Research Article

Robust Hierarchical Control for Uncertain Multivariable Hexarotor Systems

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Multirotor helicopter attracts more attention due to its increased load capacity and being highly maneuverable. However, these helicopters are uncertain multivariable systems, which pose a challenge for their robust controller design. In this paper, a robust two-loop control scheme is proposed for a hexarotor system. The resulted controller consists of a nominal controller and a robust compensator. The robust compensators are added to restrain the influences of uncertainties such as nonlinear dynamics, coupling, parametric uncertainties, and external disturbances. It is proven that the tracking errors are ultimately bounded with specified boundaries by choosing the parameters of the robust compensators. Simulation results on the hexarotor demonstrate the effectiveness of the proposed control method.

1. Introduction

Unmanned aerial vehicles (UAV) have attracted great attention in scientific research area in the last two decades (see [1–5] to mention a few). These kinds of vehicles have been widely used to perform civil and military missions such as inspection, surveillance, exploration, search, and rescue. Unmanned rotorcrafts have some advantages over the fixed-wing aircrafts due to their special way of thrust lift generated. The helicopters can hover and take off and land vertically. Furthermore, unmanned helicopters can enter the building and implement indoor exploration tasks as well as inspection missions.

Multirotors have received much interest in the automatic control field because they can outperform the conventional helicopters in many aspects. Firstly, the fixed-pitch rotors are used and lift thrusts can be altered by the control of motor rotational speeds without a swashplate [6]. Secondly, multiple rotors can increase payload and the maneuverability of the helicopter [7]. Thirdly, the usage of the multiple rotors guarantees that each individual rotor is smaller than the equivalent main rotor of a conventional helicopter for a given airframe size [8]. Multiple smaller rotors can be enclosed within a protective frame and thus increased safety

of operators. As a result of the strong points mentioned above, intensive efforts have been devoted to the tracking control problem of multirotor helicopters. Quadrotors have gained much attention in the automatic control field (see, e.g., [9–15]). In this paper, the robust motion control problem of hexarotor helicopters is investigated. Compared with quadrotors, although hexarotor helicopters may present a weight and energy consumption augmentation due to the extra motors, they can increase load capacities. Moreover, hexarotors are highly maneuverable in respect that at least four rotors can influence the dynamics of each direction. On the other hand, it results in a more fault-tolerant mechanical system for each hexarotor helicopter.

The controller design problem of a hexarotor system shares some merits with that of the quadrotor helicopter. Both kinds of helicopters can be considered as rigid bodies and thus one can apply Lagrange approach to obtain the dynamical model of the two kinds of helicopters. However, there also exist some differences between the two kinds of helicopters. The differences result from the aerodynamic forces and torques generated by the rotors. Because of two extra rotors, the torque produced in each channel is different from the quadrotors and thus affects the dynamical response

FIGURE 1: The schematic of the hexarotor helicopter.

differently. In this paper, a robust hierarchical control scheme is applied based on PD control method and the robust compensation approach. For each loop, a robust controller is designed with a nominal PD controller and a robust compensator. The robust compensators are introduced to restrain the influences of the uncertainties in both loops.

Compared to previous studies on the robust motion control problem of the multirotor helicopter, the proposed robust hierarchical control scheme can restrain disturbances in both the inner and outer loops. The tracking errors of this rotorcraft are guaranteed to be bounded with specified boundaries. In addition, this control scheme results in a linear time-invariant controller which is easily to be implemented in practical applications.

The remaining parts of this paper are organized as follows. In Section 2, the nonlinear mathematic model of the helicopter is described and problem description is presented. The nominal controllers and robust compensators in the attitude and position loops are designed in Section 3. In Section 4, the robust properties of closed-loop system by the designed control laws are proven. Simulation results are shown in Section 5 and conclusions are stated in Section 6.

For $y(t) = [y_i(t)] \in L_\infty^n$ and $h(t) = [h_{ij}(t)] \in L_1^{m \times n}$, denote that

$$\|y\|_\infty = \max_i \sup_{t \geq 0} |y_i(t)|, \quad (1)$$

$$\|H(p)\|_1 = \|h\|_1 = \max_i \left(\sum_j \int_0^\infty |h_{ij}(t)| dt \right),$$

where p is the Laplace operator, $H(p) = \ell(h(t))$, and $\ell(\cdot)$ indicates the Laplace transform.

2. Problem Description

The schematic of the hexarotor is depicted in Figure 1. Let $\alpha = \{E_x, E_y, E_z\}$ denote an inertial frame and $\beta = \{E_{bx}, E_{by}, E_{bz}\}$ is a frame attached to the helicopter body with origin in the center of the mass of the rotorcraft as shown in Figure 1. The vector $\xi = [x \ y \ z]^T$ indicates the position of the origin of the body-fixed frame β with respect to the inertial frame α .

In addition, let $\eta = [\theta \ \phi \ \psi]^T$ indicate the three Euler angles: the pitch angle θ , the roll angle ϕ , and the yaw angle ψ , which depends on the rotation from the inertial frame α to the body-fixed frame β . The mathematical model of the helicopter can be derived by the Lagrangian method as (see, e.g., [9])

$$m\ddot{\xi} = Rf + mge_3, \quad (2)$$

$$J\ddot{\eta} = -C(\eta, \dot{\eta})\dot{\eta} + \tau,$$

where m is the mass of the helicopter, $J = [J_\theta \ J_\phi \ J_\psi]$ is the inertia matrix of the helicopter relative to the frame α , g is the gravity constant, $C(\eta, \dot{\eta})$ is the Coriolis terms, $R = [-\sin\theta \ \cos\theta\sin\phi \ \cos\theta\cos\phi]^T$, $e_3 = [0 \ 0 \ 1]^T$, and f and $\tau = [\tau_\theta \ \tau_\phi \ \tau_\psi]^T$ are the external body-fixed frame force and torque respectively. f and τ can be obtained as

$$f = f_1 + f_2 + f_3 + f_4 + f_5 + f_6,$$

$$\tau_\theta = l_{rg}(f_1 - f_4) + \frac{l_{rg}(f_2 - f_3 - f_5 + f_6)}{2}, \quad (3)$$

$$\tau_\phi = \frac{\sqrt{3}l_{rg}(f_2 + f_3 - f_5 - f_6)}{2},$$

$$\tau_\psi = k_{fm}(f_1 - f_2 + f_3 - f_4 + f_5 - f_6),$$

where l_{rg} is the distance from each motor to the center of the mass of the rotorcraft, $k_{fm} > 0$ denotes the force-to-moment scaling factor, and f_i ($i = 1, 2, \dots, 6$) are the thrusts generated by the six rotors respectively. f_i ($i = 1, 2, \dots, 6$) take the following forms:

$$f_i = k_\omega \omega_i^2, \quad i = 1, 2, \dots, 6, \quad (4)$$

where k_ω is a positive constant and ω_i ($i = 1, 2, \dots, 6$) are the rotational speeds of the six rotors respectively. Define the control inputs u_i ($i = 1, 2, \dots, 4$) as

$$u_1 = \omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2 + \omega_5^2 + \omega_6^2,$$

$$u_2 = \omega_1^2 - \omega_4^2 + \frac{(\omega_2^2 - \omega_3^2 - \omega_5^2 + \omega_6^2)}{2}, \quad (5)$$

$$u_3 = \frac{\sqrt{3}(\omega_2^2 + \omega_3^2 - \omega_5^2 - \omega_6^2)}{2},$$

$$u_4 = \omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2 + \omega_5^2 - \omega_6^2.$$

In practical applications, there already exist many power distribution boards to distribute the four control inputs to the six rotors for the multirotors. Therefore, the power distribution problem is not further discussed here.

In addition, let

$$\begin{aligned}
 b_1^N &= -\frac{k_\omega^N u_{10}}{m^N}, \\
 b_2^N &= \frac{k_\omega^N u_{10}}{m^N}, \\
 b_3^N &= \frac{k_\omega^N}{m^N}, \\
 b_4^N &= \frac{l_{rg}^N k_\omega^N}{J_\theta^N}, \\
 b_5^N &= \frac{l_{rg}^N k_\omega^N}{J_\varphi^N}, \\
 b_6^N &= \frac{k_{fm}^N k_\omega^N}{J_\psi^N},
 \end{aligned} \tag{6}$$

where the superscript N denotes the nominal value of the helicopter parameter and u_{10} is the value of control input u_1 in the hovering working point which is a positive constant. Then from (2), one can obtain that

$$\begin{aligned}
 \ddot{x} &= b_1^N (\theta + q_1), \\
 \ddot{y} &= b_2^N (\phi + q_2), \\
 \ddot{z} &= b_3^N (u_1 + q_3) + g, \\
 \ddot{\theta} &= b_4^N (u_2 + q_4), \\
 \ddot{\phi} &= b_5^N (u_3 + q_5), \\
 \ddot{\psi} &= b_6^N (u_4 + q_6),
 \end{aligned} \tag{7}$$

where q_i ($i = 1, 2, \dots, 6$) are called equivalent disturbances and take the forms

$$\begin{aligned}
 q_1 &= \frac{(-\sin \theta k_\omega u_1 / m - b_1^N \theta)}{b_1^N} + d_1, \\
 q_2 &= \frac{(\cos \theta \sin \phi k_\omega u_1 / m - b_2^N \phi)}{b_2^N} + d_2, \\
 q_3 &= \frac{(\cos \theta \cos \phi k_\omega / m - b_3^N) u_1}{b_3^N} + \frac{(m - m^N) g}{b_3^N} + d_3, \\
 q_i &= \frac{(b_i - b_i^N) u_{i-2}}{b_i^N} + c_i + d_i, \quad i = 4, 5, 6,
 \end{aligned} \tag{8}$$

where d_i ($i = 1, 2, \dots, 6$) are external disturbances and c_i ($i = 1, 2, 3$) are higher order terms that result from the Coriolis terms.

Assumption 1. The pitch and roll angles are bounded among $(-\pi/2, \pi/2)$ and satisfy that $\theta \in [-\pi/2 + \delta_\theta, \pi/2 - \delta_\theta]$ and $\phi \in [-\pi/2 + \delta_\phi, \pi/2 - \delta_\phi]$ with δ_θ and δ_ϕ positive constants.

In order to avoid singularities in the Euler angle representation, the hexarotor is required to avoid overturning during the flight [16].

Assumption 2. The control input u_1 is bounded and satisfies that $u_1 \in [\delta_{u1}, 2u_{10} - \delta_{u1}]$, where δ_{u1} is a positive constant.

Assumption 3. The uncertain parameters b_i ($i = 4, 5, 6$), k_ω , and m are bounded. The nominal parameters b_i^N are negative and b_i^N ($i = 2, 3, \dots, 6$) are positive and satisfy that $b_1^N \leq -2k_\omega u_{10}/\pi/m$, $b_2^N \geq 2k_\omega u_{10}/\pi/m$, and $|b_i - b_i^N| \leq b_i^N$ ($i = 3, 4, 5, 6$).

One can set that $-b_1^N$ are b_i^N ($i = 2, \dots, 6$) with larger positive values, if b_i^N ($i = 1, 2, \dots, 6$) do not satisfy the above the assumption.

Define $\rho_1 = -|k_\omega \sin \theta u_1 / \theta / m + b_1^N| / b_1^N$, $\rho_2 = |k_\omega \cos \theta \sin \phi u_1 / \phi / m - b_2^N| / b_2^N$, $\rho_3 = |k_\omega \cos \theta \cos \phi / m + b_3^N| / b_3^N$, and $\rho_i = |b_i - b_i^N| / b_i^N$ ($i = 4, 5, 6$).

If Assumptions 1, 2, and 3 hold, one can obtain that $0 \leq \rho_i < 1$ ($i = 1, \dots, 6$).

Assumption 4. The external disturbances d_i ($i = 1, \dots, 6$) are bounded.

In the position control mode discussed here, the three positions and the yaw angle $\{x, y, z, \psi\}$ are chosen as outputs. This paper will investigate the robust controller design problem to achieve practical tracking of the prescribed reference signals r_i ($i = x, y, z, \psi$) for x, y, z , and ψ , respectively.

Assumption 5. The reference signals r_i ($i = x, y, z, \psi$) and their derivatives $\dot{r}_i$ ($i = x, y, z, \psi$) are piecewise uniformly bounded.

3. Robust PD Controller Design

Based on the PD control approach and the signal compensation method [17–21], the attitude and position controllers are designed in two steps.

From the helicopter system (7), one can see that the robust tracking control of the longitude and lateral directions can be achieved by controlling the attitude angles θ and φ appropriately, if the three positions and the yaw angle are chosen as the outputs; that is, the position controller involved x and y positions that will be designed to produce the desired attitude signals r_θ and r_φ for θ and φ to track, whereas the vertical direction (z -axis) can be dealt with independently. The attitude controller will be applied to track the references signals r_θ , r_φ , and r_ψ for the three attitude channels.

3.1. Position Controller Design. Define $u_{\xi 1} = \theta_d$, $u_{\xi 2} = \varphi_d$, $u_{\xi 3} = u_1 + g/b_1^N$, and $e_{\xi i} = [e_{\xi i 1} \ e_{\xi i 2}]^T$ ($i = 1, 2, 3$), where $e_{\xi 1 1} = x - r_x$, $e_{\xi 1 2} = \dot{e}_{\xi 1 1}$, $e_{\xi 2 1} = y - r_y$, $e_{\xi 2 2} = \dot{e}_{\xi 2 1}$, $e_{\xi 3 1} = z - r_z$, and $e_{\xi 3 2} = \dot{e}_{\xi 3 1}$. The position motion equations of the rotorcraft can be expressed as space-state forms:

$$\dot{e}_{\xi i} = A_{\xi i} e_{\xi i} + B_{\xi i} (u_{\xi i} + q_i), \quad i = 1, 2, 3, \tag{9}$$

where

$$\begin{aligned} A_{\xi i} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \\ B_{\xi i} &= \begin{bmatrix} 0 \\ b_i^N \end{bmatrix}. \end{aligned} \quad (10)$$

The control inputs $u_{\xi i}$ ($i = 1, 2, 3$) consist of two parts: the PD control input $u_{\xi i}^{\text{PD}}$ and the robust compensating input $u_{\xi i}^{\text{RC}}$; that is, the control input can be expressed as

$$u_{\xi i} = u_{\xi i}^{\text{PD}} + u_{\xi i}^{\text{RC}}. \quad (11)$$

Firstly, PD feedback control laws are constructed as

$$u_{\xi i}^{\text{PD}} = -K_{\xi i} e_{\xi i}, \quad i = 1, 2, 3, \quad (12)$$

where $K_{\xi i} = [k_{\xi i}^p \ k_{\xi i}^d]$ is selected such that $A_{\xi i} - B_{\xi i}K_{\xi i}$ is Hurwitz.

Then, the robust compensation technique is used to restrain the influences of the equivalent disturbances q_i ($i = 1, 2, 3$). The robust compensating inputs for the three channels can be obtained by the robust filters $F_{\xi i}(p)$ ($i = 1, 2, 3$) as

$$u_{\xi i}^{\text{RC}}(p) = -F_{\xi i}(p) q_i(p), \quad i = 1, 2, 3, \quad (13)$$

where $F_{\xi i}(p) = f_{\xi i} g_{\xi i} / (p + f_{\xi i})(p + g_{\xi i})$. From (9)–(13), the robust compensating inputs $u_{\xi i}^{\text{RC}}$ ($i = 1, 2, 3$) can be realized by

$$\begin{aligned} \dot{z}_{\xi i 1} &= -g_{\xi i} z_{\xi i 1} - g_{\xi i}^2 e_{\xi i} + b_i^N u_{\xi i}, \\ \dot{z}_{\xi i 2} &= -f_{\xi i} z_{\xi i 2} + (f_{\xi i} + g_{\xi i}) e_{\xi i} + z_{\xi i 1}, \\ u_{\xi i}^{\text{RC}} &= \frac{f_{\xi i} g_{\xi i} (z_{\xi i 2} - e_{\xi i})}{b_i^N}, \quad i = 1, 2, 3. \end{aligned} \quad (14)$$

3.2. Attitude Controller Design. Define $u_\eta = [u_2 \ u_3 \ u_4]$, $q_\eta = [q_4 \ q_5 \ q_6]$, and $e_\eta = [e_{\eta i}]_{6 \times 1}$, where $e_{\eta 1} = \theta - r_\theta$, $e_{\eta 2} = \dot{e}_{\eta 1}$, $e_{\eta 3} = \varphi - r_\varphi$, $e_{\eta 4} = \dot{e}_{\eta 3}$, $e_{\eta 5} = \psi - r_\psi$, and $e_{\eta 6} = \dot{e}_{\eta 5}$. The position motion equations of the rotorcraft can be expressed as space-state forms:

$$\dot{e}_\eta = A_\eta e_\eta + B_\eta (u_\eta + q_\eta), \quad (15)$$

where

$$\begin{aligned} A_\eta &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ B_\eta &= \begin{bmatrix} 0 & 0 & 0 \\ b_4^N & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b_5^N & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b_6^N \end{bmatrix}. \end{aligned} \quad (16)$$

Similar to the position controller design, the attitude control input u_η is designed with two parts: the PD control input u_η^{PD} and the robust compensating input u_η^{RC} , as follows:

$$u_\eta = u_\eta^{\text{PD}} + u_\eta^{\text{RC}} = -K_\eta e_\eta - F_\eta(p) q_\eta(p), \quad (17)$$

where $K_\eta = [K_{\eta 1} \ K_{\eta 2} \ K_{\eta 3}]$, $F_\eta(p) = \text{diag}(F_{\eta 1}(p), F_{\eta 2}(p), F_{\eta 3}(p))$, $K_{\eta i} = [k_{\eta i}^p \ k_{\eta i}^d]$, $F_{\eta i}(p) = f_{\eta i} g_{\eta i} / (p^2 + f_{\eta i} p + g_{\eta i} p + f_{\eta i} g_{\eta i})$ ($i = 1, 2, 3$). K_η is selected such that $A_\eta - B_\eta K_\eta$ is Hurwitz. The robust compensating input u_η^{RC} can be realized in a similar way as (14).

4. Robust Properties Analysis

In this section, the robust performances of the closed-loop system constructed as in the previous section will be analyzed in two steps. Firstly, it will be proven that the state of the vertical channel is bounded and the tracking error is bounded and will converge to any given neighborhood of the origin under the influences of the dynamics of other channels. Then, the proof of the robust properties of the other channels will be presented based on the analysis of the robust performances of the vertical channel.

Define $e_\xi = [e_{\xi 1} \ e_{\xi 2}]^T$, $q_\xi = [q_{\xi 1} \ q_{\xi 2}]^T$, $K_\xi = [K_{\xi 1} \ K_{\xi 2}]$, and $A_{\xi H} = \text{diag}\{A_{\xi 1H}, A_{\xi 2H}\}$. Combining (9)–(13), (15), and (17), one has

$$\max_j |e_{ij}(t)| \leq \max_j |c_{ij}^T e^{A_{\xi H} t} e_i(0)| + \mu_i \|q_i\|_\infty, \quad (18)$$

$i = \xi, \xi 3, \eta,$

where $\mu_i = \|(pI_i - A_{\xi H})^{-1} B_i(1 - F_i)\|_1$ and I_i is corresponding unit matrix. It follows that

$$\|e_i\|_\infty \leq \gamma_{e_i(0)} + \mu_i \|q_i\|_\infty, \quad i = \xi, \xi 3, \eta, \quad (19)$$

where $\gamma_{e_i(0)} = \max_j \sup_{t \geq 0} |c_{ij}^T e^{A_{\xi H} t} e_i(0)|$, and $c_{\xi j}$, $c_{\xi 3 j}$, and $c_{\eta j}$ are 4×1 , 2×1 , and 6×1 vectors with ones on the j th row and

zeros elsewhere, respectively. Define $g_\xi = \min\{g_{\xi 1}, g_{\xi 2}\}$ and $g_\eta = \min\{g_{\eta 1}, g_{\eta 2}, g_{\eta 3}\}$. If f_i and g_i ($i = \xi 1, \xi 2, \xi 3, \eta 1, \eta 2, \eta 3$) are sufficiently large and satisfy $f_i \gg g_i > 0$, from [19], there exist positive constants γ_{gi} ($i = \xi, \xi 3, \eta$) such that

$$\mu_i \leq \frac{\gamma_{gi}}{g_i}, \quad i = \xi, \xi 3, \eta. \quad (20)$$

4.1. The Robust Properties of the Vertical Channel

Theorem 6. *If Assumptions 1, 2, 3, and 5 hold, the closed-loop system of the vertical channel has robust tracking performances; that is, for any given positive constant $\varepsilon_{\xi 3}$ and any given bounded initial state, one can find positive constants $f_{\xi 3}$, $g_{\xi 3}$ with sufficiently large values and satisfying $f_{\xi 3} \gg g_{\xi 3}$, and $T_{\xi 3}$, such that, the state $e_{\xi 3}$ is bounded, and*

$$\|e_{\xi 3}(t)\| \leq \varepsilon_{\xi 3}, \quad \forall t \geq T_{\xi 3}. \quad (21)$$

Proof. Under the Assumptions 1, 3, and 4, one has that

$$\|q_3\|_\infty \leq \rho_3 \|u_{\xi 3}\|_\infty + \|d_3\|_\infty. \quad (22)$$

From (11)–(13) and (22), one can obtain two positive constants $\gamma_{e\xi 3}$ and $\gamma_{c\xi 3}$ such that

$$\|q_3\|_\infty \leq \gamma_{e\xi 3} \|e_{\xi 3}\|_\infty + \gamma_{c\xi 3}. \quad (23)$$

Then, from (19), (20), and (23), there exist positive constants ς_{q3} and $\varsigma_{e\xi 3}$ satisfying

$$\|q_3\|_\infty \leq \gamma_{e\xi 3}(0) \gamma_{e\xi 3} + \frac{\gamma_{g\xi 3} \gamma_{e\xi 3} \|q_{\xi 3}\|_\infty}{g_{\xi 3}} + \gamma_{c\xi 3} \leq \varsigma_{q3}, \quad (24)$$

$$\|e_{\xi 3}\|_\infty \leq \gamma_{e\xi 3}(0) + \frac{\gamma_{g\xi 3} \|q_{\xi 3}\|_\infty}{g_{\xi 3}} \leq \varsigma_{e\xi 3}.$$

Furthermore, from (18), (20), and (24), one has

$$\max_j |e_{\xi 3j}(t)| \leq \max_j |c_{\xi 3j}^T e^{A_{\xi 3H} t} e_{\xi 3}(0)| + \frac{\gamma_{g\xi 3} \varsigma_{q3}}{g_{\xi 3}}. \quad (25)$$

Therefore, for any given positive constant $\varepsilon_{\xi 3}$, there exists a positive constant $T_{\xi 3}$ satisfying (21). $\square$

One can see that the dynamics of other channels cannot influence the robustness property analysis of the vertical channel and, thus, the values of $f_{\xi 3}$ and $g_{\xi 3}$ can be determined before discussing the tracking performances of other channels.

Lemma 7. *There exist a positive constant ς_{u10} such that the control input u_1 satisfies*

$$\|u_1\|_\infty \leq \varsigma_{u10}. \quad (26)$$

Proof. From (11)–(13) and (24), one can obtain a positive constant ς_{u10} such that

$$\|u_1\|_\infty \leq \|K_{\xi 3}\|_1 \|e_{\xi 3}\|_\infty + \|q_3\|_\infty + \frac{\|(g - \ddot{r}_z)\|_\infty}{b_3^N} \quad (27)$$

$$\leq \varsigma_{u10}.$$

Lemma 7 follows. $\square$

Theorem 8. *If Assumptions 1 to 5 hold, the closed-loop system of the longitudinal, latitude, and three attitude channels have robust tracking performances; that is, for any given positive constants ε_ξ and ε_η and any given bounded initial state, one can obtain positive constants $i_{\xi 1}$, $i_{\xi 2}$, $i_{\eta 1}$, $i_{\eta 2}$, and $i_{\eta 3}$ ($i = f, g$) with sufficiently large values and satisfying $f_i \gg g_i$ ($i = \xi 1, \xi 2, \eta 1, \eta 2, \eta 3$), and T_ξ and T_η such that, the states e_ξ and e_η are bounded, and*

$$\begin{aligned} \max_i |e_{\xi i}(t)| &\leq \varepsilon_\xi, \quad \forall t \geq T_\xi, \\ \max_i |e_{\eta i}(t)| &\leq \varepsilon_\eta, \quad \forall t \geq T_\eta. \end{aligned} \quad (28)$$

Proof. From (8) and Lemma 7, one can obtain positive constants $\varsigma_{q\xi\eta}$, $\varsigma_{q\xi e\xi}$, and $\varsigma_{q\xi c}$ such that

$$\|q_\xi\|_\infty \leq \varsigma_{q\xi\eta} \|e_\eta\|_\infty + \varsigma_{q\xi e\xi} \|e_\xi\|_\infty + \varsigma_{q\xi c}. \quad (29)$$

From (8), (11), (12), (13), (17), (29), and Lemma 7, there exist positive constants $\varsigma_{q\eta\eta}$, $\varsigma_{q\eta e\xi}$, $\varsigma_{q\eta\eta 2}$, $\varsigma_{q\eta e\xi 2}$, $\varsigma_{q\eta\eta\xi}$, and $\varsigma_{q\eta c}$ such that

$$\begin{aligned} \|q_\eta\|_\infty &\leq \varsigma_{q\eta\eta} \|e_\eta\|_\infty + \varsigma_{q\eta e\xi} \|e_\xi\|_\infty + \varsigma_{q\eta\eta 2} \|e_\eta\|_\infty^2 \\ &\quad + \varsigma_{q\eta e\xi 2} \|e_\xi\|_\infty^2 + \varsigma_{q\eta\eta\xi} \|e_\eta\|_\infty \|e_\xi\|_\infty + \varsigma_{q\eta c}. \end{aligned} \quad (30)$$

Furthermore, from (19), (29), and (30), one can have that

$$\begin{aligned} \|e_\xi\|_\infty &\leq \gamma_{e\xi}(0) + \mu_\xi \varsigma_{q\xi\eta} \|e_\eta\|_\infty + \mu_\xi \varsigma_{q\xi e\xi} \|e_\xi\|_\infty \\ &\quad + \mu_\xi \varsigma_{q\xi c}, \\ \|e_\eta\|_\infty &\leq \gamma_{e\eta}(0) + \mu_\eta \varsigma_{q\eta\eta} \|e_\eta\|_\infty + \mu_\eta \varsigma_{q\eta e\xi} \|e_\xi\|_\infty \\ &\quad + \mu_\eta \varsigma_{q\eta c} + \mu_\eta \varsigma_{q\eta\eta 2} \|e_\eta\|_\infty^2 + \mu_\eta \varsigma_{q\eta e\xi 2} \|e_\xi\|_\infty^2 \\ &\quad + \mu_\eta \varsigma_{q\eta\eta\xi} \|e_\eta\|_\infty \|e_\xi\|_\infty. \end{aligned} \quad (31)$$

Combining (30) and (31), one can obtain positive constants $\varsigma'_{q\eta\eta}$, $\varsigma'_{q\eta\eta 2}$, and $\varsigma'_{q\eta c}$ satisfying

$$\|q_\eta\|_\infty \leq \varsigma'_{q\eta\eta} \|e_\eta\|_\infty + \varsigma'_{q\eta\eta 2} \|e_\eta\|_\infty^2 + \varsigma'_{q\eta c}. \quad (32)$$

If $\|e_\eta\|_\infty$ satisfies that

$$(\varsigma'_{q\eta\eta} + \varsigma'_{q\eta\eta 2} \|e_\eta\|_\infty) (\sqrt{\mu_\eta} + \mu_\eta) \leq 1, \quad (33)$$

then one can obtain that

$$\|q_\eta\|_\infty \leq \frac{(\gamma_{e\eta}(0) + \varsigma'_{q\eta c} / \varsigma'_{q\eta\eta})}{\sqrt{\mu_\eta}}. \quad (34)$$

It follows that

$$\begin{aligned} \max_j |e_{\eta j}(t)| &\leq \max_j |c_{\eta j}^T e^{A_{\eta H} t} e_\eta(0)| \\ &\quad + \sqrt{\mu_\eta} \left(\gamma_{e\eta}(0) + \frac{\varsigma'_{q\eta c}}{\varsigma'_{q\eta\eta}} \right). \end{aligned} \quad (35)$$

TABLE 1: Helicopter parameters.

Parameter	Value	Parameter	Value	Parameter	Value
b_1^N	-9.8	b_2^N	9.8	b_3^N	1
b_4^N	0.32	b_5^N	0.32	b_6^N	0.8

It should be noted that, from (33), one can obtain the attractive region of the travel channel

$$\|e_\eta\|_\infty \leq \frac{1}{\zeta'_{q\eta\eta 2} / (\sqrt{\mu_\eta} + \mu_\eta)} - \frac{\zeta'_{q\eta\eta}}{\zeta'_{q\eta\eta 2}}. \quad (36)$$

Define $\zeta'_{q\eta} = \gamma_{e_\eta(0)} + \zeta'_{q\eta c} / \zeta'_{q\eta\eta}$. If $e_\eta(0)$ is in the attractive region, $e_\eta(t)$ can remain inside it if

$$\gamma_{e_\eta(0)} + \sqrt{\mu_\eta} \zeta'_{q\eta} < \frac{1}{\zeta'_{q\eta\eta 2} / (\sqrt{\mu_\eta} + \mu_\eta)} - \frac{\zeta'_{q\eta\eta}}{\zeta'_{q\eta\eta 2}}. \quad (37)$$

If μ_η is sufficiently small, one can see that the above inequality holds. So if the initial state of the angles satisfies that

$$\|e_\eta(0)\|_\infty \leq \gamma_{e_\eta(0)} + \sqrt{\mu_\eta} \left(\gamma_{e_\eta(0)} + \frac{\zeta'_{q\eta c}}{\zeta'_{q\eta\eta}} \right), \quad (38)$$

one can obtain (33).

From (19), (29), and (34), one can have that

$$\|q_\xi\|_\infty \leq \frac{\zeta_{q\xi\eta} \gamma_{e_\eta(0)} + \zeta_{q\xi\eta} \sqrt{\mu_\eta} \zeta'_{q\eta} + \zeta_{q\xi\eta} \gamma_{e_\xi(0)} + \zeta_{q\xi c}}{1 - \mu_\xi \zeta_{q\xi\eta}}. \quad (39)$$

Moreover, from (18) and (39), one has that

$$\begin{aligned} \max_j |e_{\xi j}(t)| &\leq \max_j |c_{\xi j}^T e_{\xi H}^T e_\xi(0)| \\ &\leq \max_j |c_{\xi j}^T e_{\xi H}^T e_\xi(0)| \\ &\quad + \mu_\xi \frac{\zeta_{q\xi\eta} \gamma_{e_\eta(0)} + \zeta_{q\xi\eta} \sqrt{\mu_\eta} \zeta'_{q\eta} + \zeta_{q\xi\eta} \gamma_{e_\xi(0)} + \zeta_{q\xi c}}{1 - \mu_\xi \zeta_{q\xi\eta}}. \end{aligned} \quad (40)$$

If μ_ξ and μ_η are sufficiently small, then from (35) and (40), one can get positive constants T_ξ and T_η such that (28) holds. $\square$

5. Simulation Results

The reference signals of the four channels are generated by $r_i(p) = r_i^{ic}(p) / (\tau_{ri}p + 1)^2$ ($i = x, y, z, \psi$), respectively, where $r_i^{ic}(p)$ ($i = x, y, z, \psi$) are the reference input commands. Set $\tau_{rx} = 0.7$, $\tau_{ry} = 0.7$, $\tau_{rz} = 0.7$, and $\tau_{r\psi} = 0.7$. The helicopter parameters are shown in Table 1, followed by the controller parameters in Table 2.

The helicopter is required to carry out aggressive missions in harsh conditions; that is, amplitudes for the references r_x, r_y, r_z , and r_ψ are 10 m, 10 m, 10 m, and 60 deg, respectively; all of the parametric uncertainties are assumed up to 40% of the nominal values, and the rotorcraft is subject to persistent external disturbances $d_1 = 0.2$, $d_2 = 0.2$, $d_3 = 3$, $d_4 = 0.2$,

FIGURE 2: Responses of the longitudinal position and the latitudinal position with RCT.

FIGURE 3: Responses of the height and the yaw angle with RCT.

$d_5 = 0.2$, and $d_6 = 1$. The external disturbance d_3 is larger than other disturbances because the aerial vehicle is likely subject to the ground effect when it flies near the ground.

Figures 2 and 3 show the simulation results of the trajectory tracking of the desired references. The rotorcraft

TABLE 2: Controller parameters.

Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
$k_{\xi 1}^p$	2	$k_{\xi 1}^d$	1	$f_{\xi 1}$	100	$g_{\xi 1}$	20
$k_{\xi 2}^p$	2	$k_{\xi 2}^d$	1	$f_{\xi 2}$	100	$g_{\xi 2}$	20
$k_{\xi 3}^p$	10	$k_{\xi 3}^d$	25	$f_{\xi 3}$	100	$g_{\xi 3}$	20
$k_{\eta 1}^p$	20	$k_{\eta 1}^d$	100	$f_{\eta 1}$	2500	$g_{\eta 1}$	500
$k_{\eta 2}^p$	20	$k_{\eta 2}^d$	100	$f_{\eta 2}$	2500	$g_{\eta 2}$	500
$k_{\eta 3}^p$	10	$k_{\eta 3}^d$	25	$f_{\eta 3}$	100	$g_{\eta 3}$	20

-- Reference signal
— Longitudinal position

(a) Longitudinal response

-- Reference signal
— Latitudinal position

(b) Latitudinal response

FIGURE 4: Responses of the longitudinal position and the latitudinal position without RCT.

starts from $x(0) = 0$, $y(0) = 0$, $z(0) = 0$, and $\psi(0) = 0$. x , y , z , and ψ are required to follow large references r_x , r_y , r_z , and r_ψ simultaneously. Besides, from Model (2), it can be seen that there exist inter-axis couplings between channels. By the proposed robust control method, good tracking performance can be achieved for aggressive maneuvers under the influence of various uncertainties. Furthermore, the proposed robust controller is compared with the nominal controller. The corresponding responses of x , y , z , and ψ without the RCT (robust compensating technique) are shown in Figures 4 and 5. From these figures, it can be seen that the proposed control method achieves better dynamical and steady-state performance especially in the longitudinal and latitudinal directions. The tracking performance of the height and yaw angle is only improved slightly, because the nonlinear dynamical models of these two channels are simple compared to the models in the longitudinal and latitudinal channels as shown in the nonlinear model (2). Because the quadrotor is

-- Reference signal
— Height

(a) Height response

-- Reference signal
— Yaw angle

(b) Yaw response

FIGURE 5: Responses of the height and the yaw angle without RCT.

underactuated in the longitudinal and latitudinal directions, the pitch and roll angles are required to track the desired references obtained by the longitudinal and latitudinal position tracking errors. The responses of the two angles are shown in Figure 6. Good dynamical and steady-state tracking performance is also achieved for the pitch and roll angles.

6. Conclusions

A robust controller was proposed to achieve the motion control for a hexarotor system. The proposed control structure is a hierarchical scheme that consists of two control loops: an attitude loop and a position loop. Both the attitude and position control loops include a nominal controller to achieve the desired tracking performances and a robust compensator to restrain the influences of uncertainties such as nonlinear dynamics, coupling, parametric uncertainties, and external disturbances. It is proven that the tracking errors are ultimately bounded and the boundaries can be specified.

FIGURE 6: Responses of the pitch angle and the roll angle with RCT.

Simulation results on the hexarotor helicopter demonstrate the effectiveness of the proposed control method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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