Research Article

Robust Adaptive Exponential Synchronization of Stochastic Perturbed Chaotic Delayed Neural Networks with Parametric Uncertainties

Yang Fang,¹ Kang Yan,¹ and Kelin Li²

¹ School of Automation and Electronic Information, Sichuan University of Science & Engineering, Sichuan 643000, China

² Institute of Nonlinear Science and Engineering Computing, Sichuan University of Science & Engineering, Sichuan 643000, China

Correspondence should be addressed to Kelin Li; llk@suse.edu.cn

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This paper investigates the robust adaptive exponential synchronization in mean square of stochastic perturbed chaotic delayed neural networks with nonidentical parametric uncertainties. A robust adaptive feedback controller is proposed based on Gronwall's inequality, drive-response concept, and adaptive feedback control technique with the update laws of nonidentical parametric uncertainties as well as linear matrix inequality (LMI) approach. The sufficient conditions for robust adaptive exponential synchronization in mean square of uncoupled uncertain stochastic chaotic delayed neural networks are derived in terms of linear matrix inequalities (LMIs). The effect of nonidentical uncertain parameter uncertainties is suppressed by the designed robust adaptive feedback controller rapidly. A numerical example is provided to validate the effectiveness of the proposed method.

1. Introduction

Synchronization of chaotic delayed neural networks has been an intensive topic because of its promising connections with many disciplines, such as image encryption [1], image processing [2], harmonic oscillation generation [3], and secure communications [4–6]. The diverse control schemes have been proposed for the synchronization of chaotic delayed neural networks, for example, adaptive control (see, [7–13]), slide mode control [14, 15], coupling control [16], feedback control [17–21], impulsive control [22–25], and sampled-data control [26–28].

The existence of random uncertainties such as stochastic noise in the electrical circuits design of neural networks possesses an important source in what may change or destroy the synchronization. Therefore, the stochastic effects must be taken into consideration for the synchronization problem of chaotic delayed neural networks. Some works on the synchronization of stochastic perturbed chaotic neural networks have been reported in the literature; see [8, 9, 17, 29, 30] and the references therein. The authors in [29] were concerned with the problem of exponential synchronization

for stochastic jumping chaotic neural networks (SJCNNs) with mixed delays and sector nonlinearities by employed Lyapunov-Krasovskii functional and free-weighting matrix method and proposed a delay-dependent feedback controller with sector nonlinearities to achieve the synchronization in mean square in terms of linear matrix inequalities (LMIs). In [9], Zhu and Cao have derived some novel sufficient conditions achieving complete synchronization of unidirectionally coupled stochastic delayed neural networks by utilizing LaSalle invariant principle of stochastic differential delay equations and the stochastic analysis as well as the adaptive feedback control technique and LMI approach. Li et al. in [30] investigated the synchronization problem of a class of chaotic neural networks with time-varying delays and unbounded distributed delays under stochastic perturbations via Lyapunov-Krasovskii functional, drive-response concept, output coupling with delay feedback, and LMI approach, some sufficient conditions in terms of LMIs ensuring the exponential synchronization of the addressed neural networks are derived.

On the other hand, besides stochastic noise, it is well known that the effects of parametric uncertainties which

may also destroy the stability of the controlled system cannot be ignored in many applications. However, all of the abovementioned works mainly focus on the stochastic perturbed chaotic delayed neural networks without parametric uncertainties. According to the best of our knowledge, there are still few results about the synchronization of stochastic perturbed chaotic delayed neural networks with nonidentical parametric uncertainties. This is the motivation of our research in the present paper.

In this paper, the main aim is to design a robust adaptive feedback controller with the update laws of nonidentical parametric uncertainties and find some sufficient conditions in order to guarantee exponential synchronization in mean square for uncoupled chaotic delayed neural networks with stochastic perturbation and parametric uncertainties. Based on Gronwall's inequality, drive-response concept, adaptive feedback control technique, and linear matrix inequality (LMI) approach, several sufficient conditions in the form of linear matrix inequalities (LMIs) are derived to ensure exponential synchronization in mean square for uncoupled uncertain stochastic chaotic delayed neural networks. In addition, the existence of the desired controller can be validated by MATLAB LMI toolbox efficiently. The significant difference from previous results is that the nonidentical parametric uncertainties are entered into both the connection weight matrix and the delayed connection weight matrix in the drive-response systems. The task for compensating the nonidentical parametric uncertainties can be realized by the designed robust adaptive feedback controller rapidly. Moreover, we have pointed out that the LaSalle invariance principle for stochastic differential delay equation [31, Corollary 3.1] cannot be applied for the stability analysis of stochastic delayed systems without a trivial solution $e(t; 0) = 0$. Finally, a numerical example and its simulation are given to illustrate the usefulness of the given method.

Notation. Let $\mathbb{R}$ denote the set of real numbers, let $\mathbb{R}^+$ denote the set of all nonnegative real numbers, and $\mathbb{R}^n$ and $\mathbb{R}^{n \times m}$ denote the n -dimensional and $n \times m$ dimensional real spaces equipped with the Euclidean norm; $\|z\|$ is the Euclidean norm of the vector z . $\mathbb{N}$ denotes the set of positive integers. For any matrix $X \in \mathbb{R}^{n \times n}$, $X > 0$ denotes that X is a symmetric and positive definite matrix. If X_1 and X_2 are symmetric matrices, then $X_1 \leq X_2$ means that $X_1 - X_2$ is a negative semidefinite matrix. X^T and X^{-1} mean the transpose of X and the inverse of a square matrix. I denotes the identity matrix with appropriate dimensions. Let $\tau > 0$ and $C([- \tau, 0]; \mathbb{R}^n)$ denote the family of all continuous $\mathbb{R}^n$ -valued functions $\xi(\theta)$ on $[- \tau, 0]$ with the norm $\|\xi\| = \sup_{-\tau \leq \theta \leq 0} \|\xi(\theta)\|$. Let $C_{\mathcal{F}_0}^b([- \tau, 0]; \mathbb{R}^n)(t \geq 0)$ denote the family of all $\mathcal{F}_0$ measurable bounded $C([- \tau, 0]; \mathbb{R}^n)$ -valued random variables $\xi = \{\xi(\theta) : -\tau \leq \theta \leq 0\}$, such that $\int_{-\tau}^0 E|\xi(s)|^2 ds < \infty$, where $E\{\cdot\}$ stands for the correspondent expectation operator with respect to the given probability measure $\mathcal{P}$. Let $C^{1,2}(\mathbb{R}^+ \times \mathbb{R}^n; \mathbb{R}^+)$ denote the family of all nonnegative functions $V(t, x)$ on $\mathbb{R}^+ \times \mathbb{R}^n$ which are continuously twice differentiable in x and differentiable in t . The notation $\star$ always denotes the symmetric block in one symmetric matrix.

2. Model Description and Preliminaries

Consider the following chaotic delayed neural networks with time-varying parametric uncertainties:

$$\begin{aligned} dx(t) = & [-Cx(t) + (A + \Delta A_1(t)) f(x(t)) \\ & + (B + \Delta B_1(t)) f(x(t - \tau(t))) + J] dt, \end{aligned} \quad (1)$$

where $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T$ is the neuron state vector, C is a positive diagonal matrix, $A = (a_{ij}) \in \mathbb{R}^{n \times n}$ and $B = (b_{ij}) \in \mathbb{R}^{n \times n}$ are the connection weight matrix and the delayed connection weight matrix, respectively, $\Delta A_1(t), \Delta B_1(t)$ are the time-varying parametric uncertainties, J is the constant input vector, $f(x(\cdot)) = (f_1(x_1(\cdot)), f_2(x_2(\cdot)), \dots, f_n(x_n(\cdot)))^T$ is the nonlinear neuron activation function which describes the behavior in which the neurons respond to each other, and the time delay $\tau(t)$ is bounded and continuously differentiable function; that is, there exist two positive constants τ and μ , such that $0 \leq \tau(t) \leq \tau$ and $0 \leq \dot{\tau}(t) \leq \mu < 1$.

For drive system (1), we construct the following response system with nonidentical parametric uncertainties and stochastic perturbation:

$$\begin{aligned} dy(t) = & [-Cy(t) + (A + \Delta A_2(t)) f(y(t)) \\ & + (B + \Delta B_2(t)) f(y(t - \tau(t))) + u(t) + J] dt \\ & + \sigma(t, y(t) - x(t), y(t - \tau(t)) \\ & - x(t - \tau(t))) d\omega(t), \end{aligned} \quad (2)$$

where $y(t) = (y_1(t), y_2(t), \dots, y_n(t))^T$ is the neuron state vector of the response system; $\omega(t) = (\omega_1(t), \dots, \omega_n(t))^T$ is an n -dimensional Brown motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with a natural filtration $\{\mathcal{F}_t\}_{t \geq 0}$ (i.e., $\mathcal{F}_t = \sigma\{\omega(s) : 0 \leq s \leq t\}$), which satisfies $E\{d\omega(t)\} = 0$ and $E\{[d\omega(t)]^2\} = dt$; $\sigma : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$ is the noise intensity matrix; $u(t)$ is the robust adaptive controller which will be designed to compensate the nonidentical parametric uncertainties.

Let $e(t) = y(t) - x(t)$ be the synchronization error; then we can obtain the error system between systems (1) and (2) as follows:

$$\begin{aligned} de(t) = & [-Ce(t) + (A + \Delta A_2(t)) h(e(t)) \\ & + (B + \Delta B_2(t)) h(e(t - \tau(t))) \\ & + (\Delta A_2(t) - \Delta A_1(t)) f(x(t)) \\ & + (\Delta B_2(t) - \Delta B_1(t)) f(x(t - \tau(t))) + u(t)] dt \\ & + \sigma(t, e(t), e(t - \tau(t))) d\omega(t), \end{aligned} \quad (3)$$

where $h(e(\cdot)) = f(e(\cdot) + x(\cdot)) - f(x(\cdot))$.

Throughout this paper, we assume that

(H1) [32] each neuron activation function $f_i : \mathbb{R} \rightarrow \mathbb{R}$ is continuous with $f_i(0) = 0$ ($i = 1, 2, \dots, n$), and there exist positive scalars l_i , such that

$$|f_i(a) - f_i(b)| \leq l_i |a - b|, \quad (4)$$

for any $a, b \in \mathbb{R}$, $a \neq b$, and $L = \text{diag}(l_1, l_2, \dots, l_n)$;

(H2) the noise matrix $\sigma(\cdot)$ is local Lipschitz continuous and satisfies the linear growth condition as well, and $\sigma(t, 0, 0) = 0$. Moreover, there exist two real positive $n \times n$ matrices Q_1, Q_2 , such that

$$\text{trace} [\sigma^T(t, u, v) \sigma(t, u, v)] \leq u^T Q_1 u + v^T Q_2 v, \quad (5)$$

for all $u, v \in \mathbb{R}^n$;

(H3) the parametric uncertainties $\Delta A_j(t), \Delta B_j(t)$, $j = 1, 2$ are in terms of

$$\Delta A_j(t) = E_j F_j(t) H_j, \quad \Delta B_j(t) = E'_j F'_j(t) H'_j, \quad (6)$$

where E_j, E'_j, H_j , and H'_j are known real constant matrices with appropriate dimensions and $F_j(t)$ and $F'_j(t)$ are the uncertain matrices which are Lebesgue measurable in t and satisfy $F_j^T(t) F_j(t) \leq I$ and $F_j'^T(t) F'_j(t) \leq I$;

(H4) there exist unknown constants $\bar{\alpha} > 0, \bar{\beta} > 0$, such that

$$\|\Delta A_2(t) - \Delta A_1(t)\| \leq \bar{\alpha}, \quad \|\Delta B_2(t) - \Delta B_1(t)\| \leq \bar{\beta}. \quad (7)$$

Remark 1. Considering Hypothesis (H1) and the boundedness of the chaotic signals, one can get that there exist positive constants $l = \max_{1 \leq i \leq n} l_i$ and $\|x\| \leq \chi$, such that

$$\|f(x(t))\| \leq l\chi, \quad \|f(x(t - \tau(t)))\| \leq l\chi. \quad (8)$$

Remark 2. Let $e(t; \xi)$ denote the state trajectory from the initial data $e(\theta) = \xi(\theta)$ on $-\tau \leq \theta \leq 0$ in $C_{\mathcal{F}_0}^2([-\tau, 0]; \mathbb{R}^n)$. Although $\sigma(t, 0, 0) = 0$ and $f_i(0) = 0$, system (3) would not admit a unique solution or trivial solution $e(t; 0) = 0$. Thus, Hypothesis (H2) in [31] is not satisfied. Therefore, the LaSalle invariance principle for stochastic differential delay equation [31, Corollary 3.1] cannot be applied for the stability analysis of the error system (3) and other stochastic delayed systems without a trivial solution $e(t; 0) = 0$, which implies that the methods proposed in [26, 33, 34] are debatable. In order to release the restriction of the nonexistence of a trivial solution, we turn to utilize Gronwall's inequality to ensure the rigorousness of mathematical proof.

The following definition and lemmas are useful for future derivations.

Definition 3. If the function $V \in C^{1,2}(\mathbb{R}^+ \times \mathbb{R}^n; \mathbb{R}^+)$, then an operator $\mathfrak{L}V$ from $\mathbb{R}^+ \times \mathbb{R}^n$ along the trajectory of the error system (3) is defined as

$$\begin{aligned} \mathfrak{L}V(t, e(t)) &= V_t(t, e(t)) + V_e(t, e(t)) \\ &\times [-Ce(t) + (A + \Delta A_2(t))h(e(t)) \\ &\quad + (B + \Delta B_2(t))h(e(t - \tau(t))) \\ &\quad + (\Delta A_2(t) - \Delta A_1(t))f(x(t)) \\ &\quad + (\Delta B_2(t) - \Delta B_1(t))f(x(t - \tau(t))) + u(t)] \\ &\quad + \frac{1}{2} \text{trace} [\sigma^T(t, e(t), e(t - \tau(t))) V_{ee} \\ &\quad \times (t, e(t)) \sigma^T(t, e(t), e(t - \tau(t)))], \end{aligned} \quad (9)$$

where

$$\begin{aligned} V_t(t, e(t)) &= \frac{\partial V(t, e(t))}{\partial t}, \\ V_e(t, e(t)) &= \left(\frac{\partial V(t, e(t))}{\partial e_1}, \dots, \frac{\partial V(t, e(t))}{\partial e_n} \right), \\ V_{ee}(t, e(t)) &= \left(\frac{\partial^2 V(t, e(t))}{\partial e_i \partial e_j} \right)_{n \times n}. \end{aligned} \quad (10)$$

Definition 4. Systems (1) and (2) are said to be exponentially synchronized in mean square if there exist constants $\lambda > 0$ and $\mathcal{M} > 0$, such that $E\|e(t)\|^2 \leq \mathcal{M} \exp(-\lambda t)$ and $t > 0$, where λ is called the decay rate of exponential synchronization.

Lemma 5 (Gronwall's inequality (see [35])). *Let $T > 0$ and $u(\cdot)$ be a Borel measurable bounded nonnegative function on $[0, T]$. If*

$$u(t) \leq c + v \int_0^t u(s) ds, \quad \forall 0 \leq t \leq T, \quad (11)$$

for some constants c, v , then

$$u(t) \leq c \exp(vt), \quad \forall 0 \leq t \leq T. \quad (12)$$

Lemma 6 (see [36]). *If X and Y are real matrices with appropriate dimensions, then there exists a number $\varepsilon > 0$, such that*

$$X^T Y + Y^T X \leq \varepsilon X^T X + \frac{1}{\varepsilon} Y^T Y. \quad (13)$$

3. Main Results

In this section, the robust adaptive exponential synchronization in mean square for the drive system (1) and response system (2) is studied under Hypotheses (H1)–(H4). The robust adaptive feedback controller is designed as

$$\begin{aligned} u(t) &= -Ke(t) - l\chi \|e^T P\|^{-1} Pe(t) \hat{\alpha} \\ &\quad - l\chi \|e^T P\|^{-1} Pe(t) \hat{\beta}, \end{aligned} \quad (14)$$

where K is the feedback gain matrix to be scheduled; $\hat{\alpha}$ and $\hat{\beta}$ are the estimation of the bounds $\bar{\alpha}$ and $\bar{\beta}$, respectively. The update laws $\hat{\alpha}$ and $\hat{\beta}$ are designed as

$$\dot{\hat{\alpha}} = k_{\alpha} l_{\chi} \|e^T P\|, \quad \dot{\hat{\beta}} = k_{\beta} l_{\chi} \|e^T P\|, \quad (15)$$

with $k_{\alpha}, k_{\beta} > 0$.

Theorem 7. Assume that Hypotheses (H1)–(H4) hold. If there exist an $n \times n$ feedback gain matrix K , two $n \times n$ symmetric matrices $P > 0$ and $R > 0$, and four positive constants ρ, θ_1, θ_2 , and κ , such that the following conditions hold:

$$(C1) \ P \leq \rho I;$$

$$(C2)$$

$$\begin{bmatrix} \Omega_{11} & PA & PE_2 & PB & PE_2' \\ * & -\frac{1}{\theta_1} I & 0 & 0 & 0 \\ * & * & -\frac{1}{(\theta_1 \|H_2\|^2)} & 0 & 0 \\ * & * & * & -\frac{1}{\theta_2} I & 0 \\ * & * & * & * & -\frac{1}{(\theta_2 \|H_2'\|^2)} \end{bmatrix} \leq 0, \quad (16)$$

$$\frac{2}{\theta_2} L^2 + \rho Q_2 - (1 - \mu) R \leq 0,$$

where

$$\Omega_{11} = P(-K - C) + (-K - C)^T P + \frac{2}{\theta_1} L^2 + R + \rho Q_1 + \kappa I, \quad (17)$$

then the drive system (1) and response system (2) can be exponentially synchronized in mean square.

Proof. Construct a Lyapunov functional in the form of

$$V(t, e(t)) = e^T(t) P e(t) + \int_{t-\tau(t)}^t e^T(s) \text{Re}(s) ds + k_{\alpha}^{-1} \tilde{\alpha}^2 + k_{\beta}^{-1} \tilde{\beta}^2, \quad (18)$$

where $\tilde{\alpha} = \bar{\alpha} - \hat{\alpha}$, $\tilde{\beta} = \bar{\beta} - \hat{\beta}$.

From Definition 3, we obtain the operator $\mathfrak{V}(t, e(t))$

$$\begin{aligned} \mathfrak{V}(t, e(t)) &= 2e^T(t) P (-Ce(t) + (A + \Delta A_2(t)) h(e(t)) \\ &\quad + (B + \Delta B_2(t)) h(e(t - \tau(t))) \\ &\quad + (\Delta A_2(t) - \Delta A_1(t)) f(x(t)) \\ &\quad + (\Delta B_2(t) - \Delta B_1(t)) f(x(t - \tau(t))) + u(t)) \\ &\quad + \text{trace} [\sigma^T(t, e(t), e(t - \tau(t))) \\ &\quad \times P \sigma(t, e(t), e(t - \tau(t)))] \end{aligned}$$

$$\begin{aligned} &+ e^T(t) \text{Re}(t) - (1 - \dot{\tau}(t)) \\ &\times e^T(t - \tau(t)) \text{Re}(t - \tau(t)) \\ &- (2k_{\alpha}^{-1} \tilde{\alpha} \dot{\hat{\alpha}} + 2k_{\beta}^{-1} \tilde{\beta} \dot{\hat{\beta}}) \\ &= e^T(t) (-PC - C^T P + R) e(t) \\ &\quad + 2e^T(t) P (A + \Delta A_2(t)) h(e(t)) \\ &\quad + 2e^T(t) P (B + \Delta B_2(t)) h(e(t - \tau(t))) \\ &\quad + 2e^T(t) P (\Delta A_2(t) - \Delta A_1(t)) f(x(t)) \\ &\quad + 2e^T(t) P (\Delta B_2(t) - \Delta B_1(t)) \\ &\quad \times f(x(t - \tau(t))) + 2e^T(t) P u(t) \\ &\quad + \text{trace} [\sigma^T(t, e(t), e(t - \tau(t))) \\ &\quad \times P \sigma(t, e(t), e(t - \tau(t)))] \\ &\quad - (1 - \dot{\tau}(t)) e^T(t - \tau(t)) \text{Re}(t - \tau(t)) \\ &\quad - (2k_{\alpha}^{-1} \tilde{\alpha} \dot{\hat{\alpha}} + 2k_{\beta}^{-1} \tilde{\beta} \dot{\hat{\beta}}). \end{aligned} \quad (19)$$

It is clear from Lemma 6 that

$$\begin{aligned} &2e^T(t) P (A + \Delta A_2(t)) h(e(t)) \\ &\leq \theta_1 e^T(t) P A A^T P e(t) \\ &\quad + \frac{1}{\theta_1} h^T(e(t)) h(e(t)) \\ &\quad + \theta_1 e^T(t) P E_2 F_2(t) H_2 H_2^T F_2^T(t) E_2^T P e(t) \\ &\quad + \frac{1}{\theta_1} h^T(e(t)) h(e(t)) \\ &\leq \theta_1 e^T(t) P A A^T P e(t) + \frac{2}{\theta_1} e^T(t) L^2 e(t) \\ &\quad + \theta_1 \|H_2\|^2 e^T(t) P E_2 E_2^T P e(t) \\ &= e^T(t) \left[\theta_1 (P A A^T P + \|H_2\|^2 P E_2 E_2^T P) \right. \\ &\quad \left. + \frac{2}{\theta_1} L^2 \right] e(t), \\ &2e^T(t) P (B + \Delta B_2(t)) h(e(t - \tau(t))) \\ &\leq \theta_2 e^T(t) P B B^T P e(t) \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{\theta_2} h^T(e(t-\tau(t))) h(e(t-\tau(t))) \\
& + \theta_2 e^T(t) P E_2' F_2'(t) H_2' H_2'^T F_2'^T(t) E_2'^T P e(t) \\
& + \frac{1}{\theta_2} h^T(e(t-\tau(t))) h(e(t-\tau(t))) \\
& \leq \theta_2 e^T(t) P B B^T P e(t) \\
& + \frac{2}{\theta_2} e^T(t-\tau(t)) L^2 e(t-\tau(t)) \\
& + \theta_2 \|H_2'\|^2 e^T(t) P E_2' E_2'^T P e(t) \\
& = e^T(t) \left[\theta_2 (P B B^T P + \|H_2'\|^2 P E_2' E_2'^T P) \right] e(t) \\
& + \frac{2}{\theta_2} e^T(t-\tau(t)) L^2 e(t-\tau(t)), \\
& 2e^T(t) P (\Delta A_2(t) - \Delta A_1(t)) f(x(t)) \\
& + 2e^T(t) P (\Delta B_2(t) - \Delta B_1(t)) f(x(t-\tau(t))) \\
& \leq 2 \|e^T P\| \bar{\alpha} l \chi + 2 \|e^T P\| \bar{\beta} l \chi \\
& = 2 \|e^T P\| l \chi (\bar{\alpha} + \bar{\beta}).
\end{aligned} \tag{20}$$

From Hypothesis (H3), we have

$$\begin{aligned}
\text{trace} [\sigma^T P \sigma] & \leq \lambda_{\max}(P) \text{trace} [\sigma^T \sigma] \\
& \leq \rho(e^T(t) Q_1 e(t) \\
& + e^T(t-\tau(t)) Q_2 e(t-\tau(t))).
\end{aligned} \tag{21}$$

Substituting (14) and (15), (20) and (21) to (19), we obtain

$$\begin{aligned}
& \mathfrak{L}V(t, e(t)) \\
& \leq e^T(t) \Omega_1 e(t) \\
& + e^T(t-\tau(t)) \Omega_2 e(t-\tau(t)) \\
& + 2 \|e^T P\| l \chi (\bar{\alpha} + \bar{\beta}) \\
& - 2e^T(t) P (l \chi \|e^T P\|^{-1} P e(t) \hat{\alpha} \\
& + l \chi \|e^T P\|^{-1} P e(t) \hat{\beta}) \\
& - 2k_\alpha^{-1} \bar{\alpha} k_\alpha l \chi \|e^T P\| - 2k_\beta^{-1} \bar{\beta} k_\beta l \chi \|e^T P\| \\
& = e^T(t) \Omega_1 e(t) + e^T(t-\tau(t)) \Omega_2 e(t-\tau(t)) \\
& + 2 \|e^T P\| l \chi (\bar{\alpha} + \bar{\beta} - \hat{\alpha} - \hat{\beta}) - 2 \|e^T P\| l \chi (\bar{\alpha} + \bar{\beta}) \\
& = e^T(t) \Omega_1 e(t) + e^T(t-\tau(t)) \Omega_2 e(t-\tau(t)) \\
& = e^T(t) (\Omega_1 + \kappa I) e(t) + e^T(t-\tau(t)) \Omega_2 e(t-\tau(t)) \\
& - \kappa e^T(t) e(t) \\
& = e^T(t) \Omega_1' e(t) + e^T(t-\tau(t)) \Omega_2 e(t-\tau(t)) \\
& - \kappa e^T(t) e(t),
\end{aligned} \tag{22}$$

where

$$\begin{aligned}
\Omega_1 & = P(-K - C) + (-K - C)^T P \\
& + R + \rho Q_1 + \frac{2}{\theta_1} L^2 \\
& + \theta_1 (P A A^T P + \|H_2\|^2 P E_2 E_2^T P) \\
& + \theta_2 (P B B^T P + \|H_2'\|^2 P E_2' E_2'^T P),
\end{aligned} \tag{23}$$

$$\Omega_1' = \Omega_1 + \kappa I,$$

$$\Omega_2 = \frac{2}{\theta_2} L^2 + \rho Q_2 - (1 - \mu) R.$$

Considering condition (C2) of Theorem 7 and Schur complement [37], we get

$$\mathfrak{L}V(t, e(t)) \leq -\kappa e^T(t) e(t). \tag{24}$$

It follows from Itô's formula that

$$\begin{aligned}
& EV(t, e(t)) - EV(0, e(0)) \\
& = \int_0^t E \mathfrak{L}V \leq -\kappa \int_0^t E(e(s)^T e(s)) ds,
\end{aligned} \tag{25}$$

$t > 0,$

which means that

$$\begin{aligned}
& \lambda_{\min}(P) E \|e(t)\|^2 \leq EV(t, e(t)) \\
& \leq EV(0, e(0)) - \kappa \int_0^t E \|e(s)\|^2 ds.
\end{aligned} \tag{26}$$

By using Lemma 5, we have

$$E \|e(t)\|^2 \leq c \exp(vt), \tag{27}$$

where $c = EV(0, e(0))/\lambda_{\min}(P)$, $v = -\kappa/\lambda_{\min}(P)$.

On the other hand, we denote

$$\begin{aligned}
& EV(0, e(0)) = E e^T(0) P e(0) \\
& + \int_{-\tau(0)}^0 E e^T(s) \text{Re}(s) \\
& + k_\alpha^{-1} \bar{\alpha}^2(0) + k_\beta^{-1} \bar{\beta}^2(0) \\
& \leq \lambda_{\max}(P) E \sup_{-\tau \leq \theta \leq 0} \|\xi(\theta)\|^2 \\
& + \tau \lambda_{\max}(R) E \sup_{-\tau \leq \theta \leq 0} \|\xi(\theta)\|^2 \\
& + k_\alpha^{-1} \bar{\alpha}^2(0) + k_\beta^{-1} \bar{\beta}^2(0) \\
& = (\lambda_{\max}(P) + \tau \lambda_{\max}(R)) E \sup_{-\tau \leq \theta \leq 0} \|\xi(\theta)\|^2 \\
& + k_\alpha^{-1} \bar{\alpha}^2(0) + k_\beta^{-1} \bar{\beta}^2(0).
\end{aligned} \tag{28}$$

Combining (27) and (28), we finally obtain

$$\|E\|e(t)\|^2 \leq \mathcal{M} \exp\left(\frac{-\kappa}{\lambda_{\min}(P)}t\right), \quad (29)$$

where

$$\begin{aligned} \mathcal{M} = c = & \left((\lambda_{\max}(P) + \tau\lambda_{\max}(R)) E \sup_{-\tau \leq \theta \leq 0} \|\xi(\theta)\|^2 \right. \\ & \left. + k_{\alpha}^{-1} \tilde{\alpha}^2(0) + k_{\beta}^{-1} \tilde{\beta}^2(0) \right) \times (\lambda_{\min}(P))^{-1}. \end{aligned} \quad (30)$$

Therefore, by Definition 4 we see that the drive system (1) and response system (2) can be exponentially synchronized in mean square with a decay rate $\lambda = \kappa/\lambda_{\min}(P)$. The proof of Theorem 7 is completed. $\square$

Let $Y = PK$; the feedback gain matrix K can be calculated by $K = P^{-1}Y$; then, the following Theorem holds.

Theorem 8. Assume that Hypotheses (H1)–(H4) hold. If there exist an $n \times n$ matrix Y , two $n \times n$ symmetric matrices $P > 0$ and $R > 0$, and four positive constants ρ , θ_1 , θ_2 , and κ , such that the following conditions hold

$$(C1) \quad P \leq \rho I;$$

$$(C2')$$

$$\begin{bmatrix} \Omega'_{11} & PA & PE_2 & PB & PE'_2 \\ * & -\frac{1}{\theta_1}I & 0 & 0 & 0 \\ * & * & -\frac{1}{(\theta_1\|H_2\|^2)} & 0 & 0 \\ * & * & * & -\frac{1}{\theta_2}I & 0 \\ * & * & * & * & -\frac{1}{(\theta_2\|H'_2\|^2)} \end{bmatrix} \leq 0, \quad (31)$$

$$\frac{2}{\theta_2}L^2 + \rho Q_2 - (1 - \mu)R \leq 0,$$

where

$$\Omega'_{11} = -PC - Y - C^TP - Y^T + \frac{2}{\theta_1}L^2 + R + \rho Q_1 + \kappa I, \quad (32)$$

then the drive system (1) and response system (2) can be exponentially synchronized in mean square.

Substituting $K = P^{-1}Y$ to Theorem 7, the above result is easy to be obtained. So the proof of Theorem 8 is omitted.

Remark 9. Our method does not need to construct a complex Lyapunov-Krasovskii function which contains quadratic integral terms and triple integral terms. Thus, the amount

of calculation is greatly reduced. Furthermore, we do not require that the matrix $P > 0$ is diagonal (see [8, 9, 30]) just only being symmetric. This partly shows the less conservativeness of our control strategy. If the drive system (1) and response system (2) have no nonidentical parametric uncertainties, the corresponding synchronization problems have been addressed in [8, 9, 29, 30]. Therefore, our results are more general than those given in [8, 9, 29, 30].

By constructing another Lyapunov functional $V'(t, e(t)) = e^T(t)Pe(t) + \int_{t-\tau(t)}^t e^T(s)Re(s)ds$, we can derive the following corollaries by Theorem 7 and Theorem 8, respectively.

Corollary 10. Assume that Hypotheses (H1)–(H4) hold, and the feedback controller is designed as $u(t) = -K(y(t) - x(t))$. If there exist an $n \times n$ feedback gain matrix K , two $n \times n$ symmetric matrices $P > 0$ and $R > 0$, and four positive constants ρ , θ_1 , θ_2 , and κ , such that the following conditions hold

$$(i) \quad P \leq \rho I;$$

$$(ii)$$

$$\begin{bmatrix} \Xi_{11} & PA & PB \\ * & -\frac{1}{\theta_1}I & 0 \\ * & * & -\frac{1}{\theta_2}I \end{bmatrix} \leq 0, \quad (33)$$

$$\frac{2}{\theta_2}L^2 + \rho Q_2 - (1 - \mu)R \leq 0,$$

where

$$\begin{aligned} \Xi_{11} = & P(-K - C) + (-K - C)^TP \\ & + \frac{2}{\theta_1}L^2 + R + \rho Q_1 + \kappa I, \end{aligned} \quad (34)$$

then the drive system (1) and response system (2) can be exponentially synchronized in mean square.

Corollary 11. Assume that Hypotheses (H1)–(H4) hold, and the feedback controller is designed as $u(t) = -K(y(t) - x(t))$. If there exist an $n \times n$ matrix Y , two $n \times n$ symmetric matrices $P > 0$ and $R > 0$, and four positive constants ρ , θ_1 , θ_2 , and $\kappa > 0$, such that the following conditions hold

$$(i) \quad P \leq \rho I;$$

$$(ii')$$

$$\begin{bmatrix} \Xi'_{11} & PA & PB \\ * & -\frac{1}{\theta_1}I & 0 \\ * & * & -\frac{1}{\theta_2}I \end{bmatrix} \leq 0, \quad (35)$$

$$\frac{2}{\theta_2}L^2 + \rho Q_2 - (1 - \mu)R \leq 0,$$

where

$$\Xi'_{11} = -PC - Y - C^T P - Y^T + \frac{2}{\theta_1} L^2 + R + \rho Q_1 + \kappa I, \quad (36)$$

$$Y = PK,$$

then the drive system (1) and response system (2) can be exponentially synchronized in mean square.

The proof of Corollaries 10 and 11 is direct, so it is omitted.

Remark 12. Suppose response system (2) without noise perturbation; then, the synchronization issue of the drive system (1) and response system (2) converted to the synchronization of chaotic delayed neural networks with parametric uncertainties, which has been investigated in [11, 23]. In addition, the authors in [33] only studied the time invariance parametric uncertainties. Subsequently, our method has a wider application range than those results in [11, 23, 33].

4. Numerical Results

In this section, a numerical example and its simulations are presented to demonstrate the effectiveness of the proposed scheme in the previous sections. Throughout the simulations, we use the Euler-Maruyama method [38] with step 10^{-3} .

Example 13. Consider two-dimensional chaotic delayed neural networks [39] as the drive system:

$$\begin{aligned} dx(t) = & [-Cx(t) + (A + \Delta A_1(t))f(x(t)) \\ & + (B + \Delta B_1(t))f(x(t - \tau(t))) + J]dt, \end{aligned} \quad (37)$$

where the initial condition $x(s) = (0.1, 0.1)^T$, $s \in [-\tau, 0]$, $f(x) = 0.5(|x+1| - |x-1|)$, and $\tau(t) = 1$, $J = (0, 0)^T$; then, from Hypothesis (H1) and Remark 1, it is easy to verify that $L_1 = (1, 1)^T$, $L_2 = (0, 0)^T$, and $l = 1$. The parameter matrices C , A , and B and the parametric uncertainties are given as follows:

$$\begin{aligned} C &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad A = \begin{pmatrix} 1 + \frac{\pi}{4} & 20 \\ 0.1 & 1 + \frac{\pi}{4} \end{pmatrix}, \\ B &= \begin{pmatrix} -\sqrt{2} \times \frac{1.3\pi}{4} & 0.1 \\ 0.1 & -\sqrt{2} \times \frac{1.3\pi}{4} \end{pmatrix}, \quad (38) \\ \Delta A_1(t) &= 0.04 * \begin{pmatrix} \sin(t) & 0 \\ 0 & \sin(t) \end{pmatrix}, \\ \Delta B_1(t) &= 0.04 * \begin{pmatrix} \cos(t) & 0 \\ 0 & \cos(t) \end{pmatrix}; \end{aligned}$$

that is,

$$\begin{aligned} F_1(t) &= \begin{pmatrix} \sin(t) & 0 \\ 0 & \sin(t) \end{pmatrix}, \\ F'_1(t) &= \begin{pmatrix} \cos(t) & 0 \\ 0 & \cos(t) \end{pmatrix}, \quad (39) \\ E_1 &= H_1 = E'_1 = H'_1 = 0.2I. \end{aligned}$$

The corresponding response system is taken as

$$\begin{aligned} dy(t) = & [-Cy(t) + (A + \Delta A_2(t))f(y(t)) \\ & + (B + \Delta B_2(t))f(y(t - \tau(t))) \\ & + u(t) + J]dt + \sigma(t, y(t) - x(t), y(t - \tau(t)) \\ & - x(t - \tau(t)))d\omega(t), \end{aligned} \quad (40)$$

where the initial condition $y(s) = (0.2, -0.2)^T$, $s \in [-\tau, 0]$, the noise perturbation

$$\begin{aligned} \sigma(t, e(t), e(t - \tau(t))) &= \text{diag}(\sqrt{0.005}e_1(t), \sqrt{0.025}e_2(t)) \\ &+ \text{diag}(\sqrt{0.25}e_1(t - \tau(t)), \sqrt{0.005}e_2(t - \tau(t))), \end{aligned} \quad (41)$$

and $\omega(t)$ is a 2-dimensional Brownian motion satisfying $E\{d\omega(t)\} = 0$ and $E\{[d\omega(t)]^2\} = dt$; then, from Hypothesis (H2), it is easy to get $Q_1 = \text{diag}(0.01, 0.05)$ and $Q_2 = \text{diag}(0.5, 0.01)$. And the parametric uncertainties are given as

$$\begin{aligned} \Delta A_2(t) &= -0.04 * \begin{pmatrix} \sin(t) & 0 \\ 0 & \sin(t) \end{pmatrix}, \\ \Delta B_2(t) &= -0.04 * \begin{pmatrix} \cos(t) & 0 \\ 0 & \cos(t) \end{pmatrix}; \end{aligned} \quad (42)$$

that is,

$$\begin{aligned} F_2(t) &= -\begin{pmatrix} \sin(t) & 0 \\ 0 & \sin(t) \end{pmatrix}, \\ F'_2(t) &= -\begin{pmatrix} \cos(t) & 0 \\ 0 & \cos(t) \end{pmatrix}, \quad (43) \\ E_2 &= H_2 = E'_2 = H'_2 = 0.2I. \end{aligned}$$

The robust adaptive feedback controller $u(t)$ satisfies (14). The update laws $\hat{\alpha}$ and $\hat{\beta}$ are designed as (15).

FIGURE 1: Time response of state variables and the phase plots of systems (37) and (40) without the robust adaptive feedback controller $u(t)$.

By using MATLAB LMI toolbox, we can obtain the following feasible solutions to LMIs in Theorem 8:

$$\begin{aligned}
 P &= \begin{pmatrix} 0.0040 & -0.0065 \\ -0.0065 & 0.0772 \end{pmatrix}, & Y &= \begin{pmatrix} 0.5206 & 0.0019 \\ 0.0019 & 0.4607 \end{pmatrix}, \\
 R &= \begin{pmatrix} 0.4307 & -0.0002 \\ -0.0002 & 0.3292 \end{pmatrix}, \\
 \rho &= 0.2734, & \theta_1 &= 6.1797, \\
 \theta_2 &= 9.4490, & \kappa &= 0.0842.
 \end{aligned} \tag{44}$$

Thus, we can calculate the feedback gain matrix K and the decay rate, which are given as follows:

$$\begin{aligned}
 K &= P^{-1}Y = \begin{pmatrix} 149.1244 & 11.6011 \\ 12.5158 & 6.9422 \end{pmatrix}, \\
 \lambda &= \frac{\kappa}{\lambda_{\min}(P)} = 24.2757.
 \end{aligned} \tag{45}$$

We choose $\chi = 16$, $k_\alpha = k_\beta = 0.1$, $\hat{\alpha}(0) = 0.079$, and $\hat{\beta}(0) = 0.079$. Figures 1(a)–1(d) show the time response of state variables and the phase plots of systems (37) and (40) without the robust adaptive feedback controller $u(t)$ designed in (14). Figures 2(a)–2(d) depict the time response of state variables, the synchronization errors, and the phase plots of systems

FIGURE 2: Time response of state variables, the synchronization errors, and the phase plots of systems (37) and (40) with the robust adaptive feedback controller $u(t)$.

(37) and (40) with the robust adaptive feedback controller $u(t)$ designed in (14), from which we can see that the drive system (37) and response system (40) can be exponentially synchronized in mean square. Figures 3(a) and 3(b) display the estimated parameters $\hat{\alpha}$ and $\hat{\beta}$ converge asymptotically to some constants, which show the effectiveness of the proposed robust adaptive synchronization scheme.

Remark 14. The simulation results in Example 13 show that the effect of the nonidentical parametric uncertainties

between the drive system (37) and response system (40) can be suppressed rapidly via the designed robust adaptive feedback controller, which means that the proposed strategy has strong robustness against the nonidentical parametric uncertainties. Meanwhile, we have a relatively large decay rate λ .

Remark 15. Synchronization is encountered in various fields of science, in engineering and in social behavior [40, 41]. In fact, synchronization problem of networks system belongs to the category of control, which is an important branch of

FIGURE 3: The curves of the estimated parameters $\hat{\alpha}$ and $\hat{\beta}$.

synthetic theory in model control field. The authors in [42] dealt with the synchronization problem of coupled switched neural networks with mode-dependent impulsive effects and time delays by using switching analysis techniques and a comparison principle. Zhang et al. in [43] have investigated the synchronization problem for a class of nonlinear delayed dynamical networks with heterogeneous impulsive effects. The distributed synchronization problem in networks of agent systems with controllers and nonlinearities subject to Bernoulli switchings has been studied in [44], in which the advantage of distributed adaptive controllers over conventional adaptive controllers has also been validated. The abovementioned literature is interesting synthesis problems of networks and will become our future investigative directions.

5. Conclusion

In this paper, we have studied robust exponential adaptive synchronization of stochastic perturbed chaotic delayed neural networks with nonidentical parametric uncertainties. A robust adaptive feedback controller has been designed to equalize the effect of the nonidentical parametric uncertainties. A numerical example has also been exploited to depict the usefulness of the obtained results. The nonidentical parametric uncertainties, which have been taken into account, exhibited the main advantage of the proposed scheme. The simulation results confirmed that our method has a high robustness property against parameter uncertainties mismatch.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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