Research Article

Unsteady Model for Transverse Fluid Elastic Instability of Heat Exchange Tube Bundle

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From the viewpoint of practical application, based on the unsteady analytical model for transverse fluid elastic instability of tube array proposed by Yetisir and the linear attenuation function introduced by Li Ming, a new explicit model based on nonsteady state “streamtube” hypothesis is proposed and solved using complex number method. In the model, numerical integral is avoided and inappropriate aspects in Li Ming model are modified. Using the model, the fluid elastic instability analysis of a single flexible tube is made. The stability graphs for four typical types of tube array are plotted and contrasted with experimental results. It is found that the current explicit model is effective in the analysis of transverse fluid elastic instability of tube bundle.

1. Introduction

Among the various fluid excitation mechanisms of the tube bundle in heat exchange, fluid elastic instability can cause the most rapid deterioration of the tube supports and has received the most attention [1, 2]. Many efforts have been made to study this phenomenon. However the mechanism of instability is not fully understood. Connors [3] and Afshin [4] successively studied the fluid elastic instability of tube bundle induced by transverse flow with wind tunnel experiments. Based on their studies, a quasistatic model was proposed to describe fluid elastic instability and the judgment formula of critical velocity for single-phase fluid elastic instability was given. Then Blevins [5] proposed another quasistatic model for one-dimensional tube array. These models are just comprehensive analytical solutions with experimental data, not the real analytical solutions. Then Price and Paidoussis [6] and Blevins [7] put forward a quasisteady model in which the fluid influence on displacement of the tubes has been further considered. This model is the simplified form of unsteady flow model and is only suitable for high conversion velocity. Thereafter, based on the general equation of fluid mechanics, an unsteady flow model was proposed by Tanaka et al. [8, 9].

In the model, many parameters for the calculation of fluid elastic force need to be determined by experiment; therefore it was also known as the semiempirical theoretical model. Lever and Weaver [10] proposed a simplified elastic fluid system based on a large number of experimental observations and given analytical expressions of the fluid force used in one-dimension unsteady flow theory. Then Lai [11] used this method to study the dynamic characteristics of a heat exchange tube bundle. Potential function was also proposed by Paidoussis et al. [12] to solve the flow force and calculate the critical velocity. These models are called analytical model due to the advantages that they need less experimental data over semiempirical theory model. Furthermore, the flow force induced by the pipe movement in analytical model was derived by Yetisir and Weaver [13] using unsteady continuous and momentum equations. This method is essentially an unsteady analytical model with relatively few experimental data. The result based on the model is in good agreement with those from experiment. Consequently, many similar models were developed. However, the calculation of the flow force in this kind of model involves numerical integration. Therefore it is not very convenient in practical to use them.

FIGURE 2: Control volume in streamtube.

where $\forall$ denotes control volume (see Figure 2) and $\forall = Ad s$, ρ is the fluid density, and the right item denotes the sum of external forces on the control volume with unit length.

Using the derivation process similar to the above mentioned one, the momentum equation (7) can be further written as follows by substituting formulas (2) and (3) into it:

$$\begin{aligned} & \frac{\omega}{\omega_n} l_0^* U_r \int_{s_i}^{s_e} \frac{\partial u^*(s^*, t^*)}{\partial t^*} ds^* + \frac{\omega}{\omega_n} \frac{l_0^*}{A_0^*} U_r \int_{s_i}^{s_e} \frac{\partial a^*(s^*, t^*)}{\partial t^*} ds^* \\ & + \left[2l_0^* u^*(s_i^*, t^*) + \frac{l_0^*}{A_0^*} a^*(s_i^*, t^*) \right] l_0^* U_r^2 \\ & + \left[2l_0^* u^*(s_e^*, t^*) + \frac{l_0^*}{A_0^*} a^*(s_e^*, t^*) \right] l_0^* U_r^2 \\ & = -p^*(s_i^*, t^*) - p^*(s_e^*, t^*). \end{aligned} \quad (8)$$

It is worth noting that $p^* = p/(\rho d^2 \omega_n^2)$ which is not mentioned in Li's model. This is probably the main reason which leads to no density (ρ) existence in the final fluid force expression in Li's model.

Many studies have proved the fact that fluid elastic instability usually first occurs in the lift direction. Therefore, in the following part, only the fluid force calculation and fluid elastic analysis in the lift direction are made.

The force on the centre flexible tube can be obtained by integrating the pressure item on the pipe surface attached with fluid ($s_a^* \leq s \leq s_s^*$). Consider

$$F_y^* = \int_{s_a^*}^{s_s^*} p^*(s^*, t^*) \cos[\beta(s^*)] ds^*, \quad (9)$$

where $\beta(s^*)$ is the angle of the normal vector of the tube surface and $\beta(s^*) = 2s^*/Pr$. More detail about $\beta(s^*)$ is shown in literature [13].

2.3. Dynamic Equation. It is assumed that flexible tube vibrates in the lift and drags directions with the same harmonic oscillation

$$m\ddot{y} + c\dot{y} + ky = F_y a_1(s_m, t) - F_y a_2(s_m, t), \quad (10)$$

where $a_1(s_m, t)$ and $a_2(s_m, t)$, respectively, denote the area perturbations at the minimum clearances of streamtube 1 and streamtube 2.

The displacement mode of the centre pipe can be defined as follows:

$$y(t) = Y e^{i\omega t}, \quad (11)$$

where ω is vibration complex number frequency and Y is maximum vibration amplitude.

Substituting formula (11) into formula (10) and introducing the dimensionless variables, $y^* = Y/d$, $\bar{F}_y = F_y/\rho d^3 \omega_n$ can yield the following equation:

$$\begin{aligned} & \frac{m}{\rho d^2} \left[-\left(\frac{\omega}{\omega_n}\right)^2 + \frac{\delta}{\pi} \left(\frac{\omega}{\omega_n}\right) i + 1 \right] y^* \\ & = F_y^* a_1^*(s_m, t) - F_y^* a_2^*(s_m, t). \end{aligned} \quad (12)$$

Formula (12) can be further transformed as follows:

$$\frac{m}{\rho d^2} = \frac{F_y^*(U_r, \omega/\omega_n)}{1 - (\omega/\omega_n)^2 + i(\delta/\pi)(\omega/\omega_n)}, \quad (13a)$$

$$F_y^* = 2l_0^* U_r \cos \alpha F_y^*(s) = F_R^* \left(U_r, \frac{\omega}{\omega_n} \right) + i F_I^* \left(U_r, \frac{\omega}{\omega_n} \right), \quad (13b)$$

where m is the quality of the tube with unit length (including added fluid mass), ρ is fluid density, δ is damping logarithmic attenuation rate, F_y^* is the fluid resultant force applied on the unit length tube in the lift direction and is the function of frequency ratio ω/ω_n , and reduced speed $U_r = U_0/\omega_n l_0$. It is noted that the fluid resultant force F_y^* is a complex number and can be written as (13b) where F_R^* , F_I^* , respectively, represent real part and imaginary part.

According to the relation among the array geometry, fluid velocity at minimum interval U_0 , interval fluid velocity $U_p = [Pr/(Pr-1)]U_\infty$ (U_∞ is free flow velocity, Pr is tube pitch ratio, and $Pr = P/D$), and fluid continuity equation, formula (14) can be obtained:

$$U_r = \frac{U_p}{f_n D} \frac{1}{l_0^* A_0^*} \frac{\cos \alpha}{2\pi} (Pr-1). \quad (14)$$

2.4. Area Perturbation Equation. In order to obtain the explicit expression for the calculation of fluid force, the following linear attenuation function [14] is used, according to the requirement on it. For the convenience of introduction, superscript “*” will be ignored in the following derivation process:

$$\begin{aligned} f(s) &= \frac{s-s_1}{s_a-s_1}, \quad s_1 \leq s \leq s_a, \\ f(s) &= 1, \quad s_a \leq s \leq s_s. \end{aligned} \quad (15)$$

Noting the fact that perturbation attenuation only takes place on “streamtube” path, two area perturbation equations for parallel triangular tube array are presented as follows:

$$\begin{aligned} a_1(s, U_r) &= f(s) e^{i\varphi(s, U_r)} \cos \alpha, \\ \varphi(s, U_r) &= \varphi(U_r) \frac{s - s_a}{s_1 - s_a}, \\ s_1 &\leq s \leq s_a, \\ a_2 &= \cos \alpha, \quad f(s) = 1.0, \quad \varphi(s, U_r) = 0, \\ s_a &\leq s \leq s_s. \end{aligned} \quad (16)$$

For other types of tube array, area perturbation equations are similar as the above equations except for the item $\cos \alpha$.

2.5. Solving Method. For the convenience of derivation, let $\bar{s} = (s - s_a)/(s_1 - s_a)$, so that attenuation function and (16) can be rewritten, respectively, as follows:

$$\begin{aligned} f(s) &= 1 - \bar{s}, \\ a_1(s, U_r) &= a(-s_m, t) f(s) e^{i\varphi(s)} \\ &= \frac{Y}{D} e^{i\omega t} \cos \alpha (1 - \bar{s}) e^{i\bar{s}\varphi(U_r)}, \\ s_1 &\leq s \leq s_a, \\ a_2 &= \frac{Y}{D} e^{i\omega t} \cos \alpha, \quad f(s) = 1.0, \quad \varphi(s, U_r) = 0, \\ s_a &\leq s \leq s_s. \end{aligned} \quad (17)$$

To further facilitate the derivation, $\varphi(U_r)$ is briefly written as φ in the following work.

Substituting (17) into (6) yields the following expression:

$$u_1(s, U_r) = (A_1 e^{i\varphi\bar{s}} + A_2 \bar{s} e^{i\varphi\bar{s}} + A_3) \frac{Y}{D} e^{i\omega t} \cos \alpha, \quad (18)$$

where

$$\begin{aligned} A_1 &= -\frac{1}{A_0} - \frac{\omega_r}{l_0 A_0 U_r} (s_l - s_a) \left(\frac{1}{\varphi} - \frac{i}{\varphi^2} \right), \\ A_2 &= \frac{1}{A_0} + \frac{\omega_r}{l_0 A_0 U_r} (s_l - s_a) \frac{1}{\varphi}, \\ A_3 &= -\frac{\omega_r i}{l_0 A_0 U_r} (s_1 - s_a) \frac{1}{\varphi^2} e^{i\varphi}. \end{aligned} \quad (19a)$$

In the same way we have

$$\begin{aligned} u_2(s, U_r) &= (A_4 \bar{s} + A_5) \frac{Y}{D} e^{i\omega t} \cos \alpha, \\ 0 \leq \bar{s} &\leq \frac{s_s - s_a}{s_1 - s_a}, \end{aligned} \quad (19b)$$

where

$$\begin{aligned} A_4 &= -\frac{\omega_r i}{l_0 A_0 U_r} (s_l - s_a), \\ A_5 &= -\frac{\omega_r}{l_0 A_0 U_r} (s_l - s_a) \left(\frac{1}{\varphi} - \frac{i}{\varphi^2} + \frac{i}{\varphi^2} e^{i\varphi} \right) - \frac{1}{A_0}. \end{aligned} \quad (20)$$

Substituting (18) and (19b) into the integral expression for unsteady pressure (8) yields the following formula:

$$p_2(s, U_r) = (C_1 \bar{s}^2 + C_2 \bar{s} + C_3) \frac{Y}{D} e^{i\omega t} \cos \alpha, \quad (21)$$

where

$$\begin{aligned} C_1 &= -\omega_r l_0 U_r i B_2, \\ C_2 &= -\omega_r l_0 U_r i B_3 - 2l_0^2 U_r^2 A_4 - \frac{\omega_r l_0 U_r i}{A_0} (s_1 - s_a), \\ C_3 &= -\omega_r l_0 U_r i B_1 - 2l_0^2 U_r^2 A_5 - \frac{l_0^2 U_r^2}{A_0} - \frac{\omega_r l_0 U_r}{A_0} B_4, \end{aligned} \quad (22)$$

where

$$\begin{aligned} B_1 &= (s_1 - s_a) \\ &\times \left(\frac{-iA_1}{\varphi} + \frac{iA_1}{\varphi} e^{i\varphi} + \frac{iA_2}{\varphi} e^{i\varphi} + \frac{A_2}{\varphi^2} - \frac{A_2}{\varphi^2} e^{i\varphi} - A_3 \right), \\ B_2 &= \frac{(s_1 - s_a) A_4}{2}, \\ B_3 &= (s_1 - s_a) A_5, \\ B_4 &= (s_1 - s_a) \left(-\frac{i}{\varphi} - \frac{1}{\varphi^2} + \frac{1}{\varphi^2} e^{i\varphi} \right). \end{aligned} \quad (23)$$

The explicit expression for the fluid force applied on the centre moving tube is obtained by substituting formula (21) into formula (9). Consider

$$\begin{aligned} F_y &= \frac{Y}{D} e^{i\omega t} \cos \alpha \int_{s_a}^{s_s} p_2(s, U_r) \cos \left(\frac{2s}{\text{Pr}} \right) ds \\ &= \frac{Y}{D} e^{i\omega t} \cos \alpha \int_{s_a}^{s_s} (C_1 \bar{s}^2 + C_2 \bar{s} + C_3) \cos \left(\frac{2s}{\text{Pr}} \right) ds \\ &= \frac{Y}{D} e^{i\omega t} \cos \alpha \sum_{i=1}^3 C_i D_i, \end{aligned} \quad (24)$$

TABLE 1: Numerical values of geometric parameters for test arrays [1].

Layout pattern	P/d	δ	ρ (kg/m ³)	A_0/d	s_0/d	s_1/d	l_0/d	x_1/d
Square (90°)	1.25	0.012	1.112	0.25	1.25	1.25	5.00	0.05
	1.32	0.015	1.125	0.32	1.32	1.32	5.28	0.05
	1.41	0.010	1.110	0.41	1.41	1.41	5.64	0.05
Rotated triangular (60°)	1.25	0.024	1.124	0.25	0.654	0.982	2.616	0.104
	1.32	0.021	1.110	0.32	0.691	1.037	2.764	0.0985
	1.41	0.030	1.110	0.41	0.738	1.107	2.952	0.092
Rotated square (45°)	1.25	0.019	1.110	0.25	0.981	1.472	3.925	0.155
	1.41	0.015	1.110	0.41	1.107	1.660	4.427	0.137
Triangular (30°)	1.25	0.014	1.110	0.125	1.249	1.874	4.996	0.203
	1.41	0.018	1.110	0.205	1.371	2.057	5.484	0.181

where

$$\begin{aligned}
D_1 &= \left(\frac{1}{s_1 - s_a} \right)^2 \left(\frac{2s}{b^2} \cos bs - \frac{2}{b^3} \sin bs + \frac{s^2}{b} \sin bs \right) \Big|_{s_a}^{s_s} \\
&\quad - \frac{2s_a}{(s_1 - s_a)^2} \left(\frac{1}{b^2} \cos bs + \frac{s}{b} \sin bs \right) \Big|_{s_a}^{s_s} \\
&\quad + \left(\frac{s_a}{s_1 - s_a} \right)^2 \frac{1}{b} \sin bs \Big|_{s_a}^{s_s}, \\
D_2 &= \frac{1}{s_1 - s_a} \left(\frac{1}{b^2} \cos bs + \frac{s}{b} \sin bs \right) \Big|_{s_a}^{s_s} \\
&\quad - \frac{s_a}{s_1 - s_a} \frac{1}{b} \sin bs \Big|_{s_a}^{s_s}, \\
D_3 &= \int_{s_a}^{s_s} \cos \left(\frac{2s}{Pr} \right) ds \\
&= \int_{s_a}^{s_s} \cos (bs) ds = \frac{1}{b} \sin bs \Big|_{s_a}^{s_s},
\end{aligned} \tag{25}$$

where

$$a = \frac{\varphi}{s_1 - s_a}, \quad b = \frac{2}{Pr}. \tag{26}$$

In formula (24), F_y is fluid force with complex number form and can be written as $F_y = F_R + F_I i$. By substituting the fluid force into formula (13a), the relation between the reduced fluid velocity $U_p/f_n D$ and quality-damping ratio $m\delta/\rho D^2$ can be obtained.

3. Example

Using the above explicit unsteady model, fluid elastic instability analyses on four standard tube arrays are carried out and the result is compared with those from experiment in order to verify the validity of the model. Geometric parameters of the four types of tube arrays are listed in Table 1. The diameter

and first order natural frequency of the tubes in experiment are, respectively, 0.022 m and 90 Hz.

The results from the explicit analytic method and experiment are presented in Figure 3 where vertical coordinate denotes reduced fluid velocity and horizontal coordinate denotes quality-damping parameter. It is shown in the figure that for the four typical tube arrays, the results from the explicit analytic method are consistent with those from Nie's experiment [1]. For $m\delta/\rho D^2 > 5$, the results based on current method tend to be more conservative than the result of experiment, whereas, for $m\delta/\rho D^2 < 5$, the result of experiment is more conservative. But the difference between the two types of results is not great. The influence of the pitch ratio (P/d) on the result is not obvious. This is consistent with the observed phenomenon by Lever and Weaver [10].

It is shown in the above analysis that the current explicit analytic method inherits the advantage of Y-W "streamtube" model which possesses good calculation precision and needs little experiment data. Moreover, comparing with Y-W model, the unsteady explicit model presented in this paper has another advantage that no numerical integral is needed in the calculation of fluid force due to its explicit expression. Therefore, the current method is more convenient for practical application.

4. Conclusion

This paper introduces an unsteady model in detail for fluid elastic instability analysis of standard array configuration in transverse flow. Complex number solving method is used to deduce the unsteady explicit analytical model with new area perturbation equation. In the model, numerical integration similar to Y-W model is avoided and inappropriate parameters presented in Li's model are corrected. Finally the explicit expression of the model which is convenient for application is presented.

Fluid elastic instability analysis on four standard array configurations is made and the stability maps based on current model are given. A comparison of the result between the current model and those from Nie's experiment data is

FIGURE 3: Stability maps for typical tube arrays (solid black points represent Nie's experiment result and black solid lines represent the result in this paper).

made to indicate the validity of current explicit fluid elastic analytical model.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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