Research Article

Models of Relative Nearness Incidence Based on Standard Distance Entropy

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The model of grey nearness incidence cannot reflect the nearness degree of sequences correctly. Therefore, the model of relative nearness incidence of sequences, curves, and surfaces is suggested based on standard distance entropy to remove the current bottlenecks and its properties are studied. At last, three cases are exemplified to demonstrate the validity and practicability of relative nearness incidence. The proposed models have enriched the theory of grey nearness incidence, filled the defects of grey nearness incidence.

1. Introduction

In 1982, Deng created grey systems theory [1–3], and it has been a far-reaching development in the past 30 years. The subjects of grey systems theory is with part of the information known and part of the information unknown. Therefore, grey systems theory has a very broad prospect of development. In recent years, grey system theory has had successful application in many scientific fields, and it won wide recognition and attention.

As an important part of grey systems theory, grey incidence analysis is the basis of grey forecasting, grey clustering, and grey decision making. The basic idea of grey incidence analysis is based on the curve similarity of different sequences, the closer the curve, the greater the incidence degree and vice versa. Since Deng put forward the theory of grey incidence analysis, many scholars participated in related research, and construct a large number of grey incidence model [4–17]. The models of incidence degree have good applications [18–27]. In addition, scholars study the properties of some incidence degree models [28, 29].

Previous studies of the grey incidence model are mainly focused on time sequences. However, there is very few research on the grey incidence model of curves or surfaces.

In addition, many grey incidence models do not satisfy the symmetry. Finally, the grey nearness incidence may be 1 for different sequences; this is obviously unreasonable [12]. Therefore, this paper will construct the grey incidence model of sequences, curves, and surfaces based on the above-mentioned defects.

2. The Model of Grey Nearness Incidence

With the development of grey incidence theory, Liu et al. proposed the model of grey nearness incidence in 2010; the basic idea is as follows [12].

The system behavior sequence $X_i = (x_i(1), x_i(2), \dots, x_i(n))$, where $i = 1, 2, \dots, t$. X_i and X_j denote the crease of X_i and X_j else. Suppose

$$S_i - S_j = \int_1^n (X_i - X_j) dt. \quad (1)$$

Definition 1. Suppose the length of X_i and X_j is equal, $S_i - S_j$ as shown above; then the grey nearness incidence of X_i and X_j is defined as

$$\rho_{ij} = \frac{1}{1 + |S_i - S_j|}. \quad (2)$$

Theorem 2. Suppose the length of X_i and X_j is equal; then (notes: part of [12] seems to be missing, now we will correct it)

$$\rho_{ij} = (1) \times \left(1 + \left| \frac{1}{2} (x_i(1) - x_j(1)) + \sum_{k=2}^{n-1} (x_i(k) - x_j(k)) + \frac{1}{2} (x_i(n) - x_j(n)) \right| \right)^{-1}. \quad (3)$$

However, there are many defects. The grey nearness incidence may be 1 for different sequences. We will use Example 3 to illustrate it.

Example 3. The sequences are as follows:

$$\begin{aligned} X_1 &= (1, 2, 1) \\ X_2 &= (2, 1, 2). \end{aligned} \quad (4)$$

The corresponding crease of each sequence is shown in Figure 1. We know the grey nearness incidence is $\rho_{12} = 1$ based on the above-mentioned model.

This result is clearly not consistent with the fact. So the model of grey nearness incidence cannot reflect the nearness degree of sequences correctly. Therefore, the model of relative nearness incidence of sequences, curves, and surfaces is suggested based on standard distance entropy, and they can solve the above-mentioned problem.

3. Distance Entropy and Standard Distance Entropy

Definition 4. Suppose A and B are positive numbers; then the distance entropy of A and B is defined as [30]

$$H_{AB} = - \left(\frac{A}{A+B} \ln \frac{A}{A+B} + \frac{B}{A+B} \ln \frac{B}{A+B} \right). \quad (5)$$

The distance entropy satisfies the following basic properties:

- (1) $H_{AB} = H_{BA}$;
- (2) if $A^* = xA, B^* = xB$, then $H_{A^*B^*} = H_{AB}$;
- (3) $0 < H_{AB} \leq \ln 2$ and $\inf H = 0$;
- (4) $H_{AB} = \ln 2$ if and only if $A = B$.

Properties (1) and (2) are obvious; now we prove the properties (3) and (4).

Theorem 5. The maximum value of $f(x) = -[x \ln x + (1-x) \ln(1-x)]$ on $0 < x < 1$ is $\ln 2$, the minimum does not exist, and the infimum is 0.

Proof. Deriving for $f(x)$, we know

$$f'(x) = -[\ln x + 1 - \ln(1-x) - 1] = \ln \left(\frac{1-x}{x} \right). \quad (6)$$

FIGURE 1: The corresponding crease of each sequence.

Suppose $f'(x) \geq 0$; then $0 < x \leq (1/2)$, we know $f(x)$ is an increasing function on $(0, (1/2)]$. Suppose $f'(x) < 0$, then $(1/2) < x < 1$, we know $f(x)$ is an decreasing function on $((1/2), 1)$. Therefore, when $0 < x < 1$, $f(x)$ achieves maximum value when $x = (1/2)$, and the maximum value is $\max[f(x)] = f(1/2) = \ln 2$. Obviously, $f(x)$ has no minimum value in $(0, 1)$, and the infimum of $f(x)$ is $\inf f(x) = 0$. So we obtain the property (3).

During the proof process of Theorem 5, we know the closer the value of x and $(1/2)$, the larger the value of $f(x)$. Therefore, when $A = B > 0$, H_{AB} achieves the maximum value, and the maximum value is $\ln 2$. So we obtain the property (4). $\square$

Definition 6. Suppose A and B are positive numbers; then the standard distance entropy of A and B is defined as

$$\overline{H_{AB}} = \frac{H_{AB} - \inf H_{AB}}{\max H_{AB} - \inf H_{AB}} = \frac{H_{AB}}{\ln 2}. \quad (7)$$

From the definition of standard distance entropy, we know $0 < \overline{H_{AB}} \leq 1$, and the closer the value of A and B , the larger the value of $\overline{H_{AB}}$. So we can use $\overline{H_{AB}}$ to express the relative nearness degree of A and B .

The standard distance entropy satisfies the following basic properties:

- (1) $\overline{H_{AB}} = \overline{H_{BA}}$;
- (2) if $A^* = xA, B^* = xB$, then $\overline{H_{A^*B^*}} = \overline{H_{AB}}$;
- (3) $0 < \overline{H_{AB}} \leq 1$;
- (4) $\overline{H_{AB}} = 1$ if and only if $A = B$.

The properties of standard distance entropy can obtained from the properties of distance entropy, so we will not repeat it.

4. The Models of Relative Nearness Incidence Based on Standard Distance Entropy

4.1. Relative Nearness Incidence of Sequence

Definition 7. For sequences $X_i = \{x_i(1), x_i(2), \dots, x_i(n)\}$, where $x_i(t) > 0$, $i = 1, 2, \dots, m$, $t = 1, 2, \dots, n$. Then the distance entropy of $x_i(t)$ and $x_j(t)$ is defined as

$$H_{ij}(t) = - \left(\frac{x_i(t)}{x_i(t) + x_j(t)} \ln \frac{x_i(t)}{x_i(t) + x_j(t)} + \frac{x_j(t)}{x_i(t) + x_j(t)} \ln \frac{x_j(t)}{x_i(t) + x_j(t)} \right). \quad (8)$$

Standardising $H_{ij}(t)$, we know

$$\overline{H_{ij}(t)} = \frac{H_{ij}(t)}{\ln 2}. \quad (9)$$

$\overline{H_{ij}(t)}$ can express the relative nearness degree of $x_i(t)$ and $x_j(t)$; then the average of $\overline{H_{ij}(t)}$ can express the relative nearness incidence of X_i and X_j . So we have Theorem 8.

Theorem 8. Sequences $X_i = \{x_i(1), x_i(2), \dots, x_i(n)\}$ and $X_j = \{x_j(1), x_j(2), \dots, x_j(n)\}$ are constant positive; then the relative nearness incidence of X_i and X_j is

$$\begin{aligned} H_{ij} &= \frac{1}{n} \sum_{t=1}^n \overline{H_{ij}(t)} = \frac{1}{n \ln 2} \sum_{t=1}^n H_{ij}(t) \\ &= - \frac{1}{n \ln 2} \sum_{t=1}^n \left(\frac{x_i(t)}{x_i(t) + x_j(t)} \ln \frac{x_i(t)}{x_i(t) + x_j(t)} + \frac{x_j(t)}{x_i(t) + x_j(t)} \ln \frac{x_j(t)}{x_i(t) + x_j(t)} \right), \end{aligned} \quad (10)$$

where $\overline{H_{ij}(t)}$ means the standard distance entropy of $x_i(t)$ and $x_j(t)$.

4.2. Relative Nearness Incidence of Curve

Definition 9. For curves $y_i = f_i(x)$ and $y_j = f_j(x)$, where $y_i = f_i(x) > 0$ and $y_j = f_j(x) > 0$ on $x \in [a, b]$, $i = 1, 2, \dots, m$, when $x = x_0$, the distance entropy of $y_i = f_i(x)$ and $y_j = f_j(x)$ is defined as

$$\begin{aligned} H_{ij}(x_0) &= - \left(\frac{f_i(x_0)}{f_i(x_0) + f_j(x_0)} \ln \frac{f_i(x_0)}{f_i(x_0) + f_j(x_0)} + \frac{f_j(x_0)}{f_i(x_0) + f_j(x_0)} \ln \frac{f_j(x_0)}{f_i(x_0) + f_j(x_0)} \right). \end{aligned} \quad (11)$$

Standardising $H_{ij}(t)$, we know

$$\overline{H_{ij}(x_0)} = \frac{H_{ij}(x_0)}{\ln 2}. \quad (12)$$

$\overline{H_{ij}(x_0)}$ can express the relative nearness degree of $y_i = f_i(x)$ and $y_j = f_j(x)$ when $x = x_0$. So we have Theorem 10.

Theorem 10. Curves $y_i = f_i(x)$ and $y_j = f_j(x)$ and they are positive on $x \in [a, b]$; then the relative nearness incidence of $y_i = f_i(x)$ and $y_j = f_j(x)$ is

$$\begin{aligned} H_{ij}(x) &= \frac{1}{b-a} \int_a^b \overline{H_{ij}(x_0)} dx_0 \\ &= - \frac{1}{(b-a) \ln 2} \int_a^b H_{ij}(x_0) dx_0 \\ &= - \frac{1}{(b-a) \ln 2} \\ &\quad \times \int_a^b \left(\frac{f_i(x_0)}{f_i(x_0) + f_j(x_0)} \ln \frac{f_i(x_0)}{f_i(x_0) + f_j(x_0)} + \frac{f_j(x_0)}{f_i(x_0) + f_j(x_0)} \ln \frac{f_j(x_0)}{f_i(x_0) + f_j(x_0)} \right) dx_0. \end{aligned} \quad (13)$$

4.3. Relative Nearness Incidence of Surface

Definition 11. For surfaces $z_i = f_i(x, y)$ and $z_j = f_j(x, y)$, where $z_i = f_i(x, y) > 0$ and $z_j = f_j(x, y) > 0$ on $x \in [a, b]$ and $y \in [c, d]$, $i = 1, 2, \dots, m$, when $x = x_0$ and $y = y_0$, the distance entropy of $z_i = f_i(x, y)$ and $z_j = f_j(x, y)$ is defined as

$$\begin{aligned} H_{ij}(x_0, y_0) &= - \left(\frac{f_i(x_0, y_0)}{f_i(x_0, y_0) + f_j(x_0, y_0)} \ln \frac{f_i(x_0, y_0)}{f_i(x_0, y_0) + f_j(x_0, y_0)} + \frac{f_j(x_0, y_0)}{f_i(x_0, y_0) + f_j(x_0, y_0)} \right. \\ &\quad \times \ln \frac{f_j(x_0, y_0)}{f_i(x_0, y_0) + f_j(x_0, y_0)} \left. \right). \end{aligned} \quad (14)$$

Standardising $H_{ij}(x_0, y_0)$, we know

$$\overline{H_{ij}(x_0, y_0)} = \frac{H_{ij}(x_0, y_0)}{\ln 2}. \quad (15)$$

$\overline{H_{ij}(x_0, y_0)}$ can express the relative nearness degree of $z_i = f_i(x, y)$ and $z_j = f_j(x, y)$ when $x = x_0$ and $y = y_0$. So we have Theorem 12.

Theorem 12. Surfaces $z_i = f_i(x, y)$ and $z_j = f_j(x, y)$ and they are positive on $x \in [a, b]$, $y \in [c, d]$. Then the relative nearness incidence of $z_i = f_i(x, y)$ and $z_j = f_j(x, y)$ is

$$\begin{aligned}
 H_{ij}(x, y) &= \frac{1}{(b-a)(d-c)} \int_c^d \int_a^b \overline{H_{ij}(x_0, y_0)} dx_0 dy_0 \\
 &= -\frac{1}{(b-a)(d-c) \ln 2} \int_c^d \int_a^b H_{ij}(x_0, y_0) dx_0 dy_0 \\
 &= -\frac{1}{(b-a)(d-c) \ln 2} \\
 &\quad \times \int_c^d \int_a^b \left(\frac{f_i(x_0, y_0)}{f_i(x_0, y_0) + f_j(x_0, y_0)} \right. \\
 &\quad \times \ln \frac{f_i(x_0, y_0)}{f_i(x_0, y_0) + f_j(x_0, y_0)} \\
 &\quad + \frac{f_j(x_0, y_0)}{f_i(x_0, y_0) + f_j(x_0, y_0)} \\
 &\quad \left. \times \ln \frac{f_j(x_0, y_0)}{f_i(x_0, y_0) + f_j(x_0, y_0)} \right) dx_0 dy_0.
 \end{aligned} \tag{16}$$

Theorem 13. The relative nearness incidence of sequences, curves, and surfaces satisfies the following properties:

- (1) $0 < H_{ij} \leq 1$, $0 < H_{ij}(x) \leq 1$, $0 < H_{ij}(x, y) \leq 1$;
- (2) $H_{ij} = H_{ji}$, $H_{ij}(x) = H_{ji}(x)$, $H_{ij}(x, y) = H_{ji}(x, y)$;
- (3) $H_{ij} = 1$ if and only if $x_i(t) = x_j(t)$; $H_{ij}(x) = 1$ if and only if $f_i(x) = f_j(x)$; $H_{ij}(x, y) = 1$ if and only if $f_i(x, y) = f_j(x, y)$;
- (4) if $X'_i = aX_i$, $X'_j = aX_j$, then $H'_{ij} = H_{ij}$; if $f'_i(x) = af_i(x)$, $f'_j(x) = af_j(x)$, $H'_{ij}(x) = H_{ij}(x)$; if $f'_i(x, y) = af_i(x, y)$, $f'_j(x, y) = af_j(x, y)$, then $H'_{ij}(x, y) = H_{ij}(x, y)$.

The above-mentioned properties are obvious, so we will not repeat them here.

5. Examples

In order to illustrate the validity and practicability of the relative nearness incidence, we have the following 3 examples.

Example 1. Suppose

$$\begin{aligned}
 X_1 &= (0.5, 0.9, 1.2, 2.0) \\
 X_2 &= (0.4, 0.5, 1.6, 2.3) \\
 X_3 &= (0.6, 0.9, 1.8, 2.2).
 \end{aligned} \tag{17}$$

As seen in the above, we know

$$\begin{aligned}
 H_{12}(1) &= 0.6870, & H_{12}(2) &= 0.6518, \\
 H_{12}(3) &= 0.6829, & H_{12}(4) &= 0.6907, \\
 H_{13}(1) &= 0.6890, & H_{13}(2) &= \ln 2, \\
 H_{13}(3) &= 0.6730, & H_{13}(4) &= 0.6920.
 \end{aligned} \tag{18}$$

So

$$\begin{aligned}
 H_{12} &= \frac{1}{4 \ln 2} (0.6870 + 0.6518 + 0.6829 + 0.6907) = 0.9783 \\
 H_{13} &= \frac{1}{4 \ln 2} (0.6890 + \ln 2 + 0.6730 + 0.6920) = 0.9908.
 \end{aligned} \tag{19}$$

Then

$$H_{12} < H_{13}. \tag{20}$$

So the relative nearness incidence of X_1 and X_2 is weaker than the relative nearness incidence of X_1 and X_3 .

In order to compare the relative nearness incidence of sequences with the relative degree of grey incidence of sequences, we put the data of Example 2 into the formula of relative degree of grey incidences. It is easy to know that $\gamma_{12} = 0.8425$ and $\gamma_{13} = 0.9844$. So

$$\gamma_{12} < \gamma_{13}. \tag{21}$$

Then the relative degree of grey incidences of X_1 and X_2 is weaker than the relative degree of grey incidences of X_1 and X_3 . Therefore, the relative nearness incidence of sequences and the relative degree of grey incidences of sequences are harmonious.

Example 2. Suppose $f_1(x) = x$, $f_2(x) = x^2$, $f_3(x) = x^3$, where $x \in [0.5, 2]$. As seen in the above, we know

$$\begin{aligned}
 H_{12}(x_0) &= -\left(\frac{x_0}{x_0 + x_0^2} \ln \frac{x_0}{x_0 + x_0^2} + \frac{x_0^2}{x_0 + x_0^2} \ln \frac{x_0^2}{x_0 + x_0^2} \right) \\
 H_{13}(x_0) &= -\left(\frac{x_0}{x_0 + x_0^3} \ln \frac{x_0}{x_0 + x_0^3} + \frac{x_0^3}{x_0 + x_0^3} \ln \frac{x_0^3}{x_0 + x_0^3} \right).
 \end{aligned} \tag{22}$$

Then

$$\begin{aligned}
 H_{12}(x) &= \frac{1}{(2-0.5) \ln 2} \int_{0.5}^2 H_{12}(x_0) dx_0 \\
 &= -\frac{1}{(2-0.5) \ln 2} \\
 &\quad \times \int_{0.5}^2 \left(\frac{x_0}{x_0 + x_0^2} \ln \frac{x_0}{x_0 + x_0^2} + \frac{x_0^2}{x_0 + x_0^2} \ln \frac{x_0^2}{x_0 + x_0^2} \right) dx_0 \\
 &= 0.9705
 \end{aligned}$$

$$\begin{aligned}
H_{13}(x) &= \frac{1}{(2-0.5)\ln 2} \int_{0.5}^2 H_{13}(x_0) dx_0 \\
&= -\frac{1}{(2-0.5)\ln 2} \\
&\quad \times \int_{0.5}^2 \left(\frac{x_0}{x_0+x_0^3} \ln \frac{x_0}{x_0+x_0^3} + \frac{x_0^3}{x_0+x_0^3} \ln \frac{x_0^3}{x_0+x_0^3} \right) dx_0 \\
&= 0.8931.
\end{aligned} \tag{23}$$

Obviously, when $x \in [0.5, 2]$, then $H_{12}(x) > H_{13}(x)$. So the relative nearness incidence of $f_1(x)$ and $f_2(x)$ is stronger than the relative nearness incidence of $f_1(x)$ and $f_3(x)$.

Example 3. Suppose $f_1(x, y) = x^2 + y$, $f_2(x, y) = x + y$, $f_3(x, y) = x^2 + 2y^2$, where $x \in [0.3, 1.5]$, $y \in [0.2, 1.3]$. As seen in the above, we know

$$\begin{aligned}
H_{12}(x_0, y_0) &= - \left(\frac{x_0^2 + y_0}{(x_0^2 + y_0) + (x_0 + y_0)} \ln \frac{x_0^2 + y_0}{(x_0^2 + y_0) + (x_0 + y_0)} \right. \\
&\quad \left. + \frac{x_0 + y_0}{(x_0^2 + y_0) + (x_0 + y_0)} \ln \frac{x_0 + y_0}{(x_0^2 + y_0) + (x_0 + y_0)} \right) \\
H_{13}(x_0, y_0) &= - \left(\frac{x_0^2 + y_0}{(x_0^2 + y_0) + (x_0^2 + 2y_0^2)} \ln \frac{x_0^2 + y_0}{(x_0^2 + y_0) + (x_0^2 + 2y_0^2)} \right. \\
&\quad \left. + \frac{x_0^2 + 2y_0^2}{(x_0^2 + y_0) + (x_0^2 + 2y_0^2)} \ln \frac{x_0^2 + 2y_0^2}{(x_0^2 + y_0) + (x_0^2 + 2y_0^2)} \right).
\end{aligned} \tag{24}$$

Then

$$\begin{aligned}
H_{12}(x, y) &= \frac{1}{(1.5-0.3)(1.3-0.2)\ln 2} \\
&\quad \times \int_{0.2}^{1.3} \int_{0.3}^{1.5} H_{12}(x_0, y_0) dx_0 dy_0 \\
&= -\frac{1}{(1.5-0.3)(1.3-0.2)\ln 2} \\
&\quad \times \int_{0.2}^{1.3} \int_{0.3}^{1.5} \left(\frac{x_0^2 + y_0}{(x_0^2 + y_0) + (x_0 + y_0)} \right. \\
&\quad \times \ln \frac{x_0^2 + y_0}{(x_0^2 + y_0) + (x_0 + y_0)} \\
&\quad + \frac{x_0 + y_0}{(x_0^2 + y_0) + (x_0 + y_0)} \\
&\quad \times \ln \frac{x_0 + y_0}{(x_0^2 + y_0) + (x_0 + y_0)} \left. \right) dx_0 dy_0 \\
&= 0.9937
\end{aligned}$$

$$\begin{aligned}
H_{13}(x, y) &= \frac{1}{(1.5-0.3)(1.3-0.2)\ln 2} \\
&\quad \times \int_{0.2}^{1.3} \int_{0.3}^{1.5} H_{13}(x_0, y_0) dx_0 dy_0 \\
&= -\frac{1}{(1.5-0.3)(1.3-0.2)\ln 2} \\
&\quad \times \int_{0.2}^{1.3} \int_{0.3}^{1.5} \left(\frac{x_0^2 + y_0}{(x_0^2 + y_0) + (x_0^2 + 2y_0^2)} \right. \\
&\quad \times \ln \frac{x_0^2 + y_0}{(x_0^2 + y_0) + (x_0^2 + 2y_0^2)} \\
&\quad + \frac{x_0^2 + 2y_0^2}{(x_0^2 + y_0) + (x_0^2 + 2y_0^2)} \\
&\quad \times \ln \frac{x_0^2 + 2y_0^2}{(x_0^2 + y_0) + (x_0^2 + 2y_0^2)} \left. \right) dx_0 dy_0 \\
&= 0.9769.
\end{aligned} \tag{25}$$

Obviously, when $x \in [0.3, 1.5]$ and $y \in [0.2, 1.3]$, then $H_{12}(x, y) > H_{13}(x, y)$. So the relative nearness incidence of $f_1(x, y)$ and $f_2(x, y)$ is stronger than the relative nearness incidence of $f_1(x, y)$ and $f_3(x, y)$.

6. Conclusion

The model of grey nearness incidence proposed by Liu et al. cannot reflect the nearness degree of sequences correctly. Therefore, the model of relative nearness incidence of sequences, curves, and surfaces is suggested based on the standard distance entropy to remove the current bottlenecks. The proposed models have enriched the theory of grey nearness incidence, filled the defects of grey nearness incidence. However, only two positive numbers can calculate its standard distance entropy. Therefore, there are many defects of relative nearness incidence based on standard distance entropy; our future research will focus on it.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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