Research Article

Improved Results on Robust Stability for Systems with Interval Time-Varying Delays and Nonlinear Perturbations

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This paper investigated delay-dependent robust stability criteria for systems with interval time-varying delays and nonlinear perturbations. A delay-partitioning approach is used in this paper, the delay-interval is partitioned into multiple equidistant subintervals, a new Lyapunov-Krasovskii (L-K) functional contains some triple-integral terms, and augment terms are introduced on these intervals. Then, by using integral inequalities method together with free-weighting matrix approach, a new less conservative delay-dependent stability criterion is formulated in terms of linear matrix inequalities (LMIs), which can be easily solved by optimization algorithms. Numerical examples are given to show the effectiveness and the benefits of the proposed method.

1. Introduction

Time-delay phenomena are ubiquitous in many practical systems such as communication systems, nuclear reactors, aircraft stabilization, and process control systems, which are often major sources of instability and poor performance. Hence, stability analysis and stabilization of systems with time-delays have received considerable attention in the past few decades [1–14]. Recently, many researchers pay attention to the interval time-varying delay, wherein the delay varies in a range for which the lower bound is not restricted to be zero. A typical example with interval time delay is the network control systems [2].

The basic framework for stability analysis and synthesis of stabilizing controllers is L-K functional and linear matrix inequality (LMI). Under this framework, an important issue was to enlarge the feasible region of stability criteria. To derive the delay-dependent stability conditions, many methods have been reported in the literature. For example, the free-weighting matrix method was used in [3–6]; Jensen's integral inequality method was adopted in [7–13]. Recently, inspired by the discretized Lyapunov method, delay-partitioning approach was proposed in [14, 15], wherein the delay-interval was uniformly divided into multiple segments, choosing proper functional with different weighted

matrices corresponding to different segments. A new technique called delay-central point method was proposed in [16]. Based on the delay-central point method and decomposition technique, [17] proposed a less conservative stability criterion for computing the maximum allowable bound of the delay range. As an extension of delay-central point method, a new delay-partitioning approach was proposed in [18] for the uncertain stochastic systems with interval time-varying delay. Referring to the nonlinearities, as time delays, also can cause instability and poor performance of practical systems. Therefore, the stability problem of time-delay systems with nonlinear perturbations has received increasing attention [19–24]. A descriptor model transformation was employed in [19]. The free-weighting matrices approach was adopted in [20, 21]. Recently, a less conservative delay-dependent stability criterion was provided in [22] by partitioning the delay-interval into two segments of equal length and evaluating the time-derivative of a candidate L-K functional in each segment of the delay-interval. The main advantage of the method [22] is that more information on the variation interval of the delay is employed, but we can employ information of time-delay much more if we partition the delay-interval into more segments.

Inspired by the idea of [18, 22], in this paper, we divide the variation interval of the delay into N parts with equal

length and construct a new L-K functional with triple-integral terms and augment terms for this delay-interval. Based on integral inequalities method together with free-weighting matrix approach, a new delay-dependent stability criterion for the system is formulated in terms of linear matrix inequalities, which can be easily calculated by using MATLAB LMI control toolbox. Numerical examples are given to illustrate the effectiveness and less conservatism of the proposed method.

2. Problem Description and Preliminaries

Consider the following neutral system with mixed time-varying delays and nonlinear perturbations:

$$\begin{aligned}\dot{x}(t) &= \mathbf{A}x(t) + \mathbf{B}x(t-h(t)) \\ &\quad + f_1(x(t), t) + f_2(x(t-h(t)), t), \\ x(\theta) &= \phi(\theta), \quad \dot{x}(\theta) = \varphi(\theta), \\ \forall \theta &\in [-\max(h_M, \tau), 0],\end{aligned}\quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $\mathbf{A}$ and $\mathbf{B}$ are constant matrices with appropriate dimensions, and $h(t)$ is a time-varying delay satisfying

$$0 \leq h_m \leq h(t) \leq h_M, \quad \dot{h}(t) \leq \mu, \quad \forall t \geq 0, \quad (2)$$

where h_m and h_M represent the lower and upper bounds of the time-varying delay $h(t)$, respectively, μ is the bound on the delay-derivative, and initial condition $\varphi(t)$ is a continuous vector-valued function. The functions $f_1(x(t), t)$ and $f_2(x(t-h(t)), t)$ are unknown nonlinear perturbations with respect to the current state $x(t)$ and in the delay state $x(t-h(t))$, respectively. They satisfy $f_1(0, t) = 0$, $f_2(0, t) = 0$, and

$$\begin{aligned}f_1^T(x(t), t) f(x(t), t) &\leq \alpha^2 x^T(t) x(t), \\ f_2^T(x(t-h(t)), t) f_2(x(t-h(t)), t) \\ &\leq \beta^2 x^T(t-h(t)) x(t-h(t)),\end{aligned}\quad (3)$$

where $\alpha \geq 0$, $\beta \geq 0$ are given constants; for simplicity we denote $f_1 = f_1(x(t), t)$, $f_2 = f_2(x(t-h(t)), t)$.

Before moving on, we need the following instrumental lemma.

Lemma 1 (see [10]). *For any scalar $h(t) \geq 0$ and constant matrix $\mathbf{Q} \in \mathbb{R}^{n \times n}$, $\mathbf{Q} = \mathbf{Q}^T > 0$, the following inequality holds:*

$$\begin{aligned}-\int_{t-h(t)}^t \dot{x}^T(s) \mathbf{Q} \dot{x}(s) ds &\leq h(t) \zeta^T(t) \mathbf{V} \mathbf{Q}^{-1} \mathbf{V}^T \zeta(t) \\ &\quad + 2\zeta^T(t) \mathbf{V} [x(t) - x(t-h(t))],\end{aligned}\quad (4)$$

where

$$\begin{aligned}\zeta^T(t) &= \begin{bmatrix} x^T(t) & x^T(t-h_i) & x^T(t-h(t)) & x^T(t-h_{i+1}) \\ \dot{x}^T(t) & \int_{t-h_i}^t x^T(s) ds & \int_{t-h_{i+1}}^{t-h_i} x^T(s) ds & f_1^T & f_2^T \end{bmatrix}. \\ (5)\end{aligned}$$

$\mathbf{V}$ is a free-weighting matrix with appropriate dimensions.

Lemma 2 (see [9]). *For any constant matrix $\mathbf{W} \in \mathbb{R}^{n \times n}$, $\mathbf{W} = \mathbf{W}^T > 0$, a scalar function $h := h(t) > 0$ and a vector-valued function $\dot{x} : [-h, 0] \rightarrow \mathbb{R}^n$, such that the following integrations are well defined; then*

$$\begin{aligned}-h \int_{t-h}^t \dot{x}^T(s) \mathbf{W} \dot{x}(s) ds &\leq \zeta_1^T(t) \begin{bmatrix} -\mathbf{W} & \mathbf{W} \\ \mathbf{W} & -\mathbf{W} \end{bmatrix} \zeta_1(t), \\ -\frac{h^2}{2} \int_{-h}^0 \int_{t+\theta}^t \dot{x}^T(t) \mathbf{W} \dot{x}(t) ds &\leq \zeta_2^T(t) \begin{bmatrix} -\mathbf{W} & \mathbf{W} \\ \mathbf{W} & -\mathbf{W} \end{bmatrix} \zeta_2(t)\end{aligned}\quad (6)$$

with $\zeta_1^T(t) = [x^T(t) \ x^T(t-h)]$ and $\zeta_2^T(t) = [hx^T(t) \ \int_{t-h}^t x^T(s) ds]$.

Lemma 3 (see [25]). *Suppose $\gamma_1 \leq \gamma(t) \leq \gamma_2$, where $\gamma(\cdot) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$. Then, for any constant matrices Ξ_1 , Ξ_2 , and Ω with proper dimensions, the following matrix inequality*

$$\Omega + (\gamma(t) - \gamma_1) \Xi_1 + (\gamma_2 - \gamma(t)) \Xi_2 < 0 \quad (7)$$

holds, if and only if

$$\begin{aligned}\Omega + (\gamma_2 - \gamma_1) \Xi_1 &< 0, \\ \Omega + (\gamma_2 - \gamma_1) \Xi_2 &< 0.\end{aligned}\quad (8)$$

3. Main Result

In this section, we study the delay-dependent robust stability of system (1) based on the delay-partitioning approach.

Theorem 4. *For given positive scalars h_m , h_M , μ , α , and β system (1) with uncertainty (3) is asymptotically stable if there exist symmetric matrices*

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} & \mathbf{P}_{13} \\ * & \mathbf{P}_{22} & \mathbf{P}_{23} \\ * & * & \mathbf{P}_{33} \end{bmatrix} \quad (9)$$

and $\mathbf{Q}_1, \mathbf{Q}_2, \mathbf{Q}_3, \mathbf{Z}_2, \mathbf{Z}_3, \mathbf{R}_2, \mathbf{R}_3$, any matrices $\mathbf{K}_a, \mathbf{V}_a, \mathbf{W}_a$ ($a = 1, 2$), and scalars $\varepsilon_i \geq 0$ ($i = 1, 2$) such that the following LMIs hold:

$$\begin{bmatrix} \Phi & \sqrt{h_\delta} \mathbf{W} \\ * & -\mathbf{Z}_3 \end{bmatrix} < 0, \quad i = 1, 2, \dots, N, \quad (10)$$

$$\begin{bmatrix} \Phi & \sqrt{h_\delta} \mathbf{V} \\ * & -\mathbf{Z}_3 \end{bmatrix} < 0, \quad i = 1, 2, \dots, N, \quad (11)$$

where $\Phi = (\Phi_{i,j})_{9 \times 9}$ with

$$\begin{aligned}
 \Phi_{11} &= \mathbf{P}_{12} + \mathbf{P}_{12}^T + \mathbf{Q}_1 + \mathbf{Q}_2 + \mathbf{Q}_3 - \mathbf{Z}_2 - h_i^2 \mathbf{R}_2 \\
 &\quad - h_\delta^2 \mathbf{R}_3 + \mathbf{K}_1 \mathbf{A} + \mathbf{A}^T \mathbf{K}_1^T + \varepsilon_1 a^2, \\
 \Phi_{12} &= \mathbf{Z}_2 - \mathbf{P}_{12} + \mathbf{P}_{13}, \quad \Phi_{13} = \mathbf{K}_1 \mathbf{B}, \\
 \Phi_{14} &= -\mathbf{P}_{13}, \quad \Phi_{15} = \mathbf{P}_{11} - \mathbf{K}_1 + \mathbf{A}^T \mathbf{K}_2^T, \\
 \Phi_{16} &= \mathbf{P}_{22}^T + h_i \mathbf{R}_2, \quad \Phi_{17} = \mathbf{P}_{32}^T + h_\delta \mathbf{R}_3, \\
 \Phi_{18} &= \Phi_{19} = \mathbf{K}_1, \quad \Phi_{22} = -\mathbf{Q}_2 - \mathbf{Z}_2 + \mathbf{V}_1 + \mathbf{V}_1^T, \\
 \Phi_{23} &= -\mathbf{V}_1 + \mathbf{V}_2^T, \quad \Phi_{24} = \Phi_{25} = 0, \\
 \Phi_{26} &= -\mathbf{P}_{22}^T + \mathbf{P}_{23}^T, \quad \Phi_{27} = -\mathbf{P}_{32}^T + \mathbf{P}_{33}^T, \\
 \Phi_{28} &= \Phi_{29} = 0, \\
 \Phi_{33} &= -(1 - \mu) \mathbf{Q}_1 - \mathbf{V}_2 - \mathbf{V}_2^T + \mathbf{W}_1 + \mathbf{W}_1^T + \varepsilon_2 \beta^2, \\
 \Phi_{34} &= -\mathbf{W}_1 + \mathbf{W}_2^T, \quad \Phi_{35} = \mathbf{B}^T \mathbf{K}_2^T, \\
 \Phi_{36} &= \dots = \Phi_{39} = 0, \quad \Phi_{44} = -\mathbf{Q}_3 - \mathbf{W}_2 - \mathbf{W}_2^T, \\
 \Phi_{45} &= 0, \quad \Phi_{46} = -\mathbf{P}_{23}^T, \\
 \Phi_{47} &= -\mathbf{P}_{33}^T, \quad \Phi_{48} = \Phi_{49} = 0, \\
 \Phi_{55} &= h_i^2 \mathbf{Z}_2 + h_\delta \mathbf{Z}_3 + \frac{1}{4} h_i^4 \mathbf{R}_2 \\
 &\quad + \frac{1}{4} (h_{i+1}^2 - h_i^2)^2 \mathbf{R}_3 - \mathbf{K}_2 - \mathbf{K}_2^T, \\
 \Phi_{56} &= \mathbf{P}_{21}^T, \quad \Phi_{57} = \mathbf{P}_{31}^T, \\
 \Phi_{58} &= \Phi_{59} = \mathbf{K}_2, \quad \Phi_{66} = -\mathbf{R}_2, \\
 \Phi_{67} &= \dots = \Phi_{69} = 0, \\
 \Phi_{77} &= -\mathbf{R}_3, \quad \Phi_{78} = \Phi_{79} = 0, \\
 \Phi_{88} &= -\varepsilon_1 \mathbf{I}, \quad \Phi_{89} = 0, \quad \Phi_{99} = -\varepsilon_2 \mathbf{I}, \\
 h_\delta &= h_{i+1} - h_i = \frac{(h_M - h_m)}{N}, \\
 h_i &= h_1 + \frac{(i-1)(h_M - h_m)}{N}, \\
 \mathbf{K} &= [\mathbf{K}_1^T \ 0 \ 0 \ 0 \ \mathbf{K}_2^T \ 0 \ 0 \ 0 \ 0]^T, \\
 \mathbf{V} &= [0 \ \mathbf{V}_1^T \ \mathbf{V}_2^T \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T, \\
 \mathbf{W} &= [0 \ 0 \ \mathbf{W}_1^T \ \mathbf{W}_2^T \ 0 \ 0 \ 0 \ 0 \ 0]^T.
 \end{aligned} \tag{12}$$

Proof. First, we decompose the delay-interval $[h_m, h_M]$ into N equidistant subinterval, where N is a given integer; that is,

$$\begin{aligned}
 [h_m, h_M] \\
 = [h_1, h_2] \cup [h_2, h_3] \cup \dots \cup [h_i, h_{i+1}] \cup \dots \cup [h_N, h_{N+1}].
 \end{aligned} \tag{13}$$

Then, the L-K functional corresponding to the time-variation $h(t) \in [h_2, h_3]$ is chosen as

$$\begin{aligned}
 V_2(t) &= \xi^T(t) \mathbf{P} \xi(t) + \int_{t-h(t)}^t x^T(s) \mathbf{Q}_1 x(s) ds \\
 &\quad + \int_{t-h_2}^t x^T(s) \mathbf{Q}_2 x(s) ds + \int_{t-h_3}^t x^T(s) \mathbf{Q}_3 x(s) ds \\
 &\quad + h_2 \int_{-h_2}^0 \int_{t+\theta}^t \dot{x}^T(s) \mathbf{Z}_2 \dot{x}(s) ds d\theta \\
 &\quad + \int_{-h_3}^{-h_2} \int_{t+\theta}^t \dot{x}^T(s) \mathbf{Z}_3 \dot{x}(s) ds d\theta \\
 &\quad + \frac{h_2^2}{2} \int_{-h_2}^0 \int_{\theta}^0 \int_{t+\lambda}^t \dot{x}^T(s) \mathbf{R}_2 \dot{x}(s) ds d\lambda d\theta \\
 &\quad + \frac{(h_3^2 - h_2^2)}{2} \int_{-h_3}^{-h_2} \int_{\theta}^0 \int_{t+\lambda}^t \dot{x}^T(s) \mathbf{R}_3 \dot{x}(s) ds d\lambda d\theta
 \end{aligned} \tag{14}$$

with $\xi^T(t) = [x^T(t) \ \int_{t-h_2}^t x^T(s) ds \ \int_{t-h_3}^{t-h_2} x^T(s) ds]$.

The time-derivative of the L-K functional along the trajectory of (1) is given by

$$\begin{aligned}
 \dot{V}_2(t) &= 2\xi^T(t) \mathbf{P} \dot{\xi}(t) + x^T(t) (\mathbf{Q}_1 + \mathbf{Q}_2 + \mathbf{Q}_3) x(t) \\
 &\quad - (1 - \mu) x^T(t - h(t)) \mathbf{Q}_1 x(t - h(t)) \\
 &\quad - x^T(t - h_2) \mathbf{Q}_2 x(t - h_2) - x^T(t - h_3) \mathbf{Q}_3 x(t - h_3) \\
 &\quad + \dot{x}^T(t) [h_2^2 \mathbf{Z}_2 + (h_3 - h_2) \mathbf{Z}_3] \dot{x}(t) \\
 &\quad - h_2 \int_{t-h_2}^t \dot{x}^T(s) \mathbf{Z}_2 \dot{x}(s) ds - \int_{t-h_3}^{t-h_2} \dot{x}^T(s) \mathbf{Z}_3 \dot{x}(s) ds \\
 &\quad + \frac{1}{4} h_2^4 \dot{x}^T(t) \mathbf{R}_2 \dot{x}(t) + \frac{1}{4} (h_3^2 - h_2^2) \dot{x}^T(t) \mathbf{R}_3 \dot{x}(t) \\
 &\quad - \frac{h_2^2}{2} \int_{-h_2}^0 \int_{t+\theta}^t \dot{x}^T(s) \mathbf{R}_2 \dot{x}(s) ds d\theta \\
 &\quad - \frac{(h_3^2 - h_2^2)}{2} \int_{-h_3}^{-h_2} \int_{t+\theta}^t \dot{x}^T(s) \mathbf{R}_3 \dot{x}(s) ds d\theta.
 \end{aligned} \tag{15}$$

Note that

$$-\int_{t-h_3}^{t-h_2} \dot{x}^T(s) \mathbf{Z}_3 \dot{x}(s) ds = -\int_{t-h_3}^{t-h(t)} \dot{x}^T(s) \mathbf{Z}_3 \dot{x}(s) ds - \int_{t-h(t)}^{t-h_2} \dot{x}^T(s) \mathbf{Z}_3 \dot{x}(s) ds. \quad (16)$$

From Lemmas 1 and 2, we have

$$\begin{aligned} & -\int_{t-h(t)}^{t-h_2} \dot{x}^T(s) \mathbf{Z}_3 \dot{x}(s) ds \\ & \leq (h(t) - h_2) \zeta^T(t) \mathbf{V} \mathbf{Z}_3^{-1} \mathbf{V}^T \zeta(t) \\ & \quad + 2\zeta^T(t) \mathbf{V} [x(t - h_2) - x(t - h(t))], \\ & -\int_{t-h_3}^{t-h(t)} \dot{x}^T(s) \mathbf{Z}_3 \dot{x}(s) ds \\ & \leq (h_3 - h(t)) \zeta^T(t) \mathbf{W} \mathbf{Z}_3^{-1} \mathbf{W}^T \zeta(t) \\ & \quad + 2\zeta^T(t) \mathbf{W} [x(t - h(t)) - x(t - h_3)], \\ & -h_2 \int_{t-h_2}^t \dot{x}^T(s) \mathbf{Z}_2 \dot{x}(s) ds \\ & \leq \begin{bmatrix} x(t) \\ x(t - h_2) \end{bmatrix}^T \begin{bmatrix} -\mathbf{Z}_2 & \mathbf{Z}_2 \\ * & -\mathbf{Z}_2 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - h_2) \end{bmatrix}, \\ & -\frac{h_2^2}{2} \int_{-h_2}^0 \int_{t+\theta}^t \dot{x}^T(s) \mathbf{R}_2 \dot{x}(s) ds d\theta \\ & \leq \begin{bmatrix} h_2 x(t) \\ \int_{t-h_2}^t x(s) ds \end{bmatrix}^T \begin{bmatrix} -\mathbf{R}_2 & \mathbf{R}_2 \\ \mathbf{R}_2 & -\mathbf{R}_2 \end{bmatrix} \begin{bmatrix} h_2 x(t) \\ \int_{t-h_2}^t x(s) ds \end{bmatrix}, \\ & -\frac{(h_3^2 - h_2^2)}{2} \int_{-h_3}^{-h_2} \int_{t+\theta}^t \dot{x}^T(s) \mathbf{R}_3 \dot{x}(s) ds d\theta \\ & \leq \begin{bmatrix} (h_3 - h_2) x(t) \\ \int_{t-h_3}^{t-h_2} x(s) ds \end{bmatrix}^T \begin{bmatrix} -\mathbf{R}_3 & \mathbf{R}_3 \\ \mathbf{R}_3 & -\mathbf{R}_3 \end{bmatrix} \begin{bmatrix} (h_3 - h_2) x(t) \\ \int_{t-h_3}^{t-h_2} x(s) ds \end{bmatrix}. \end{aligned} \quad (18)$$

From (3), we can obtain, for any scalars $\varepsilon_1 > 0, \varepsilon_2 > 0$,

$$\begin{aligned} & \varepsilon_1 [\alpha^2 x^T(t) x(t) - f_1^T(x(t), t) f_1(x(t), t)] \geq 0, \\ & \varepsilon_2 [\beta^2 x^T(t - h(t)) x(t - h(t)) \\ & \quad - f_2^T(x(t - h(t)), t) f_2(x(t - h(t)), t)] \geq 0. \end{aligned} \quad (19)$$

From the system (1), we have the following equation:

$$\begin{aligned} & 2[x^T(t) \mathbf{K}_1 + \dot{x}^T(t) \mathbf{K}_2] \\ & \times [\mathbf{A}x(t) + \mathbf{B}x(t - h(t)) \\ & \quad + f_1(x(t), t) + f_2(x(t - h(t)), t) - \dot{x}(t)] = 0. \end{aligned} \quad (20)$$

By substituting (17)~(20) in (15) and defining an augmented state vector,

$$\begin{aligned} \zeta^T(t) = & \begin{bmatrix} x^T(t) & x^T(t - h_i) & x^T(t - h(t)) & x^T(t - h_{i+1}) \\ \dot{x}^T(t) & \int_{t-h_{i+1}}^t x^T(s) ds & \int_{t-h_{i+1}}^{t-h_i} x^T(s) ds & f_1^T & f_2^T \end{bmatrix}. \end{aligned} \quad (21)$$

Then the time-derivative $\dot{V}_2(x(t), t)$ can be expressed as follows:

$$\begin{aligned} \dot{V}_2(x(t), t) \leq & \zeta^T(t) (\Phi + (h_3 - h(t)) \mathbf{W} \mathbf{Z}_3^{-1} \mathbf{W}^T \\ & + (h(t) - h_2) \mathbf{V} \mathbf{Z}_3^{-1} \mathbf{V}^T) \zeta(t). \end{aligned} \quad (22)$$

One can see that if $h(t) \in [h_2, h_3]$,

$$\Phi + (h_3 - h(t)) \mathbf{W} \mathbf{Z}_3^{-1} \mathbf{W}^T + (h(t) - h_2) \mathbf{V} \mathbf{Z}_3^{-1} \mathbf{V}^T < 0. \quad (23)$$

Then $\dot{V}(t) < -\varepsilon \|x(t)\|^2$ for some scalar $\varepsilon > 0$, from which we conclude that system (1) is asymptotically stable according to L-K stability theory [1].

Applying Lemma 3 to (23) yields the following inequality:

$$\begin{aligned} & \Phi + (h_3 - h(t)) \mathbf{W} \mathbf{Z}_3^{-1} \mathbf{W}^T < 0, \\ & \Phi + (h(t) - h_2) \mathbf{V} \mathbf{Z}_3^{-1} \mathbf{V}^T < 0. \end{aligned} \quad (24)$$

By Schur complement on (24), we can obtain (10) and (11) with $i = 2$.

Without loss of generality, when $h(t) \in [h_i, h_{i+1}]$ ($i = 1, 3, \dots, N$), construct the following L-K functional:

$$\begin{aligned} V_i(t) = & \xi^T(t) \mathbf{P} \xi(t) + \int_{t-h(t)}^t x^T(s) \mathbf{Q}_1 x(s) ds \\ & + \int_{t-h_i}^t x^T(s) \mathbf{Q}_2 x(s) ds + \int_{t-h_{i+1}}^t x^T(s) \mathbf{Q}_3 x(s) ds \\ & + h_i \int_{-h_i}^0 \int_{t+\theta}^t \dot{x}^T(s) \mathbf{Z}_2 \dot{x}(s) ds d\theta \\ & + \int_{-h_{i+1}}^{-h_i} \int_{t+\theta}^t \dot{x}^T(s) \mathbf{Z}_3 \dot{x}(s) ds d\theta \\ & + \frac{h_i^2}{2} \int_{-h_i}^0 \int_{\theta}^0 \int_{t+\lambda}^t \dot{x}^T(s) \mathbf{R}_2 \dot{x}(s) ds d\lambda d\theta \\ & + \frac{(h_{i+1}^2 - h_i^2)}{2} \int_{-h_{i+1}}^{-h_i} \int_{\theta}^0 \int_{t+\lambda}^t \dot{x}^T(s) \mathbf{R}_3 \dot{x}(s) ds d\lambda d\theta \end{aligned} \quad (25)$$

with $\xi^T(t) = [x^T(t) \int_{t-h_i}^t x^T(s)ds \int_{t-h_{i+1}}^{t-h_i} x^T(s)ds]$, and $\mathbf{P}, \mathbf{Q}_1, \mathbf{Q}_2, \mathbf{Q}_3, \mathbf{Z}_2, \mathbf{Z}_3, \mathbf{R}_2, \mathbf{R}_3$ are the same matrix variables used in $V_2(t)$. Now, define an augmented state vector:

$$\zeta^T(t) = \begin{bmatrix} x^T(t) & x^T(t-h_i) & x^T(t-h(t)) & x^T(t-h_{i+1}) \\ \dot{x}^T(t) & \int_{t-h_{i+1}}^t x^T(s)ds & \int_{t-h_{i+1}}^{t-h_i} x^T(s)ds & f_1^T \ f_2^T \end{bmatrix}. \quad (26)$$

Using the same method, we have the conditions (10) and (11) as that for $h(t) \in [h_i, h_{i+1}]$ ($i = 1, 3, \dots, N$). The proof is completed. $\square$

Remark 5. As an extension of the method used in [18, 22], we divide the delay-interval into N subintervals, constructing a new L-K functional that contains some triple-integral terms and augment term for each delay-interval. This treatment makes us employ more information on the time delay and yields less conservative delay-range bounds.

Remark 6. If there is no perturbation, that is, $f = 0, g = 0$, then the stability problem of system (1) is reduced to analyze the stability of the system:

$$\begin{aligned} \dot{x}(t) &= \mathbf{A}x(t) + \mathbf{B}x(t-h(t)), \\ x(\theta) &= \phi(\theta), \quad \forall \theta \in [-h_M, 0]. \end{aligned} \quad (27)$$

According to Theorem 4, we can obtain the following corollary for the delay-dependent stability of system (27).

Corollary 7. For given positive scalars, h_m, h_M, μ , system (27) with uncertainty (10) is asymptotically stable if there exist real symmetric positive definite matrices

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} & \mathbf{P}_{13} \\ * & \mathbf{P}_{22} & \mathbf{P}_{23} \\ * & * & \mathbf{P}_{33} \end{bmatrix} \quad (28)$$

and $\mathbf{Q}_1, \mathbf{Q}_2, \mathbf{Q}_3, \mathbf{Z}_2, \mathbf{Z}_3, \mathbf{R}_2, \mathbf{R}_3$, any matrices $\mathbf{K}_a, \mathbf{V}_a, \mathbf{W}_a$ ($a = 1, 2$), and scalars $\varepsilon_i \geq 0$ ($i = 1, 2$) such that the following LMIs hold:

$$\begin{aligned} \begin{bmatrix} \bar{\Phi} & \sqrt{h_\delta} \bar{\mathbf{W}} \\ * & -\mathbf{Z}_3 \end{bmatrix} &< 0, \quad i = 1, 2, \dots, N, \\ \begin{bmatrix} \bar{\Phi} & \sqrt{h_\delta} \bar{\mathbf{V}} \\ * & -\mathbf{Z}_3 \end{bmatrix} &< 0, \quad i = 1, 2, \dots, N, \end{aligned} \quad (29)$$

where $\bar{\Phi} = (\Phi_{i,j})_{7 \times 7}$ with

$$\begin{aligned} \bar{\Phi}_{11} &= \mathbf{P}_{12} + \mathbf{P}_{12}^T + \mathbf{Q}_1 + \mathbf{Q}_2 + \mathbf{Q}_3 - \mathbf{Z}_2 \\ &\quad - h_i^2 \mathbf{R}_2 - h_\delta^2 \mathbf{R}_3 + \mathbf{K}_1 \mathbf{A} + \mathbf{A}^T \mathbf{K}_1^T, \\ \bar{\Phi}_{12} &= \mathbf{P}_{12}, \quad \bar{\Phi}_{13} = \mathbf{P}_{13}, \quad \bar{\Phi}_{14} = \mathbf{P}_{14}, \\ \bar{\Phi}_{15} &= \mathbf{P}_{15}, \quad \bar{\Phi}_{16} = \mathbf{P}_{16}, \quad \bar{\Phi}_{17} = \mathbf{P}_{17}, \\ \bar{\Phi}_{22} &= \mathbf{P}_{22}, \quad \bar{\Phi}_{23} = \mathbf{P}_{23}, \quad \bar{\Phi}_{24} = \bar{\Phi}_{25} = 0, \\ \bar{\Phi}_{26} &= \mathbf{P}_{26}, \quad \bar{\Phi}_{27} = \mathbf{P}_{27}, \end{aligned}$$

$$\bar{\Phi}_{33} = -(1-\mu) \mathbf{Q}_1 - \mathbf{V}_2 - \mathbf{V}_2^T + \mathbf{W}_1 + \mathbf{W}_1^T,$$

$$\bar{\Phi}_{34} = \Phi_{34}, \quad \bar{\Phi}_{35} = \Phi_{35}, \quad \bar{\Phi}_{36} = \bar{\Phi}_{37} = 0,$$

$$\bar{\Phi}_{44} = \Phi_{44}, \quad \bar{\Phi}_{45} = 0, \quad \bar{\Phi}_{46} = \Phi_{46},$$

$$\bar{\Phi}_{47} = \Phi_{47}, \quad \bar{\Phi}_{55} = \Phi_{55}, \quad \bar{\Phi}_{56} = \Phi_{56},$$

$$\bar{\Phi}_{57} = \Phi_{57}, \quad \bar{\Phi}_{66} = \Phi_{66}, \quad \Phi_{67} = 0,$$

$$\Phi_{77} = -\mathbf{R}_3,$$

$$h_\delta = h_{i+1} - h_i = \frac{(h_M - h_m)}{N},$$

$$h_i = h_1 + \frac{(i-1)(h_M - h_m)}{N},$$

$$\bar{\mathbf{K}} = [\mathbf{K}_1^T \ 0 \ 0 \ 0 \ \mathbf{K}_2^T \ 0 \ 0]^T,$$

$$\bar{\mathbf{V}} = [0 \ \mathbf{V}_1^T \ \mathbf{V}_2^T \ 0 \ 0 \ 0 \ 0]^T,$$

$$\bar{\mathbf{W}} = [0 \ 0 \ \mathbf{W}_1^T \ \mathbf{W}_2^T \ 0 \ 0 \ 0]^T.$$

(30)

4. Numerical Examples

Example 1. Consider the following neutral time-delay system with

$$\mathbf{A} = \begin{bmatrix} -1.2 & 0.1 \\ -0.1 & -1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -0.6 & 0.7 \\ -1 & -0.8 \end{bmatrix}. \quad (31)$$

For given values of α, β , and μ , we apply Theorem 4 to calculate the maximal allowable value h_M that guarantees that the asymptotical stability of the system is listed in Table 1. From Table 1, it is easy to see that our proposed stability criterion gives much less conservative results than those in [21, 22].

Example 2. Consider the system (27) with the following matrices:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}. \quad (32)$$

For given h_m and μ , we calculate the allowable upper bound of h_M that guarantees the asymptotical stability of system (27). Using different methods, computational results are obtained and these are listed in Table 2. From Table 2, it can be seen that our results are less conservative than the existing criteria.

5. Conclusion

This paper studies the problem of robust delay-dependent stability for a class of linear systems with interval time-varying delay and nonlinear perturbations. Based on the delay-partitioning approach, the less conservative delay-dependent stability conditions are derived. The reduction in

TABLE 1: Admissible upper bounds h_2 for various μ and $h_m = 0, 0.5, 1$.

h_m	α, β	$\alpha = 0, \beta = 0.1$			$\alpha = 0.1, \beta = 0.1$		
	μ	0.5	0.9	1.1	0.5	0.9	1.1
0	[21]	1.442	1.280	1.280	1.284	1.209	1.209
	[22]	1.443	1.408	1.408	1.287	1.279	1.279
	Theorem 4 ($N = 2$)	1.6299	1.4938	1.4938	1.4743	1.3943	1.3943
	Theorem 4 ($N = 3$)	1.6866	1.6866	1.6866	1.5304	1.5304	1.5304
0.5	[21]	1.442	1.338	1.338	1.284	1.245	1.245
	[22]	1.558	1.558	1.558	1.384	1.384	1.384
	Theorem 4 ($N = 2$)	1.6618	1.6618	1.6618	1.5272	1.5272	1.5272
	Theorem 4 ($N = 3$)	1.8599	1.8599	1.8599	1.6622	1.6622	1.6622
1	[21]	1.543	1.543	1.543	1.408	1.408	1.408
	[22]	1.760	1.760	1.760	1.532	1.532	1.532
	Theorem 4 ($N = 2$)	1.8824	1.8824	1.8824	1.7059	1.7059	1.7059
	Theorem 4 ($N = 3$)	2.0650	2.0650	2.0650	1.8188	1.8188	1.8188

TABLE 2: Maximum bounds h_M for a given h_m .

μ	Method	$h_m = 1$	$h_m = 2$	$h_m = 3$	$h_m = 4$	$h_m = 5$
0.3	[4]	2.2125	2.4091	3.3342	4.2799	5.2393
	[8]	2.2474	2.4798	3.3896	4.3250	5.2773
	[18] ($N = 2$)	2.3564	3.0484	3.8779	4.7481	5.6475
	Corollary 7 ($N = 2$)	2.5378	3.0804	3.9171	4.7933	5.6967
	[18] ($N = 3$)	2.7077	3.4408	4.2307	5.0463	5.9319
	Corollary 7 ($N = 3$)	2.7469	3.4900	4.2899	5.1313	6.0030

the conservatism of the proposed stability criteria is mainly attributed to the new L-K functional which contains some triple-integral terms and augment terms for each divided segment. Numerical examples have illustrated the effectiveness of the proposed method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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