Finite-Time Stability Analysis for a Class of Continuous Switched Descriptor Systems

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Finite-time stability has more practical application values than the classical Lyapunov asymptotic stability over a fixed finite-time interval. The problems of finite-time stability and finite-time boundedness for a class of continuous switched descriptor systems are considered in this paper. Based on the average dwell time approach and the multiple Lyapunov functions technique, the concepts of finite-time stability and boundedness are extended to continuous switched descriptor systems. In addition, sufficient conditions for the existence of state feedback controllers in terms of linear matrix inequalities (LMIs) are obtained with arbitrary switching rules, which guarantee that the switched descriptor system is finite-time stable and finite-time bounded, respectively. Finally, two numerical examples are presented to illustrate the reasonableness and effectiveness of the proposed results.

1. Introduction

Switched systems are a special class of hybrid systems, which consist of a collection of continuous or discrete-time subsystems together with a switching rule that orchestrates switching between these subsystems to achieve the control objectives [1]. Descriptor systems are also referred to as singular systems, implicit systems, or differential-algebraic systems, which are also a natural representation of dynamic systems and can describe physical systems better than the normal linear systems. Descriptor systems have been widely applied in many practical systems such as networks, power systems, electrical circuits, economics mathematical modeling, and many other fields [2, 3]. In actual control systems, switching phenomenon of descriptor systems is ubiquitous. Nevertheless, because of the switching between multiple descriptor subsystems and the algebraic constraints in descriptor model, it is inevitably difficult to analyze and synthesize for switched descriptor systems.

Up to now, much attention has been mainly focused on system stability and reliability [2–5], $H\infty$ control [6–8], cost guaranteed control [9–11], system controllability, and reachability [12, 13] for switched descriptor systems. Generally, most of existing results related to stability and performance criteria of switched descriptor systems are based on the classical Lyapunov asymptotic stability, which is defined as an infinite time interval. However, in many actual systems, such as network control systems, the practical system state does not exceed some bound during some time interval and we need to avoid saturations and the excitation of nonlinear dynamics. In this case, the asymptotic stability is not enough for practical applications, because the system could be Lyapunov asymptotically stable but it possesses undesirable transient performances. Then the concept of finite-time stability was firstly put forward in [14], which concerns the boundedness of the system state over a fixed finite-time interval. To a certain degree, the development of the finite-time stability theory is parallel with the development of Lyapunov asymptotic stability.

In recent years, the abundant studies on finite-time stability of switched systems [15–20] and descriptor systems [21–24] have been obtained. Lin et al. [15] gave some results on finite-time boundedness and finite-time weighted $L_2$-gain for a class of switched delay systems with time-varying exogenous disturbances. Sufficient conditions which ensure that the switched system with time-time is finite-time bounded and has finite-time weighted $L_2$-gain were proposed based on the average dwell time approach and the multiple Lyapunov
functions technique. In [23], the issue of robust finite-time stabilization of descriptor stochastic systems with time-varying norm-bounded disturbance and parametric uncertainties via static output feedback was discussed. Suppose that the state vector is not available for feedback; a static output feedback controller in terms of restricted LMI s was provided to guarantee the underlying closed-loop descriptor stochastic system finite-time stabilization with a prescribed \( H_{\infty} \) r-disturbance attenuation over the given finite-time interval. Meanwhile, an illustrative example was employed to verify the efficiency of the proposed method. In view of the importance of practical application, we need to pay great attention to the research on finite-time stability for switched descriptor systems compared with Lyapunov asymptotic stability. Few works that deal with the finite-time stability for this type of systems have been reported. Based on the different multiple Lyapunov functions, the papers [25, 26] focused on the discrete-time switched descriptor systems and switched descriptor systems with time-varying delay, respectively.

The paper is organized as follows. Firstly, the concepts of finite-time stability and finite-time boundedness for normal systems are expanded to continuous switched descriptor systems. Then, based on the state transfer matrix method, the sufficient and necessary condition of finite-time stability for this kind of system is given. Moreover, we tackle the problems of state feedback finite-time stabilization and finite-time boundedness; the sufficient conditions for the existence of controllers are obtained with arbitrary switching rules, which guarantee that the closed-loop systems are finite-time stable and finite-time bounded, respectively. Detailed proofs are presented by using the multiple Lyapunov functions and the average dwell-time approach. Finally, two examples are presented to show the validity of the developed methodology. Our research results are totally different from those previous results and important supplements for stability study for switched descriptor systems.

2. Problem Description and Preliminaries

Consider a class of switched descriptor system as follows:

\[
\begin{align*}
E x(t) &= A_{\sigma(t)} x(t) + B_{\sigma(t)} u_{\sigma(t)}(t) + G_{\sigma(t)} w(t), \\
\dot{w}(t) &= F_{\sigma(t)} w(t),
\end{align*}
\]

where \( x(t) \in \mathbb{R}^n \) is the system state, \( u_{\sigma}(t) \in \mathbb{R}^p \) is the control input, and \( w(t) \in \mathbb{R}^q \) is the exogenous disturbance signal and satisfies the constraint \( w^T(0)w(0) \leq d, d \geq 0 \); the switching signal \( \sigma(t) : \mathbb{Z}^+ \to \{1, 2, \ldots, m\} \) is a piecewise constant and right continuous function; \( m \) is the number of subsystems; \( \sigma(t) = i \) represents that the \( i \)th subsystem is activated. \( E, A_j, B_j, G_j, \) and \( F \) are known constant matrices with appropriate dimension, and it is assumed that \( \operatorname{rank} E = r \leq n \).

Remark 1. In view of the special structure of descriptor systems, the initial condition is given as \( Ex(t_0) = \dot{Ex}_0 \). Corresponding to the switching signal \( \sigma(t) \), the switching sequence is defined as follows:

\[
\{x_{q_i}(t_0, t_0), \ldots, (i_k, t_k), \ldots | i_k \in \{1, 2, \ldots, m\}, k = 0, 1, \ldots \},
\]

which means that the \( i_k \)th subsystem is activated when \( t \in [t_k, t_{k+1}) \) and \( x_{q_i}, t_0 \) are the initial state and initial time, respectively. In addition, we can transform the descriptor matrix of different form into the singular matrix “E” of system (1) by nonsingular transformation.

Assumption 2. The initial state of system (1) discussed is the consistent initial state, and the system state does not jump at the switching moment.

Now, we give the definitions of finite-time stability and finite-time boundedness for the continuous switched descriptor systems.

Definition 3. Continuous switched descriptor system

\[
\begin{align*}
\dot{x}(t) &= A_{\sigma(t)} x(t) + G_{\sigma(t)} w(t), \\
\dot{w}(t) &= F_{\sigma(t)} w(t), \quad w^T(0)w(0) \leq d
\end{align*}
\]

is said to be finite-time bounded with respect to \((c_1, c_2, d, R, T_f, \sigma)\), if the following formula:

\[
x^T(0)E^T R Ex(t) \leq c_1 \Rightarrow x^T(t)E^T R Ex(t) \leq c_2, \quad \forall t \in [0, T_f],
\]

holds, where \( c_2 > c_1 > 0, d \geq 0, T_f > 0, \) and \( R > 0 \).

Definition 4. Continuous switched descriptor system

\[
\dot{x}(t) = A_{\sigma(t)} x(t)
\]

is said to be finite-time stable with respect to \((c_1, c_2, R, T_f, \sigma)\), if the following formula:

\[
x^T(0)E^T R Ex(t) \leq c_1 \Rightarrow x^T(t)E^T R Ex(t) \leq c_2, \quad \forall t \in [0, T_f],
\]

holds, where \( c_2 > c_1 > 0, T_f > 0, \) and \( R > 0 \).

Definition 5 (see [15]). For any \( t_f \geq t_0 \geq 0, \) if \( N_{\sigma(t)}(t_0, t_f) \leq N_0 + (t_f - t_0)/\tau_a \), holds for given \( N_0 \geq 0, \tau_a > 0, \) where \( N_{\sigma(t)}(t_0, t_f) \) denotes the switching number of \( \sigma(t) \) over \([t_0, t_f]\), then the constant \( \tau_a \) is called the average dwell time and \( N_0 \) is the flutter bound. As commonly used in the previous literature, we choose \( N_0 = 0 \).

Remark 6. Finite-time stability for norm switched descriptor systems refers to the fact that the state of slow subsystem is less than a given upper bound. According to regularity of systems, the state of fast subsystem is also less than a given upper bound.
3. Main Results

Firstly, the sufficient and necessary condition of finite-time stability for system (5) is given by applying the state transition matrix method.

Theorem 7. Given positive constants $c_1, c_2$, and $T_f$, the system (5) is finite-time stable with respect to $(c_1, c_2, R, T_f, \sigma)$, if and only if

$$
\Phi_\sigma^T(t, 0) E^T R \Phi_\sigma(t, 0) \leq \frac{c_2}{c_1} E^T R E, \quad \forall t \in [0, T_f),
$$

(7)

where $\Phi_\sigma(t, 0) = \Phi_{i_1}(t, t_1) \Phi_{i_2}(t_1, t_{i_2}) \cdots \Phi_{i_d}(t_d, 0)$ is the state transition matrix.

Proof. The following proof can be divided into two cases.

(a) Sufficiency. Since $\Phi_\sigma(t, 0)$ is the state transition matrix of system (5), then

$$
x(t) = \Phi_\sigma(t, 0) x(0), \quad t \in [0, T_f),
$$

(8)

where $\Phi_\sigma(t, 0) = \Phi_{i_1}(t, t_1) \Phi_{i_2}(t_1, t_{i_2}) \cdots \Phi_{i_d}(t_d, 0)$. In view of $x^T(0) E^T R E x(0) \leq c_1$, one obtains

$$
x^T(t) E^T R E x(t) = x^T(0) \Phi_\sigma^T(t, 0) E^T R \Phi_\sigma(t, 0) x(0).
$$

(9)

On the other hand, from (7) and $x^T(0) E^T R E x(0) \leq c_1$, we have

$$
x^T(t) E^T R E x(t) \leq x^T(0) \frac{c_2}{c_1} E^T R E x(0) \leq c_2.
$$

(10)

Thus, the system (5) is finite-time stable according to Definition 4.

(b) Necessity. Suppose that the system (5) is finite-time stable with respect to $(c_1, c_2, R, T_f, \sigma)$. By using the reduction to absurdity, if there exists $E x(t^*) \neq 0$, $t^* \in [0, T_f)$, such that

$$
x^T(t^*) E^T R E x(t^*) \geq x^T(t^*) \frac{c_2}{c_1} E^T R E x(t^*) > 0.
$$

(11)

Let $\lambda = \sqrt{c_1 / x^T(t^*) E^T R E x(t^*)}$, $x_0 = \lambda x(t^*)$; then we get

$$
x_0^T E^T R E x_0 = c_1. \quad \text{By virtue of (11), we have}
$$

$$
x_0^T \Phi_\sigma^T(t^*, 0) E^T R E \Phi_\sigma(t^*, 0) x_0 \geq x_0^T \frac{c_2}{c_1} E^T R E x_0.
$$

(12)

Meanwhile, according to (8), we can obtain

$$
x^T(t^*) E^T R E x(t^*) = x_0^T \Phi_\sigma^T(t^*, 0) E^T R \Phi_\sigma(t^*, 0) x_0 \geq c_2.
$$

(13)

Noticing that it is inconsistent with the hypothesis that the system (5) is finite-time stable with respect to $(c_1, c_2, R, T_f, \sigma)$, the proof is completed. \qed

Remark 8. From Theorem 7 we can obtain the sufficient and necessary condition, which guarantees that the switched descriptor system (5) is finite-time stable. However, it is difficult and inconvenient to calculate state transition matrix and design controller. Thus, it is difficult to apply in the actual systems.

Theorem 9. For any $i, j \in \{1, 2, \ldots, m\}$, if there exist nonsingular matrices $P_i$, matrices $Q_i > 0$, $Z_i > 0$, and scalars $\alpha \geq 0, \mu \geq 1$ such that

$$
P_i E = E^T P_i^T \geq 0,
$$

(14)

$$
\begin{bmatrix}
P_i A_i + A_i^T P_i - \alpha P_i E & P_i G_i \\
G_i^T P_i & F_i^T Q_i + Q_i F_i - \alpha Q_i
\end{bmatrix} < 0,
$$

(15)

$$
P_i E = E^T R_1/2 Z_i R_1/2 E,
$$

(16)

$$
P_i E \leq \mu P_i E, \quad Q_i \leq \mu Q_i,
$$

(17)

$$
\frac{\lambda_i c_1 + \lambda_i d}{\lambda_i c_2} < e^{-\alpha T_f}.
$$

(18)

And the average dwell time of the switching signal $\sigma$ satisfies

$$
\tau_a > \tau^* = \frac{T_f \ln \mu}{\ln(\lambda_2 c_2) - \ln(\lambda_1 c_1 + \lambda_3 d) - \alpha T_f},
$$

(19)

where $\lambda_1 = \max_{i=1,2,\ldots,m} |\lambda_{\text{max}}(Z_i)|$, $\lambda_2 = \min_{i=1,2,\ldots,m} |\lambda_{\text{min}}(Z_i)|$, and $\lambda_3 = \max_{i=1,2,\ldots,m} |\lambda_{\text{max}}(Q_i)|$; then system (3) is regular, pulse-free, and finite-time bounded with respect to $(c_1, c_2, d, R, T_f, \sigma)$.

Proof. First, from (15) we have

$$
P_i A_i + A_i^T P_i - \alpha P_i E < 0.
$$

(20)

Considering rank $E = r$ and assuming that there exist invertible matrices $M$, $N$, such that $E = M E N = \text{diag}(I_r, 0)$, then it follows from (14) and (20) that

$$
\begin{bmatrix}
P_i A_i + A_i^T P_i - \alpha P_i E \\
P_i G_i & F_i^T Q_i + Q_i F_i - \alpha Q_i
\end{bmatrix} < 0,
$$

(21)

$$
P_i E = E^T R_1/2 Z_i R_1/2 E,
$$

(22)

where $E = M E N = \text{diag}(I_r, 0)$, $\overline{A}_i = MA_i N, \overline{P}_i = N^T P_i M^{-1}$. According to (21), we can obtain $\overline{P}_i$ as follows:

$$
\overline{P}_i = \begin{bmatrix} P_{i_1} & P_{i_2} \\ 0 & P_{i_3} \end{bmatrix}, \quad P_{i_1} > 0, \quad \text{det}(P_{i_3}) \neq 0.
$$

(23)

Correspondingly, suppose that $\overline{A}_i = \begin{bmatrix} A_{i_1} & A_{i_2} \\ A_{i_3} & A_{i_4} \end{bmatrix}$, and it follows from (22) that

$$
\begin{bmatrix}
# & # \\
# & P_{i_3} A_{i_2} + A_{i_4}^T P_{i_3}^T
\end{bmatrix} < 0,
$$

(24)

where we do not need to know the expression of # and it does not affect the following discussion. It follows from (24)
that $P_{i3}A_{i22} + A_{i22}^TP_{i3} < 0$; namely, $A_{i22}$ is nonsingular. Then, there exists a scalar $s$ such that $\det(sE - \overline{A}_i) \neq 0$ and, for $\forall s$, $\deg \det(sE - \overline{A}_i) = \text{rank } \overline{E}$ holds. Thus, the system (3) is regular and pulse-free.

In the following, we will prove that system (3) is finite-time bounded with respect to $(c_1, c_2, d, R, T_f, \sigma)$. Choose the multiple Lyapunov function as

$$V(t, x(t), w(t)) = V_{\sigma(t)}(t)$$

$$= x^T(t) P_{\sigma(t)} x(t) + w^T(t) Q_{\sigma(t)} w(t).$$

(25)

Then, let $V(t, x(t), w(t)) = V(t)$ and $V_{\sigma}(t) = V_{\sigma}(t)$. By virtue of (14) and switched sequence (2), when $t \in [t_k, t_{k+1})$, taking derivative of $V(t)$ with respect to $t$ along the trajectory of the system (3) yields

$$\dot{V}(t) = \dot{V}_{\sigma(t)}(t)$$

$$= \begin{bmatrix} x(t) \\ w(t) \end{bmatrix}^T$$

$$\begin{bmatrix} P_{\sigma(t)} A_{\sigma(t)} + A_{\sigma(t)}^T P_{\sigma(t)} \\ G_{\sigma(t)} F_{\sigma(t)} \\ F_{\sigma(t)} Q_{\sigma(t)} + Q_{\sigma(t)} F_{\sigma(t)} \\ x(t) \\ w(t) \end{bmatrix}.$$ 

(26)

Then, it follows from (15) that

$$\dot{V} < \alpha V = \alpha V_{\sigma(t)}(t).$$

(27)

Integrating both sides of (27) from $t_k$ to $t$ gives

$$V(t) < e^{\alpha(t-t_k)} V_{\sigma(t_k)}(t_k).$$

(28)

Then, together with (17), at the switched moment $t_k$ we derive

$$V(t) < e^{\alpha(t-t_k)} \mu V_{\sigma(t_k)}(t_k).$$

(29)

For any $t \in [0, T_f)$, assuming that $N$ is the switched number of systems between $[0, T_f)$, one can obtain $N_{\sigma}(0, t) \leq N$. Considering $\mu \geq 1$ and using (17) together with (29), based on iterative method, we have

$$V(t) < e^{\alpha(t)} \mu^N V_{\sigma(0)}(0).$$

(30)

Noticing that $N \leq T_f/r_T$, then

$$V(t) < e^{\alpha(t)} \mu^{T_f/r_T} V_{\sigma(0)}(0).$$

(31)

On the other hand, it follows from (16) that

$$V(t) \geq x^T(t) P_{\sigma(t)} E x(t)$$

$$\geq \lambda_{\min}(Z_{\sigma}) x^T(t) E^T R E x(t)$$

$$\geq \lambda_2 x^T(t) E^T R E x(t),$$

$$V_{\sigma(0)}(0)$$

$$= x^T(0) P_{\sigma(0)} E x(0) + w^T(0) Q_{\sigma(0)} w(0)$$

$$\leq \lambda_{\max}(Z_{\sigma(0)}) x^T(0) E^T R E x(0) + \lambda_{\max}(Q_{\sigma(0)}) d$$

$$\leq \lambda_1 x^T(0) E^T R E x(0) + \lambda_3 d,$$

(32)

when $x^T(0) E^T R E x(0) \leq c_1$; putting together (31)-(32), it can easily be verified that

$$x^T(t) E^T R E x(t) < e^{\alpha(t)} \mu^{T_f/r_T} \left( \frac{\lambda_1 c_1 + \lambda_3 d}{\lambda_2} \right).$$

(33)

According to (18), we get

$$\ln(\lambda_2 c_2) - \ln(\lambda_1 c_1 + \lambda_3 d) - \alpha T_f > 0.$$  

(34)

It follows from (19) and (34) that

$$\mu^{T_f/r_T} \left( \frac{\lambda_1 c_1 + \lambda_3 d}{\lambda_2} \right) < e^{-\alpha T_f c_2}.$$  

(35)

By virtue of (33) and (35), we can obtain

$$x^T(t) E^T R E x(t) < e^{\alpha(t)} e^{-\alpha T_f c_2} = e^{\alpha(t-T_f)} c_2;$$

then, considering $t < T_f$ and $\alpha \geq 0$, we have

$$x^T(t) E^T R E x(t) < c_2.$$  

(37)

Thus, the system (3) is finite-time bounded with respect to $(c_1, c_2, d, R, T_f, \sigma)$, and the proof is completed.

\textbf{Remark 10.} Since different Lyapunov functions can be constructed for different subsystems, so the multiple Lyapunov functions method is an effective and flexible design tool. Now, the multiple Lyapunov functions have been employed and discussed to study the stability and performance of switched or hybrid systems such as [27–29]. In this paper, the function $V(t)$ in the proof of Theorem 9 is taken as the multiple Lyapunov functions. Compared with the classical Lyapunov function for switched systems of asymptotical stability, there is really no requirement of negative definiteness or negative semidefiniteness on $V(t)$. Actually, if we limit the constants $\alpha < 0$ and the exogenous disturbance $w(t) = 0$, then $V(t)$ will be a negative definite function. In this case, the system (1) is asymptotically stable on the infinite interval $[0, +\infty)$. We can find the detailed proof in [29].

In order to design controller conveniently, the following conclusion is given to satisfy the condition of Theorem 9.
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Theorem 11. For any \(i, j \in \{1, 2, \ldots, m\}\), if there exist non-singular matrix \(X_{ij}\), matrices \(Q_i > 0, Z_i > 0\), and scalars \(\alpha \geq 0\), \(\mu \geq 1\) such that (18) and (19) hold and
\[
EX_i = X_i^TE^T \geq 0, \tag{38}
\]
\[
\begin{bmatrix}
A_iX_i + X_i^T A_i^T - \alpha EX_i & G_i \\
G_i^T & F_i^TQ_i + Q_iF_i - \alpha Q_i
\end{bmatrix} < 0, \tag{39}
\]
\[
X_i^{-1}E = E^TR_i^{-1}Z_iR_i^{-1}E, \tag{40}
\]
\[
(1 - 2\mu)EX_i + \mu EX_j \leq 0, \quad Q_i \leq \alpha Q_j, \tag{41}
\]
then system (3) is regular, pulse-free, and finite-time bounded with respect to \((\epsilon_1, \epsilon_2, d, R, T_j, \sigma)\).

Proof. Multiply both sides of (38) separately by \(X_i^{-T}\) and \(X_i^{-1}\). Let \(X_i^{-T} = P_i\); we can obtain that (14) holds. Similarly, multiply both sides of (39) separately by \(diag[X_i^{-T}, I]\) and \(diag[X_i^{-1}, I]\). Let \(X_i^{-T} = P_i\); then (15) holds. On the other hand, let \(X_i = P_i^T\), it follows from (38) and (41) that
\[
EP_i^{-T} - \alpha EP_i^{-T} - \alpha P_i^{-1}E^T + \alpha EP_i^{-T} \leq 0,
\]
\[
Q_i \leq \alpha Q_j. \tag{42}
\]
Now, by virtue of \(-\alpha P_i^{-1}P_iE^T \leq -\alpha P_i^{-1}E^T - \alpha P_i^{-1}E^T + \alpha EP_i^{-T}\), it follows from (42) that \(EP_i^{-T} - \alpha P_i^{-1}P_iE^T \leq 0, Q_i \leq \alpha Q_j\). Obviously, the previous equation is equivalent to (17), so (41) can ensure that \(\Omega_i\) holds. From Theorem 9, it is easy to obtain that system (3) is regular, pulse-free, and finite-time bounded with respect to \((\epsilon_1, \epsilon_2, d, R, T_j, \sigma)\). \(\square\)

In the following, we give the following conclusion about the finite-time boundedness problem of system (1) via the action of the state feedback controller \(u_o(t) = K_o x(t)\).

Theorem 12. For any \(i, j \in \{1, 2, \ldots, m\}\), if there exist non-singular matrices \(X_{ij}, L_i, C_i\), matrices \(Q_i > 0, Z_i > 0\), and scalars \(\alpha \geq 0, \mu \geq 1\) such that (18), (19), and (38)-(41) hold and
\[
\begin{bmatrix}
A_iX_i + X_i^T A_i^T + B_i L_i + L_i^T B_i^T - \alpha EX_i & G_i \\
G_i^T & F_i^TQ_i + Q_iF_i - \alpha Q_i
\end{bmatrix} < 0, \tag{43}
\]
then the system (1) is regular, pulse-free, and finite-time bounded with respect to \((\epsilon_1, \epsilon_2, d, R, T_j, \sigma)\) via the action of the state feedback controller \(u_o(t) = L_iX_i^{-1}x(t)\).

Proof. According to the proof of Theorem 11, first replace \(A_i\) with \(A_i + B_iK_i\) and let \(K_iX_i = L_i\), and then it is easy to obtain the condition of Theorem 12.

Now, in order to solve by means of the LMI toolbox conveniently, we will process (38) and (40). According to rank \(E = r\), there exist invertible matrices \(M, N\) such that \(E = MEN = diag[L, 0]\). Let \(X_i = N^{-1}X_iM^T\) and from (38) \(\bar{X}_i\) is given as
\[
\bar{X}_i = \begin{bmatrix}
X_{i1} & 0 \\
X_{i3} & X_{i4}
\end{bmatrix}, \tag{44}
\]
where \(X_{i1} > 0\), det(\(X_{i4}\)) \neq 0, and \(X_{i3}\) is a matrix with appropriate dimension. In addition, let \(\Psi = N\begin{bmatrix}0 \\
I_{n-r}\end{bmatrix}\) and we can obtain \(E\Psi = 0\). Based on the above discussion, the following equation holds:
\[
X_i = N\begin{bmatrix}
X_{i1} & 0 \\
X_{i3} & X_{i4}
\end{bmatrix}M^T
= N\begin{bmatrix}
X_{i1} & 0 \\
0 & X_{i4}
\end{bmatrix}M^T\tag{45}
= N\Omega_iN^TE^T + \Psi Y_iM^T,
\]
where \(\Omega_i = \text{diag}[X_{i1}, \sigma] > 0\) and \(Y_i = [X_{i3} X_{i4}]\). It is obvious that \(X_i = N\Omega_iN^TE^T + \Psi Y_iM^T\) satisfies (41) and (43). Let \(Z_i = R_i^{-1/2}M^T\Omega_i^{-1}MR_i^{-1/2}\), and then one obtains that (40) holds.

Equation (18) can be guaranteed by the following LMIs. For any \(i, j, k \in \{1, 2, \ldots, m\}\), there exist scalars \(\epsilon_1, \epsilon_2\) such that
\[
\epsilon_1 I < R_i^{-1/2}M^T\Omega_i^{-1}MR_i^{-1/2} < I,
\]
\[
Q_i < \epsilon_2 I, \tag{46}
\]
\[
\begin{bmatrix}
-\epsilon_1^T \epsilon_2 + \epsilon_2^2 - \sqrt{\epsilon_1}\cr
-\sqrt{\epsilon_1} - \epsilon_1
\end{bmatrix} < 0.
\]

Theorem 13. For any \(i, j \in \{1, 2, \ldots, m\}\), if there exist matrices \(\Omega_i = \text{diag}[X_{i1}, \sigma] > 0, Q_i > 0, L_i, Y_i\), and scalars \(\alpha \geq 0, \mu \geq 1\) such that (19) and (46) hold and
\[
(1 - 2\mu)EN\Omega_i N^TE^T + \mu E\Omega_i N^TE^T \leq 0,
\]
\[
Q_i \leq \alpha Q_j, \tag{47}
\]
then the switched descriptor system (1) is regular, pulse-free, and finite-time bounded with respect to \((\epsilon_1, \epsilon_2, d, R, T_j, \sigma)\) via the action of the state feedback controller \(u_o(t) = L_i\beta^{-1}(\Omega_i, Y_i)x(t)\), where
\[
\begin{bmatrix}
\Xi_i & G_i \\
G_i^T & F_i^TQ_i + Q_iF_i - \alpha Q_i
\end{bmatrix} < 0,
\]
and the matrix \(\beta(\Omega_i, Y_i) = N\Omega_iN^TE^T + \Psi Y_iM^T\) is non-singular; matrices \(M, N\) satisfy \(MEN = \text{diag}[L, 0]\), \(\Psi = N[0, I_{n-r}]^T\).
Remark 14. If there exists $E = I$, $F_i = 0$, we can obtain the sufficient condition of finite-time boundedness for switched systems with constant disturbance based on the above conclusion. If there exists $w(t) = 0$, and $V(t)$ is taken as common Lyapunov function, then the system is of consistent finite-time boundedness with arbitrary switching signals.

Theorem 15. Consider the switched descriptor system as follows:

$$E \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u_{\sigma(t)}(t).$$  \hspace{1cm} (49)

For any $i, j \in \{1, 2, \ldots, m\}$, if there exist matrices $\Omega_i = \text{diag}(X_{ij}, \#) > 0$, $L_i$, $Y_i$, and scalars $\alpha \geq 0, \mu \geq 1$ such that

$$(1 - 2\mu) EN\Omega_i N^T E^T + \mu EN\Omega_i N^T E^T \leq 0,$$

\begin{align*}
A_i^T
B_i L_i + L_i^T B_i - \alpha E \beta(\Omega_i, Y_i) < 0, \hspace{1cm} (50)
\end{align*}

$$\varepsilon_1 I < R^{1/2}M^{-1}\dot{\Omega}_i M^{-1} R^{1/2} < \varepsilon_2 I,$$

$$\varepsilon_2 c_1 < c_2,$$

and the average dwell time satisfies

$$\tau_a > \tau_a^* = \frac{T_f \ln \mu}{\ln(\lambda_2 c_2) - \ln(\lambda_1 c_1)} - \alpha T_f,$$  \hspace{1cm} (51)

then the switched descriptor system (49) is regular, pulse-free, and finite-time stable with respect to ($c_1, c_2, d, R, T_f, \alpha$) via the action of the state feedback controller $u_i(t) = L_i \beta^{-1}(\Omega_i, Y_i) x(t)$, where $\beta(\Omega_i, Y_i) = N\dot{\Omega}_i N^T E^T + \Psi Y_i M^{-1} \dot{r}$ is nonsingular and matrices $M, N$ satisfy $M E N = \text{diag}[I, 0], \Psi = N[0 I_{n-r}]^T$.

4. Numerical Simulations

Example 16. Consider the continuous switched descriptor system (1) with parameters as follows:

$$A_1 = \begin{bmatrix}
-2.1 & 1.0 & -0.3 \\
-0.8 & -0.4 & 0.4 \\
0.8 & 0.6 & 1.0
\end{bmatrix},$$

$$B_1 = \begin{bmatrix}
1 \\
-1 \\
1
\end{bmatrix},$$

$$G_1 = \begin{bmatrix}
0.2 & 0.1 \\
-0.1 & 0.3 \\
0.1 & -0.1
\end{bmatrix},$$

$$F_1 = \begin{bmatrix}
0 & 0.1 \\
-0.1 & 0
\end{bmatrix},$$

$$A_2 = \begin{bmatrix}
-2.6 & -0.5 & 0.5 \\
0.6 & 0.5 & -0.8 \\
-0.5 & 0.2 & 1.0
\end{bmatrix},$$

$$B_2 = \begin{bmatrix}
-1 \\
1 \\
1
\end{bmatrix},$$

$$G_2 = \begin{bmatrix}
-0.1 & 0.2 \\
0.2 & -0.1 \\
-0.1 & 0.3
\end{bmatrix},$$

$$F_2 = \begin{bmatrix}
0 & 0.1 \\
-0.1 & 0
\end{bmatrix}.$$  \hspace{1cm} (52)

The values of $E, \alpha, c_1, c_2, d, T_f$, and $R$ are given as follows: $\alpha = 0.3, \mu = 1.2, E = \text{diag}[1, 1, 0], c_1 = 2, c_2 = 10, d = 3, T_f = 1$, and $R = I$.

Then, according to Theorem 13, we get

$$\Omega_1 = \begin{bmatrix}
0.5249 & 0.0266 & 0 \\
0.0266 & 0.4635 & 0 \\
0 & 0 & 0.4749
\end{bmatrix},$$

$$\Omega_2 = \begin{bmatrix}
0.5353 & 0.0275 & 0 \\
0.0275 & 0.4787 & 0 \\
0 & 0 & 0.4749
\end{bmatrix},$$

$$Y_1 = [-0.5388 -0.2418 -0.5585],$$

$$Y_2 = [1.2591 -0.2093 -1.1868],$$

$$L_1 = [0.2230 0.4453 -0.2745],$$

$$L_2 = [0.0427 -1.1710 0.3543],$$

$$Q_1 = \begin{bmatrix}
2.1001 & 0 & 0 \\
0 & 2.1001 & 0
\end{bmatrix},$$

$$Q_2 = \begin{bmatrix}
2.1001 & 0 & 0 \\
0 & 2.1001 & 0
\end{bmatrix}.$$  \hspace{1cm} (53)

In the following, we have by calculating

$$\beta(\Omega_1, Y_1) = \begin{bmatrix}
0.5453 & 0.0266 & 0 \\
0.0266 & 0.4635 & 0 \\
-0.5388 & -0.2418 & -0.5585
\end{bmatrix},$$

$$\beta(\Omega_2, Y_2) = \begin{bmatrix}
0.5353 & 0.0275 & 0 \\
0.0275 & 0.4787 & 0 \\
1.2591 & -0.2093 & -1.1868
\end{bmatrix}. $$  \hspace{1cm} (54)

Then we can obtain the designed state feedback controllers as follows:

$$u_1(t) = L_1 \beta^{-1}(\Omega_1, Y_1) x(t) = \begin{bmatrix}
0.8376 \\
1.1691 \\
0.4914
\end{bmatrix} x(t),$$

$$u_2(t) = L_2 \beta^{-1}(\Omega_2, Y_2) x(t) = \begin{bmatrix}
0.7566 \\
-2.6204 \\
-0.2985
\end{bmatrix} x(t). $$  \hspace{1cm} (55)

Furthermore, we can obtain $\lambda_1 = 2.1949, \lambda_2 = 1.8078, \lambda_3 = 2.1001$, and $\tau_0 > \tau_a^* = 0.2517$. For any switching signal $\sigma(t)$ with average dwell time $\tau_a > 0.2517$, the switched descriptor system (1) is finite-time stable.

When $w(t) = \begin{bmatrix}
\sin(0.1t) \\
\cos(0.1t)
\end{bmatrix}$, the trajectory of $x^T(t) E^T R E x(t)$ is presented in Figure 1 and the switching signal is shown in Figure 2.
Example 17. Consider the continuous switched descriptor system (49) with parameters as follows:

\[
A_1 = \begin{bmatrix} -1 & -2 & 1 \\ 2 & -1 & 3 \\ 0 & 1 & -2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -1 \\ -9 \\ 1 \end{bmatrix}, \\
A_2 = \begin{bmatrix} -2 & 1 & -1 \\ 1 & -1 & 0 \\ 2 & -2 & -1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 3 \\ -8 \\ -1 \end{bmatrix}.
\] (56)

Choosing the matrix \( E = \text{diag}[1, 1, 0] \) and letting \( \alpha = 0.1, \mu = 1.2, c_1 = 1, c_2 = 5, T_f = 6, \) and \( R = I, \) by virtue of Theorem 15, the feasible solutions are given as

\[
\Omega_1 = \begin{bmatrix} 33.4670 & -2.1549 & 0 \\ -2.1549 & 37.2884 & 0 \\ 0 & 0 & 36.9815 \end{bmatrix}, \\
\Omega_2 = \begin{bmatrix} 31.4690 & 0.5435 & 0 \\ 0.5435 & 38.9031 & 0 \\ 0 & 0 & 36.9815 \end{bmatrix}, \\
Y_1 = \begin{bmatrix} 6.8444 & 21.3449 & 15.4656 \\ -1.3833 & -31.4843 & 32.2021 \end{bmatrix}, \\
Y_2 = \begin{bmatrix} 3.7061 & 5.1168 & 5.1099 \\ 11.9850 & -1.8186 & -5.5726 \end{bmatrix}.
\] (57)

Now, it can be obtained as

\[
\beta(\Omega_1, Y_1) = \begin{bmatrix} 33.4670 & -2.1549 & 0 \\ -2.1549 & 37.2884 & 0 \\ 6.8444 & 21.3449 & 15.4656 \end{bmatrix}, \\
\beta(\Omega_2, Y_2) = \begin{bmatrix} 31.4690 & 0.5435 & 0 \\ 0.5435 & 38.9031 & 0 \\ -1.3833 & -31.4843 & 32.2021 \end{bmatrix}.
\] (58)

Then we can obtain the designed controllers as follows:

\[
u_1(t) = L_1 \beta^{-1}(\Omega_1, Y_1) x(t) = \begin{bmatrix} 0.0400 \\ -0.0496 \\ 0.3304 \end{bmatrix} x(t), \\
u_2(t) = L_2 \beta^{-1}(\Omega_2, Y_2) x(t) = \begin{bmatrix} 0.3766 \\ -0.1921 \\ -0.1731 \end{bmatrix} x(t).
\] (59)

By calculating, we have \( \lambda_1 = 0.0318, \lambda_2 = 0.0257, \) and \( \tau_a > \tau^* = 1.3758. \) For any switching signal \( \sigma(t) \) with average dwell time \( \tau_a > 1.3758, \) the system (49) is finite-time stable with respect to \( (c_1, c_2, d, R, T_f, \sigma) \) via the action of the state feedback controller. Figure 3 shows the trajectory of \( x^T(t)E^TREx(t), \) and Figure 4 shows the switching signal.

If the switching is too frequent, it is possible that the whole system is not finite-time stable. For instance, given the switching signal as follows:

\[
\sigma(t) = \begin{cases} 
1, & x_2x_3 - x_1x_2 < 0, \\
2, & x_2x_3 - x_1x_2 \geq 0,
\end{cases}
\] (60)

the whole system is not finite-time stable any more. The trajectory of \( x^T(t)E^TREx(t), \) is shown in Figure 5 and the switching signal \( \sigma(t) \) is shown in Figure 6.

5. Conclusion

In this paper, the issues of finite-time stability and finite-time boundedness for a class of continuous switched descriptor systems have been studied. The sufficient and necessary condition of finite-time stability for switched descriptor systems is presented by applying the state transition matrix method.
The obtained condition has certain theoretical value, but it also has two disadvantages in practical application. First, it is difficult to calculate the state transition matrix; on the other hand, it is inconvenient to design controller. In order to solve these problems, based on the average dwell time approach and the multiple Lyapunov function technique, the existence of state feedback controllers is proposed with arbitrary switching rules, which guarantee that the switched descriptor systems are finite-time stable and finite-time bounded, respectively. More possible future works are to consider output feedback stabilization for the uncertain switched descriptor systems with time-varying delay.

**Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

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