Research Article

Mobile Robots Path Planning Using the Overall Conflict Resolution and Time Baseline Coordination

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This paper aims at resolving the path planning problem in a time-varying environment based on the idea of overall conflict resolution and the algorithm of time baseline coordination. The basic task of the introduced path planning algorithms is to fulfill the automatic generation of the shortest paths from the defined start poses to their end poses with consideration of generous constraints for multiple mobile robots. Building on this, by using the overall conflict resolution, within the polynomial based paths, we take into account all the constraints including smoothness, motion boundary, kinematics constraints, obstacle avoidance, and safety constraints among robots together. And time baseline coordination algorithm is proposed to process the above formulated problem. The foremost strong point is that much time can be saved with our approach. Numerical simulations verify the effectiveness of our approach.

1. Introduction

Since the 1990s, industrial robots have played an important role in the robotics research [1]. One foremost research topic of industrial robots is mobile robots' path planning [2, 3], wherein the paths connecting the start and end poses of robots are planned to assist completing their task. In industrial application, to suit the actual situation, there is no denial that the nonholonomic constraint [4, 5] which means the mobile robots subjecting to the requirements of rolling without slipping should be taken into consideration. Further, in the traverse process of mobile robots, owing to the mutual influence of robots and obstacles moving and stationary in a time-varying environment, the conflicts would be induced inevitably. Thus, our concerned path planning problems become much more difficult since the planned paths must take above geometric and nonholonomic constraints and conflict-avoidance constraints into account. Then, how to plan the shortest paths for multiple mobile robots while explicitly considering smoothness, kinematics constraints,

motion boundary, obstacle avoidance, and safety constraints among mobile robots becomes our mainly focused problem.

During the past decades, various path planning methodologies for mobile robots have been developed [2, 6–12]. We have presented the detailed introduction in [2] with a sense of hierarchy. In [6, 13], differential evolution and honey bee mating optimization algorithms are used to resolve the cooperative multirobot path planning problem, respectively. However, too many broken lines exist in the planned paths, which cannot be directly used in the real scenario. With those ill-smooth paths as depicted in [14], the roller slip would be triggered in the motion process of mobile robots. Then, the slip would endanger the safety of mobile robots and furthermore lead to the meaninglessness of those paths.

As pointed in [15] the planned path should be smooth to accord with the practical applications. With the smooth paths, we can account for the continuity [16] and the kinematics constraints sufficiently. Contrarily, if with the ill-smooth paths, the jerks would be triggered during the motion process. Given that polynomials are the set of highly smooth

functions, there is no doubt that polynomial based paths are quite smooth [15]. Hence, we adopt the polynomial based paths to represent their motion in our research.

Generally in many works like [2, 9, 10], they focus on two-phase approach for path planning problems. Firstly, the path for each mobile robot is reached with some certain algorithm. Following that, by using the predefined principles, the conflicts among the planned paths are resolved in the second phase. However, approaches of this category cannot be illustrated analytically and do not work sometimes. Consequently, the sound approach is disposing of all the conflicts as a whole not using the two-phase approach.

To avoid the two-phase methods, the studies like [2, 7–11] adopted quartic Bezier curves to represent the paths of mobile robots. However, approaches in [7, 8] strictly restrict the paths to the fourth-order Bezier curves and require the initial and final velocities of each mobile robot, which would be impractical for the realistic application. Comparatively, as a natural continuation of our previous work in [2], to avoid the deadlock of the two-phase conflict dissolution approach, in our objective function, we eliminate the overall conflicts of the focused problem. When it comes to the objective function, we set the summation of the shortest paths of all mobile robots as the goal function and take generous constraints into account. Eventually, we formulate the problem as one time-varying nonlinear programming problem (TNLPP). To solve the TNLPP, we devise the time baseline coordination method. Within this method, we choose the least motion time of all mobile robots as the time baseline; by using this parameter we can fulfill their motion coordination. The detailed illustration is presented in Section 4.

This paper is organized into seven sections. In Section 2, the shortest polynomial based paths for multiple mobile robots are defined. In Section 3, modeling for path planning problems with overall confliction is introduced explicitly. In Section 4, time baseline coordination algorithm is detailed to resolve the path planning problems. In Section 5, simulation results are presented to demonstrate the effectiveness of our approaches. Section 6 discusses results of cases in Section 5. Section 7 concludes this paper.

2. Problem Formulation

Suppose that one mobile robot A_i ($i = 1, 2, \dots, J$) is represented by a r_i -radius circle with center $x_i(t) = (x_{1i}(t), x_{2i}(t))$ and orientation $\theta_i(t)$, $t \in [0, t_{fi}]$, where m is the number of mobile robots, x_i is one parameterized polynomial with unknown index and used to denote the path of A_i , and t_{fi} is a given real number as final moving time of A_i . The poses of mobile robots at time t are illustrated as $P_i(t) = (x_i(t), \theta_i(t))$. Then, we should plan the shortest and conflict-free paths for all mobile robots A_i ($i = 1, 2, \dots, J$) from the start pose P_{i0} to the end pose P_{if} , where $P_{i0} = P_i(0)$ and $P_{if} = P_i(t_f)$.

As sensors or cameras are used to detect the positions of obstacles [17], the paths of obstacles can be duly reached. Meanwhile, suppose the obstacle O_k ($k = 1, 2, \dots, q$) is a r_k -radius circle with center $\alpha_k(t) = (\alpha_{1k}(t), \alpha_{2k}(t))$, $t \in [0, t_k]$, where q is the number of the obstacles, $\alpha_k(\cdot)$ is the path of O_k

and expressed with known parameterized polynomial, and t_k is a given real number as final motion time of O_k .

2.1. Polynomial Based Paths. To detail x_i which is the path of A_i and equals $(x_{1i}(t), x_{2i}(t))$, we set $x_{1i}(t) = x_{1i}$, $x_{2i}(t) = x_{2i}$, and the unknown indices of x_{1i} and x_{2i} are M and N , respectively. The unknown coefficients of x_{1i} and x_{2i} are represented by $[a_{0i} \ a_{1i} \ \dots \ a_{mi} \ \dots \ a_{Mi}]$ and $[b_{0i} \ b_{1i} \ \dots \ b_{ni} \ \dots \ b_{Ni}]$ ($1 \leq m \leq M, 1 \leq n \leq N$), respectively. Then, the path of A_i is given in the following form, where $t \in [0, t_{fi}]$:

$$\begin{aligned} x_{1i} &= a_{0i} + a_{1i}t + \dots + a_{mi}t^m + \dots + a_{Mi}t^M, \\ x_{2i} &= b_{0i} + b_{1i}t + \dots + b_{ni}t^n + \dots + b_{Ni}t^N. \end{aligned} \quad (1)$$

Suppose that $A^i = [a_{0i} \ a_{1i} \ \dots \ a_{mi} \ \dots \ a_{Mi}]$, $B^i = [b_{0i} \ b_{1i} \ \dots \ b_{ni} \ \dots \ b_{Ni}]$, $T_A^i = [t^0 \ t \ \dots \ t^m \ \dots \ t^M]$, and $T_B^i = [t^0 \ t \ \dots \ t^n \ \dots \ t^N]$, $t \in [0, t_{fi}]$. Then, the path x_i can be

$$\begin{bmatrix} x_{1i} \\ x_{2i} \end{bmatrix} = \begin{bmatrix} A^i & 0 \\ 0 & B^i \end{bmatrix} \begin{bmatrix} T_A^i \\ T_B^i \end{bmatrix}. \quad (2)$$

Within path x_i , the velocities in the direction of x_{1i} and x_{2i} are $\dot{x}_{1i}(t)$ and $\dot{x}_{2i}(t)$, respectively. The accelerations in the direction of x_{1i} and x_{2i} are $\ddot{x}_{1i}(t)$ and $\ddot{x}_{2i}(t)$, respectively. Consequently, the path length of A_i can be

$$s_i = \int_0^{t_{fi}} \sqrt{\dot{x}_{1i}^2(t) + \dot{x}_{2i}^2(t)} dt = \int_0^{t_{fi}} \|\dot{x}_i(t)\|_2 dt. \quad (3)$$

Analogously, for the path of O_k featured by α_k , we set $\alpha_{1k}(t) = \alpha_{1k}$, $\alpha_{2k}(t) = \alpha_{2k}$, and the known indices of α_{1k} and α_{2k} are L and U , respectively. The known coefficients of α_{1k} and α_{2k} are represented by $[a_{0Ok} \ a_{1Ok} \ \dots \ a_{lOk} \ \dots \ a_{LOk}]$ and $[b_{0Ok} \ b_{1Ok} \ \dots \ b_{uOk} \ \dots \ a_{UOk}]$ ($1 \leq l \leq L, 1 \leq u \leq U$), respectively. Then the path of O_k is given in the following form, where $t \in [0, t_k]$:

$$\begin{aligned} \alpha_{1k} &= a_{0Ok} + a_{1Ok}t + \dots + a_{lOk}t^l + \dots + a_{LOk}t^L, \\ \alpha_{2k} &= b_{0Ok} + b_{1Ok}t + \dots + b_{uOk}t^u + \dots + b_{UOk}t^U. \end{aligned} \quad (4)$$

Suppose that $A_O^k = [a_{0Ok} \ a_{1Ok} \ \dots \ a_{lOk} \ \dots \ a_{LOk}]$, $B_O^k = [b_{0Ok} \ b_{1Ok} \ \dots \ b_{uOk} \ \dots \ a_{UOk}]$, $T_A^k = [t^0 \ t \ \dots \ t^l \ \dots \ t^L]$, and $T_B^k = [t^0 \ t \ \dots \ t^u \ \dots \ t^U]$, $t \in [0, t_k]$. Then, the path α_k can be

$$\begin{bmatrix} \alpha_{1k} \\ \alpha_{2k} \end{bmatrix} = \begin{bmatrix} A_O^k & 0 \\ 0 & B_O^k \end{bmatrix} \begin{bmatrix} T_A^k \\ T_B^k \end{bmatrix}. \quad (5)$$

2.2. Generous Constraints. To plan the effective and reasonable paths for multiple mobile robots, when achieving our objective function, generous constraints including kinematics, motion boundaries, and obstacle avoidance should be satisfied. For this end, to probe into the essence of each constraint, we detail each constraint sequentially.

2.2.1. Kinematics Constraints. Kinematics constraints indicate that the velocity and acceleration along with the path should be bound to their maximum allowed intervals to prevent robots from slipping. Then, the kinematics constraints can be

$$v_i(t) \leq v_{\max i}, \quad a_i(t) \leq a_{\max i}, \quad t \in [0, t_{fi}], \quad i \in [1, J]. \quad (6)$$

In the kinematics constraints, $v_i(t)$ and $a_i(t)$ are denoted by $\dot{x}_i(t)$ and $\ddot{x}_i(t)$, respectively.

2.2.2. Motion Boundary. X is used to represent the two-dimensional environment boundary. The constraints imposed by the motion boundary should be satisfied as below:

$$\begin{aligned} P_i(0) &= (x_{0i}, \theta_{0i}), \\ P_i(t_{fi}) &= (x_{fi}, \theta_{fi}), \\ X_{0i}, X_{fi} &\in X, \\ i &\in [1, J]. \end{aligned} \quad (7)$$

2.2.3. Obstacle Avoidance Constraints. Obstacle avoidance constraints guarantee the least safe distance between the current moving robot and all the obstacles. Suppose that d_{oik} and $d_{\text{obs},ik}$ are the real distance and the preset safe distance between A_i and O_k , respectively. d_{oik} is represented as below. For $1 \leq i \leq J$, $1 \leq k \leq q$,

$$d_{oik}(t) = \|x_i(t) - \alpha_k(t)\|_2 - (r_i + r_k). \quad (8)$$

Then, the obstacle avoidance constraints for A_i and O_k ($1 \leq i \leq J$, $1 \leq k \leq q$) in X should be

$$d_{\text{obs},ik} - d_{oik}(t) \leq 0, \quad t \in [0, \max(t_{fi}, t_k)]. \quad (9)$$

2.2.4. Safety Constraints among Mobile Robots. Within the safety constraints among mobile robots, all the robots can be kept away from each other with distances larger than the predefined safety value. Thus, the collisions between mobile robots can be averted triumphantly with ease.

Suppose d_{ij} and $d_{s,ij}$ are the real distance and the preset safe distance between A_i and A_j , respectively. For $1 \leq i \leq J$, $i \neq j$, d_{ij} can be

$$d_{ij}(t) = \|x_i(t) - x_j(t)\|_2 - (r_i + r_j). \quad (10)$$

Then, the safety constraints among A_i in X should be satisfied as below ($1 \leq i \leq J$):

$$d_{s,ij} - d_{ij}(t) \leq 0, \quad t \in [0, \max(t_{fi}, t_{fj})]. \quad (11)$$

2.3. Goal Function. Our goal function is to minimize the summation of all the paths length, which can be

$$\min \sum_{i=1}^J s_i. \quad (12)$$

Similar to our work in [2], we equally divide the time interval $[0, t_{fi}]$ into F parts, where h denotes the value of each

part, $h = t_{fi}/F$, $0 \leq f \leq F$; set $E_i(t) = \|\dot{x}_i(t)\|_2$. Consequently, after the transformation, the goal function can be

$$\min \sum_{i=1}^J \frac{h \left(2 \sum_{f=1}^F E_i(fh) - E(0) - E_i(t_{fi}) \right)}{2}. \quad (13)$$

3. Problem Modeling with the Overall Conflict Resolution

3.1. The Meaning of the Overall Conflict Resolution. Generally in the path planning problems, the conflicts including the collisions among mobile robots owing to the crossover of their planned paths and the crashes between mobile robots and obstacles occurred within the motion boundary. To fulfill the conflict-free path planning, the strategies or methods adopted to remove and avoid the collisions can be named conflict resolution strategies. Therein, we propose the overall conflict resolution.

In our overall conflict resolution, we consider all the constraints demonstrated above in the process of modeling. By using the overall conflict resolution, the phase of how to bring forward some ideal conflict resolution strategies in the multiphase approaches is deleted. However, compared with the path planning model of the multiphase methods, the founded model based on overall conflict resolution is much more difficult and harder to actualize.

3.2. Problem Modeling. As none of the above constraints can be violated, with overall conflict resolution, for $1 \leq i, j \leq J$, $i \neq j$, $1 \leq k \leq q$, we consider all the constraints as a whole in our objective function as below:

$$\min \sum_{i=1}^J \frac{h \left(2 \sum_{f=1}^F E_i(fh) - E(0) - E_i(t_{fi}) \right)}{2}$$

$$\text{s.t.} \quad v_i(t) \leq v_{\max i} \quad t \in [0, t_{fi}]$$

$$a_i(t) \leq a_{\max i} \quad i \in [1, J]$$

$$P_i(0) = (x_{0i}, \theta_{0i})$$

$$P_i(t_{fi}) = (x_{fi}, \theta_{fi})$$

$$X_{0i}, X_{fi} \in X$$

$$i \in [1, J]$$

$$d_{s,ij} - d_{ij}(t) \leq 0, \quad t \in [0, \max(t_{fi}, t_{fj})]$$

$$d_{\text{obs},ik} - d_{oik}(t) \leq 0, \quad t \in [0, \max(t_{fi}, t_k)].$$

(14)

Based on the overall conflict resolution strategy, our focused path planning problem is transformed to one objective function while subjects to all above constraints. Substantially, the concerned problem is one time-varying nonlinear programming problem with multiple nonlinear constraints. In (14), conflicts among all the mobile robots and obstacles

Input: Start and end poses P_{i0} and P_{if} , respectively. Unknown path $x_i(t)$, a known path $\alpha_k(t)$, motion time t_{fi} and t_k , and number of divided time interval F , temporary time variable $T_{\min}$.

Output: Parameters A^i , B^i , T_A^i and T_B^i which are used to represent the path of A_i .

Begin

$T_{\min} = 0;$

foreach t_{fi} of A_i **do**

$T_{\min} = \min t_{fi};$

end foreach

foreach A_i in the set of multiple mobile robots

Map motion time interval $[0, t_{fi}]$ of A_i into $[0, t_{fi}/T_{\min}]$ with the multiples of $1/T_{\min}$.

Map constraints of kinematics and obstacle-avoidance of A_i into corresponding forms with respect to $[0, t_{fi}/T_{\min}]$.

end foreach

Modify the objective function with all the adjustments, then reach the newly TNLPPs named TNLPPs_{new}.

Call $\text{Trap}_{\text{TNLPPs}}(\text{TNLPPs}_{\text{new}})$

foreach A_i **do**

Multiples $A^{i'}$, $B^{i'}$, $T_A^{i'}$ and $T_B^{i'}$ with $T_{\min}$.

end foreach

return A^i , B^i , T_A^i and T_B^i .

End

ALGORITHM 1: $\text{Time}_{\text{Coor}}(P_{i0}, P_{if}, x_i(t), t_{fi}, \alpha_k(t), t_k, F, T_{\min})$.

moving and static are considered explicitly, which evades the general multilevel conflicts. Thus, compared with the multiphase approaches, since path planning for all mobile robots just needs to be calculated only once, it is evident that much time can be saved by using our approach. Logically, with this model, the path planning efficiency is improved evidently. However, all above advantages are gained at the cost of the upgrading of the resolve difficulty of our model. To entirely dissolve those conflicts, in the following section we propose the time baseline coordination approach.

4. Optimal Solution to Path Planning Using the Time Baseline Coordination

In the path planning problems, suppose that all the mobile robots commence moving at the same time. In this section, we illustrate the time baseline coordination at first, following that analyze the equivalence of this approach, and finally put forward the implementation of our subtle approach.

4.1. Brief Introduction of the Time Baseline Coordination. As depicted in Section 2, the final motion time of A_i and O_k is t_{fi} and t_k , respectively. Suppose that the baseline time is $T_{\min}$. The essence of our time baseline coordination is selecting the least motion time of A_i denoted as $T_{\min}$ to coordinate the motion of all the mobile robots.

In the time baseline coordination, we demonstrate the general operating procedures concisely.

Firstly, with $T_{\min}$, we can transform all the velocities, the accelerations, and so forth into $[0, t_{fi}/T_{\min}]$.

Following that, we renew the corresponding constraints in our goal function.

Thirdly, using the algorithm of our former work in [2], we solve then the transformed problems.

And then, with multiple $1/T_{\min}$, we map the resolution into $[0, t_{fi}]$ corresponding to the first step.

Finally, we can reach the final solution eventually.

4.2. Time Baseline Coordination Algorithm $\text{Time}_{\text{Coor}}$. To illustrate the proposed approach definitely, we devise the following time baseline coordination algorithm called $\text{Time}_{\text{Coor}}$ (see Algorithm 1).

The algorithm $\text{Trap}_{\text{TNLPPs}}$ is our previous work in [2], which is proposed to solve the time-varying nonlinear programming problem. Brief flow of algorithm $\text{Trap}_{\text{TNLPPs}}$ is presented in the Appendix, and a comprehensive procession can be found in [2].

4.3. Equivalence of the Time Baseline Coordination Algorithm. To proof the equivalence of our approach, several equivalent descriptions (which are abbreviated as Equ) are presented as below.

Equ-1. The motion boundary has not varied after mapping.

Proof. X is the given motion boundary, which has little relation with the time. $\square$

Equ-2. The ration of the transformed velocities to the former is $T_{\min}$.

Proof. As shown in Equ-1, there is no change in the trajectory of A_i , while the motion time is mapped into $[0, t_{fi}/T_{\min}]$. Thus, with theorem of integral, the division between the current velocities and the former is $T_{\min}$. $\square$

Equ-3. The ration of the transformed accelerations to the former is $T_{\min}^2$.

Proof. Based on Equ-1 and Equ-2, according to the relationship between velocities and accelerations, with theorem of integral, the division between the current accelerations and the former is $(T_{\min})^2$. $\square$

Equ-4. The ration of the velocities in the final solution to the velocities in the mapped solution is $1/T_{\min}$.

Proof. Based on Equ-1 and Equ-2, there is no change in the trajectory of A_i , while the motion time is transformed from $[0, t_{fi}/T_{\min}]$ to $[0, t_{fi}]$. With theorem of integral, the division between the velocities in the final solution and the velocities in the mapped solution is $1/T_{\min}$. $\square$

Equ-5. The ration of the accelerations in the final solution to the accelerations in the mapped solution is $1/(T_{\min})^2$.

Proof. Based on Equ-1 and Equ-2, according to the relationship between velocities and accelerations, with theorem of integral, the division between the accelerations in the final solution and the accelerations in the mapped solution is $(T_{\min})^2$. $\square$

With the overall conflict resolution, when all constraints are considered, paths for all mobile robots are promptly achieved based on algorithm $\text{Time}_{\text{Coor}}$.

5. Simulation

Simulations are carried out to show the effectiveness of our approach. Four curves have been used increasingly as metrics to evaluate the performance of the proposed approach [2, 7, 8, 18]. They are the curve of velocities relative to time, the curve of acceleration relative to time, the curve of the safety distances relative to time, and the curve of planned paths relative to their motion time. Experiments are conducted under several different time-varying scenarios.

In the simulation, we first consider the case without obstacles and then the latter with static and moving obstacles. In the following figures of Section 5, the scales are same and all quantities conform to one certain unit system, for instance, meters and meters per second.

5.1. Path Planning for Multiple Mobile Robots without Obstacles. When it comes to the case of multiple mobile robots moving synchronously in the environment without obstacles, for $1 \leq i, j \leq J, i \neq j$, the essential objective function can be

$$\begin{aligned} \min \quad & \sum_{i=1}^J \frac{h(2 \sum_{f=1}^F E_i(fh) - E(0) - E_i(t_{fi}))}{2} \\ \text{s.t.} \quad & v_i(t) \leq v_{\max i} \quad t \in [0, t_{fi}] \\ & a_i(t) \leq a_{\max i} \quad t \in [0, t_{fi}] \\ & P_i(0) = (x_{0i}, \theta_{0i}) \\ & P_i(t_{fi}) = (x_{fi}, \theta_{fi}) \end{aligned}$$

TABLE 1: Initial information of mobile robots A_i ($i = 1, 2, 3$) in Section 5.1.1.

A_i	P_{i0}	P_{if}	v_0	v_f	$v_{\max}$
A_1	$[0.2, 1.4, -\pi/4]^T$	$[1.4, 0.2, -\pi/4]^T$	0.4	0.4	0.8
A_2	$[1.4, 0.2, 3\pi/4]^T$	$[0.2, 1.4, 3\pi/4]^T$	0.4	0.5	0.8
A_3	$[0.2, 0.2, \pi/4]^T$	$[1.4, 1.4, \pi/4]^T$	0.4	0.4	0.8

$$X_{0i}, X_{fi} \in X$$

$$i \in [1, J]$$

$$d_{s:ij} - d_{ij}(t) \leq 0, \quad t \in [0, \max(t_{fi}, t_{fj})].$$

(15)

5.1.1. Path Planning for Three Mobile Robots without Obstacles.

In the first case, to test the validity of our approach, we choose the data listed in Table 1 from recently published work [8] to examine the effectiveness of our algorithm. Approach in [8] was based on fourth-order Bezier curve.

In [8], the safety distances, the maximum motion time, and the maximum accelerations are given as $d_{s:ij} = 0.35$, $T_{\max i} = 5$, and $a_{\max i} = 0.5$ ($i, j = 1, 2, 3, i \neq j$), respectively.

Within our approach, in order to fulfill the coordination, the minimized motion time of all the mobile robots A_i ($i = 1, 2, 3$) is chosen as the time baseline which is used for coordination. Subsequently, applying the coordination algorithm $\text{Time}_{\text{Coor}}$, for $t \in [0, 5]$, paths of A_i ($i = 1, 2, 3$) are obtained as below:

$$\begin{aligned} A^1 &= [0.2 \quad 0.2828 \quad -0.166 \quad 0.097 \quad -0.021 \quad 0.001]; \\ B^1 &= [1.4 \quad -0.2828 \quad 0.198 \quad -0.073 \quad 0.007]; \\ A^2 &= [1.4 \quad -0.2828 \quad 0.518 \quad -0.052 \quad 0.017 \quad -0.002]; \\ B^2 &= [0.2 \quad 0.283 \quad -0.127 \quad 0.041 \quad -0.004]; \\ A^3 &= [0.2 \quad 0.2828 \quad 0.216 \quad -0.116 \quad 0.019 \quad -0.001]; \\ B^3 &= [1.4 \quad 0.2828 \quad 0.145 \quad -0.065 \quad 0.007]; \\ T_A^i &= [1 \quad t_i \quad t_i^2 \quad t_i^3 \quad t_i^4 \quad t_i^5]; \\ T_B^i &= [1 \quad t_i \quad t_i^2 \quad t_i^3 \quad t_i^4 \quad t_i^5]. \end{aligned}$$

Corresponding to Figures 1(a), 1(b), and 1(c), our planned paths evolve from the perspective of three different time intervals $[0, 2.3]$, $[2.3, 5]$, and $[0, 5]$. In Figures 1(a), 1(b), and 1(c), it is shown that each mobile robot smoothly moves from its initial pose to final pose without any collision. The paths of [8] in the time interval $[0, 5]$ are shown in Figure 1(d). Thus, our approach based on $\text{Time}_{\text{Coor}}$ can manipulate the multiple mobile robots path planning problems successfully.

The variation curves of velocities and accelerations along the time are shown in Figures 2(a) and 2(b), respectively. Curves in Figure 2 are sufficient to the constraints in Table 1 and all other requirements in [8] fully. Then the planned paths meet the constraints of kinematics.

Figure 3 illustrates that the variation of safety distances d_{12} , d_{13} , and d_{23} is satisfied with the predefined requirements in this case.

FIGURE 1: Planned paths evolution of A_i and paths of [8], $i = 1, 2, 3$. (a) In $[0, 2.3]$, (b) in $[2.3, 5]$, (c) in $[0, 5]$, and (d) in $[0, 5]$.

In our approach, the polynomials are general that are more flexible than the polynomials of [8] which are limited to four. Meanwhile, with approach in [8], prior to path planning, the initial and final velocities of each mobile robot should be specified in detail. Generally, in practice, the velocities belong to some certain intervals, not the constant predefined value. Contrastively, our approach is much more suitable for the general multiple mobile robots path planning problems.

Compared to our prior work in [2], the paths of A_i ($i = 1, 2, 3$) should be calculated for three times in the path planning phase. In our approach, it only takes a third of the time in the first phase of [2] to reach the conflict-free solution. Furthermore, the planned paths of A_1 and A_2 in

[2] are almost overlapped, and the conflicts would exist in the planned paths of A_i ($i = 1, 2, 3$); the phase of conflict coordination should be applied which can be ineluctable. Synthetically, from the perspective of calculational time, the proposed approach excels the approach of [2] remarkably.

5.1.2. Path Planning for Four Mobile Robots without Obstacles.

To validate the effectiveness and feasibility of our algorithm under much more complex time-varying environment, the simulation scenario is presented with four mobile robots moving synchronously bound to the limited area. For this case, the essential objective function can be (15), wherein the

FIGURE 2: (a) v - t , (b) a - t in Section 5.1.1. FIGURE 3: Variation curve of safety distance d - t in Section 5.1.1.

maximum number of the robots equals 4 and the test data is listed in Table 2.

The preset safety distances are $d_{s,ij} = 0.25$ ($i = 1, j = 2, 3$), $d_{s,ij} = 0.25$ ($i = 2, j = 3, 4$), $d_{s,14} = 0.035$, and $d_{s,34} = 0.25$.

In the time baseline coordination method, we select the minimized motion time of A_i ($i = 1, 2, 3, 4$) as the coordinate factor in the function (15). Then, after applying

TABLE 2: Initial information of mobile robots A_i ($i = 1, 2, 3, 4$) in Section 5.1.2.

A_i	P_{i0}	P_{if}	$T_{\max}$	$v_{\max}$	$a_{\max}$
A_1	$[0.8, 0.8, \pi/4]^T$	$[0.2, 0.2, -3\pi/4]^T$	5	0.8	0.5
A_2	$[1.4, 0.2, 3\pi/4]^T$	$[0.2, 1.4, \pi/2]^T$	5	0.8	0.5
A_3	$[0.2, 1.4, -\pi/4]^T$	$[1.4, 0.2, 0]^T$	5	0.8	0.5
A_4	$[0.2, 0.2, \pi/4]^T$	$[1.4, 1.4, \pi/4]^T$	5	0.8	0.5

the coordination algorithm $\text{Time}_{\text{Coor}}$, the paths of A_i ($i = 1, 2, 3, 4$) are reached as below:

$$x_{11}(t) = 0.8 - 0.2819t + 0.0407t^2 - 0.00167t^3,$$

$$x_{21}(t) = 0.8 - 0.2689t + 0.03t^2 + 0.00141t^3 \\ - 0.000257t^4,$$

$$t \in [0, 5],$$

$$x_{12}(t) = 1.4 - 0.291t - 0.00735t^2 + 0.0035t^3,$$

$$x_{22}(t) = 0.2 + 0.472t - 0.04731t^2 - 0.00136t^3 \\ + 0.000311t^4,$$

$$t \in [0, 5],$$

$$x_{13}(t) = 0.2 + 0.149t + 0.0044t^2 + 0.00276t^3,$$

FIGURE 4: Planned paths evolution of A_i ($i = 1, 2, 3, 4$) in Section 5.1.2. (a) In $[0, 1.15]$, (b) in $[1.15, 2.4]$, (c) in $[2.4, 3.4]$, (d) in $[3.4, 5]$, and (e) in $[0, 5]$.

$$\begin{aligned}
 x_{23}(t) &= 1.4 - 0.3353t + 0.0513t^2 - 0.00812t^3 \\
 &\quad + 0.00036t^4, \\
 t &\in [0, 5],
 \end{aligned}$$

$$\begin{aligned}
 x_{24}(t) &= 0.2 + 0.3003t - 0.048t^2 + 0.01446t^3 \\
 &\quad - 0.001453t^4, \\
 t &\in [0, 5].
 \end{aligned}$$

$$x_{14}(t) = 0.2 + 0.326t - 0.02121t^2 + 0.00079t^3, \quad (16)$$

FIGURE 5: (a) v - t , (b) a - t in Section 5.1.2. FIGURE 6: Variation curve of safety distance d - t in Section 5.1.2.

Figure 4 demonstrates the paths evolvement from the perspective of five different time intervals $[0, 1.15]$, $[1.15, 2.4]$, $[2.4, 3.4]$, $[3.4, 5]$, and $[0, 5]$. In Figure 4, the subfigures from (a) to (e) show that each mobile robot smoothly moves from its start pose to end pose without any collision.

The variation curves of velocities and acceleration along with time are shown in Figures 5(a) and 5(b), respectively. Curves in Figure 5 are sufficient to the requirements of the

constraints in Table 2 fully. Thus, the planned paths are consistent with the constraints of kinematics.

Furthermore, Figure 6 illustrates that the variation of safety distances $d_{12}, d_{13}, d_{14}, d_{23}, d_{24}$, and d_{34} lives up to the predefined requirements quite well.

In a word, with above verification, it is substantiated that our approach is effective for path planning problems with even much more time-varying environment.

5.2. Path Planning for Mobile Robots with Static and Moving Obstacles. The path planning optimization function for multiple mobile robots with static and moving obstacles is the same as (14). To test the effectiveness of algorithm $\text{Time}_{\text{Coor}}$ for multiple mobile robots path planning with stationary and moving obstacles, we use the data in Table 3 and the information of two stationary obstacles O_1, O_2 and one moving obstacle O_3 . For O_1 and O_2 , $\alpha_1 = (0.5, 0.6)$, $\alpha_2 = (0.8, 0.9)$, $d_{\text{obs},1k} = 0.05$, $d_{\text{obs},2k} = 0.03$, $r_{\text{obs}k} = 0$, and $k = 1, 2$. Trajectory of O_3 is one curve from left to right, which is expressed as below and safety distance is set as $d_{\text{obs},i3} = 0.16$, $i = 1, 2$:

$$\begin{aligned}\alpha_{31}(t) &= 0.27 + 0.02t + 0.0043t^2, \\ \alpha_{32}(t) &= 0.6 + 0.01t + 0.001t^2 - 0.00008t^3, \quad (17) \\ t &\in [0, 10].\end{aligned}$$

With our approach, for $t_1 \in [0, 5]$ and $t_2 \in [0, 10]$, the expressions of the planned paths are given as below and corresponding curves with time intervals $[0, 2.1]$ $[2.1, 4]$ $[4, 10]$, and $[0, 10]$ are shown in Figure 7. It demonstrates that, with

FIGURE 7: Planned paths of A_1 and A_2 with different time intervals in Section 5.2. (a) In $[0, 2.1]$, (b) in $[2.1, 4]$, (c) in $[4, 10]$, and (d) in $[0, 10]$.

our planned paths, there is no collision among the motion process of the two mobile robots and the three obstacles static and moving:

$$\begin{aligned}
 x_{11}(t_1) &= 0.2 + 0.0711t_1 + 0.0095t_1^2 + 0.0017t_1^3, \\
 x_{21}(t_1) &= 1 - 0.08t_1 - 0.04t_1^2 + 0.01t_1^4 - 0.0006t_1^4, \\
 x_{12}(t_2) &= 1 + 0.0011t_2 - 0.009t_2^2 + 0.0005t_2^3, \\
 x_{22}(t_2) &= 0.2 + 0.25t_2 - 0.011t_2^2 - 0.003t_2^3 + 0.0002t_2^4.
 \end{aligned} \tag{18}$$

TABLE 3: Initial information of mobile robots A_i ($i = 1, 2$) in Section 5.2.

A_i	P_{i0}	P_{if}	$T_{\max}$	$v_{\max}$	$a_{\max}$
A_1	$[0.2, 1, -\pi/4]^T$	$[1, 0.5, -3\pi/4]^T$	5	0.3	0.4
A_2	$[1, 0.2, -3\pi/4]^T$	$[0.6, 1, -3\pi/4]^T$	10	0.25	0.4

The v - t and a - t curves are given in Figures 8(a) and 8(b), respectively. From Figure 8, it is verified that the velocity and acceleration constraints are satisfied, and then the kinematics constraints are met as well.

FIGURE 8: (a) v - t , (b) a - t in Section 5.2. FIGURE 9: Variation curves of safety distance d - t in Section 5.2.

Distances variation curves d_{12} , $d_{\text{obs},11}$, $d_{\text{obs},12}$, $d_{\text{obs},13}$, $d_{\text{obs},21}$, $d_{\text{obs},22}$, and $d_{\text{obs},23}$ between mobile robots A_1 and A_2 , A_i and O_k ($i = 1, 2, k = 1, 2, 3$) are shown in Figure 9. In Figure 9, the safety distance constraints are satisfied as all the distances are in accordance with the corresponding requirements.

With this case, it is verified that our approach is effective for multiple mobile robots path planning under the time-varying environment with stationary and moving obstacles.

6. Discussion

In the case of Section 5.1.1, from Figure 1, it is shown that the initial and final poses of A_1 and A_2 are interchanged with each other. Above situations are common in the path planning problems which belong to the classical deadlock problems. If using one multiphase approach, it is hard to present the concerted ideal solution. However, with our overall conflict resolution and time baseline coordination algorithm, we can dispose this problem perfectly. Meanwhile, in the case of Section 5.1.2, from Figures 4, 5, and 6, we can assert that our approach can handle the path planning problems under the complex time-varying environment. Consequently, those numerical case studies substantiate that our proposed approach would do great deeds for robotic industrial production.

In the case of Section 5.2, from Figure 7, although the trajectory of the moving obstacles almost traverses all the boundaries, no conflict among the planned paths exists. From this case, it is firmly verified that our approach can resolve the path planning problem under the congestive situations with static and moving obstacles which occupied the motion area.

In all the numerical cases, we consider the poses constraints of mobile robots, which have not been achieved in most literatures such as [19]. Thus, our algorithm validates for the multiple mobile robots path planning problems with poses' constraints.

7. Conclusions

The path planning problem for multiple mobile robots under the time-varying environment with stationary and moving obstacles has been studied. The considered objective of our

Input: Admissible error between two consecutive values of path length function: ϵ , number of the interval $[0, t_{fi}]$ equal parts: F , and coefficient of polynomials: m and n .

Output: Optimal path parameters of A_i : A^i, B^i, T_A^i and T_B^i , path length s_i .

- (1) Set $m = 1, n = 1$.
- (2) NLPP(ϵ, F, m, n)
- (3) **If** NLPP is infeasible
- (4) **then** $m++, n++$
- (5) Transfer to (2)
- (6) **else** optimal path parameters A^i, B^i, T_A^i and T_B^i , path length s_i^0
- (7) **end if**
- (8) $m++, n++$
- (9) NLPP(ϵ, F, m, n)
- (10) Assign the results of (9) to A^i, B^i, T_A^i and T_B^i, s_i^1
- (11) **if** $\text{abs}(s_i^0, s_i^1) > \epsilon$
- (12) **then** Transfer to (8)
- (13) **else** optimal path length value $s_i = s_i^1$
- (14) **return** optimal path parameters A^i, B^i, T_A^i and T_B^i , path length s_i

ALGORITHM 2: Trap_{TNLPPs}(ϵ, F, m, n).

formulated TNLPPs is to simultaneously minimize all the paths' lengths of mobile robots while subjecting to generous constraints. The algorithm consists of the overall conflict resolution and time baseline coordination is suggested to solve this problem. With overall conflict resolution, all conflicts among the planned paths are removed in our objective function. By using the time baseline coordination method, we can attain the high-quality planned paths. With our approach, all the paths just need to be calculated only once; there is no need to calculate the path for each mobile robot one by one. Furthermore, the phase of conflict coordination is not required any more. Numerical examples under various scenarios are utilized to validate the efficiency of our approach. Moreover, since generous metaheuristics have been proposed and developed in recent years, we can integrate our algorithm with those heuristics methods to optimize the analytical solving process in the near future.

Appendix

See Algorithm 2.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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