Research Article

Blow-Up Phenomena for Certain Nonlocal Evolution Equations and Systems

Mohamed Jleli and Bessem Samet

Department of Mathematics, College of Science, King Saud University, P.O. Box 2455, Riyadh 11451, Saudi Arabia

Correspondence should be addressed to Bessem Samet; bessem.samet@gmail.com

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We provide sufficient conditions for the nonexistence of global positive solutions to the nonlocal evolution equation

$$u_{tt}(x,t) = (J * u - u)(x,t) + u^{p}(x,t), \quad (x,t) \in \mathbb{R}^N \times (0,\infty), \quad (u(x,0), u_t(x,0)) = (u_0(x), u_1(x)), \quad x \in \mathbb{R}^N,$$

where $J : \mathbb{R}^N \to \mathbb{R}_+$ is a compactly supported nonnegative function with unit integral, $p > 1$, and $(u_0,u_1) \in L^1_{\text{loc}}(\mathbb{R}^N; \mathbb{R}_+) \times L^1_{\text{loc}}(\mathbb{R}^N; \mathbb{R}_+)$. Next, we deal with global nonexistence for certain nonlocal evolution systems. Our method of proof is based on a duality argument.

1. Introduction

In [1], García-Melián and Quirós considered the nonlocal diffusion problem:

$$u_t (x,t) = (J * u - u)(x,t) + u^{p}(x,t), \quad (x,t) \in \mathbb{R}^N \times (0,\infty), \quad u(x,0) = u_0(x), \quad x \in \mathbb{R}^N,$$

where $J : \mathbb{R}^N \to \mathbb{R}_+$ is a compactly supported nonnegative function with unit integral, $p > 1$, and $u_0 \in L^1(\mathbb{R}^N; \mathbb{R}_+) \cap L^\infty(\mathbb{R}^N; \mathbb{R}_+)$. Equation (1) may model a variety of biological, epidemiological, ecological, and physical phenomena involving media with properties varying in space [2, 3]; similar equations appear, for example, in Ising systems with Glauber dynamics [4]. In [1] the authors proved that (1) has a critical exponent:

$$p_c = 1 + \frac{2}{N},$$

which is the Fujita exponent for the classical nonlinear heat equation $u_t = \Delta u + u^p$ [5]. More precisely, they proved that if $1 < p \leq p_c$, the solution blows up in finite time for any nonnegative and nontrivial initial data $u_0 \in L^1(\mathbb{R}^N; \mathbb{R}_+) \cap L^\infty(\mathbb{R}^N; \mathbb{R}_+)$; if $p > p_c$, there exist global solutions for small initial data $u_0 \in L^1(\mathbb{R}^N; \mathbb{R}_+) \cap L^\infty(\mathbb{R}^N; \mathbb{R}_+)$. Very recently, Yang [6] considered the nonlinear coupled nonlocal diffusion system:

$$u_t (x,t) = (J * u - u)(x,t) + v^p(x,t), \quad (x,t) \in \mathbb{R}^N \times (0,\infty),$$

$$v_t (x,t) = (J * v - v)(x,t) + u^q(x,t), \quad (x,t) \in \mathbb{R}^N \times (0,\infty),$$

where $p, q > 1$ and $(u_0, v_0) \in L^\infty(\mathbb{R}^N; \mathbb{R}_+) \times L^\infty(\mathbb{R}^N; \mathbb{R}_+)$. Equation (3) can serve as a model for the processes of heat diffusion and combustion in two-component continua with nonlinear heat conduction and volumetric release [7]. In this case, Yang established that the critical Fujita curve is given by

$$(pq)^* = 1 + \frac{2}{N} \max \{p + 1, q + 1\},$$

which is also the Fujita curve for the coupled heat system $u_t = \Delta u + v^p$ and $v_t = \Delta v + u^q$, obtained by Escobedo and Herrero [8].
In this paper, we are first concerned with the following evolution problem:

\[ u_t (x, t) = (J * u - u)(x, t) + v^p(x, t), \]

\[ (x, t) \in \mathbb{R}^N \times (0, \infty), \tag{5} \]

\[ (u(x, 0), u_t(x, 0)) = (u_0(x), u_1(x)), \quad x \in \mathbb{R}^N, \]

where \( J : \mathbb{R}^N \to \mathbb{R}_+, \)

\[ p, q > 1 \]

\[ u(x, 0), v(x, 0) \in \mathbb{R}^N \times (0, \infty), \]

where \( J : \mathbb{R}^N \to \mathbb{R}_+, \)

\[ p, q > 1 \]

\[ (u(x, 0), v(x, 0)) = (u_0(x), v_0(x)), \quad x \in \mathbb{R}^N, \]

\[ (u_t(x, 0), v_t(x, 0)) = (u_1(x), v_1(x)), \quad x \in \mathbb{R}^N, \]

\[ (u(x, 0), v(x, 0)) = (u_0(x), v_0(x)), \quad x \in \mathbb{R}^N, \]

\[ (u_t(x, 0), v_t(x, 0)) = (u_1(x), v_1(x)), \quad x \in \mathbb{R}^N, \]

where \( p, q > 1. \) For each system, we find a bound on \( u \) leading to the absence of global nontrivial solutions. Our method of proof is based on a duality argument developed by Mitidieri and Pokhozhaev [9, 10].

### 2. Main Results

Through this paper, \( \mathbb{R}_+ = [0, \infty), \mathbb{Q} = \mathbb{R}^N \times (0, \infty), \) and \( J : \mathbb{R}^N \to \mathbb{R}_+ \) is a continuous function satisfying the following conditions:

1. \( J \) is symmetric; that is, \( J(z) = J(-z), \) for every \( z \in \mathbb{R}^N . \)
2. \( \int_{\mathbb{R}^N} J(z)dz = 1 . \)
3. \( A(J) := \int_{\mathbb{R}^N} J(z)|z|^2\,dz < \infty. \)

The following lemmas will be used later.

#### Lemma 1

Let \( a, b, \varepsilon > 0 \) and \( p > 1. \) Then

\[ ab \leq \varepsilon a^p + c_\varepsilon b^{p/(p - 1)}, \tag{8} \]

where \( c_\varepsilon = (p - 1)p^{-1}(\varepsilon p)^{1/(p - 1)}. \)

#### Lemma 2

(see [11]). Let \( X, Y, A, B, C, D \) be nonnegative functions and let \( \alpha_i, \beta_i, i = 1, 2, 3, \) be positive reals such that \( \alpha_2 < \alpha_1, \beta_2 < \beta_1, \beta_3 \geq 1, \) and \( \alpha_3 \beta_3 < \alpha_1 \beta_1. \) If

\[ X^{\alpha_1} \leq AX^{\alpha_2} + BY^{\beta_2}, \]

\[ Y^{\beta_3} \leq CX^{\alpha_3} + DY^{\beta_3}, \tag{9} \]

then

\[ X^{\alpha_1} \leq L \left[ A^{\alpha_2/\alpha_1} + B^{\beta_2/\beta_3} B^{\beta_3} \right], \]

\[ + \left( B^{\beta_3} C^{\beta_3} \right)^{\alpha_3/\alpha_1 - \alpha_2/\alpha_3} \tag{10} \]

for some constant \( L > 0. \)

#### 2.1. A Nonexistence Result for (5)

The definition of solutions we adopt for (5) is as follows.

#### Definition 3

Let \( (u_0, u_1) \in L^1_{loc}(\mathbb{Q}; \mathbb{R}_+ \times \mathbb{R}^N; \mathbb{R}_+). \) We say that \( u \) is a global weak solution to (5) if \( u \in L^p_{loc}(\mathbb{Q}; \mathbb{R}_+) \) and

\[ \int_{\mathbb{Q}} u^p \psi \, dx \, dt + \int_{\mathbb{R}^N} u_1(x) \psi(x, 0) \, dx = \int_{\mathbb{Q}} (J * u - u)(x, t) \psi \, dx \, dt \]

\[ - \int_{\mathbb{Q}} (J * u - u)(x, t) \psi \, dx \, dt \]

\[ + \int_{\mathbb{R}^N} u_0(x) \psi_0(x, 0) \, dx, \tag{11} \]

for every regular test function \( \psi \geq 0 \) with \( \psi(\cdot, t \geq T) \equiv 0. \)

Our first main result is given by the following theorem.

#### Theorem 4

Suppose that one of the following conditions hold:

\[ N = 1 < p \]

or

\[ N \geq 2, \]

\[ 1 < p < \frac{N + 1}{N - 1}. \tag{13} \]

Then (5) admits no global weak solutions other than the trivial one.

#### Proof

Suppose that \( u \) is a nontrivial global weak solution to (5). As a test function, we take

\[ \psi(x, t) = \xi(x) \phi(t/R), \quad (x, t) \in \mathbb{Q}, \tag{14} \]

where \( R > 0 \) is large enough,

\[ \xi(x) = \exp \left( \frac{-1}{R^2} \right), \quad x \in \mathbb{R}^N. \tag{15} \]
\( \omega \gg 1 \), and \( \phi : \mathbb{R}_+ \to [0, 1] \) is given by
\[
\phi(\sigma) = \begin{cases} 
1 & \text{if } 0 \leq \sigma \leq 1, \\
0 & \text{if } \sigma \geq 2.
\end{cases}
\]
From the definition of \( \phi \), clearly we have
\[
\varphi_t(x, 0) = 0, \quad x \in \mathbb{R}^N,
\]
which yields
\[
\int_{\mathbb{R}^N} u_0(x) \varphi_t(x, 0) \, dx = 0.
\]
Writing
\[
\int_{Q} u \varphi_t \, dx \, dt = \int_{Q} \left( u \varphi^{1/p} \right) \left( \varphi^{-1/p} \varphi_t \right) \, dx \, dt
\]
and applying Lemma 1, we obtain
\[
\int_{Q} u \varphi_t \, dx \, dt \leq \varepsilon \int_{Q} \varphi \, dx \, dt + c_1 \int_{Q} \varphi^{-p/(p-1)} |\varphi_t|^p \, dx \, dt,
\]
for some \( \varepsilon > 0 \), where \( p' = p/(p-1) \). On the other hand,
\[
\int_{Q} \varphi^{-p/p'} |\varphi_t|^p \, dx \, dt = \omega^p R^{N+2p'} \int_{Q} \exp \left( \frac{-|x|^2}{R^2} \right) \varphi^{p'} \, dx \, dt,
\]
where
\[
h(\sigma) = (\omega - 1) \varphi'(\sigma) + \varphi(\sigma) \varphi''(\sigma), \quad \sigma \geq 0.
\]
Using the change of variable \( x = R y \) and \( t = R s \), we obtain
\[
\int_{Q} \varphi^{-p/p'} |\varphi_t|^p \, dx \, dt = \omega^p R^{N+1-2p'} \int_{Q} \exp \left( \frac{-|y|^2}{R^2} \right) \cdot \varphi^{p'} \, dx \, dt,
\]
The above equality with (20) yields
\[
\int_{Q} u \varphi_t \, dx \, dt \leq \varepsilon \int_{Q} \varphi \, dx \, dt + c_1 R^{N+1-2p'},
\]
for some constant \( c_1 > 0 \). Next, we have
\[
\int_{Q} (J * u) \varphi \, dx \, dt
\]
Using the symmetry of \( J \) and Fubini’s theorem, we obtain
\[
\int_{Q} (J * u) \varphi \, dx \, dt = \int_{Q} (J * \varphi) u \, dx \, dt.
\]
Using the property (J1), we obtain
\[
(J * \varphi - \varphi)(x, t) = \phi \omega(t/R) \int_{R^N} J(z) \cdot (\exp \left( \frac{-|x+z|^2}{R^2} \right) - \exp \left( \frac{-|x|^2}{R^2} \right)) \, dz
\]
By the property (J2) and the definition of \( \varphi \), we have
\[
(J * \varphi - \varphi)(x, t) = \phi \omega \left( \frac{t}{R} \right) \int_{R^N} J(z) \cdot (\exp \left( \frac{-1}{R^2} \left( 2 \sum_{i=1}^{N} x_i z_i + |z|^2 \right) \right) - 1) \, dz.
\]
The property (J3) and the inequality $e^x \geq x + 1$ yield
\[
(J * \varphi - \varphi)(x, t) \\
\geq -\frac{\varphi(x, t)}{R^2} \int_{\mathbb{R}^N} J(z) \left( \frac{2}{N} \sum_{i=1}^{N} x_i z_i + |z|^2 \right) dz
\]
\[
= -\frac{\varphi(x, t)}{R^2} \int_{\mathbb{R}^N} J(z) |z|^2 dz = -A(J) R^{-2} \varphi(x, t).
\]
From this, we have
\[
-\int_{Q} (J * u - u) \varphi \, dx \, dt \leq A(J) R^{-2} \int_{Q} u \varphi \, dx \, dt.
\]
Writing
\[
\int_{Q} u \varphi \, dx \, dt = \int_{Q} u^\frac{1}{p} \varphi^{1/p'} \, dx \, dt,
\]
using Hölder’s inequality and Lemma 1, we obtain
\[
-\int_{Q} (J * u - u) \varphi \, dx \, dt \leq A(J)
\]
\[
\cdot R^{-2} \left( \int_{Q} u^p \varphi \, dx \, dt \right)^{\frac{1}{p'}} \left( \int_{Q} \varphi \, dx \, dt \right)^{\frac{1}{p'}}
\]
\[
\leq \delta \int_{Q} u^p \varphi \, dx \, dt + c_3 A(J) R^{-2} p' \int_{Q} \varphi \, dx \, dt
\]
\[
\leq \delta \int_{Q} u^p \varphi \, dx \, dt + c_3 A(J) R^{p'}
\]
\[
\cdot R^{N - 1 - 2p'} \left( \int_{\mathbb{R}^N} \exp \left( -\frac{|x|^2}{R^2} \right) dx \right) \left( \int_{0}^{\infty} \phi^{\omega} \left( \frac{t}{R} \right) dt \right)
\]
\[
\leq \delta \int_{Q} u^p \varphi \, dx \, dt + c_3 A(J) R^{p'}
\]
for some $\delta > 0$. We get
\[
-\int_{Q} (J * u - u) \varphi \, dx \, dt
\]
\[
\leq \delta \int_{Q} u^p \varphi \, dx \, dt + c_2 R^{N - 1 - 2p'},
\]
for some constant $c_2 > 0$. Consequently, it follows from (11), (18), (24), and (34) that
\[
(1 - \varepsilon - \delta) \int_{Q} u^p \varphi \, dx \, dt \leq (c_1 + c_2) R^{N + 1 - 2p'}.
\]
For $\varepsilon = \delta = 1/4$, we obtain
\[
\int_{Q} u^p \varphi \, dx \, dt \leq c R^{N + 1 - 2p'},
\]
where $c = 2(c_1 + c_2)$. Observe that if one of conditions (12) or (13) is satisfied, then $N + 1 - 2p' < 0$. In this case, letting $R \rightarrow \infty$ in the above inequality and using the monotone convergence theorem, we obtain
\[
\int_{Q} u^p \, dx \, dt = 0,
\]
which is a contradiction. The proof is finished.

2.2. A Nonexistence Result for System (6). The definition of solutions we adopt for (6) is as follows.

Definition 5. Let $(u_i, v_i) \in L^1_{\text{loc}}(\mathbb{R}^N; \mathbb{R}_+) \times L^p_{\text{loc}}(\mathbb{R}^N; \mathbb{R}_+), i = 0, 1$. We say that the pair $(u, v)$ is a global weak solution to (6) if $(u, v) \in L^1_{\text{loc}}(Q; \mathbb{R}_+) \times L^p_{\text{loc}}(Q; \mathbb{R}_+), (J * u, J * v) \in L^1_{\text{loc}}(Q; \mathbb{R}_+) \times L^1_{\text{loc}}(Q; \mathbb{R}_+)$, and
\[
\int_{Q} u^p \varphi \, dx \, dt + \int_{\mathbb{R}^N} u_0(x) \varphi_1(x, 0) \, dx
\]
\[
= \int_{Q} u^p \varphi_1 \, dx \, dt - \int_{Q} (J * u - u) \varphi \, dx \, dt
\]
\[
+ \int_{\mathbb{R}^N} u_0(x) \varphi_1(x, 0) \, dx;
\]
\[
\int_{Q} u^p \varphi_1 \, dx \, dt + \int_{\mathbb{R}^N} \psi_1(x) \varphi_0(x, 0) \, dx
\]
\[
= \int_{Q} v^q \varphi_1 \, dx \, dt - \int_{Q} (J * v - v) \varphi \, dx \, dt
\]
\[
+ \int_{\mathbb{R}^N} \psi_0(x) \varphi_0(x, 0) \, dx;
\]
for every regular test function $\varphi \geq 0$ with $\varphi(\cdot, t \geq T) \equiv 0$.

We have the following result.

Theorem 6. Let $p, q > 1$. Suppose that
\[
1 \leq N 
\]
\[
< 1 + \frac{2}{pq - 1} \max \{p + 1, q + 1\}.
\]
Then (6) admits no global weak solutions other than the trivial one.

Proof. Suppose that $(u, v)$ is a nontrivial global weak solution to (6). As a test function, we take the function $\varphi$ defined by (14). From the definition of $\varphi$, we have
\[
\int_{\mathbb{R}^N} u_0(x) \varphi_1(x, 0) \, dx = \int_{\mathbb{R}^N} \psi_0(x) \varphi_0(x, 0) \, dx = 0.
\]
Writing
\[
\int_{Q} u^p \varphi_1 \, dx \, dt
\]
\[
= \left( \int_{Q} u^q \varphi_1 \, dx \, dt \right)^{1/q} \left( \int \varphi_1^{p/q} \, dx \, dt \right)^{1/q},
\]
and using Hölder’s inequality, we obtain
\[
\int_{Q} u^p \varphi_1 \, dx \, dt
\]
\[
\leq \left( \int_{Q} u^q \varphi_1 \, dx \, dt \right)^{1/q} \left( \int \varphi_1^{p/q} \, dx \, dt \right)^{1/q},
\]
where \( q' = q/(q - 1) \). On the other hand, from (23), we have

\[
\int_{Q} \phi^{q'/q}(\nu_{n})^q \, dx \, dt = \omega^q \int_{Q} \exp\left(-\frac{|y|^2}{2}\right) \cdot \phi^{q' - 2/q(q - 1)}(s) \, dy \, ds,
\]

which yields

\[
\int_{Q} u \not\equiv u \phi \, dx \, dt \leq C \int_{Q} u \phi \, dx \, dt
\]

for some constant \( C > 0 \). As consequence, from (38), (45), and (47), it follows that

\[
\int_{Q} v \phi \, dx \, dt \leq C \int_{Q} \phi \, dx \, dt
\]

for some constant \( C \) > 0. Combining (48) with (49), we obtain

\[
\int_{Q} v \phi \, dx \, dt \leq CR^3
\]

for some constant \( C \) > 0, where

\[
\lambda_{1} = -2 - \frac{2}{q} + \frac{N + 1}{pq} (q - 1),
\]

\[
\lambda_{2} = -2 - \frac{2}{p} + \frac{N + 1}{pq} (q - 1).
\]

Observe that (40) is equivalent to \( \lambda_{i} < 0, i = 1, 2 \). Under this condition, letting \( R \to \infty \) in (50), we get

\[
\int_{Q} v \phi \, dx \, dt = \int_{Q} u \phi \, dx \, dt = 0,
\]

which is a contradiction.

\( \square \)

**Remark 7.** Taking \( u = v \) and \( p = q \) in Theorem 6, we obtain the result given by Theorem 4 for (5).

### 2.3. A Nonexistence Result for System (7).

The definition of solutions we adopt for (7) is as follows.

**Definition 8.** Let \((u, v) \in L_{loc}^{1}(\mathbb{R}^{N}; \mathbb{R}) \times L_{loc}^{1}(\mathbb{R}^{N}; \mathbb{R}), i = 1, 2 \). We say that the pair \((u, v)\) is a global weak solution to (6) if \((u, v) \in L_{loc}^{1}(\mathbb{R}^{N}; \mathbb{R}) \times L_{loc}^{1}(\mathbb{R}^{N}; \mathbb{R}), u \not\equiv 0, v \not\equiv 0, (J * u, J * v) \in L_{loc}^{1}(\mathbb{R}^{N}; \mathbb{R}) \times L_{loc}^{1}(\mathbb{R}^{N}; \mathbb{R}), \) and

\[
\int_{Q} u \phi \, dx \, dt + \int_{Q} u \phi \, dx \, dt = \int_{Q} u \phi \, dx \, dt - \int_{Q} (J * u) \phi \, dx \, dt
\]

for every regular test function \( \phi \geq 0 \) with \( \phi(x, t) \equiv 0 \).

We have the following result.

**Theorem 9.** Let \( p, q > 1 \). If

\[
1 \leq N < \max \left\{ \Theta_{1}, \Theta_{2} \right\},
\]

where

\[
\Theta_{1} = \min \left\{ \frac{p + 1}{p - 1}, \frac{q + 1}{q - 1}, \frac{pq + 2p + 1}{pq - 1} \right\},
\]

\[
\Theta_{2} = \min \left\{ \frac{p + 1}{p - 1}, \frac{q + 1}{q - 1}, \frac{pq + 2q + 1}{pq - 1} \right\},
\]

then (7) admits nonglobal weak solutions.
Proof. As before, we argue by contradiction. Suppose that 
\((u, v)\) is a nontrivial global weak solution to (7). As a test function, we take the function \(\varphi\) defined by (14). From (45), we have
\[
\int_Q u \varphi_t \, dx \, dt \leq c_1 R^{(N+1)/p - 2} \left( \int_Q u^p \varphi \, dx \, dt \right)^{1/p}. \tag{57}
\]
From (47), we have
\[
- \int_Q (J * u - u) \varphi \, dx \, dt 
\leq c_2 R^{-2 + (N+1)/q} \left( \int_Q \varphi^q \, dx \, dt \right)^{1/q}. \tag{58}
\]
Using (53), (57), and (58), we get
\[
\int_Q u^p \varphi \, dx \, dt \leq c_1 R^{(N+1)/p - 2} \left( \int_Q u^p \varphi \, dx \, dt \right)^{1/p} + c_2 R^{-2 + (N+1)/q} \left( \int_Q \varphi^q \, dx \, dt \right)^{1/q}. \tag{59}
\]
Similarly, using (54), (57), and (58), we get
\[
\int_Q \varphi^q \, dx \, dt \leq d_1 R^{(N+1)/q - 2} \left( \int_Q \varphi^q \, dx \, dt \right)^{1/q} + d_2 R^{-2 + (N+1)/p} \left( \int_Q u^p \varphi \, dx \, dt \right)^{1/p}. \tag{60}
\]
Here, \(c_i, d_i, i = 1, 2\), are some positive constants. Set
\[
X = \left( \int_Q u^p \varphi \, dx \, dt \right)^{1/p}, \quad Y = \left( \int_Q \varphi^q \, dx \, dt \right)^{1/q}, \tag{61}
\]
we obtain from (59) and (60) the following system:
\[
X^p \leq c_1 R^{(N+1)/p - 2} X + c_2 R^{-2 + (N+1)/q} Y, \quad Y^q \leq d_1 R^{(N+1)/q - 2} X + d_2 R^{(N+1)/p - 2} Y. \tag{62}
\]
Using Lemma 2, we obtain
\[
X^{pq} \leq c \left( R^{\lambda_1} + R^{\lambda_2} + R^{\lambda_3} \right), \tag{63}
\]
where
\[
\frac{p - 1}{q} \lambda_1 = (N + 1) (p - 1) - 2p, \quad \frac{q - 1}{q} \lambda_2 = (N + 1) (q - 1) - 2q, \quad \frac{pq - 1}{pq} \lambda_3 = -2q + (N + 1) (q - 1) \tag{64}
\]
\[
+ \frac{(N + 1) (p - 1) - 2p}{p}.
\]
Similarly, we have
\[
Y^{pq} \leq c \left( R^{\mu_1} + R^{\mu_2} + R^{\mu_3} \right), \tag{65}
\]
where
\[
\frac{q - 1}{p} \mu_1 = (N + 1) (q - 1) - 2q, \quad \frac{p - 1}{p} \mu_2 = (N + 1) (p - 1) - 2p, \quad \frac{pq - 1}{pq} \mu_3 = -2p + (N + 1) (p - 1) \tag{66}
\]
\[
+ \frac{(N + 1) (q - 1) - 2q}{q}.
\]
It is not difficult to observe that condition (55) is equivalent to \(\lambda_i < 0, i = 1, 2, 3\), or \(\mu_i < 0, i = 1, 2, 3\). In both cases, letting \(R \to \infty\) in (63) or in (65), we obtain \(XY = 0\), which is a contradiction. \(\square\)

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References


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