Research Article

Active Power Measurement Based on Multiwavelet Transforms

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This paper discusses a new method for calculating active power in the multiwavelet domain. When the voltage and current waveforms are analyzed using multiwavelet, the active power can be calculated by simply adding the products of the multiwavelet coefficients without having to reconstruct the signals back to the time domain first and then using the traditional integration. From the simulation result, we can see that the results using multiwavelet are better than the ones using wavelet and Fourier Transforms no matter which prefilter is used.

1. Introduction

Active power is important for many purposes such as designing power system equipment, setting tariffs, developing measurement meters, and designing compensation devices for improving the quality of the electric energy. Under sinusoidal conditions, the definitions of active power work well. However, due to the recent widespread use of nonlinear loads, the voltage and current waveforms become nonsinusoidal and therefore their traditional definitions become unsuitable. As a result, many attempts have been made to define active power under this new situation [1–5].

Wavelet is an effective tool for nonstational signal processing and has been used in the measurement of active power [6–8]. However, scalar wavelets cannot contain orthogonality, symmetry, compact support, and higher order of vanishing moments simultaneously. Multiwavelet transform is a new concept in the framework of wavelet transform but has some important differences. It simultaneously possesses orthogonality, compact support, higher order of vanishing moments, and symmetry. It has become a tool for power quality study recently [9–11].

In this paper, a new approach to measure active power based on multiwavelet transforms is studied. In Section 2, we introduce the multiwavelet transform. In Section 3, a new approach to measure active power based on multiwavelet transforms is discussed. In Section 4, an example is given to illustrate validity of our method. At last a conclusion is given in Section 5.

2. Multiwavelet Transform

Unlike a scalar wavelet, a multiwavelet has several mother wavelet functions (scaling functions) which are used to expand a given function [12]. Let Φ denote a compactly supported orthonormal scaling vector $\Phi = (\phi_1, \dots, \phi_r)^T$, where r is the number of scalar scaling functions. Then $\Phi(t)$ satisfies a two-scaling dilation equation of the form

$$\Phi(t) = \sum_k H_k \Phi(2t - k) \quad (1)$$

for some finite sequence H of $r \times r$ matrices. Furthermore, the integer shifts of the components of Φ form an orthonormal system; that is,

$$\langle \phi_n(\cdot - k), \phi_{n'}(\cdot - k') \rangle = \delta_{n-n', k-k'}. \quad (2)$$

Let V_0 denote the closed span of $\{\phi_n(\cdot - k) \mid k \in \mathbb{Z}, n = 1, 2, \dots, r\}$ and define $V_j = \{f(2^j \cdot) \mid f \in V_0\}$. Then $(V_j)_{j \in \mathbb{Z}}$ is a multiresolution analysis of $L^2(\mathbb{R})$ [13]. Note that we choose the increasing convention $V_{j-1} \subset V_j$.

Let W_j denote the orthogonal complement of V_j in V_{j+1} . Then there exists an orthogonal multiwavelet $\Psi = (\psi_1, \dots, \psi_r)^T$ such that $\{\psi_n(\cdot - k) \mid n = 1, 2, \dots, r \text{ and } k \in \mathbb{Z}\}$ forms an orthogonal basis of W_0 . Since $W_0 \subset V_1$, there exists a sequence G of $r \times r$ matrices such that

$$\Psi(t) = \sum_k G_k \Phi(2t - k). \quad (3)$$

Let $f \in V_0$; then f can be written as a linear combination of the basis in V_0 . Consider

$$f(t) = \sum_{k \in \mathbb{Z}} \mathbf{c}_0^*(k) \Phi(t-k) \quad (4)$$

for some sequence $\mathbf{c}_0 \in l_2(\mathbb{Z})^r$ of vector coefficients. The superscript $*$ stands for the complex conjugate transposition. Here $l_2(\mathbb{Z})^r$ denotes the space of finite energy vector sequences $\mathbf{c}$ with the norm

$$\|\mathbf{c}\| = \left(\sum_{\substack{i=1,\dots,r \\ n \in \mathbb{Z}}} |\mathbf{c}_i(n)|^2 \right)^{1/2}. \quad (5)$$

Because $V_0 = V_{-1} \oplus W_{-1}$ [14], where the symbol $\oplus$ denotes an orthogonal direct sum, f can be written as a linear combination of the basis functions of V_{-1} and W_{-1}

$$f(t) = \sum_{k \in \mathbb{Z}} \sqrt{2} \mathbf{c}_1^*(k) \Phi(2t-k) + \sum_{k \in \mathbb{Z}} \sqrt{2} \mathbf{d}_1^*(k) \Psi(2t-k). \quad (6)$$

The coefficients $\mathbf{c}_1$ and $\mathbf{d}_1$ are related to $\mathbf{c}_0$ via the following decomposition and reconstruction algorithm:

$$\begin{aligned} \mathbf{c}_1(k) &= \sum_n H(n) \mathbf{c}_0(2k+n), \\ \mathbf{d}_1(k) &= \sum_n G(n) \mathbf{c}_0(2k+n), \end{aligned} \quad (7)$$

$$\mathbf{c}_0(k) = \sum_n H^*(k-2n) \mathbf{c}_1(n) + \sum_n G^*(k-2n) \mathbf{d}_1(n).$$

Define $\Phi_{l,k}(t) = 2^{l/2} \Phi(2^l t - k)$ and $\Psi_{l,k}(t) = 2^{l/2} \Psi(2^l t - k)$; then $\{\Phi_{l,k} : k \in \mathbb{Z}\}$ and $\{\Psi_{l,k} : k \in \mathbb{Z}\}$ are stable bases of V_l and W_l , respectively. And (6) can be written as

$$f(t) = \sum_{k \in \mathbb{Z}} \mathbf{c}_1^*(k) \Phi_{1,k}(t) + \sum_{k \in \mathbb{Z}} \mathbf{d}_1^*(k) \Psi_{1,k}(t). \quad (8)$$

Because $V_0 = V_{-1} \oplus W_{-1} = (V_{-2} \oplus W_{-2}) \oplus W_{-1} = \dots = V_{-m} \oplus W_{-m} \oplus \dots \oplus W_{-1}$, where m is the levels for decomposition, f can be written as

$$f(t) = \sum_{k \in \mathbb{Z}} \mathbf{c}_m^*(k) \Phi_{m,k}(t) + \sum_{l=-m}^{-1} \sum_{k \in \mathbb{Z}} \mathbf{d}_l^*(k) \Psi_{l,k}(t). \quad (9)$$

In general, for a signal $f(t) \in V_n$, $n \in \mathbb{Z}$, it can be expressed as a linear combination of the basis in V_n . Consider

$$f(t) = \sum_k \mathbf{c}_n^*(k) \Phi_{n,k}(t). \quad (10)$$

Similar to (9),

$$f(t) = \sum_k \mathbf{c}_l^*(k) \Phi_{l,k}(t) + \sum_{j=l}^{n-1} \sum_k \mathbf{d}_j^*(k) \Psi_{j,k}(t), \quad (11)$$

where $l = n - (\text{levels for decomposition})$ and $\mathbf{c}_l^*(k)$, $\mathbf{d}_j^*(k)$ are row vectors.

Multiwavelets are a new addition to the body of wavelet theory. Realizable as matrix-valued filter banks leading to wavelet bases, multiwavelets offer simultaneous orthogonality, symmetry, and short support, which is not possible with scalar two-channel wavelet systems. The DGHM multiwavelets which are developed by Donovan et al. [15] are a very popular multiwavelet. They are shown in Figure 1.

Remark 1. It is important to note that the orthogonality conditions for the DGHM scaling function and the associated multiwavelet remain valid even when restricted to a compact interval. This is not true in general. It is actually another favorable property of this multiwavelet system.

The DGHM scaling functions satisfy (1) with four coefficients:

$$\begin{aligned} H_0 &= \begin{pmatrix} \frac{3}{10} & \frac{2\sqrt{2}}{5} \\ -\frac{\sqrt{2}}{40} & -\frac{3}{20} \end{pmatrix}, & H_1 &= \begin{pmatrix} \frac{3}{10} & 0 \\ \frac{9\sqrt{2}}{40} & \frac{1}{2} \end{pmatrix}, \\ H_2 &= \begin{pmatrix} 0 & 0 \\ \frac{9\sqrt{2}}{40} & -\frac{3}{20} \end{pmatrix}, & H_3 &= \begin{pmatrix} 0 & 0 \\ -\frac{\sqrt{2}}{40} & 0 \end{pmatrix}. \end{aligned} \quad (12)$$

And the multiwavelets satisfy (1) with four coefficients as

$$\begin{aligned} G_0 &= \begin{pmatrix} -\frac{\sqrt{2}}{40} & -\frac{3}{40} \\ -\frac{1}{20} & -\frac{3\sqrt{2}}{20} \end{pmatrix}, & G_1 &= \begin{pmatrix} \frac{9\sqrt{2}}{40} & -\frac{1}{2} \\ \frac{9}{20} & 0 \end{pmatrix}, \\ G_2 &= \begin{pmatrix} \frac{9\sqrt{2}}{40} & -\frac{3}{20} \\ -\frac{9}{20} & \frac{3\sqrt{2}}{20} \end{pmatrix}, & G_3 &= \begin{pmatrix} -\frac{\sqrt{2}}{40} & 0 \\ \frac{1}{20} & 0 \end{pmatrix}. \end{aligned} \quad (13)$$

GHM multiple scaling functions and multiwavelets are very popular with many advantages over scalars. They are either symmetric or antisymmetric, orthogonal, supported on $[0, 1]$ and $[0, 2]$, respectively, with approximation order 2. However, it is impossible for scalar wavelets [16].

Let $\mathbf{c}_j(k) = (c_{1,j,k}, c_{2,j,k})$ and $\mathbf{d}_j(k) = (d_{1,j,k}, d_{2,j,k})$. By dilations (1) and (3), we have the following recursive relationship between the coefficients $(c_{1,j,k}, c_{2,j,k})^T$ and $(d_{1,j,k}, d_{2,j,k})^T$:

$$\begin{aligned} \begin{pmatrix} c_{1,j-1,k} \\ c_{2,j-1,k} \end{pmatrix} &= \sum_n H_{n-2k} \begin{pmatrix} c_{1,j,n} \\ c_{2,j,n} \end{pmatrix}, & j, k \in \mathbb{Z} \\ \begin{pmatrix} d_{1,j-1,k} \\ d_{2,j-1,k} \end{pmatrix} &= \sum_n G_{n-2k} \begin{pmatrix} c_{1,j,n} \\ c_{2,j,n} \end{pmatrix}, & j, k \in \mathbb{Z}. \end{aligned} \quad (14)$$

FIGURE 1: GHM multiple scaling functions (top) and multiwavelets (bottom).

Moreover,

$$\begin{pmatrix} c_{1,j,k} \\ c_{2,j,k} \end{pmatrix} = \sum_n \left(H_{n-2k}^T \begin{pmatrix} c_{1,j-1,n} \\ c_{2,j-1,n} \end{pmatrix} + G_{n-2k}^T \begin{pmatrix} d_{1,j-1,n} \\ d_{2,j-1,n} \end{pmatrix} \right), \quad j, k \in \mathbb{Z}. \quad (15)$$

Due to their matrix-valued filter banks, multiwavelets differ from scalar wavelets in requiring two or more input streams. That means the multiwavelet decomposition and reconstruction are a multi-input and multioutput (MIMO) system. It is necessary to preprocess the input signal before decomposition and postprocess the output signal after reconstruction [17]. Postfilter is just the inverse matrix of prefilter.

Thus, the above decomposition and reconstruction can be represented in Figure 2. $Q(\omega)$ is the prefilter and $P(\omega)$ is the postfilter.

3. Active Power Representation Using Multiwavelet Transform

It is known that the active power is calculated by averaging the voltage and current product with a running window over

one cycle of the fundamental frequency

$$p = \frac{1}{T} \int_0^T u(t) i(t) dt, \quad (16)$$

where $u(t)$ and $i(t)$ are instantaneous voltage and current, and T is the period of the specified fundamental frequency.

Let $t = Tx$, $u'(x) = u(Tx)$, and $i'(x) = i(Tx)$. Then (16) becomes

$$p = \int_0^1 u'(x) i'(x) dx. \quad (17)$$

Suppose that the instantaneous voltage $u'(x)$ and current $i'(x)$ have been discretized and expressed using scaling functions and multiwavelets over a running unit interval $0 \leq x \leq 1$. Consider

$$\begin{aligned} u'(x) &= \sum_j \mathbf{c}_{J_0,j}^* \Phi_{J_0,j}(x) + \sum_j \mathbf{d}_{J_0,j}^* \Psi_{J_0,j}(x), \\ i'(x) &= \sum_j \mathbf{a}_{K_0,j}^* \Phi_{K_0,j}(x) + \sum_j \mathbf{b}_{K_0,j}^* \Psi_{K_0,j}(x), \end{aligned} \quad (18)$$

where J_0 and K_0 are the coarsest resolution level of the decomposition; $\mathbf{c}_{J_0,j}^*$ and $\mathbf{a}_{J_0,j}^*$ are the approximation coefficients that represent the smoothed part of the signal; $\mathbf{d}_{J_0,j}^*$ and

FIGURE 2: Multiwavelets (a) decomposition and (b) reconstruction ($r = 2$).

$\mathbf{b}_{J_0,j}^*$ are the detail coefficients that represent the oscillatory part of the same signal.

Substituting (18) into (17), we have

$$p = \int_0^1 \left[\sum_j \mathbf{c}_{J_0,j}^* \Phi_{J_0,j}(x) + \sum_j \mathbf{d}_{J_0,j}^* \Psi_{J_0,j}(x) \right] \times \left[\sum_j \mathbf{a}_{K_0,j}^* \Phi_{K_0,j}(x) + \sum_j \mathbf{b}_{K_0,j}^* \Psi_{K_0,j}(x) \right] dx. \quad (19)$$

Define $\mathbf{c}_{J_0,j}^* = (c_{J_0,j,1}, c_{J_0,j,2})^T$, $\mathbf{d}_{J_0,j}^* = (d_{J_0,j,1}, d_{J_0,j,2})^T$, $\mathbf{a}_{K_0,j}^* = (a_{K_0,j,1}, a_{K_0,j,2})^T$, and $\mathbf{b}_{K_0,j}^* = (b_{K_0,j,1}, b_{K_0,j,2})^T$; then (19) can be rewritten as

$$p = \int_0^1 \sum_{j,k} (c_{J_0,j,1} a_{K_0,k,1} \phi_{J_0,j,1}(x) \phi_{K_0,k,1}(x) + c_{J_0,j,1} a_{K_0,k,2} \phi_{J_0,j,1}(x) \phi_{K_0,k,2}(x) + c_{J_0,j,2} a_{K_0,k,1} \phi_{J_0,j,2}(x) \phi_{K_0,k,1}(x) + c_{J_0,j,2} a_{K_0,k,2} \phi_{J_0,j,2}(x) \phi_{K_0,k,2}(x)) dx + \int_0^1 \sum_{j,k} (c_{J_0,j,1} b_{K_0,k,1} \phi_{J_0,j,1}(x) \psi_{K_0,k,1}(x) + c_{J_0,j,1} b_{K_0,k,2} \phi_{J_0,j,1}(x) \psi_{K_0,k,2}(x) + c_{J_0,j,2} b_{K_0,k,1} \phi_{J_0,j,2}(x) \psi_{K_0,k,1}(x) + c_{J_0,j,2} b_{K_0,k,2} \phi_{J_0,j,2}(x) \psi_{K_0,k,2}(x)) dx + \int_0^1 \sum_{j,k} (d_{J_0,j,1} a_{K_0,k,1} \psi_{J_0,j,1}(x) \phi_{K_0,k,1}(x) + d_{J_0,j,1} a_{K_0,k,2} \psi_{J_0,j,1}(x) \phi_{K_0,k,2}(x) + d_{J_0,j,2} a_{K_0,k,1} \psi_{J_0,j,2}(x) \phi_{K_0,k,1}(x) + d_{J_0,j,2} a_{K_0,k,2} \psi_{J_0,j,2}(x) \phi_{K_0,k,2}(x)) dx + \int_0^1 \sum_{j,k} (d_{J_0,j,1} b_{K_0,k,1} \psi_{J_0,j,1}(x) \psi_{K_0,k,1}(x) + d_{J_0,j,1} b_{K_0,k,2} \psi_{J_0,j,1}(x) \psi_{K_0,k,2}(x) + d_{J_0,j,2} b_{K_0,k,1} \psi_{J_0,j,2}(x) \psi_{K_0,k,1}(x) + d_{J_0,j,2} b_{K_0,k,2} \psi_{J_0,j,2}(x) \psi_{K_0,k,2}(x)) dx. \quad (20)$$

We further assume the following.

- (1) The signal data is zero outside the decomposition window so that the integration range can be extended to the whole real space $\mathbb{R}$.
- (2) The voltage and current signals are already in the same approximation space.
- (3) The voltage and current waveforms have been decomposed to the same coarsest level; that is, $J_0 = K_0$ in (20).
- (4) The multiwavelet analysis of voltage and current signals employs the same type of scaling functions and orthogonal multiwavelets.

Remark 2. Assumption (1) can be satisfied very easy. For example, the periodic signals such as $\sin(\omega t)$ can satisfy assumption (1).

Assumptions (2)–(4) are decided by us. Let the voltage signal and the current signal be $\sin(\omega t)$ and $\sin(\omega t + \varphi)$, respectively. They are in the space V_0 and have been decomposed to the same coarsest level V_{-5} using the same multiwavelet GHM.

Expand (20) and take into account the following orthogonal properties:

$$\begin{aligned}\langle \Phi_{j,k}, \Phi_{m,n} \rangle &= \begin{cases} I, & j = m, \quad k = n \\ 0, & \text{others} \end{cases} \\ \langle \Psi_{j,k}, \Psi_{m,n} \rangle &= \begin{cases} I, & j = m, \quad k = n \\ 0, & \text{others} \end{cases} \\ \langle \Phi_{j,k}, \Psi_{m,n} \rangle &= 0.\end{aligned}\quad (21)$$

We get

$$\begin{aligned}p \mid V_{J_0} &= \sum_j (c_{J_0,j,1} a_{J_0,j,1} + c_{J_0,j,2} a_{J_0,j,2} + d_{J_0,j,1} b_{J_0,j,1} + d_{J_0,j,2} b_{J_0,j,2}) \\ &= \sum_j c_{J_0,j} \mathbf{a}_{J_0,j}^* + \sum_j d_{J_0,j} \mathbf{b}_{J_0,j}^*.\end{aligned}\quad (22)$$

Remark 3. The orthogonal properties (21) can effectively reduce or eliminate the aliasing phenomenon. They ensure the orthogonality of the frequency band.

Using (14), we have

$$\begin{aligned}p \mid V_J &= \sum_j c_{J_0,j} \mathbf{a}_{J_0,j}^* + \sum_{k=J_0}^J \sum_j d_{k,j} \mathbf{b}_{k,j}^* \\ &= p_\Phi + p_{\Omega_0} + p_{\Omega_1} + \cdots + p_{\Omega_{J-1}},\end{aligned}\quad (23)$$

where $p_\Phi = \sum_j c_{J_0,j} \mathbf{a}_{J_0,j}^*$ and $p_{\Omega_k} = \sum_j d_{J_0+k,j} \mathbf{b}_{J_0+k,j}^*$ with $k = 0, 1, \dots, J - J_0$.

From (23), we can see that, in the multiwavelet domain, the power delivered is the addition of the power calculated at each decomposition level, and the power at a decomposition level is the addition of the products of the corresponding multiwavelet coefficients.

It is interesting to note that (23) resembles to a certain extent the formula for calculating active power by wavelet [7] and by Fourier series source [18].

4. Numerical Example

A numerical example is considered in this section. All computations in this section are carried out by Matlab 7.8.0.347.

The example considers the case of sinusoidal voltage ($f_1 = 50$ Hz) and a nonsinusoidal current containing the fundamental, third, and fifth harmonic components

$$\begin{aligned}u(t) &= 100 \sin(314t), \\ i(t) &= 10 \sin(314t) + 2 \sin(942t) + 0.3 \sin(1570t).\end{aligned}\quad (24)$$

In the field of power systems, it is generally considered up to the 50th harmonic signals; that is, $f_{\text{hmax}} = f_1 \times 50 = 2500$ Hz. According to the Nyquist sampling theorem, the sampling frequency is greater than or equal to 2 times the

highest frequency; that is, $f_s \geq 2 \times f_{\text{hmax}} = 5000$ Hz. So $f_s = 6400$ Hz is selected as the sampling frequency. You can also choose the other frequencies f_s to be greater than 5000 Hz.

After sampling the voltage and current waveforms with a sampling frequency ($f_s = 6400$ Hz), a prefilter is used to produce the initial coefficients. Then the multiwavelet transform is applied in order to get the approximations and the details.

Since we want to extract the fundamental signal in original signal, it needs the fundamental signal to be in the center of the frequency bank. The lowest decomposition approximation frequency range is 0–100 Hz, which means that the width of approximation frequency bank is 100 Hz. The width of approximation frequency bank is $B_a = f_s/2^{l+1}$, where l is the decomposition level. So we have $l = \log_2(f_s/B_a) - 1 = 5$. If it is further decomposed to level 6, the fundamental signal is then at the boundary of frequency range 0–50 Hz and 50–100 Hz. If the decomposition level is 4, then the approximation frequency range of the signal is 0–200 Hz, resulting in 3rd harmonic signal that also falls in this frequency range, and the fundamental signal cannot be separated. Thus five wavelet levels are chosen, and this represents the best number of levels for decomposition; therefore, each frequency component of the analyzed waveform can be extracted in a single band.

The voltage and current signals are analyzed using multiwavelet GHM, Daubechies (Db4, Db10), and Fourier Transform (FT). When using wavelet and multiwavelet, the boundary should be handled first. There are many handle methods such as zero padding, symmetric extension, and periodic extension. Zero padding does not preserve orthogonality. It is shown in [19] that the finite multiwavelet transform based on symmetric extension will then preserve the symmetry across scales as in the scalar case. The periodic extension approach will work for multiwavelets and will preserve orthogonality unless the data are truly periodic. Because our data are periodic, here we are using the periodic extension. When using multiwavelet, it also needs a prefilter to transfer the scalar signal into a vector. We are using the two popular prefilters, one is the oversampling [20] and other is GHM.init [17].

The calculation results of the active power using GHM with oversampling prefilter, GHM with GHM.init prefilter, Db4, Db10, and FT are listed in Table 1. We can see that an active power of about 500 W is obtained, and the difference is too small. Furthermore, we can see that the active power measured using multiwavelet no matter which prefilter is used is better than using wavelet (Db4 and Db10) and Fourier Transforms.

Remark 4. From the results, we can see that the active power measured using multiwavelet no matter which prefilter is used is better than using wavelet (Db4 and Db10) and Fourier Transforms. However, multiwavelet transform needs prefiltering and postfiltering for one-dimensional signals. Calculation of multiwavelet transform is greater than wavelet transform. Fast algorithm for multiwavelet transform will be our next work.

TABLE 1: Active power calculation results.

Method name	Prefilter	Theory values	Calculated values	Error
GHM	Oversampling	500	500.3804	7.6×10^{-4}
GHM	GHM.init	500	500.3755	7.5×10^{-4}
db4	\	500	501.8237	3.6×10^{-3}
db10	\	500	499.4937	1.0×10^{-3}
FT	\	500	501.2168	2.4×10^{-3}

5. Conclusions

A new method of active power measurement was mathematically examined in the paper. It is shown that the active power can be calculated by simply adding the products of multiwavelet coefficients without having to spend time on synthesizing the processed coefficients back to time-domain signals first and then using the traditional integration. The new method is convenient and beneficial when multiwavelet transform has to be performed in the true real-time environment or to compress electric data of large volume. The new method is better than wavelet and Fourier Transforms.

Conflict of Interests

The authors declare that there is no conflict of interests concerning the publication of this paper.

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