in the poroelastic cylinder are large in comparison due to the influences of rotation and fluid;
- (iii) special cases are considered as motion independent on z and motion independent on θ in poroelastic medium, as well as in the isotropic case for very large wave numbers and dispersion curves for longitudinal mode and flexural mode ($n = 0, 1$), respectively;
- (iv) the results presented in this paper should prove to be useful for researchers in material science and designers of new materials and bones;
- (v) the study of the phenomenon of rotation, wave number, and thickness is also used to improve the conditions of bones.

Appendices

A. Coefficients of $M'_{ij}(r_k)$

The coefficients $M'_{ij}(r_k)$ ($i, j = 1, 2, \dots, 8, k = 1, 2$) are given as follows:

$$\begin{aligned}
 M'_{11}(r_1) &= \left\{ 2N \left[\frac{n(n-1)}{r_1^2} - \alpha_1^2 \right] \right. \\
 &\quad \left. + [(Q+R)\delta_1^2 - (A+Q)](\alpha_1^2) \right\} \\
 &\quad \times J_n(\beta_1 r_1) + \frac{2N\beta_1}{r_1} J_{n+1}(\beta_1 r_1) \\
 M'_{12}(r_1) &= \left\{ 2N \left[\frac{n(n-1)}{r_1^2} - \alpha_1^2 \right] \right. \\
 &\quad \left. + [(Q+R)\delta_1^2 - (A+Q)](\alpha_1^2) \right\} \\
 &\quad \times Y_n(\beta_1 r_1) + \frac{2N\beta_1}{r_1} Y_{n+1}(\beta_1 r_1), \\
 M'_{13}(r_1) &= \left\{ 2N \left[\frac{n(n-1)}{r_1^2} - \alpha_1^2 \right] \right. \\
 &\quad \left. + [(Q+R)\delta_1^2 - (A+Q)](\alpha_1^2) \right\} \\
 &\quad \times J_n(\beta_1 r_1) + \frac{2N\beta_1}{r_1} J_{n+1}(\beta_1 r_1), \\
 M'_{14}(r_1) &= \left\{ 2N \left[\frac{n(n-1)}{r_1^2} - \alpha_1^2 \right] \right. \\
 &\quad \left. + [(Q+R)\delta_1^2 - (A+Q)](\alpha_1^2) \right\} \\
 &\quad \times Y_n(\beta_1 r_1) + \frac{2N\beta_1}{r_1} J_{n+1}(\beta_1 r_1), \\
 M'_{15}(r_1) &= \frac{2Nn(n-1)}{r_1^2} J_n(\beta_3 r_1) - \frac{2Nn\beta_3}{r_1} J_{n+1}(\beta_3 r_1), \\
 M'_{16}(r_1) &= \frac{2Nn(n-1)}{r_1^2} Y_n(\beta_3 r_2) - \frac{2Nn\beta_3}{r_1} Y_{n+1}(\beta_3 r_1), \\
 M'_{21}(r_1) &= \frac{2Nn(1-n)}{r_1^2} Y_n(\beta_1 r_1) + \frac{2Nn\beta_1}{r_1} J_{n+1}(\beta_1 r_1), \\
 M'_{22}(r_1) &= \frac{2Nn(1-n)}{r_1^2} Y_n(\beta_1 r_1) + \frac{2Nn\beta_1}{r_1} Y_{n+1}(\beta_1 r_1), \\
 M'_{23}(r_1) &= \frac{2Nn(1-n)}{r_1^2} J_n(\beta_2 r_1) + \frac{2Nn\beta_2}{r_1} J_{n+1}(\beta_2 r_1), \\
 M'_{24}(r_1) &= \frac{2Nn(1-n)}{r_1^2} Y_n(\beta_2 r_1) + \frac{2Nn\beta_2}{r_1} Y_{n+1}(\beta_2 r_1),
 \end{aligned}$$

$$\begin{aligned}
 M'_{25}(r_1) &= \left[\frac{2Nn(1-n)}{r_1^2} + N\alpha_3^2 \right] J_n(\beta_3 r_1) \\
 &\quad - \frac{2N\beta_3}{r_1} J_{n+1}(\beta_3 r_1), \\
 M'_{26}(r_1) &= \left[\frac{2Nn(1-n)}{r_1^2} + N\alpha_3^2 \right] Y_n(\beta_3 r_1) \\
 &\quad - \frac{2N\beta_3}{r_1} Y_{n+1}(\beta_3 r_1),
 \end{aligned}$$

$$M'_{37}(r_1) = \frac{-Nn\beta_3}{r_1} J_n(\beta_3 r_1) - N(\alpha_3^2) J_{n+1}(\beta_3 r_1),$$

$$M'_{38}(r_1) = \frac{-Nn\beta_3}{r_1} Y_n(\beta_3 r_1) - N(\alpha_3^2) Y_{n+1}(\beta_3 r_1),$$

$$M'_{37}(r_2) = \frac{-Nn\beta_3}{r_2} J_n(\beta_3 r_2) - N(\alpha_3^2) J_{n+1}(\beta_3 r_2),$$

$$M'_{38}(r_2) = \frac{-Nn\beta_3}{r_2} Y_n(\beta_3 r_2) - N(\alpha_3^2) Y_{n+1}(\beta_3 r_2),$$

$$M'_{41}(r_1) = (R\delta_1^2 - Q)(\alpha_1^2) J_n(\beta_1 r_1),$$

$$M'_{42}(r_1) = (R\delta_1^2 - Q)(\alpha_1^2) Y_n(\beta_1 r_1),$$

$$M'_{43}(r_1) = (R\delta_2^2 - Q)(\alpha_1^2) J_n(\beta_2 r_1),$$

$$M'_{44}(r_1) = (R\delta_2^2 - Q)(\alpha_2^2) J_n(\beta_2 r_1),$$

$$M'_{45}(r_1) = 0,$$

$$M'_{46}(r_1) = 0,$$

$$M'_{41}(r_2) = (R\delta_1^2 - Q)(\alpha_1^2) J_n(\beta_1 r_2),$$

$$M'_{42}(r_2) = (R\delta_1^2 - Q)(\alpha_1^2) Y_n(\beta_1 r_2),$$

$$M'_{43}(r_2) = (R\delta_2^2 - Q)(\alpha_1^2) J_n(\beta_2 r_2),$$

$$M'_{44}(r_2) = (R\delta_2^2 - Q)(\alpha_2^2) J_n(\beta_2 r_2),$$

$$M'_{45}(r_2) = 0,$$

$$M'_{46}(r_2) = 0.$$

(A.1)

B. Coefficients of $M''_{ij}(r_k)$

The coefficients $M''_{ij}(r_k)$ ($i, j = 1, 2, \dots, 8, k = 1, 2$) are given as follows:

$$\begin{aligned}
 M''_{11}(r_1) &= \left\{ -2N\alpha_1^2 + [(Q+R)\delta_1^2 - (A+Q)] \right. \\
 &\quad \left. \times (k^2 + \alpha_1^2) \right\} \\
 &\quad \times J_n(\beta_1 r_1) + \frac{2N\beta_1}{r_1} J_{n+1}(\beta_1 r_1),
 \end{aligned}$$

$$M''_{12}(r_1) = \{-2N\alpha_1^2 + [(Q+R)\delta_1^2 - (A+Q)] \\ \times (k^2 + \alpha_1^2)\}$$

$$\times Y_n(\beta_1 r_1) + \frac{2N\beta_1}{r_1} Y_{n+1}(\beta_1 r_1),$$

$$M''_{13}(r_1) = \{-2N\alpha_2^2 + [(Q+R)\delta_2^2 - (A+Q)] \\ \times (k^2 + \alpha_2^2)\}$$

$$\times J_n(\beta_2 r_1) + \frac{2N\beta_2}{r_1} J_{n+1}(\beta_2 r_1),$$

$$M''_{14}(r_1) = \{-2N\alpha_2^2 + [(Q+R)\delta_2^2 - (A+Q)] \\ \times (k^2 + \alpha_2^2)\}$$

$$\times Y_n(\beta_2 r_1) + \frac{2N\beta_2}{r_1} Y_{n+1}(\beta_2 r_1),$$

$$M''_{17}(r_1) = 2Nik\beta_3 J_n(\beta_3 r_1) - \frac{2Nik}{r_1} J_{n+1}(\beta_3 r_1),$$

$$M''_{18}(r_1) = 2Nik\beta_3 Y_n(\beta_3 r_1) - \frac{2Nik}{r_1} Y_{n+1}(\beta_3 r_1),$$

$$M''_{25}(r_1) = -\frac{2N\beta_3}{r_1} J_{n+1}(\beta_3 r_1),$$

$$M''_{26}(r_1) = [N\alpha_3^2] Y_n(\beta_3 r_1) - \frac{2N\beta_3}{r_1} Y_{n+1}(\beta_3 r_1),$$

$$M''_{25}(r_2) = -\frac{2N\beta_3}{r_2} J_{n+1}(\beta_3 r_2),$$

$$M''_{26}(r_2) = [N\alpha_3^2] Y_n(\beta_3 r_2) - \frac{2N\beta_3}{r_1} Y_{n+1}(\beta_3 r_2),$$

$$M''_{31}(r_1) = -2Nik\beta_1 J_{n+1}(\beta_1 r_1),$$

$$M''_{32}(r_1) = -2Nik\beta_1 Y_{n+1}(\beta_1 r_1),$$

$$M''_{33}(r_1) = -2Nik\beta_2 J_{n+1}(\beta_2 r_1),$$

$$M''_{34}(r_1) = -2Nik\beta_2 Y_{n+1}(\beta_2 r_1),$$

$$M''_{37}(r_1) = -N(k^2 - \alpha_3^2) J_{n+1}(\beta_3 r_1),$$

$$M''_{38}(r_1) = -N(k^2 - \alpha_3^2) Y_{n+1}(\beta_3 r_1),$$

$$M''_{41}(r_1) = (R\delta_1^2 - Q)(k^2 + \alpha_1^2) J_n(\beta_1 r_1),$$

$$M''_{42}(r_1) = (R\delta_1^2 - Q)(k^2 + \alpha_1^2) Y_n(\beta_1 r_1),$$

$$M''_{43}(r_1) = (R\delta_2^2 - Q)(k^2 + \alpha_1^2) J_n(\beta_2 r_1),$$

$$M''_{44}(r_1) = (R\delta_2^2 - Q)(k^2 + \alpha_2^2) J_n(\beta_2 r_1),$$

$$M''_{47}(r_1) = 0,$$

$$M''_{48}(r_1) = 0,$$

$$M''_{11}(r_2) = \{-2N\alpha_1^2 + [(Q+R)\delta_1^2 - (A+Q)] \\ \times (k^2 + \alpha_1^2)\}$$

$$\times J_n(\beta_1 r_2) + \frac{2N\beta_1}{r_2} J_{n+1}(\beta_1 r_2),$$

$$M''_{12}(r_2) = \{2N\alpha_1^2 + [(Q+R)\delta_1^2 - (A+Q)] \\ \times (k^2 + \alpha_1^2)\}$$

$$\times Y_n(\beta_1 r_2) + \frac{2N\beta_1}{r_2} Y_{n+1}(\beta_1 r_2),$$

$$M''_{13}(r_2) = \{-2N\alpha_2^2 + [(Q+R)\delta_2^2 - (A+Q)] \\ \times (k^2 + \alpha_2^2)\}$$

$$\times J_n(\beta_2 r_2) + \frac{2N\beta_2}{r_2} J_{n+1}(\beta_2 r_2),$$

$$M''_{14}(r_2) = \{-2N\alpha_2^2 + [(Q+R)\delta_2^2 - (A+Q)] \\ \times (k^2 + \alpha_2^2)\}$$

$$\times Y_n(\beta_2 r_2) + \frac{2N\beta_2}{r_2} J_{n+1}(\beta_2 r_2),$$

$$M''_{17}(r_2) = 2Nik\beta_3 J_n(\beta_3 r_2) - \frac{2Nik}{r_2} J_{n+1}(\beta_3 r_2),$$

$$M''_{18}(r_2) = 2Nik\beta_3 Y_n(\beta_3 r_2) - \frac{2Nik}{r_2} Y_{n+1}(\beta_3 r_2),$$

$$M''_{31}(r_2) = -2Nik\beta_1 J_{n+1}(\beta_1 r_2),$$

$$M''_{32}(r_2) = -2Nik\beta_1 Y_{n+1}(\beta_1 r_2),$$

$$M''_{33}(r_2) = -2Nik\beta_2 J_{n+1}(\beta_2 r_2),$$

$$M''_{34}(r_2) = -2Nik\beta_2 Y_{n+1}(\beta_2 r_2),$$

$$M''_{37}(r_2) = -N(k^2 - \alpha_3^2) J_{n+1}(\beta_3 r_2),$$

$$M''_{38}(r_2) = -N(k^2 - \alpha_3^2) Y_{n+1}(\beta_3 r_2),$$

$$M''_{41}(r_2) = (R\delta_1^2 - Q)(k^2 + \alpha_1^2) J_n(\beta_1 r_2),$$

$$M''_{42}(r_2) = (R\delta_1^2 - Q)(k^2 + \alpha_1^2) Y_n(\beta_1 r_2),$$

$$M''_{43}(r_2) = (R\delta_2^2 - Q)(k^2 + \alpha_1^2) J_n(\beta_2 r_2),$$

$$M''_{44}(r_2) = (R\delta_2^2 - Q)(k^2 + \alpha_2^2) J_n(\beta_2 r_2),$$

$$M''_{47}(r_2) = 0,$$

$$M''_{48}(r_2) = 0.$$

(B.1)

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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