A hesitant fuzzy linguistic term set (HFLTS), allowing experts using several possible linguistic terms to assess a qualitative linguistic variable, is very useful to express people's hesitancy in practical decision-making problems. Up to now, little research has been done on the comparison and distance measure of HFLTSs. In this paper, we present a comparison method for HFLTSs based on pairwise comparisons of each linguistic term in the two HFLTSs. Then, a distance measure method based on the pairwise comparison matrix of HFLTSs is proposed, and we prove that this distance is equal to the distance of the average values of HFLTSs, which makes the distance measure much more simple. Finally, the pairwise comparison and distance measure methods are utilized to develop two multicriteria decision-making approaches under hesitant fuzzy linguistic environments. The result analysis shows that our methods in this paper are more reasonable.

1. Introduction

Since Zadeh introduced fuzzy sets [1] in 1965, several extensions of this concept have been developed, such as type-2 fuzzy sets [2, 3] and interval type-2 fuzzy sets [4], type-n fuzzy sets [5], intuitionistic fuzzy sets [6, 7] and interval-valued intuitionistic fuzzy sets [8], vague sets [9] (vague sets are intuitionistic fuzzy sets [10]), fuzzy multisets [11, 12], nonstationary fuzzy sets [13], Cloud models [14–18] (Cloud models are similar to nonstationary fuzzy sets and type-2 fuzzy sets), and hesitant fuzzy sets [19, 20]. In the real world, there are many situations in which problems must deal with qualitative aspects represented by vague and imprecise information. So, in these situations, often the experts are more accustomed to express their assessments using linguistic terms rather than numerical values. In [21–23], Zadeh introduced the concept of linguistic variable as "a variable whose values are not numbers but words or sentences in a natural or artificial language." Linguistic variable provides a means of approximate characterization of phenomena which are too complex or too ill defined to be amenable to description in conventional quantitative ways. Since then, fuzzy sets and linguistic variables have been widely used in describing linguistic information as they can efficiently represent people's qualitative cognition of an object or a concept [24]. Thus, linguistic approaches have been so far used successfully in a wide range of applications, such as information retrieval [25–28], data mining [29], clinical diagnosis [30, 31], and subjective evaluation [32–37], especially in decision-making [38–49]. Usually, linguistic terms (words) are represented by fuzzy sets [50], type-2 fuzzy sets [51], interval type-2 fuzzy sets [52–54], 2-tuple linguistic model [40, 55], and so forth. In these linguistic models, an expert generally provides a single linguistic term as an expression of his/her knowledge. However, just as Rodriguez et al. [56] pointed out, the expert may think of several terms at the same time or look for a more complex linguistic term that is not defined in the linguistic term set to express his/her opinion. In order to cope with this situation, they recently introduced the concept of hesitant fuzzy linguistic term sets (HFLTSs) [56] under the idea of hesitant fuzzy sets introduced in [19, 20].

Similarly to a hesitant fuzzy set which permits the membership having a set of possible values, an HFLTS allows
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an expert hesitating among several values for a linguistic variable. For example, when people assess a qualitative criterion, they prefer to use a linguistic one such as "between medium and very high" which contains several linguistic terms \{medium, high, very high\}, rather than a single linguistic term. In practical decision-making process, uncertainty and hesitancy are usually unavoidable problems. The HFLTSs can deal with such uncertainty and hesitancy more objectively, and thus it is very necessary to develop some theories about HFLTSs.

Comparisons and distance measures used for measuring the deviations of different arguments are fundamentally important in a variety of applications. In the existing literature, there are a number of studies on distance measures for intuitionistic fuzzy sets [57–60], interval-valued intuitionistic fuzzy sets [61], hesitant fuzzy sets [62, 63], linguistic values [64, 65], and so forth. Nevertheless, an HFLTS is a linguistic term subset, and the comparison among these elements is not simple. In [56], Rodriguez et al. introduced the concept of envelope for an HFLTS and then ranked HFLTSs using the distance measure of HFLTSs [67]. Consequently, it is necessary to develop some comparison methods and ranking methods which are based on the comparisons and distance measures of HFLTSs. Up to now, just a few research has been done on the distance measure of HFLTSs [67]. Consequently, it is very necessary to develop some comparison methods and distance measure methods for HFLTSs. In [67], to calculate the distance of two HFLTSs, Liao et al. extend the shorter HFLTS by adding any value in it until it has the same length of the longer one according to the decision-maker's preferences and actual situations. In this paper, we present a new comparison method of HFLTSs based on pairwise comparisons of each linguistic term in the two HFLTSs. Then, a distance measure method based on the pairwise comparison matrix of HFLTSs is proposed without adding any value. Finally, we utilize the comparison method and distance measure method to develop some approaches to solve the multicriteria decision-making problems under hesitant fuzzy linguistic environments.

The rest of the paper is organized as follows. In Section 2, the concepts of hesitant fuzzy sets and HFLTSs are introduced; also the defects of the previous comparison method for HFLTSs are analyzed according to an example. Section 3 describes the comparison and distance measure of HFLTSs based on the proposed pairwise comparison method. In Section 4, a multicriteria decision-making problem is shown to illustrate the detailed processes and effectiveness of two ranking methods which are based on the comparisons and distance measures of HFLTSs, respectively. Finally, Section 5 draws our conclusions and presents suggestions for future research.

2. Preliminaries

2.1. Hesitant Fuzzy Sets. Hesitant fuzzy sets (HFSs) were first introduced by Torra [19] and Torra and Narukawa [20]. The motivation is that when determining the membership degree of an element into a set, the difficulty is not because we have a margin of error (such as an interval) but because we have several possible values.

Definition 1 (see [19]). Let \( X \) be a fixed set; a hesitant fuzzy set (HFS) on \( X \) is in terms of a function \( h \) that when applied to \( X \) returns a subset of \([0, 1]\).

To be easily understood, Zhu et al. [68] represented the HFS as the following mathematical symbol:

\[
E = \{(x, h(x)) \mid x \in X\},
\]  

(1)

where \( h(x) \) is a set of some values in \([0, 1]\), denoting the possible membership degrees of the element \( x \in X \) to the set \( E \). Liao et al. [67] called \( h(x) \) a hesitant fuzzy element (HFE).

Example 2. Let \( h = \{0.2, 0.3, 0.4\} \); then \( h \) is an HFE.

Definition 3 (see [69]). For an HFE \( h \), the score function of \( h \) is defined as

\[
s(h) = \frac{1}{\#h} \sum_{y \in h} y,
\]  

(2)

where \( \#h \) is the number of the elements in \( h \).

For two HFEs \( h_1 \) and \( h_2 \), if \( s(h_1) > s(h_2) \), then \( h_1 \) is superior to \( h_2 \), denoted by \( h_1 \succ h_2 \); if \( s(h_1) = s(h_2) \), then \( h_1 \) is indifferent with \( h_2 \), denoted by \( h_1 \sim h_2 \).

Example 4. Assume that we have three HFEs, \( h_1 = \{0.2, 0.3, 0.4\}, h_2 = \{0.2, 0.35, 0.5\}, \) and \( h_3 = \{0.3, 0.4\} \); then according to the score function of HFE, (2), and Definition 3, we have \( s(h_1) = (0.2 + 0.3 + 0.4)/3 = 0.3 \), \( s(h_2) = (0.2 + 0.35 + 0.5)/3 = 0.35 \), and \( s(h_3) = (0.3 + 0.4)/2 = 0.35 \). Thus, \( s(h_2) > s(h_3) > s(h_1) \); that is, the ranking is \( h_2 \sim h_3 \succ h_1 \).

The concept of HFS is very useful to express people's hesitancy in daily life. So, since it was introduced, more and more decision-making theories and methods under hesitant fuzzy linguistic environment have been developed [56, 62, 63, 67–73].

2.2. Hesitant Fuzzy Linguistic Term Sets. Similarly to the HFS, an expert may hesitate among several linguistic terms, such as “between medium and very high” or “lower than medium,” to assess a qualitative linguistic variable. To deal with such situations, Rodriguez et al. [56] introduced the concept of hesitant fuzzy linguistic term sets (HFLTSs).

Definition 5 (see [56]). Suppose that \( S = \{s_0, \ldots, s_g\} \) is a finite and totally ordered discrete linguistic term set, where \( s_i \) represents a possible value for a linguistic variable. An HFLTS, \( H_S \), is defined as an ordered finite subset of the consecutive linguistic terms of \( S \).

It is required that the linguistic term set \( S \) should satisfy the following characteristics:

1. the set is ordered: \( s_i \succ s_j \), if and only if \( i > j \);
2. there is a negation operator: \( \text{Neg}(s_i) = s_{g-i} \).
Example 6. Let $S$ be a linguistic term set, $S = \{s_0 : n \text{ (nothing)}, s_1 : vl \text{ (very low)}, s_2 : l \text{ (low)}, s_3 : m \text{ (medium)}, s_4 : h \text{ (high)}, s_5 : p \text{ (perfect)}\};$ two different HFLTSs might be $H^1_S = \{s_1 : vl, s_2 : l, s_3 : m\}$ and $H^2_S = \{s_3 : m, s_4 : h\}.$

Definition 7. One defines the number of linguistic terms in the HFLTS $H_S$ as the cardinality of $H_S$, denoted by $|H_S|$. In Example 6, $|H^1_S| = 3$ and $|H^2_S| = 2$.

Definition 8 (see [56]). The lower bound $H_{\leq}$ and upper bound $H_{\geq}$ of the HFLTS $H_S$ are defined as $H_{\leq} = \min\{|s_i| \mid s_i \in H_S\}$ and $H_{\geq} = \max\{|s_i| \mid s_i \in H_S\}$.

Definition 9 (see [56]). The envelope of the HFLTS $H_S$, $env(H_S)$, is defined as the linguistic interval $[\text{Ind}(H_{\leq}), \text{Ind}(H_{\geq})]$, where Ind provides the index of the linguistic term; that is, $\text{Ind}(s_j) = i$. In Example 6, $env(H^1_S) = [s_1, s_3] = [1, 3]$ and $env(H^2_S) = [s_2, s_4] = [3, 4]$.

Based on the definition of envelope, Rodriguez et al. [56] compare two HFLTSs using the comparison method between two numerical intervals introduced by Wang et al. [66].

Definition 10 (see [66]). Letting $A = [a_1, a_2]$ and $B = [b_1, b_2]$ be two preferences, the degree of preference $D(A > B)$ over $B$ (or $A > B$) is defined as

$$P(A > B) = \frac{\max(0, a_2 - b_1) - \max(0, a_1 - b_2)}{(a_2 - a_1) + (b_2 - b_1)}$$

and the preference degree of $B$ over $A$ (or $A > B$) is defined as

$$P(B > A) = \frac{\max(0, b_2 - a_1) - \max(0, b_1 - a_2)}{(a_2 - a_1) + (b_2 - b_1)}.$$

Example 11. Let $H^1_S = \{s_1, s_2, s_3\}$, $H^2_S = \{s_3, s_4\}$, and $H^3_S = \{s_3, s_6\}$ be three different HFLTSs on $S$. According to Definition 9, we have $env(H^2_S) = [1, 3]$, $env(H^2_S) = [3, 4]$, and $env(H^3_S) = [5, 6]$. The preference degrees calculated by Definition 10, (3), and (4) are

$$P(H^1_S > H^2_S) = 0, \quad P(H^2_S > H^1_S) = 1;$$
$$P(H^1_S > H^3_S) = 0, \quad P(H^3_S > H^1_S) = 1;$$
$$P(H^2_S > H^3_S) = 0, \quad P(H^3_S > H^2_S) = 1.$$

From Example 11 mentioned above, it can be observed that when we compare two HFLTSs using the preference degree method, there exist two defects as follows:

1. The result $P(H^2_S > H^1_S) = 1$ indicates that $H^2_S$ is absolutely superior to $H^1_S$. In fact, both $H^1_S$ and $H^2_S$ contain the linguistic term $s_3$. It means that the value of a linguistic variable may be equal in these two cases. Thus, it is unreasonable to say that $H^2_S$ is absolutely superior to $H^1_S$.

2. The result $P(H^2_S > H^3_S) = 1$ means that when compared with $H^2_S$, the two HFLTSs $H^1_S$ and $H^3_S$ are identical. In fact, $H^3_S$ is more superior to $H^1_S$ compared to $H^2_S$ to $H^2_S$. Thus, using the preference degree method to compare HFLTSs may result in losing some important information.

Based on the analysis mentioned above, we think that it is not suitable to compare discrete linguistic terms in HFLTSs using the comparison method for continuous numerical intervals. By the definition of an HFLTS, we know that every linguistic term in it is a possible value of the linguistic information. And noting that, the two HFLTSs for comparing may have different lengths. So, when comparing two HFLTSs, it needs pairwise comparisons of each linguistic term in them.

3. Comparison and Distance Measure of HFLTSs

3.1. Distance between Two Single Linguistic Terms. Let $s_i, s_j \in S$ be two linguistic terms. Xu [64] defined the deviation measure between $s_i$ and $s_j$ as follows:

$$d(s_i, s_j) = \frac{|i - j|}{T},$$

where $T$ is the cardinality of $S$; that is, $T = |S|$.

If only one preestablished linguistic term set $S$ is used in a decision-making model, we can simply consider [49, 65]:

$$d(s_i, s_j) = |i - j| = s_{|i-j|}.$$

Definition 12. Letting $s_i, s_j \in S$ be two single linguistic terms, then we call

$$d(s_i, s_j) = s_i - s_j = i - j$$

the distance between $s_i$ and $s_j$.

The distance measure between $s_i$ and $s_j$ has a definite physical implication and reflects the relative position and distance between $s_i$ and $s_j$. If $d(s_i, s_j) = 0$, then $s_i = s_j$. If $d(s_i, s_j) > 0$, then $s_i > s_j$. If $d(s_i, s_j) < 0$, then $s_i < s_j$.

Theorem 13. Letting $s_i, s_j, s_k \in S$ be three linguistic terms, then

1. $d(s_i, s_j) = -d(s_j, s_i)$;
2. $\langle|S| - 1\rangle \leq d(s_i, s_j) \leq \langle|S| - 1\rangle$;
3. $d(s_i, s_k) = d(s_i, s_j) + d(s_j, s_k)$.

Proof. They are straightforward and thus omitted.

3.2. Comparison of HFLTSs. The comparison of HFLTSs is necessary in many problems, such as ranking and selection. However, an HFLTS is a linguistic term subset which contains several linguistic terms, and the comparison among HFLTSs is not simple. Here, a new comparison method of HFLTSs, which is based on pairwise comparisons of each linguistic term in the two HFLTSs, is put forward.

Definition 14. Letting $H^1_S$ and $H^2_S$ be two HFLTSs on $S$, then one defines the pairwise comparison matrix between $H^1_S$ and $H^2_S$ as follows:

$$C(H^1_S, H^2_S) = [d(s_i, s_j)]_{|H^1_S| \times |H^2_S|}, \quad s_i \in H^1_S, s_j \in H^2_S.$$
Remark 15. The number of linguistic terms in the two HFLTSs, \( H^1_S \) and \( H^2_S \), may be unequal; that is, \( |H^1_S| \neq |H^2_S| \). To deal with such situations, usually it is necessary to extend the shorter one by adding the stated value several times in it [62, 63], while our pairwise comparison method does not require this step.

Remark 16. From Definition 14, we have \([C(H^1_S, H^2_S)] = -[C(H^1_S, H^2_S)]^T\), where \( T \) is the transpose operator of matrix.

Example 17. Let \( H^1_S = \{s_1, s_2, s_3\} \) and \( H^2_S = \{s_2, s_3, s_4, s_5\} \) be two HFLTSs on \( S \). According to Definition 14, the comparison matrix \( C \) between \( H^1_S \) and \( H^2_S \) is

\[
\begin{bmatrix}
 s_2 & s_3 & s_4 & s_5 \\
 -1 & -2 & -3 & -4 \\
 0 & -1 & -2 & -3 \\
 1 & 0 & -1 & -2 \\
\end{bmatrix}.
\]

Definition 18. Letting \( C = C(H^1_S, H^2_S) \) be the pairwise comparison matrix between \( H^1_S \) and \( H^2_S \), the preference relations of \( H^1_S \) and \( H^2_S \), the preference relations of \( H^1_S \) and \( H^2_S \) are defined as follows:

\[
p(H^1_S > H^2_S) = \frac{\sum_{mn} C_{mn}}{\sum_{mn} C_{mn} + |S| - 1},
\]

\[
p(H^1_S = H^2_S) = \frac{\sum_{mn} C_{mn}}{|S| - 1 + \sum_{mn} C_{mn}},
\]

\[
p(H^1_S < H^2_S) = \frac{\sum_{mn} C_{mn}}{|S| - 1 + \sum_{mn} C_{mn}}.
\]

It is obvious that \( p(H^1_S > H^2_S) + p(H^1_S = H^2_S) + p(H^1_S < H^2_S) = 1 \). We say that \( H^1_S \) is superior to \( H^2_S \) with the degree of \( p(H^1_S > H^2_S) \), denoted by \( H^1_S \succ H^2_S \), \( H^1_S = H^2_S \), denoted by \( H^1_S \cong H^2_S \); and \( H^1_S \) is inferior to \( H^2_S \) with the degree of \( p(H^1_S < H^2_S) \), denoted by \( H^1_S \prec H^2_S \).

Considering Example 17, by Definition 18, (10), the preference relations of \( H^1_S \) and \( H^2_S \) were calculated as \( p(H^1_S > H^2_S) = 1/22 \), \( p(H^1_S = H^2_S) = 2/22 \), and \( p(H^1_S < H^2_S) = 19/22 \). Thus, the comparison results are \( H^1_S \succ 1/22 H^2_S \), \( H^1_S \cong 2/22 H^2_S \), and \( H^1_S \prec 19/22 H^2_S \).

3.3. Distance Measure of HFLTSs

Definition 19. Letting \( C = C(H^1_S, H^2_S) \) be the pairwise comparison matrix between \( H^1_S \) and \( H^2_S \), the distance between \( H^1_S \) and \( H^2_S \) is defined as the average value of the pairwise comparison matrix:

\[
d(H^1_S, H^2_S) = \frac{1}{|H^1_S| \times |H^2_S|} \sum_{mn} |C_{mn}|.
\]

Considering Example 17, one has \( d(H^1_S, H^2_S) = (-18)/(3 \times 4) = -1.5 \).

To preserve all the given information, the discrete linguistic term set \( S \) is extended to a continuous term set \( \bar{S} = [s_\alpha | \alpha \in [-q, q]] \), where \( q \) is a sufficiently large positive number. If \( s_\alpha \in S \), then we call \( s_\alpha \) an original linguistic term; otherwise, we call \( s_\alpha \) a virtual linguistic term.

Remark 20. In general, the decision-maker uses the original linguistic terms to express his/her qualitative opinions, and the virtual linguistic terms can only appear in operations.

Definition 21. The average value of an HFLTS \( H_S \) is defined as

\[
\text{Aver} (H_S) = \frac{1}{|H_S|} \sum_{s \in H_S} s_j.
\]

This definition is similar to the score function of an HFE, Definition 3.

Considering Example 17, we have \( \text{Aver}(H^1_S) = s_{1+2+3}/3 = s_2 = 2 \), and \( \text{Aver}(H^2_S) = s_{(2+3+4+5)/4} = s_3 = 3.5 \).

Theorem 22. Letting \( H_S \) be an HFLTS on \( S \), then

\[
0 \leq H_- \leq \text{Aver} (H_S) \leq H_+ \leq (|S| - 1).
\]

Proof. It is straightforward and thus omitted.

Theorem 23. Letting \( H^1_S \) and \( H^2_S \) be two HFLTSs on \( S \), the distance between \( H^1_S \) and \( H^2_S \) defined by the average value of their pairwise comparison matrix is equal to the distance of the two average values of \( H^1_S \) and \( H^2_S \); that is, the distance between \( H^1_S \) and \( H^2_S \) can be easily obtained by

\[
d(H^1_S, H^2_S) = \text{Aver} (H^1_S) - \text{Aver} (H^2_S).
\]
Considering Example 17, we have \( d(H_1^S, H_2^S) = Aver(H_1^S) - Aver(H_2^S) = 2 - 3.5 = -1.5 \).

By Theorem 23, we can easily obtain the following corollary.

**Corollary 24.** Letting \( H_1^S, H_2^S \) and \( H_3^S \) be three HFLTSs on \( S \), then

1. \( (|S| - 1) \leq d(H_1^S, H_2^S) \leq (|S| - 1) \);
2. \( d(H_1^S, H_2^S) = -d(H_2^S, H_1^S) \);
3. \( d(H_1^S, H_3^S) = d(H_1^S, H_2^S) + d(H_2^S, H_3^S) \).

**Proof.** They are straightforward and thus omitted. \( \square \)

If \( d(H_1^S, H_2^S) > 0 \) (or \( Aver(H_1^S) > Aver(H_2^S) \)), then we say that \( H_1^S \) is superior to \( H_2^S \) with the distance of \( d(H_1^S, H_2^S) \), denoted by \( H_1^S \succ H_2^S \); if \( d(H_1^S, H_2^S) = 0 \) (or \( Aver(H_1^S) = Aver(H_2^S) \)), then we say that \( H_1^S \) is indifferent to \( H_2^S \), denoted by \( H_1^S \sim H_2^S \); if \( d(H_1^S, H_2^S) < 0 \) (or \( Aver(H_1^S) < Aver(H_2^S) \)), then we say that \( H_1^S \) is inferior to \( H_2^S \) with the distance of \( d(H_1^S, H_2^S) \), denoted by \( H_1^S \prec H_2^S \).

### 4. Multicriteria Decision-Making Models Based on Comparisons and Distance Measures of HFLTSs

In this section, two new methods are presented for ranking and choice from a set of alternatives in the framework of multicriteria decision-making using linguistic information. One is based on the comparisons and preference relations of HFLTSs and the other is based on the distance measure of HFLTSs. We adopt Example 5 in [56] (Example 25 in our paper) to illustrate the detailed processes of the two methods.

**Example 25** ([see [56]]. Let \( X = \{ x_1, x_2, x_3 \} \) be a set of alternatives, \( C = \{ c_1, c_2, c_3 \} \) a set of criteria defined for each alternative, and \( S = \{ s_0 : n \) (nothing), \( s_1 : v.l. \) (very low), \( s_2 : l \) (low), \( s_3 : m \) (medium), \( s_4 : h \) (high), \( s_5 : v.h. \) (very high), \( s_6 \) : p (perfect)\) the linguistic term set that is used to generate the linguistic expressions. The assessments that are provided in such a problem are shown in Table 1 and they are transformed into HFLTSs as shown in Table 2.

### 4.1. Multicriteria Decision-Making Based on the Comparisons of HFLTSs

**Step 1.** Considering each criterion \( c_i \) (\( i = 1, 2, 3 \)), calculate the preference degrees between all the alternatives \( x_j \) (\( j = 1, 2, 3 \)).

Considering criterion \( c_1 \), \( H_S^{x_1} = \{ s_1, s_2, s_3 \} \), \( H_S^{x_2} = \{ s_2, s_3 \} \) and \( H_S^{x_3} = \{ s_4, s_5, s_6 \} \), so the preference degrees about criterion \( c_1 \) calculated using the comparison method of HFLTSs as described in Section 3.2 are \( p_{c_1}(x_1 > x_2) = 1/7, p_{c_1}(x_1 > x_3) = 2/7, p_{c_1}(x_2 > x_3) = 4/7; p_{c_1}(x_1 > x_3) = 0/27, p_{c_1}(x_1 = x_3) = 0/27, p_{c_1}(x_1 < x_3) = 27/27; p_{c_1}(x_2 > x_3) = 0/15, p_{c_1}(x_2 = x_3) = 0/15, p_{c_1}(x_2 < x_3) = 15/15. \)

**Step 2.** Aggregate the preference relations using the weighted average method: \( p(x_j > x_i) = \text{sum}(w_i \times p_{c_i}(x_j > x_i)), p(x_j = x_i) = \text{sum}(w_i \times p_{c_i}(x_j = x_i)), \) and \( p(x_j < x_i) = \text{sum}(w_i \times p_{c_i}(x_j < x_i)) \), where \( w_i \) is the weight of criterion \( c_i \), and \( \sum w_i = 1 \). In this paper, \( w_i = 1/3, i = 1, 2, 3 \). Thus, the final preference relations are \( (x_1 > x_2) = 15/21, p(x_1 = x_2) = 2/21, p(x_1 < x_2) = 4/21, p(x_1 > x_3) = 1/3, p(x_1 = x_3) = 1/12, p(x_1 < x_3) = 7/12; p(x_2 > x_3) = 1/3, p(x_2 = x_3) = 0, p(x_2 < x_3) = 2/3). \)

**Step 3.** Rank the alternatives using the nondominance choice degree method as described in [56]. From the results of Step 2, it can be easily obtained that

\[
P_D^S = \begin{pmatrix} - & 11/21 & 0 \\ 0 & 0 & - \\ 11/21 & 1 & - \\ 1 & 4 & 3 \\ 0 & 3/4 & - \end{pmatrix}.
\]

Thus, \( \text{NDD}_1 = \min[(1 - 0), (1 - 1/4)] = 3/4, \text{NDD}_2 = \min[(1 - 11/21), (1 - 1/3)] = 10/21, \) and \( \text{NDD}_3 = \min[(1 - 0), (1 - 0)] = 1 \). Finally, the ranking of alternatives is \( x_3 > x_1 > x_2 \).
The authors declare that there is no conflict of interests regarding the publication of this paper.

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