Research Article

The Finite Element Numerical Modelling of 3D Magnetotelluric

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The ideal numerical simulation of 3D magnetotelluric was restricted by the methodology complexity and the time-consuming calculation. Boundary values, the variation of weighted residual equation, and the hexahedral mesh generation method of finite element are three major causes. A finite element method for 3D magnetotelluric numerical modeling is presented in this paper as a solution for the problem mentioned above. In this algorithm, a hexahedral element coefficient matrix for magnetotelluric finite method is developed, which solves large-scale equations using preconditioned conjugate gradient of the first-type boundary conditions. This algorithm is verified using the homogeneous model, and the positive landform model, as well as the low resistance anomaly model.

1. Introduction

Both 1D and 2D MT numerical simulation technologies have brought benefits to economy with their maturity. However, they are both the approximate processing of a 3D numerical simulation, which is the closest to the real geological body model [1–9].

With the development in computer science and computing technology, numerical simulation of 3D MT data has become the research focus. Shizhe [10] proposed the conjecture and theoretical basis of “Hexahedral mesh generation method for finite element” and “Boundary element method in 3D numerical simulation” [11]. However, there are only a few publications regarding successful scientific research on 3D numerical simulations [12–15]. The major difficulties lie in three aspects: (1) insufficient computing capability due to limited CPU memory; (2) immature parallel algorithm on multicore CPU's; (3) limited boundary element 3D MT numerical simulation on pure terrain uniform earth. Shu and Chen [16] proposed a 3D forward method in horizontal electric dipole under the magnetotelluric response by using a vector finite element method. After combining scalar finite elements and vector finite elements together,

Mitsuhata and Uchida [17] conducted forward computation and error analysis for the two 3D COMMEMI models using T_Ω formula, which verified the correctness of the program as well. Under the conditions of tetrahedral elements, Linping and Shikun [18] derived the analytical expression for the underground varying conductivity to calculate the three-dimensional magnetotelluric finite element equation. Nam et al. [19] proposed the 3D magnetotelluric response simulation based on hexahedral-vector finite element method and simulated the 3D terrain using isoparametric element method. As for the underground complex geological body, only a few papers are available from public sources [20–27].

This paper deduces the hexahedral element coefficient matrix of the magnetotelluric finite element method based on Xu Shizhe's work. It uses preconditioned conjugate gradient method to solve large-scale equations under the first-type boundary conditions. This algorithm can significantly improve the computation efficiency. Via computing analysis of homogeneous model, undulating terrain model, and 3D model, the 3D low resistivity abnormal morphology can be clearly reflected. As the frequency decreases, the errors between the calculation results and analytical solutions are found gradually decreasing.

2. The Boundary Problem of 3D MT

A 3D geology space model is shown in Figure 1. The space is divided into two layers, air and underground, by an undulating ($A'B'C'D'$). The surface of ABCD is the infinite boundary of upper air which is about 100 km height, and the surface of EFGH is infinite boundary of underground which is about 30 km beneath ground. In addition, there are four infinite vertical boundary surfaces: ABFE, BFGC, DCGH and ADHE, all of which are about 30 km away from the center area of measurement.

The source of natural alternating electromagnetic field is located on the surface of ABCD. When the electromagnetic wave passes through the atmosphere and then enters the underground, the whole 3D geology space becomes the study field of MT method due to continuous electric field on the tangential direction and magnetic fields on the normal direction.

2.1. The Boundary Value Problem of MT Method. The electromagnetic waves from the magnetotelluric field source vertically incident into the underground as harmonic plane waves with a time harmonic factor $e^{-i\omega t}$. Maxwell coupled magnetotelluric field equations in frequency domain as

$$\begin{aligned}\nabla \times E &= i\omega\mu H, \\ \nabla \times H &= (\sigma - i\omega\epsilon) E,\end{aligned}\quad (1)$$

where E is the electric vector, H is the magnetic vector, ω is angular frequency, μ is the medium permeability when the permeability of air is 1.2567×10^{-6} H/m, σ is the medium conductivity or the reciprocal of medium resistivity, ϵ is the dielectric medium constant, and the dielectric constant of air is 8.85×10^{-12} F/m.

Curling both sides of the electric equations (1), the differential electric equation can be obtained as follows:

$$\begin{aligned}\nabla \times (\nabla \times E) - k^2 E &= 0, \\ \nabla \times (\nabla \times H) - k^2 H &= 0,\end{aligned}\quad (2)$$

where

$$k^2 = i\omega\mu\sigma + \omega^2\epsilon\mu. \quad (3)$$

The equations of electric and magnetic have the same form that can be written in one unification equation. Substituting E and H for U yields

$$\nabla \times (\nabla \times U) - k^2 U = 0. \quad (4)$$

The vector U can be rewritten to the vector form with three components:

$$U = U_x e_x + U_y e_y + U_z e_z, \quad (5)$$

where, e_x , e_y , and e_z are the unit vectors in the directions of x , y , and z .

2.1.1. When It Is Working in TE Model

- In the surface of ABCD shown in Figure 1, the field source is the electric vector E_x . As the ratio of electric and magnetic, U_x has no impact to the final result. Therefore, U_x can be either zero or nonzero value. Here $U_x = 1$. U_y and U_z are all zero as well since there are no components of U in x and y directions.
- In the four vertical boundaries shown in Figure 1, the propagation direction of EM wave in magnetic field is vertical downward, being perpendicular to normal direction of the boundary, which is $E \times H \perp \Gamma$. Therefore, $\partial U / \partial n$ equals zero.
- In the surface of EFGH in Figure 1, the underground can be assumed as a homogenous medium with a consideration that this surface is beyond the study area. The electromagnetic field attenuates in a normal way, which follows the index law during downward propagation. The vector U has no value in directions y and z .

$$U_x = ce^{-\chi z}, \quad U_y = 0, \quad U_z = 0, \quad (6)$$

where c is a constant factor with any nonzero value, $\chi = \sqrt{-i\omega\mu\sigma}$, and σ is the homogenous conductivity below the infinite surface EFGH. Derivative of (6) is obtained as follows:

$$\frac{\partial U_x}{\partial n} = \frac{\partial U_x}{\partial z} = -\chi U_x. \quad (7)$$

2.1.2. When It Is Working in TM Model. The field source is one component of the magnetic vector H_x . When magnetic field passes through the atmosphere, attenuation barely happens. Therefore, the boundary conditions of ABCD can be replaced by $A'B'C'D'$'s in the TE model.

3. The Variation Problem of Weighted Residual Equation

Both sides of (4) are multiplied by δU and integrated on the whole region, showing the result as follows:

$$\int_{\Omega} [\nabla \times (\nabla \times U) - k^2 U] \cdot \delta U d\Omega = 0, \quad (8)$$

where

$$\delta U = \delta U_x e_x + \delta U_y e_y + \delta U_z e_z. \quad (9)$$

According to the vector identities

$$(\nabla \times A) \cdot B = \nabla \cdot (A \times B) + (\nabla \times B) \cdot A. \quad (10)$$

Equation (8) will be

$$\begin{aligned}\int_{\Omega} \nabla \times (\nabla \times U) \cdot \delta U d\Omega &= \int_{\Omega} \nabla \cdot [(\nabla \times U) \times \delta U] d\Omega \\ &+ \int_{\Omega} \nabla \times \delta U \cdot \nabla \times U d\Omega.\end{aligned}\quad (11)$$

FIGURE 1: 3D space of the study.

In which,

$$\int_{\Omega} \nabla \cdot [(\nabla \times U) \times \delta U] d\Omega = \int_{\Gamma} (\nabla \times U) \times \delta U \cdot d\Gamma. \quad (12)$$

According to the boundary condition

- (a) in the surfaces ABCD and A'B'C'D', $\delta U = 0$ due to fixed U in these surfaces;
- (b) in the four infinite vertical surfaces, $(\nabla \times U) \times \delta U \cdot d\Gamma = 0$, because the reflection of the anomalous field can be ignored. Then the electric field U and δU have one component x only;
- (c) in the surface EFGH, $(\nabla \times U) \times \delta U = -(\partial U_x / \partial z) \delta U_x e_z$.

Substituting all the boundary conditions in (12) yields

$$\begin{aligned} \int_{\Omega} \nabla \times \delta U \cdot \nabla \times U d\Omega - \int_{\Omega} k^2 U \cdot \delta U d\Omega \\ + \int_{EFGH} \chi U_x \delta U_x d\Gamma = 0. \end{aligned} \quad (13)$$

4. Hexahedral Mesh Generation

Hexahedral element mesh is used to make Figure 1 into a 3D geology space as shown in Figure 2. Obviously, the entire space of 3D geology is meshed into the amount of hexahedral elements.

The vertices of each hexahedral element are sorted as shown in Figure 3.

5. Unit Analysis

Using the finite element method to solve differential equations requires a mesh representation of the target area, such

FIGURE 2: Hexahedral mesh.

as the widely used hexahedral mesh. Figure 4(a) is the parent element of the hexahedron and Figure 4(b) is the subelement. The coordinate correspondences between them are

$$x = x_0 + \frac{a}{2}\xi, \quad y = y_0 + \frac{b}{2}\eta, \quad z = z_0 + \frac{c}{2}\zeta, \quad (14)$$

where x_0 , y_0 , and z_0 is the center coordinate of sub element; a , b , c is the lengths of three edges. The differential relations between them are

$$dx = \frac{a}{2}d\xi, \quad dy = \frac{b}{2}d\eta, \quad dz = \frac{c}{2}d\zeta. \quad (15)$$

FIGURE 3: The vertices of hexahedral mesh.

The shape function is defined as

$$N_i = \frac{1}{8} (1 + \xi_i \xi) (1 + \eta_i \eta) (1 + \zeta_i \zeta), \quad (16)$$

where ξ_i , η_i , ζ_i are the coordinates of vertex i in the parent unit.

The interpolation function is

$$E_x = N^T E_x, \quad E_y = N^T E_y, \quad E_z = N^T E_z, \quad (17)$$

where $N^T = (N_1, \dots, N_8)$, $E_x^T = (E_{x1}, \dots, E_{x8})$, $E_y^T = (E_{y1}, \dots, E_{y8})$, $E_z^T = (E_{z1}, \dots, E_{z8})$. Therefore

$$E = N^T (E_x e_x + E_y e_y + E_z e_z) \quad (18)$$

$$\delta E = N^T (\delta E_x e_x + \delta E_y e_y + \delta E_z e_z),$$

where δE_x , δE_y , and δE_z are the variations of E_x , E_y , and E_z .

6. The Form of Unit Coefficient Matrix

The regional integral of (13) can be decomposed into the integral of each unit:

$$\sum_{\Omega} \int_e \nabla \times \delta E \cdot \nabla \times E d\Omega - \sum_{\Omega} \int_{\Omega} k^2 E \cdot \delta E d\Omega + \sum_{EFGH} \int_{1234} \chi E_x \delta E_x d\Gamma = 0. \quad (19)$$

In one unit e

$$\begin{aligned} \nabla \times E &= E_e^T (\alpha e_x + \beta e_y + \gamma e_z), \\ \nabla \times \delta E &= \delta E_e^T (\alpha e_x + \beta e_y + \gamma e_z), \end{aligned} \quad (20)$$

where

$$E_e^T = (E_x^T, E_y^T, E_z^T), \quad \delta E_e^T = (\delta E_x^T, \delta E_y^T, \delta E_z^T)$$

$$\alpha^T = \left[0, -\left(\frac{\partial N}{\partial z}\right)^T, \left(\frac{\partial N}{\partial y}\right)^T \right],$$

$$\beta^T = \left[-\left(\frac{\partial N}{\partial z}\right)^T, 0, \left(\frac{\partial N}{\partial x}\right)^T \right],$$

$$\gamma^T = \left[-\left(\frac{\partial N}{\partial y}\right)^T, \left(\frac{\partial N}{\partial x}\right)^T, 0 \right],$$

$$\left(\frac{\partial N}{\partial x}\right)^T = \left(\frac{\partial N_1}{\partial x}, \dots, \frac{\partial N_8}{\partial x}\right) \quad \frac{\partial N_i}{\partial x} = \frac{\partial N_i}{\partial \xi} \frac{2}{a},$$

$$\left(\frac{\partial N}{\partial y}\right)^T = \left(\frac{\partial N_1}{\partial y}, \dots, \frac{\partial N_8}{\partial y}\right) \quad \frac{\partial N_i}{\partial y} = \frac{\partial N_i}{\partial \eta} \frac{2}{b},$$

$$\left(\frac{\partial N}{\partial z}\right)^T = \left(\frac{\partial N_1}{\partial z}, \dots, \frac{\partial N_8}{\partial z}\right) \quad \frac{\partial N_i}{\partial z} = \frac{\partial N_i}{\partial \zeta} \frac{2}{c},$$

$$i = 1, \dots, 8.$$

(21)

Substituting (20) into the first integral of (19)

$$\int_e \nabla \times \delta E \cdot \nabla \times E d\Omega = \delta E_e^T K_{1e} E_e, \quad (22)$$

where

$$K_{1e} = \int_e (\alpha \alpha^T + \beta \beta^T + \gamma \gamma^T) d\Omega. \quad (23)$$

Its matrix form is

$$K_{1e} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix},$$

$$K_{11} = \int_e \left[\left(\frac{\partial N}{\partial y}\right) \left(\frac{\partial N}{\partial y}\right)^T + \left(\frac{\partial N}{\partial z}\right) \left(\frac{\partial N}{\partial z}\right)^T \right] d\Omega,$$

$$K_{22} = \int_e \left[\left(\frac{\partial N}{\partial x}\right) \left(\frac{\partial N}{\partial x}\right)^T + \left(\frac{\partial N}{\partial z}\right) \left(\frac{\partial N}{\partial z}\right)^T \right] d\Omega,$$

$$K_{33} = \int_e \left[\left(\frac{\partial N}{\partial x}\right) \left(\frac{\partial N}{\partial x}\right)^T + \left(\frac{\partial N}{\partial y}\right) \left(\frac{\partial N}{\partial y}\right)^T \right] d\Omega, \quad (24)$$

$$K_{21} = - \int_e \left(\frac{\partial N}{\partial x}\right) \left(\frac{\partial N}{\partial y}\right)^T d\Omega, \quad K_{12} = (K_{21})^T,$$

$$K_{31} = - \int_e \left(\frac{\partial N}{\partial x}\right) \left(\frac{\partial N}{\partial z}\right)^T d\Omega, \quad K_{13} = (K_{31})^T,$$

$$K_{32} = - \int_e \left(\frac{\partial N}{\partial y}\right) \left(\frac{\partial N}{\partial z}\right)^T d\Omega, \quad K_{23} = (K_{32})^T.$$

FIGURE 4: (a) The parent unit of hexahedron; (b) The subelement of hexahedron.

The expressions of the elements in each matrix are listed where
as

$$\begin{aligned}
 K11_{ij} &= \int_e \left(\frac{\partial N_i}{\partial \eta} \frac{\partial N_j}{\partial \eta} \frac{4}{b^2} + \frac{\partial N_i}{\partial \zeta} \frac{\partial N_j}{\partial \zeta} \frac{4}{c^2} \right) \frac{abc}{8} d\xi \eta \zeta \\
 &= \frac{1}{36} \left(\frac{ac}{b} \beta_{ij} + \frac{ab}{c} v_{ij} \right), \\
 K22_{ij} &= \int_e \left(\frac{\partial N_i}{\partial \xi} \frac{\partial N_j}{\partial \xi} \frac{4}{a^2} + \frac{\partial N_i}{\partial \zeta} \frac{\partial N_j}{\partial \zeta} \frac{4}{c^2} \right) \frac{abc}{8} d\xi \eta \zeta \\
 &= \frac{1}{36} \left(\frac{bc}{a} \alpha_{ij} + \frac{ab}{c} v_{ij} \right), \\
 K33_{ij} &= \int_e \left(\frac{\partial N_i}{\partial \xi} \frac{\partial N_j}{\partial \xi} \frac{4}{a^2} + \frac{\partial N_i}{\partial \eta} \frac{\partial N_j}{\partial \eta} \frac{4}{b^2} \right) \frac{abc}{8} d\xi \eta \zeta \\
 &= \frac{1}{36} \left(\frac{bc}{a} \alpha_{ij} + \frac{ac}{b} \beta_{ij} \right), \\
 K21_{ij} &= - \int_e \frac{\partial N_i}{\partial \xi} \frac{\partial N_j}{\partial \eta} \frac{c}{2} d\xi \eta \zeta \\
 &= - \frac{c}{24} m_{ij}, \\
 K31_{ij} &= - \int_e \frac{\partial N_i}{\partial \xi} \frac{\partial N_j}{\partial \zeta} \frac{b}{2} d\xi \eta \zeta \\
 &= - \frac{b}{24} n_{ij}, \\
 K32_{ij} &= - \int_e \frac{\partial N_i}{\partial \eta} \frac{\partial N_j}{\partial \zeta} \frac{a}{2} d\xi \eta \zeta \\
 &= - \frac{a}{24} l_{ij},
 \end{aligned} \tag{25}$$

$$\alpha_{ij} = \begin{vmatrix} 4 & 2 & 1 & 2 & -4 & -2 & -1 & -2 \\ & 4 & 2 & 1 & -2 & -4 & -2 & -1 \\ & & 4 & 2 & -1 & -2 & -4 & -2 \\ & & & 4 & -2 & -1 & -2 & -4 \\ & & & & 4 & 2 & 1 & 2 \\ & & & & & 4 & 2 & 1 \\ & & & & & & 4 & 2 \\ & & & & & & & 4 \end{vmatrix},$$

$$\beta_{ij} = \begin{vmatrix} 4 & 2 & -2 & -4 & 2 & 1 & -1 & -2 \\ & 4 & -4 & -2 & 1 & 2 & -2 & -1 \\ & & 4 & 2 & -1 & -2 & 2 & 1 \\ & & & 4 & -2 & -1 & 1 & 2 \\ & & & & 4 & 2 & -2 & -4 \\ & & & & & 4 & -4 & -2 \\ & & & & & & 4 & 2 \\ & & & & & & & 4 \end{vmatrix},$$

$$\gamma_{ij} = \begin{vmatrix} 4 & -4 & -2 & 2 & 2 & -2 & -1 & 1 \\ & 4 & -4 & -2 & 2 & 2 & -2 & -1 \\ & & 4 & -4 & -2 & 2 & 2 & -2 \\ & & & 4 & -4 & -2 & 2 & 2 \\ & & & & 4 & -4 & -2 & 2 \\ & & & & & 4 & 2 & -2 \\ & & & & & & 4 & -4 \\ & & & & & & & 4 \end{vmatrix},$$

$$m_{ij} = \begin{vmatrix} 2 & 1 & -1 & -2 & 2 & 1 & -1 & -2 \\ 1 & 2 & -2 & -1 & 1 & 2 & -2 & -1 \\ 1 & 2 & -2 & -1 & 1 & 2 & -2 & -1 \\ 2 & 1 & -1 & -2 & 2 & 1 & -1 & -2 \\ -2 & -1 & 1 & 2 & -2 & -1 & 1 & 2 \\ -1 & -2 & 2 & 1 & -1 & -2 & 2 & 1 \\ -1 & -2 & 2 & 1 & -1 & -2 & 2 & 1 \\ -2 & -1 & 1 & 2 & -2 & -1 & 1 & 2 \end{vmatrix},$$

$$n_{ij} = \begin{bmatrix} 2 & -2 & -1 & 1 & 2 & -2 & -1 & 1 \\ 2 & -2 & -1 & 1 & 2 & -2 & -1 & 1 \\ 1 & -1 & -2 & 2 & 1 & -1 & -2 & 2 \\ 1 & -1 & -2 & 2 & 1 & -1 & -2 & 2 \\ -2 & 2 & 1 & -1 & -2 & 2 & 1 & -1 \\ -2 & 2 & 1 & -1 & -2 & 2 & 1 & -1 \\ -1 & 1 & 2 & -2 & -1 & 1 & 2 & -2 \\ -1 & 1 & 2 & -2 & -1 & 1 & 2 & -2 \end{bmatrix},$$

$$l_{ij} = \begin{bmatrix} 2 & -2 & -2 & 2 & 1 & -1 & -1 & 1 \\ 2 & -2 & -2 & 2 & 1 & -1 & -1 & 1 \\ -2 & 2 & 2 & -2 & -1 & 1 & 1 & -1 \\ -2 & 2 & 2 & -2 & -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 & 2 & -2 & -2 & 2 \\ 1 & -1 & -1 & 1 & 2 & -2 & -2 & 2 \\ -1 & 1 & 1 & -1 & -2 & 2 & 2 & -2 \\ -1 & 1 & 1 & -1 & -2 & 2 & 2 & -2 \end{bmatrix}.$$

(26)

Substituting (18) into the second integral of (19):

$$\int_e k^2 E \cdot \delta E d\Omega = \delta E_e^T K_{2e} E_e, \quad (27)$$

where

$$K_{2e} = \begin{bmatrix} K_{11} & 0 & 0 \\ 0 & K_{22} & 0 \\ 0 & 0 & K_{33} \end{bmatrix},$$

$$K_{11} = K_{22} = K_{33} = \int_e k^2 NN^T d\Omega,$$

$$K_{11}_{ij} = \int_e k^2 N_i N_j \frac{abc}{8} d\xi d\eta d\zeta, \quad i, j = 1, \dots, 8$$

$$= \frac{1}{144} abck^2 o_{ij} \quad (28)$$

$$o_{ij} = \begin{bmatrix} 6 & 3 & 3 & 6 & 2 & 1 & 1 & 2 \\ & 6 & 6 & 3 & 1 & 2 & 2 & 1 \\ & & 6 & 3 & 1 & 2 & 2 & 1 \\ & & & 6 & 2 & 1 & 1 & 2 \\ & & & & 6 & 3 & 3 & 6 \\ & & & & & 6 & 6 & 3 \\ & & & & & & 6 & 3 \\ & & & & & & & 6 \end{bmatrix}.$$

When the boundary surface unit 1234 is in the boundary surface EFGH, the boundary integral is

$$\int_{1234} \chi E_x \delta E_z d\Gamma = \delta E_e^T K_{3e} E_e, \quad (29)$$

where

$$K_{3e} = \begin{bmatrix} K_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$K_{11} = \int_{1234} \chi NN^T d\Gamma,$$

$$K_{11}_{ij} = \int_{1234} \chi N_i N_j \frac{ab}{4} d\xi d\eta, \quad i, j = 1, \dots, 4 \quad (30)$$

$$= \frac{1}{36} ab \chi p_{ij},$$

$$p_{ij} = \begin{bmatrix} 4 & 2 & 1 & 2 \\ & 4 & 2 & 1 \\ & & 4 & 2 \\ & & & 4 \end{bmatrix},$$

where N_i, N_j are the shape functions of quadrilateral plane element rather than the shape functions of hexahedral plane element. Its form is shown in the following:

$$N_i = \frac{1}{4} (1 + \xi_i \xi) (1 + \eta_i \eta). \quad (31)$$

Substituting (14) into (22) and (27), (31) into (29), respectively, each element of the matrix can then be calculated.

7. The Integration of the Coefficient Matrix

Integrating all the hexahedral elements obtained above gives the integration of the coefficient matrix which is shown below:

$$\sum \partial U_e^T (K_{1e} - K_{2e} + K_{3e}) U_e = \sum \partial U_e^T K_e U_e$$

$$= \partial U^T \sum K_e U = \partial U^T K U = 0. \quad (32)$$

The arbitrary ∂U^T is not always zero, which gives the following large-scale equations:

$$K U = 0, \quad (33)$$

where K is the coefficient matrix and U is the unknown variables.

8. The Computation of Apparent Resistivity

The expansions of (2) in the Cartesian coordinate system are presented below:

$$H_x = \frac{1}{i\omega\mu} \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right),$$

$$H_y = \frac{1}{i\omega\mu} \left(\frac{\partial E_z}{\partial z} - \frac{\partial E_z}{\partial x} \right), \quad (34)$$

$$H_z = \frac{1}{i\omega\mu} \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_z}{\partial y} \right).$$

FIGURE 5: Homogeneous model.

FIGURE 6: Positive landform model.

When E_x , E_y , and E_z of each vertex have been obtained, H_x , H_y , and H_z can be computed from the approximate values of each vertex's differential quotient using the difference method. Therefore, the apparent resistivity equation is derived as:

$$\rho_{xy} = \frac{i}{\omega\mu} \left(\frac{E_z}{H_y} \right)^2. \quad (35)$$

9. Example Analysis

9.1. Homogeneous Model. Figure 5 shows a homogeneous model: the resistivity is $100 \Omega \cdot m$; the grid size is $9000 \times 9000 m^2$; the dot pitch is 1000 m; the line spacing is 1000 m. The measuring frequency is 1000 Hz, 500 Hz, 100 Hz, and 50 Hz, and the observation points are the five points of the midline (2000 m, 3000 m, 4000 m, 5000 m, and 6000 m).

Table 1 lists the numerical computation results. According to the results, the average of the relative error is 2.81% in 1000 Hz and 0.16% in 100 Hz. Gradually decreasing relative error is observed as the frequency decreases.

9.2. Positive Landform Model. In Figure 6, a positive landform model has a resistivity of $100 \Omega \cdot m$ and the topographic fall is 60 m; the grid size is $9000 \times 9000 m^2$; the dot pitch is 1000 m; and the line spacing is 1000 m. The measuring frequencies are 1000 Hz and 100 Hz, and the observation points are the nine points on the midline.

Results of the numerical computation are shown in Figure 7. The highest value is the pseudoabnormality in the positive landform. The distortion error is 39.5% at 1000 Hz

FIGURE 7: The result of numerical computation.

FIGURE 8: The model with low resistance body.

and 12.8% at 100 Hz. With a decreasing frequency, the effect of topographic distortion decreases gradually.

9.3. The Model with Low Resistance Body. Figure 8 shows a model containing a low resistance body with $10 \Omega \cdot m$ resistivity; resistivity of the rest of the model is $100 \Omega \cdot m$; The horizontal geometry of the anomalous body is $3000 m \times 3000 m$; upper surface is 250 m away from the ground and its height is 750 m; the grid size is $9000 \times 9000 m^2$; the dot pitch is 1000 m; the line spacing is 1000 m (Figure 9). The measuring frequencies are 1000 Hz and 100 Hz, and the observation points are the nine points of the mid-line.

The result of numerical computation is shown in Figure 10 with the curves showing low resistivity abnormality at both 1000 Hz and 100 Hz.

FIGURE 9: (a) Horizontal top view. (b) Positive side view.

FIGURE 10: The result of numerical computation.

TABLE 1: The result of numerical computation.

X/Km	Fre/Hz	Rho/Ω·m	Phase/(°)	RE/(%)
2000	1000	97.17	44.67	2.83
2000	500	98.26	44.66	1.74
2000	100	100.17	44.82	0.17
2000	50	99.36	45.11	0.64
3000	1000	97.16	44.68	2.84
3000	500	98.24	44.66	1.76
3000	100	100.14	44.83	0.14
3000	50	99.29	45.13	0.71
4000	1000	97.17	44.68	2.83
4000	500	98.24	44.66	1.76
4000	100	100.13	44.83	0.13
4000	50	99.23	45.14	0.77
5000	1000	97.19	44.68	2.81
5000	500	98.26	44.66	1.74
5000	100	100.15	44.84	0.15
5000	50	99.26	45.12	0.74
6000	1000	97.25	44.67	2.75
6000	500	98.34	44.65	1.66
6000	100	100.21	44.85	0.21
6000	50	99.4	45.09	0.6

10. Conclusions and Recommendations

During the research of magnetotelluric finite element numerical simulation algorithm, (1) the boundary values of 3D magnetotelluric numerical simulations are analyzed; (2) the preconditioned conjugate gradient is adopted for solving large-scale equations under the first-type boundary conditions; (3) the stability and the iterative convergence speed are ensured in 3D magnetotelluric forward calculations. This paper provides an effective tool for theoretical studies on distribution and characteristics of magnetotelluric field response of a 3D geology structure. Meanwhile, it also has established critical foundation for future 3D inversion algorithm research.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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