Research Article

Stabilization of a Class of Continuous-Time Switched Systems with State Constraints via a Mode-Dependent Switching Method

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The stability and the stabilization problems for a class of continuous-time switched systems with state constraints via a mode-dependent switching method are investigated. The paper presents an improved average dwell time method, which considers different decay rates of a Lyapunov function related to each of the active subsystems according to whether the saturations occur or not, respectively. It is shown that the improved average dwell time method is less conservative than the common average dwell time method. Based on the improved average dwell time method, the sufficient conditions and state feedback controllers for stabilization of the switched system are derived. A numerical example is given to illustrate the proposed approach.

1. Introduction

A switched system is a special class of hybrid systems that consists of a finite number of subsystems and a logical rule that orchestrates switching between these subsystems [1–3]. In the last few years, due to their success in practical applications and importance in theory development, switched systems have received much attention [4–6]. Switched systems arise in many engineering applications, such as aeronautics and astronautics systems [7, 8], mechanical systems [9], and networked control systems [10]. Owing to some safety consideration or inherent limits of devices, the majority of the practical systems have states constraints.

In the study of switched systems, one basic research topic is the issue of stability which has attracted most of the attention [11–14]. Many techniques have been studied in the study of switched systems, for example, common Lyapunov function, multiple Lyapunov functions, and switched Lyapunov functions [15–17]. However, the average dwell time method is generally recognized to have more flexibility in stabilization for switched systems [18–23].

Up to now, there exist many literatures related to stabilization of switched systems with state constraints [24–27]. However, [28] studies the stability and stabilization of switched linear systems with mode-dependent average dwell time. Reference [29] investigates the stabilization of a class of

switched systems with state constraints. It is noteworthy that both of the above literatures did not discuss the stability and stabilization of switched systems with state constraints based on MDADT. In short, according to the author, the problems of stability and stabilization for continuous-time switched systems with state constraints based on mode-independent average dwell time have not been addressed in the existing literatures.

All the above observations give rise to the question of how to design the MDADT switching to stabilize the continuous-time switched systems with state constraints. This inspires us for this study.

Thus, it is necessary to investigate the stabilization problem for a class of continuous-time switched systems with state constraints based on MDADT, which is an important property for a switched system. The main contributions of this paper are given as follows: (i) an improved average dwell time method is proposed, which is less conservative than the MDADT [28] and the ADT [29]; (ii) the sufficient conditions and state feedback controllers for stabilization of continuous-time switched systems with state constraints under MDADT switching are derived.

The remainder of this paper is organized as follows. Section 2 is the problem formulation and preliminaries. Main results are given in Section 3, including sufficient condition concerning controller design and an iterative algorithm.

Finally, a numerical example is given in Section 4. Concluding remarks are drawn in Section 5.

Notations. In this paper, the notation used is standard. $\mathbb{Z}^+$ denotes the set of nonnegative integers, $\mathbb{R}^n$ represents the n -dimensional space, $\|\cdot\|$ stands for the Euclidean vector norm, and I is the identity matrix. Symbol $*$ denotes the symmetry elements in symmetric matrices. $\mathcal{C}^1$ means the space of continuously differentiable functions.

2. Problem Formulation and Preliminaries

Consider a class of state-constrained continuous-time switched systems given by

$$\dot{x}(t) = h(A_{\sigma(t)}x(t) + B_{\sigma(t)}u_{\sigma(t)}(t)), \quad (1)$$

where $x(t) \in \mathbb{R}^n$ and $u_{\sigma(t)}(t)$, respectively, denote the state vector and the control input. The two-matrix pair (A_p, B_p) , $\forall \sigma(t) = p \in \mathcal{L}$, represents the p th subsystem or p th mode of (1); symbol $h(\cdot)$ denotes the saturation function. $\sigma(t)$ is a piecewise constant function of time, which takes its values in the finite set $\mathcal{L} = \{1, 2, \dots, N\}$; $N > 1$ is the number of subsystems. In addition, for a switching sequence $0 < t_1 < \dots < t_{i-1} < t_i < \dots$, symbol t_i means the moment of the i th switching. When $t \in (t_{i-1}, t_i)$, we say the $\sigma(t_{i-1})$ th subsystem is active.

Generally speaking, the ideal state feedback is treated as $u_{\sigma(t)}(t) = F_{\sigma(t)}x(t)$, where $F_{\sigma(t)}$ is the controller gain to be determined. Then, the resulting closed-loop system is described by

$$\dot{x}(t) = \begin{cases} \mathcal{A}_p x(t), & \text{non-constrained,} \\ \overline{\mathcal{A}}_p x(t), & \text{constrained,} \end{cases} \quad (2)$$

where $\mathcal{A}_p := A_p + B_p F_p$, which represents the closed-loop system matrix of the p th subsystem.

About the saturation function $h(\cdot)$, we transform it into the vertex of a convex hull to handle the saturations [30].

Symbol U_n denotes the set of $n \times n$ diagonal matrices. The diagonal elements of U_n are 0 or 1. Assuming that every element of U_n is marked as U_s , $U_s^- = I - U_s$, $s \in \{1, 2, \dots, 2^{n-1}, 2^n\}$. Therefore, we have

$$h(Ax) \in \text{co}\{U_s(Ax) + U_s^-L\}, \quad (3)$$

and the diagonal elements of row diagonally dominant matrix L are negative.

Now, let us first revisit the following definition and lemmas for later development.

Definition 1 (see [19]). For a switching signal $\sigma(t)$ and each $t_2 \geq t_1 \geq 0$, let $N_{\sigma(t)}(t_2, t_1)$ denote the number of discontinuities of $\sigma(t)$ in the open interval (t_1, t_2) . We say that $\sigma(t)$ has an average dwell time τ_a if there exist two positive numbers N_0 and τ_a such that

$$N_{\sigma(t)}(t_2, t_1) \leq N_0 + \frac{t_2 - t_1}{\tau_a}, \quad \forall t_2 \geq t_1 \geq 0. \quad (4)$$

Remark 2. Definition 1 means that if there exists a positive number τ_a such that a switching signal has the ADT property, the ADT between any two consecutive switching signals is no smaller than a common constant τ_a for all subsystems.

Lemma 3 (see [29]). Consider a continuous-time state-constrained switched system:

$$\dot{x}(t) = f_{\sigma(t)}(x(t)), \quad x(t) \in \Omega \subset \mathbb{R}^n, \quad \sigma(t) = p \in \mathcal{P}, \quad (5)$$

with $f_p(0) = 0$, $\forall p \in \mathcal{P}$. Assume that all trajectories remain inside Ω . Let α , β , and μ be some given constants satisfying $\alpha > \beta > 0$, $\mu \geq 1$. If there exist some piecewise continuous functions $V_p(x(t)) : \Omega \rightarrow \mathbb{R}$, for all $p \in \mathcal{P}$, such that

$$\varphi_1(\|x(t)\|) \leq V_p(x(t)) \leq \varphi_2(\|x(t)\|), \quad \forall x(t) \in \Omega, \quad (6)$$

$$\dot{V}_p(x(t)) \leq \begin{cases} -\alpha V_p(x(t)), & \forall t \in \mathcal{T}_f(t_i, t_{i+1}), \\ -\beta V_p(x(t)), & \forall t \in \mathcal{T}_s(t_i, t_{i+1}), \end{cases} \quad (7)$$

and $\forall(\sigma(t_i) = p, \sigma(t_i^-) = q) \in \mathcal{P} \times \mathcal{P}$, $p \neq q$,

$$V_p(x(t_i)) \leq \mu V_q(x(t_i)), \quad \forall x(t) \in \Omega, \quad (8)$$

where φ_1 and φ_2 are some class $\mathcal{K}$ functions, then the state-constrained switched system (5) is GUAS for any switching signal with ADT

$$\tau_a \geq \tau_a^* = \frac{\ln \mu}{\zeta}, \quad (9)$$

$$\frac{\mathcal{T}_f(t_0, t)}{\mathcal{T}_s(t_0, t)} \geq \frac{\zeta - \beta}{\alpha - \zeta} > 0.$$

Lemma 4 (see [28]). Consider the continuous-time switched system $\dot{x}(t) = f_{\sigma(t)}(x(t))$, $\sigma(t) \in \mathcal{P}$, and let $\lambda_p > 0$ and $\mu_p > 1$ be given constants. Supposing that there exist positive definite $\mathcal{C}^1$ functions $V_{\sigma(t)} : \mathbb{R}^n \rightarrow \mathbb{R}$ and two class $\mathcal{K}_\infty$ functions κ_{1p} and κ_{2p} , $p \in \mathcal{S}$,

$$\kappa_{1p}(\|x(t)\|) \leq V_p(x(t)) \leq \kappa_{2p}(\|x(t)\|), \quad (10)$$

$$\dot{V}_p(x(t)) \leq -\lambda_p V_p(x(t)),$$

and $\forall(\sigma(t_i) = p, \sigma(t_i^-) = q) \in \mathcal{S} \times \mathcal{S}$, $p \neq q$,

$$V_p(x(t_i)) \leq \mu_p V_q(x(t_i)), \quad (11)$$

then the system is GUAS for any switching signal with MDADT:

$$\tau_{ap} \geq \tau_{ap}^* = \frac{\ln \mu_p}{\lambda_p}. \quad (12)$$

3. Main Results

In this section, we first establish the stability conclusion for the continuous-time switched systems with state constraints based on MDADT in Lemma 7. The following definition is important for our result.

Definition 5 (see [28]). For a switching signal $\sigma(t)$ and each $T \geq 0$, let $N_{\sigma p}(T, 0)$ denote the switching numbers that the p th subsystem is activated over the interval $[0, T]$ and let $T_p(T, 0)$ denote the total running time of the p th subsystem over the interval $[0, T]$, $p \in \mathcal{S}$. We say that $\sigma(t)$ has a mode-dependent average dwell time τ_{ap} if there exist two positive numbers N_{0p} (we call N_{0p} the mode-dependent chatter bounds here) and τ_{ap} such that

$$N_{\sigma p}(T, 0) \leq N_{0p} + \frac{T_p(T, 0)}{\tau_{ap}}, \quad \forall T \geq 0. \quad (13)$$

Remark 6. Definition 5 constructs a novel set of switching signals with a MDADT property: it means that if there exist positive numbers τ_{ap} , $p \in \mathcal{S}$, such that a switching signal has the MDADT property, we only require that the average time among the intervals associated with the p th subsystem is larger than τ_{ap} (note that the intervals here are not adjacent), where $N_{\sigma}(T, 0) = \sum_{p=1}^M N_{\sigma p}(T, 0)$.

Next, Lemma 7 presents the stability result for the continuous-time state-constrained switched systems under MDADT switching.

Lemma 7. Consider the following switched systems:

$$\dot{x}(t) = f_{\sigma(t)}(x(t)), \quad (14)$$

where $x(t) \in \Phi \subset \mathbb{R}^n$, and let $\lambda_p > \gamma_p > 0$ and $\mu_p \geq 1$, $p \in \mathcal{L}$, be given constants. Supposing that there exist $\mathcal{C}^1$ function

$V_{\sigma(t)}(x(t)) : \Phi \rightarrow \mathbb{R}$ and some class $\mathcal{W}$ functions w_{1p} and w_{2p} , $\forall p \in \mathcal{L}$, then

$$w_{1p}(\|x(t)\|) \leq V_p(x(t)) \leq w_{2p}(\|x(t)\|), \quad (15)$$

$$\dot{V}_p(x(t)) \leq -\lambda_p V_p(x(t)), \quad \forall t \in R_p(t_{i-1}, t_i); \quad (16)$$

$$\dot{V}_p(x(t)) \leq -\gamma_p V_p(x(t)), \quad \forall t \in S_p(t_{i-1}, t_i),$$

and $\forall(\sigma(t_i) = p, \sigma(t_i^-) = q) \in \mathcal{L} \times \mathcal{L}$, $p \neq q$, $i \in \mathbb{Z}^+$,

$$V_p(x(t_i)) \leq \mu_p V_q(x(t_i)) \quad (17)$$

then the switched system (14) is GUAS under any MDADT switching signal

$$\tau_{ap} \geq \tau_{ap}^* = \frac{\ln \mu_p}{\rho}, \quad (18)$$

where $\lambda_p R_p(t_0, t) + \gamma_p S_p(t_0, t) \geq \rho T_p$.

Proof. From the above, we know that the system states can be divided into the saturated state and the nonsaturated state. For example, when $t > t_s \in S_p(t_{i-1}, t_i)$, that is to say, the system state is saturated, then we can conclude from that that

$$V_p(x(t)) - e^{-\gamma_p(t-t_s)} V_p(x(t_s)) < 0. \quad (19)$$

The inequality for the nonsaturated state can be obtained using similar techniques. Therefore, when $t > t_i$, together with (17) and (19), it implies

$$\begin{aligned} V_{\sigma(t_i)}(x(t)) &\leq e^{-\lambda_{\sigma(t_i)} R_{\sigma(t_i)}(t_i, t)} e^{-\gamma_{\sigma(t_i)} S_{\sigma(t_i)}(t_i, t)} V_{\sigma(t_i)}(x(t_i)) \leq \mu_{\sigma(t_i)} e^{-\lambda_{\sigma(t_i)} R_{\sigma(t_i)}(t_i, t)} e^{-\gamma_{\sigma(t_i)} S_{\sigma(t_i)}(t_i, t)} V_{\sigma(t_{i-1})}(x(t_{i-1}^-)) \\ &\leq \mu_{\sigma(t_i)} e^{-\lambda_{\sigma(t_i)} R_{\sigma(t_i)}(t_i, t) - \lambda_{\sigma(t_{i-1})} R_{\sigma(t_{i-1})}(t_{i-1}, t_i)} e^{-\gamma_{\sigma(t_i)} S_{\sigma(t_i)}(t_i, t) - \gamma_{\sigma(t_{i-1})} S_{\sigma(t_{i-1})}(t_{i-1}, t_i)} V_{\sigma(t_{i-1})}(x(t_{i-1})) \\ &\leq \mu_{\sigma(t_i)} \mu_{\sigma(t_{i-1})} e^{-\lambda_{\sigma(t_i)} R_{\sigma(t_i)}(t_i, t) - \lambda_{\sigma(t_{i-1})} R_{\sigma(t_{i-1})}(t_{i-1}, t_i)} e^{-\gamma_{\sigma(t_i)} S_{\sigma(t_i)}(t_i, t) - \gamma_{\sigma(t_{i-1})} S_{\sigma(t_{i-1})}(t_{i-1}, t_i)} V_{\sigma(t_{i-2})}(x(t_{i-2}^-)) \leq \dots \\ &\leq \prod_{p=1}^M \mu_p^{N_{\sigma p}} e^{-\sum_{p=1}^M [\lambda_p R_p(t_0, t) + \gamma_p S_p(t_0, t)]} V_{\sigma(t_0)}(x(t_0)). \end{aligned} \quad (20)$$

In summary, this implies

$$\begin{aligned} V_{\sigma(t_i)}(x(t)) &\leq \prod_{p=1}^M \mu_p^{N_{\sigma p}} e^{-\rho T_p} V_{\sigma(t_0)}(x(t_0)) \\ &\leq \prod_{p=1}^M \mu_p^{N_{0p}} \mu_p^{T_p/\tau_{ap}} e^{-\rho T_p} V_{\sigma(t_0)}(x(t_0)) \\ &\leq \prod_{p=1}^M \mu_p^{N_{0p}} \left(\mu_p^{1/\tau_{ap}} e^{-\rho} \right)^{T_p} V_{\sigma(t_0)}(x(t_0)). \end{aligned} \quad (21)$$

Therefore, supposing that the switching signal satisfies

$$\tau_{ap} \geq \tau_{ap}^* = \frac{\ln \mu_p}{\rho}, \quad (22)$$

we have $\mu_p^{1/\tau_{ap}} e^{-\rho} \leq \mu_p^{\rho/\ln \mu_p} e^{-\rho} = 1$. With $V_{\sigma(t_i)}(x(t))$ convergence to zero along with $t \rightarrow \infty$, the asymptotic stability also can be derived. $\square$

Remark 8. In Lemma 7, assume that $\lambda_p > \gamma_p$, and symbols λ_p and γ_p respectively denote the different decay rates of a Lyapunov function related to each of the active subsystems according to whether the saturations occur or not. For convenience of understanding, $R_p(t_{i-1}, t_i)$ and $S_p(t_{i-1}, t_i)$ represent separately the total time of the p th subsystem during which the state is saturated or nonsaturated within $[t_{i-1}, t_i]$. In addition, we define $R_p(t_0, t) = \sum_{f=1}^i R_p(t_{f-1}, t_f) + R_p(t_i, t)$ and $S_p(t_0, t) = \sum_{f=1}^i S_p(t_{f-1}, t_f) + S_p(t_i, t)$.

Remark 9. It can be seen from Lemma 3 that the parameters α , β , and μ are the same for all subsystems according to whether the saturations occur or not. However, the parameters λ_p , γ_p , and μ_p prescribed in Lemma 7 are mode-dependent; in other words, symbol λ_p and γ_p , respectively, denote the different decay rates of a Lyapunov function related to each of the active subsystems according to whether the saturations occur or not. Therefore, we can conclude that $\tau_{ap}^* \leq \tau_a^*$, from (7)–(9) and (16)–(18), and the mode-dependent features would reduce the conservativeness existing in Lemma 3.

Remark 10. It is worth nothing that the improved ADT (18) is always smaller than ADT (9) and (12). What is more, if we choose $\lambda_p = \gamma_p$ and $\lambda_p = \alpha$, $\gamma_p = \beta$, and $\mu_p = \mu$ then (18) will reduce to (12) and (9), respectively. Therefore, Lemmas 3 and 4 can be regarded as two special cases of Lemma 7. It is clear that Lemma 7 presents a more general stability criterion than Lemmas 3 and 4 which corresponds to the special case of $\lambda_p = \gamma_p$ and $\lambda_p = \alpha$, $\gamma_p = \beta$, $\mu_p = \mu$, and $\forall p \in \mathcal{L}$.

In total, we can infer from Remarks 9 and 10 that the MDADT switching signal has great flexibility superiorities for a switched system with state constraints.

Now, based on the consequences obtained above, we present the stability condition for continuous-time switched systems with state constraints based on MDADT.

Theorem 11. Consider the following switched system:

$$\dot{x}(t) = h(A_{\sigma(t)}x(t)), \quad (23)$$

where $\sigma(t) = p \in \mathcal{L}$, and let $\lambda_p > \gamma_p > 0$ and $\mu_p \geq 1$, $p \in \mathcal{L}$, be given constants. $\forall(p, q) \in \mathcal{L} \times \mathcal{L}$, $p \neq q$ and $U_s \neq I$. Supposing that there exist matrices $Q_p > 0$ and row diagonally dominant matrices L_p , where $|l_{ii}| > \sum_{j=1, j \neq i}^n |l_{ij}|$ and $l_{ii} < 0$, $\forall i \in \mathcal{N}$,

$$\begin{aligned} A_p^T Q_p + Q_p A_p + \lambda_p Q_p &< 0, \\ (U_s A_p + U_p^- L_p)^T Q_p + Q_p (U_s A_p + U_s^- L_p) + \gamma_p Q_p &< 0, \end{aligned} \quad (24)$$

$$Q_p - \mu_p Q_q < 0;$$

therefore, the continuous-time switched system with state constraints (23) is GUAS with MDADT satisfying (18).

Proof. Here, consider the following Lyapunov function:

$$V_p(x(t)) = x^T(t) Q_p x(t), \quad (25)$$

where $\forall \sigma(t) = p \in \mathcal{L}$. From Lemma 7, we know that the system states can be divided into the saturated state and the nonsaturated state. Therefore, when $t \in R_p(t_{i-1}, t_i)$, in other

words, the system state is nonsaturated, we can conclude from that that

$$\begin{aligned} \dot{V}_p(x(t)) + \lambda_p V_p(t) &= x^T(t) (\lambda_p Q_p + A_p^T Q_p + Q_p A_p) x(t) < 0, \\ V_p(x(t_i)) - \mu_p V_q(x(t_i)) &= x^T(t_i) (Q_p - \mu_p Q_q) x(t_i) < 0. \end{aligned} \quad (26)$$

The inequality for the saturated state can be obtained using similar techniques:

$$\begin{aligned} \dot{V}_p(x(t)) + \gamma_p V_p(t) &= x^T(t) (\gamma_p Q_p + \tilde{A}_p^T Q_p + Q_p \tilde{A}_p) x(t) < 0, \end{aligned} \quad (27)$$

where $\tilde{A}_p = U_s A_p + U_s^- L_p$. By simplifying the above three inequalities, we can get (24). Therefore, the continuous-time switched system with state constraints (23) is GUAS with MDADT satisfying (18). $\square$

Next, we introduce the condition of stabilizing controller for continuous-time switched systems with state constraints based on MDADT.

Theorem 12. Consider the same switched system (1), where $\sigma(t) = p \in \mathcal{L}$, and let $\lambda_p > \gamma_p > 0$ and $\mu_p \geq 1$, $p \in \mathcal{L}$, be given constants. $\forall(p, q) \in \mathcal{L} \times \mathcal{L}$, $p \neq q$ and $U_s \neq I$. Suppose that there exist matrices $P_p > 0$ and $P_q > 0$ and row diagonally dominant matrices L_p and T_p , where $|l_{ii}| > \sum_{j=1, j \neq i}^n |l_{ij}|$ and $l_{ii} < 0$, $\forall i \in \mathcal{N}$,

$$P_p A_p^T + T_p^T B_p^T + A_p P_p + B_p T_p + \lambda_p P_p < 0, \quad (28)$$

$$\begin{aligned} P_p A_p^T U_s^T + T_p^T B_p U_s^T + U_p L_p U_s^{-T} + U_s A_p P_p \\ + U_s B_p T_p + U_s^- L_p P_p + \gamma_p P_p < 0, \end{aligned} \quad (29)$$

$$P_p - \mu_p P_q < 0; \quad (30)$$

then, the continuous-time switched system with state constraints (1) is GUAS with MDADT satisfying (18). Therefore, if (28), (29), and (30) have a solution, the controller gains can be obtained by

$$F_p = T_p P_p^{-1}. \quad (31)$$

Proof. From the closed-loop system (2) and Theorem 11, we have

$$(A_p + B_p F_p)^T Q_p + Q_p (A_p + B_p F_p) < -\lambda_p Q_p, \quad (32)$$

$$\begin{aligned} [U_s (A_p + B_p F_p) + U_s^- L_p]^T Q_p \\ + Q_p [U_s (A_p + B_p F_p) + U_s^- L_p] < -\gamma_p Q_p, \end{aligned} \quad (33)$$

$$Q_p < \mu_p Q_q. \quad (34)$$

For simplifying the above inequalities, we can pre- and post-multiply Q_p^{-1} to (32) and (33). Next, we define $T_p = F_p Q_p^{-1}$ and $P_p = Q_p^{-1}$, such that (28), (29), and (30) are proved to be established. Therefore, if (28), (29), and (30) have a solution, the controller gains can be obtained by $F_p = T_p P_p^{-1}$. Then, the continuous-time switched system with state constraints (1) is GUAS with MDADT satisfying (18). $\square$

Remark 13. Theorem 12 is a prolongation of [29, Theorem 3], which deals with the stabilization problem of continuous-time switched systems with state constraints. From Remarks 9 and 10, using $\lambda_p, \gamma_p, \mu_p$ under MDADT switching to design feasible controllers has more flexibility and less conservativeness.

Finally, we give an iterative LMI algorithm for verifying the sufficient conditions of theorems for the continuous-time case. Here, we briefly describe the iterative LMI algorithm: Step 1, select a $E_p > 0$ and solve P_p and T_p from the Lyapunov equation (28); Step 2, using P_p and T_p obtained previously, solve the LMI optimization problem (29) for L_p and ξ ; Step 3, using L_p obtained in the previous step, solve the LMI optimization problems (28) and (29) for P_p and T_p and ξ ; Step 4, if $\xi \leq 0$, system (2) is GUAS at the origin. And the current F_p is the calculated feedback again. Otherwise, no result can be obtained. A different E_p may be selected and the algorithm may be repeated from Step 1. For more details on the iterative LMI algorithm, please refer to the reference literature [30].

4. Numerical Example

In this section, a numerical example of the continuous-time switched system with state constraints based on MDADT is presented to show the effectiveness of the developed approaches.

Example. Consider the following switched systems:

$$\dot{x}(t) = h(A_{\sigma(t)}x(t)), \quad (35)$$

where $\sigma(t) \in \mathcal{L} = \{1, 2\}$ and

$$A_1 = \begin{bmatrix} -2.0 & 0.1 & -0.5 \\ -0.4 & -0.9 & 0.7 \\ 0.3 & -1.0 & -0.4 \end{bmatrix}, \quad (36)$$

$$A_2 = \begin{bmatrix} -0.7 & -1.5 & 0.5 \\ 2.0 & -1.0 & 1.2 \\ 1.8 & 1.0 & -1.8 \end{bmatrix}.$$

We set the parameters $\lambda_1 = 0.3$, $\gamma_1 = 0.1$, and $\mu_1 = 1.4$; $\lambda_2 = 0.2$, $\gamma_2 = 0.14$, and $\mu_2 = 1.5$; $\rho = 0.15$ and $R_1(0, t)/S_1(0, t) = 0.4$; $R_2(0, t)/S_2(0, t) = 0.2$. In Theorem 11,

$$Q_1 = \begin{bmatrix} 32.1545 & -3.8173 & 8.9418 \\ -3.8173 & 22.8047 & -4.0990 \\ 8.9418 & -4.0990 & 22.1684 \end{bmatrix},$$

FIGURE 1: $\tau_{a1} = 2.5$ and $\tau_{a2} = 2.8$.

FIGURE 2: Saturated trajectory of system (35).

$$Q_2 = \begin{bmatrix} 41.1823 & -4.5734 & 12.2986 \\ -4.5734 & 21.8687 & -4.0101 \\ 12.2986 & -4.0101 & 21.1007 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} -3.8256 & 0.4542 & -1.0639 \\ 0.6506 & -3.8869 & -0.6986 \\ -1.4754 & 0.6764 & -3.6579 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} -3.8032 & -0.4224 & -1.1358 \\ 0.8064 & -3.8559 & 0.7071 \\ -1.9884 & 0.6484 & -3.4116 \end{bmatrix}. \quad (37)$$

When the MDADT switching signal satisfies $\tau_{a1} \geq 2.4549$ and $\tau_{a2} \geq 2.6273$, the switched system with state constraints (35) is GUAS. So, we give $\tau_{a1} = 2.5$ and $\tau_{a2} = 2.8$ in Figure 1. Figure 2 shows that despite $x(t)$ has occurred saturations, with the initial state $x_0 = [-1, 1, -1]^T$ and the MDADT switching signal, the continuous-time switched system with state constraints is GUAS. From the example, we demonstrate the effectiveness of our proposed method.

5. Conclusion

The stability and the stabilization problems for a class of continuous-time switched systems with state constraints via a mode-dependent switching method have been investigated in continuous-time context. An improved average dwell time method is proposed. It is shown that the improved average dwell time method is less conservative than the common average dwell time method. Then, based on the improved average dwell time method, the sufficient conditions and state feedback controllers for stabilization of the switched system are derived. Finally, a numerical example is given to illustrate the proposed approach.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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