Research Article

A New High Order Fuzzy ARMA Time Series Forecasting Method by Using Neural Networks to Define Fuzzy Relations

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Linear time series methods are researched under 3 topics, namely, AR (autoregressive), MA (moving averages), and ARMA (autoregressive moving averages) models. On the other hand, the univariate fuzzy time series forecasting methods proposed in the literature are based on fuzzy lagged (autoregressive (AR)) variables, having not used the error lagged (moving average (MA)) variables except for only two studies in the fuzzy time series literature. Not using MA variables could cause the model specification error in solutions of fuzzy time series. For this reason, this model specification error should be eliminated. In this study, a solution algorithm based on artificial neural networks has been proposed by defining a new high order fuzzy ARMA time series forecasting model that contains fuzzy MA variables along with fuzzy AR variables. It has been pointed out by the applications that the forecasting performance could have been increased by the proposed method in accordance with the fuzzy AR models in the literature since the proposed method is a high order model and also utilizes artificial neural networks to identify the fuzzy relation.

1. Introduction

The first fundamental definitions on fuzzy time series in the literature have been made by Song and Chissom [1] in accordance with the fuzzy set theory of Zadeh [2]. Song and Chissom [1, 3, 4] divided fuzzy time series into two separate series, namely, time variant and time invariant. A vast majority of the studies in the literature are methodologies that have been proposed for solving time invariant fuzzy time series. The context of this study is also comprised of time invariant fuzzy time series.

The fuzzy time series models developed in the literature are AR (autoregressive) models only based on the fuzzy lagged values of time series, which are divided into two groups, namely, first order fuzzy time series forecasting methods and high order fuzzy time series forecasting methods. Among studies mentioned in the literature, studies of Song and Chissom [1, 3, 4], Chen [5], Huarng [6], Huarng and Yu [7, 8], Cheng et al. [9], Yolcu et al. [10], Yu and Huarng [11], Egrioglu et al. [12] and Sadaei and Lee [13] all used first order models for fuzzy time series solutions, while the studies of Chen [14], Chen and Chung [15], Jilani and Burney [16], Lee et al. [17], Aladag et al. [18], Egrioglu et al. [19, 20] and Sadaei

et al. [21] used high order models. In addition, because some fuzzy time series contain seasonal relations, seasonal fuzzy time series forecasting methods also have been developed. Some of seasonal fuzzy time series models could be seen in the studies of Chang [22], Song [23], and Egrioglu et al. [24].

The majority of the studies in the literature are approaches towards improving the stages of fuzzification, identification of fuzzy relations, and defuzzification in order to increase forecasting performance. Regarding the stage of fuzzification, the approaches of Huarng [6] based on distribution and average, the optimization-based approach of Yolcu et al. [10], and the ratio-based approaches of Huarng and Yu [8] have brought a systematic view on the identification of interval length. Along with these, the approach of dividing the universe of discourse into equal intervals proposed initially by Song and Chissom [1] still maintains its validity and is used commonly in many studies. For the second stage called identification of fuzzy relations, this is also very effective in the forecasting performances of models, and different approaches have been proposed in the literature. Song and Chissom [1, 3, 4] firstly have proposed approaches based on complex matrix operations in their studies. Thus, Chen [5] has proposed in his study a new approach that contains

simpler operations where fuzzy logic group relation tables are without the need for the complex matrix operations of Song and Chissom [1, 3]. Also, Chen [14] has brought a new approach by using the fuzzy logic group relation tables also in high order fuzzy time series models. Additionally, studies where fuzzy relations have identified via artificial neural networks have commonly observed. Studies where artificial neural networks have also been used for identifying fuzzy relations may be listed as the studies of Huarng and Yu [7], Aladag et al. [18], Egrioglu et al. [19, 24, 26], Yu and Huarng [11] and Yolcu et al. [25]. In the vast majority of studies within the literature, the centralization method has been used during the defuzzification stage.

The fuzzy time series forecasting methods developed in the literature have focused on the AR model. There has not been any study using MA and ARMA models except the studies of Egrioglu et al. [20], Uslu et al. [27], and Alpaslan et al. [28] until 2013 year. These studies [20, 27, 28] on the other hand are methods that have been proposed for the solution of seasonal time series. However, Egrioglu et al. [29] and Kocak [30] developed two different methods in the fuzzy ARMA type time series for the solution of nonseasonal time series in 2013 year. In these studies, Egrioglu et al.'s [29] study is the first of first order fuzzy ARMA type time series models based on particle swarm optimization in the fuzzy time series literature. Kocak's [30] study also is the second of first order fuzzy ARMA type models based on the group relation tables which are similar to Chen's approaches [5, 14] in identifying fuzzy relation.

All of the models expect these two studies [29, 30] in the literature have focused issues such as using the partitions of universe of discourse, membership order, model order, and artificial intelligence methods; however, no study has been taken notice of the model construction of fuzzy time series model for nonseasonal fuzzy time sets, whereas there are 3 different models used in the solutions of linear time series; namely, they are AR (autoregressive), MA (moving averages), and ARMA (autoregressive moving averages). Therefore, inclusion of only AR variables in the fuzzy time series forecasting models proposed in the literature could cause model specification error. Because modelling of many of the real-life time series requires using MA variables, thus, using only AR variables in the solution of many fuzzy time series might cause to be identified of fuzzy relations incorrectly and it also cause to the decrease of the forecasting performance.

In this study, ARMA type fuzzy time series models have been defined as a new model for eliminating the above-mentioned adverse effects. A solution algorithm has been developed for high degree fuzzy ARMA(p, q) time series forecasting model where fuzzy relations are identified via artificial neural networks. The proposed method has been considered to be a significant study due to be the first of high order fuzzy ARMA models in the fuzzy time series literature. The advantages of the proposed algorithm are as follows.

- (i) By also using MA variables along with AR variables in solutions, the model specification error is eliminated, and a more realistic approach is obtained for real-life time series. Thus, forecasting performance has

significantly increased according to fuzzy AR models in the literature.

- (ii) Since high degree fuzzy time series models take into consideration also the model order, usually the forecasting performance could be obtained to be better than first order methods. With the proposed method on the other hand, a forecasting performance can be obtained considerably higher than performances of fuzzy AR models in the literature because of the fact that both the AR and the MA variables are high order.
- (iii) Using artificial neural networks has simplified the identification of fuzzy relations according to some methods [1, 5, 14] and has been effective in increasing the forecasting performance.

In the second section of the study, the basic definitions of fuzzy time series have been presented, and basic information on artificial neural networks has been provided in the third section. In the fourth section, definitions with regard to the proposed method and the solution algorithm of the proposed method have been given. In the fifth section, results obtained through applying the proposed method and some other methods in the literature on Istanbul Stock Exchange (IMKB) national 100 index time series have been presented. In the sixth section, the results obtained from this paper have been discussed.

2. Definition of Fuzzy Time Series

Main time series definitions developed using AR variables are listed below.

Definition 1. Let $Y(t)$, ($t = \dots, 0, 1, 2, \dots$) be a real value time series, let U be the universe of discourse in compliance with $Y(t)$, and let $f_j(t)$ (fuzzy subsets) be the subset functions identified in accordance with subintervals (u_j) of U . If $F(t)$ is a collection of $f_1(t), f_2(t), \dots, F(t)$ is called a fuzzy time series [1, 3, 4].

Definition 2. Let $R(t, t-1)$ be the fuzzy relation between $F(t-1)$ and $F(t)$. For any t value, if there is the equal of $R(t, t-1) = R(t-1, t-2)$, then $R(t, t-1)$ is independent from t . In that case, $F(t)$ is called the time invariant fuzzy time series [1].

Definition 3. If $F(t)$ is only affected by $F(t-1)$, then the fuzzy relation between $F(t-1)$ and $F(t)$ is expressed as

$$F(t-1) \longrightarrow F(t) \quad (1)$$

and is defined as a first order fuzzy time series forecasting model. The expression in (1), showing the fuzzy relation between $R(t, t-1)$, $F(t-1)$, and $F(t)$, can also be expressed as

$$F(t) = F(t-1) \circ R(t, t-1). \quad (2)$$

The " $\circ$ " operator in (2) had been determined as the max-min operator by Song and Chissom [1].

FIGURE 1: Multilayer FFANN with one output neuron.

Definition 4. If $F(t)$ is affected by $F(t-1), F(t-2), \dots, F(t-p)$ which are lagged fuzzy time series, then the fuzzy relation between $F(t)$ and the $F(t-1), F(t-2), \dots, F(t-p)$ fuzzy time series may be expressed as

$$F(t-p), \dots, F(t-2), F(t-1) \longrightarrow F(t) \quad (3)$$

and is called the p th order fuzzy time series forecasting model [14].

3. Feedforward Artificial Neural Networks (FFANN)

Artificial neural networks (ANN) can be defined as algorithms developed by inspiration from biological neural networks in order to provide the skills of the human brain such as producing new information and discovery through learning [31, 32]. ANN, besides being an intuitive technique aiming optimization, is used for the purposes of classification, estimation, and modelling in identification of functional relations.

There are two fundamental architectures in the contemporary ANN. These are feedback ANN and feedforward ANN. Feedback ANN (FBANN) is used for solution exploration for optimisation problems. Feedforward ANN (FFANN), on the other hand, is used commonly in identifying relations between variables. A FFANN architecture may be seen in Figure 1.

Main components of the FFANN seen in Figure 1 are as follows.

3.1. Architecture Structure. A FFANN network consists of one input layer, one output layer, and one or more hidden layers. Each layer has one or more neurons. Connections are unidirectional and forward oriented. The neuron values on the input layer are sent to the neurons on the hidden layer. The neuron values on each layer transmit value to the neuron on the following layer. Thus, the neuron values on the output layer are obtained in the final stage.

3.2. Learning Algorithm. Learning of FFANN is a process of adjusting weights as to perform a desired process. Learning algorithms used in FFANNs aim to specify the weights that

would minimise the error. Studies conducted on gradient-based approaches are still used for the learning of nonlinear events through FFANN. Backpropagation learning, one of these approaches, is commonly used in our day. One of the backpropagation algorithms, Levenberg-Marquardt algorithm [33], has certain positive qualities that eliminate some limitations of its predecessor methods of gradient decrease and Newton [34]. Therefore, Levenberg-Marquardt algorithm has been used while fuzzy relations are determined with FFANN in this study.

3.3. Activation Function. Inputs coming to the neurons on the hidden layer or output layers are obtained that the outputs of neurons on the previous layer are multiplied with relevant weights and summed up. The information received at these neurons is processed through an activation function to obtain the output of the neuron. Activation function provides curvilinear matchup. Therefore, nonlinear activation functions are used for hidden layer units. The nonlinear activation function most commonly used in the analysis of fuzzy time series is the logistic activation function. For this reason, logistic activation function has been used while fuzzy relations are determined with FFANN in this study.

4. The Proposed Method

Not utilizing MA variables in the fuzzy time series models has caused model specification error. Thus, it is necessary to develop fuzzy ARMA models that also include MA variables for the solution of fuzzy time series. Due to this necessity, the high order fuzzy ARMA time series forecasting method based on artificial neural networks has been proposed within the context of this study. The proposed method could be considered to be leading for future studies due to being the first of high order fuzzy ARMA model in the fuzzy time series literature. The high order fuzzy ARMA time series model to be used in the solution algorithm of the proposed method is as below.

Definition 5. Let there be $F(t)$, n observed fuzzy time series and $\varepsilon(t)$, fuzzy error series obtained from $F(t)$. As $p, q \leq n$, if $F(t)$ is affected from both the lagged $F(t-1), F(t-2), \dots, F(t-p)$ fuzzy time series and the lagged $\varepsilon(t-1), \varepsilon(t-2), \dots, \varepsilon(t-q)$ fuzzy time series, then the fuzzy relation between $F(t)$ fuzzy time series, $F(t-1), F(t-2), \dots, F(t-p)$ fuzzy time series, and $\varepsilon(t-1), \varepsilon(t-2), \dots, \varepsilon(t-q)$ fuzzy error series could be expressed with the below expression:

$$F(t-p), \dots, F(t-2), F(t-1), \varepsilon(t-q), \dots, \varepsilon(t-2), \varepsilon(t-1) \longrightarrow F(t) \quad (4)$$

and is defined as high order ARMA(p, q) fuzzy time series forecasting model. Here, p gives the order of AR variable, and q gives the order of MA variable.

In expression (4) within Definition 5, when the value 1 is put in p and q , the first order fuzzy ARMA(1, 1) time series forecasting model given in Definition 6 below is obtained.

TABLE 1: An example for the determination of fuzzy relations for the fuzzy AR(3) model.

$F(t-3)$	$F(t-2)$	$F(t-1)$	$F(t)$	Input 1	Input 2	Input 3	Target
A5	A1	A4	A3	5	1	4	3
A1	A4	A3	A2	1	4	3	2
A4	A3	A2	A6	4	3	2	6
A3	A2	A6	A5	3	2	6	5

Definition 6. Let $F(t)$ be a fuzzy time series and let $\varepsilon(t)$, $F(t)$ be the fuzzy error series obtained from the fuzzy time series. If $F(t)$ is affected by one lagged $F(t-1)$ and one lagged $\varepsilon(t-1)$ fuzzy error series, the fuzzy relation between $F(t)$, $F(t-1)$ and $\varepsilon(t-1)$ fuzzy could be expressed as

$$F(t-1), \varepsilon(t-1) \longrightarrow F(t) \quad (5)$$

and is defined as the first order autoregressive moving averages (ARMA(1,1)) fuzzy time series forecasting model [29, 30].

The fuzzy ARMA(p, q) model given in expression (4) can be considered a general high order model that also covers the first order fuzzy ARMA(1,1) model given in expression (5). In this study, an algorithm has been proposed for solving the high order fuzzy ARMA(p, q) time series forecasting model defined in expression (4). In the proposed algorithm, the fuzzy AR(p) model defined in (3) has been forecasted, and the errors are calculated by using time series and defuzzified forecasts obtained from fuzzy AR(p) model. Then, the fuzzy ARMA(p, q) model given in (4) has been forecasted. The algorithm of the proposed approach is given below.

4.1. The Algorithm of the Proposed Method

Step 1 (the universe of discourse (U) and subintervals (u_i , $i = 1, 2, \dots, b$) are defined). Starting and ending points of U are defined. Then the universal set is divided into subintervals (u_i) according to appropriate interval length. Definition of interval length is important to the researcher. It should not be forgotten that the interval length to be determined affects the number of subintervals (b). If the smallest value of the time series is taken as $Y_{\min}$, largest value as $Y_{\max}$, and two arbitrary values as D_1 and D_2 , the universal set can be defined as the closed interval of

$$U = [Y_{\min} - D_1, Y_{\max} + D_2]. \quad (6)$$

u_i subintervals determined for $i = 1, 2, \dots, b$ are the subintervals of the universal set U , which is defined as

$$U = \{u_1, u_2, \dots, u_b\}. \quad (7)$$

Step 2. For the time series, fuzzy sets are defined according to the universe of discourse (U) and the partitions (u_i). These fuzzy sets are expressed as

$$A_i = \frac{f_{A_i}(u_1)}{u_1} + \frac{f_{A_i}(u_2)}{u_2} + \dots + \frac{f_{A_i}(u_b)}{u_b}, \quad (8)$$

for $i = 1, 2, \dots, b$.

FIGURE 2: The architectural structure of FFANN for fuzzy AR(3) model.

Here, for $i = 1, 2, \dots, b$,

$$f_{A_i}(u_i) = \begin{cases} 1, & k = i \\ 0.5, & k = i - 1, i + 1 \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

Step 3 (time series $Y(t)$ is fuzzified). Subintervals (u_i) where each observation occurs are defined. Then the fuzzy set A_i where the defined subinterval has the highest membership value is defined. The fuzzy value of the observation is this A_i .

Step 4 (for the fuzzy AR(p) model, fuzzy relations are determined via feedforward artificial neural network (FFANN)). For the purposes of identifying relations via FFANN, the lagged fuzzy time series variables $F(t-p), \dots, F(t-2), F(t-1)$ are the inputs of FFANN, and $\hat{F}(t)$ is the output of FFANN, where the fuzzy time series $F(t)$ is the target value. The training samples of FFANN are comprised of the sequence numbers of the fuzzy sets. For example, let the fuzzy time series observations be, respectively, A5, A1, A4, A3, A2, A6, and A5. In Table 1, the inputs and target value of the artificial neural network for this example as per fuzzy AR(3) model are displayed.

The architectural structure of FFANN used in this step for fuzzy relation identification regarding the example given in Table 1 is shown in Figure 2.

For the training of FFANN, the Levenberg-Marquardt learning algorithm is utilised, and the following logistic

activation function is used for the calculation of each neuron value:

$$f(\text{net}) = \frac{1}{1 + \exp(-\text{net})}. \quad (10)$$

Step 5 (fuzzy forecasts are obtained). The output of FFANN of which architectural structure is given in Figure 2 is the sequence numbers of the fuzzy sets. The fuzzy sets with these sequence numbers form the fuzzy forecasts. For example, if the output of the artificial neural network is obtained as 5 when rounded up or down, the fuzzy forecast will be found as A_5 .

Step 6 (defuzzification process is executed). In this step, centralisation method is utilised. When the fuzzy forecast is A_i , the defuzzified forecast becomes the middle point of the u_i interval that has the highest membership value within A_i .

Step 7. Errors are calculated by taking the differences between the observed time series values and the defuzzified forecast values obtained in Step 6. Real values of the time series are $Y(t)$ and the defuzzified forecast values obtained in Step 6 are $\hat{Y}(t)$; the error series $e(t)$ is calculated as follows:

$$e(t) = Y(t) - \hat{Y}(t). \quad (11)$$

For example, if the error values of the first 3 observations for the fuzzy AR(3) model are assumed as 0, the errors from the 4th to the last observation are calculated with Formula (11).

Step 8 (for the errors, the universe of discourse is defined as (V) and subintervals are defined as v_j ($j = 1, 2, \dots, c$)). Starting and ending points of V are defined. Then V is divided into subintervals according to appropriate interval length. If the smallest value of the time series is taken as $e_{\min}$, largest value as $e_{\max}$, and two arbitrary values as D_1 and D_2 , the universe of discourse can be defined as the closed interval of

$$V = [e_{\min} - D_1, e_{\max} + D_2]. \quad (12)$$

The v_i subintervals determined for $j = 1, 2, \dots, c$ are the subintervals of the universal set V , expressed as

$$V = \{v_1, v_2, \dots, v_c\}. \quad (13)$$

Step 9 (fuzzy sets based on the universe of discourse (V) and partitions (v_j) are defined for the errors). The fuzzy sets are expressed, based on the explanations in Definition 1, as follows:

$$B_j = \frac{f_{B_j}(v_1)}{v_1} + \frac{f_{B_j}(v_2)}{v_2} + \dots + \frac{f_{B_j}(v_c)}{v_c}, \quad (14)$$

for $j = 1, 2, \dots, c$.

Here, for $j = 1, 2, \dots, c$,

$$f_{B_j}(v_j) = \begin{cases} 1, & k = j \\ 0.5, & k = j - 1, j + 1 \\ 0, & \text{otherwise.} \end{cases} \quad (15)$$

FIGURE 3: The architectural structure of FFANN used for fuzzy ARMA(3, 2) model.

Step 10 (error series $e(t)$ is fuzzified). Subintervals (v_j) for each error value are determined. Then the fuzzy set B_j where the determined subinterval has the highest membership value is defined. Fuzzy value of the error value is this B_j fuzzy set of which value was determined.

Step 11 (fuzzy relations for the fuzzy ARMA(p, q) model are determined via feedforward artificial neural network (FFANN)). For the purposes of identifying relations via FFANN, the lagged fuzzy time series variables $F(t-p), \dots, F(t-2), F(t-1)$ and the fuzzy error series variables $\varepsilon(t-p), \dots, \varepsilon(t-2), \varepsilon(t-1)$ are the inputs of FFANN, and $\hat{F}(t)$ is the output of the artificial neural network where the fuzzy time series $F(t)$ is the target value. Learning examples of FFANN are comprised of the sequence numbers of the fuzzy sets. For example, let the fuzzy time series observations be, respectively, $A_5, A_1, A_4, A_3, A_2, A_6$, and A_5 and let the fuzzy error series observations be, respectively, $B_3, B_3, B_3, B_2, B_4, B_3, B_5$, and B_6 . In Table 2, the inputs and target value of FFANN for this example as per fuzzy ARMA(3, 2) model are displayed.

The architectural structure of the feedforward artificial neural network used in this step for fuzzy relation identification regarding the example given in Table 2 is shown in Figure 3.

For the training of FFANN, the Levenberg-Marquardt learning algorithm is utilised, and the logistic activation function given in expression (10) is used for the calculation of each neuron value.

Step 12 (fuzzy forecasts are obtained). The output of FFANN of which architectural structure is given in Figure 3 is the sequence numbers of the fuzzy sets. The fuzzy sets with these sequence numbers form the fuzzy forecasts. For example, if

TABLE 2: An example for the determination of fuzzy relations for fuzzy ARMA(3,2) model.

$F(t-3)$	$F(t-2)$	$F(t-1)$	$F(t)$	$\varepsilon(t-2)$	$\varepsilon(t-1)$	Input 1	Input 2	Input 3	Input 4	Input 5	Target
A5	A1	A4	A3	B3	B3	5	1	4	3	3	3
A1	A4	A3	A2	B3	B2	1	4	3	3	2	2
A4	A3	A2	A6	B2	B4	4	3	2	2	4	6
A3	A2	A6	A5	B4	B3	3	2	6	4	3	5

the output of FFANN is obtained as 5 when rounded up or down, the fuzzy forecast will be found as A_5 .

Step 13 (defuzzification process is executed). In this step, centralisation method is utilised. When the fuzzy forecast obtained in Step 12 is A_i , the defuzzified forecast becomes the middle point of the u_i interval that has the highest membership value within A_i .

5. Application

For the application of the proposed method and the methods in literature, by increasing one unit the number of fuzzy sets of times series and the number of fuzzy sets of error series that are between 5 and 35, a total of 31 different numbers of fuzzy sets have been tried. If the method required calculation according to interval length, the interval lengths have been determined as to have the number of fuzzy sets comply with the interval of 5–35, using the formula below:

$$\text{Interval Length} = \frac{\max(\text{data}) - \min(\text{data})}{\text{Number of Fuzzy Sets}}. \quad (16)$$

The number of hidden layer units of the feedforward artificial neural networks used in the 4th and 11th steps of the proposed method has been varied between 1 and 10. Levenberg-Marquardt has been used for the training of the artificial neural network, and the logistic activation function given in (10) has been used for the hidden layer units and output layer. Forecasts that make the value of the RMSE (root mean square error) given in (17) minimum have been obtained as the best results of the method:

$$\text{RMSE} = \left(\frac{\sum_{i=1}^T (y_i - \hat{y}_i)^2}{T} \right)^{1/2}. \quad (17)$$

According to RMSE scale, the values of MAPE (mean average percentage error) and DA (Direction Accuracy) values are also calculated as given in the below equations of (18) for the best results of all methods:

$$\text{MAPE} = \frac{1}{T} \sum_{i=1}^T \left| \frac{y_i - \hat{y}_i}{y_i} \right|,$$

$$\text{DA} = \frac{1}{T} \sum_{i=1}^T a_i, \quad a_i = \begin{cases} 1 & \text{if } (y_{i+1} - y_i)(\hat{y}_{i+1} - y_i) > 0 \\ 0 & \text{otherwise.} \end{cases} \quad (18)$$

We divide the data of Istanbul Stock Exchange (IMKB) national 100 index (IMKB) into three parts to compare the

FIGURE 4: The time series graph of data set 1, the period between 03.10.2008 and 31.12.2008.

FIGURE 5: The time series graph of data set 2, the period between 01.10.2009 and 31.12.2009.

results to ones to be obtained from the methods in the literature. Two different sizes for the test data were tried out to every data set. Consequently 6 different analyses have been done. The periods, 03/10/2008–31/12/2008, 01/10/2009–31/12/2009, and 01/10/2010–23/12/2010 of IMKB were referred as data sets 1, 2, and 3, respectively. The time series graphs of the data sets are given in Figures 4, 5, and 6, respectively.

By taking the last 7 observations in the first solution of the data belonging to data set 1, data set 2, and data set 3 with the methods in the literature, and taking the last 15 observations in the second solution as test set, a total of 6 different applications have been realised. For the solution of the data with the AR models in the literature, consider the following.

FIGURE 6: The time series graph of data set 3, the period between 01.10.2010 and 23.12.2010.

- (i) During the fuzzification stage of Song-Chissom's AR(1) model [1], 31 different results have been obtained from increasing the fuzzy set number between 5 and 35. The test set forecast that has the minimum RMSE value from these results has been determined as the best result of the method.
- (ii) For each of Chen's AR(1) [5] and Chen's AR(p) ($p = 1, 2, \dots, 5$) [14] models, the RMSE values have been found for different results with the interval length being increased 100 units between 400 and 2700 for data set 1, between 200 and 1600 for data set 2, and between 300 and 2300 for data set 3. The test set forecasts that have the minimum RMSE values among these results have been determined as the best results of the methods.
- (iii) For Aladag et al.'s AR(p) ($p = 1, 2, \dots, 5$) [18] model, the RMSE values have been found for different results with the interval length being increased 100 units between 400 and 2700 for data set 1, between 200 and 1600 for data set 2, and between 300 and 2300 for data set 3 and by increasing the number of units for hidden layers of the feedforward artificial neural network 1 unit between 1 and 10. The test set forecasts that have the minimum RMSE values among these results have been determined as the best results of the methods.
- (iv) By using the optimal interval lengths calculated via the distribution-based approach and average-based approach of Huarng [6] and the average-based approaches, solution has been applied as per the first order fuzzy time series forecast method of Chen [5]. Thus, the best results of the test set from the distribution based approach and the average-based approaches have been obtained via a single trial.
- (v) In the application of the ratio-based approach of Huarng and Yu [8], alpha parameter has been taken 0.50. Then, the best result for the test set of the method has been obtained in one trial.
- (vi) The best forecasts of Cheng et al. [9] and Yolcu et al. [25] methods have been taken from the study of Yolcu et al. [25].

Processes which were made for the solution of the data with using the proposed method are as follows.

- (i) During the fuzzification stage in Step 1 for the IMKB time series, different trials have been done with the interval length being increased 100 units between 400 and 2700 for data set 1, between 200 and 1600 for data set 2, and between 300 and 2300 for data set 3.
- (ii) While different trials have been made in Step 1, different forecasts have been obtained for AR(p) ($p = 1, 2, \dots, 5$) model, by increasing the number of hidden layers of the feedforward artificial neural network by 1 unit between 1 and 10 and trying different interval lengths in Step 4.
- (iii) Among the different forecasts, the forecast that has minimum RMSE value for test set is determined as the best result of AR(p).
- (iv) For the best forecast of the time series, the error series ($e(t)$) has been calculated by using Formula (11) in Step 7.
- (v) During the fuzzification stage of errors, 31 different trials have been done by increasing the number of fuzzy sets belonging to $e(t)$ error series between 5 and 35 by 1 unit.
- (vi) For the identification of fuzzy relations in Step 11 for the second time, forecasts with different performances have been obtained for ARMA(p, q) (p and $q = 1, 2, \dots, 5$) model by increasing the number of units for hidden layers of the feedforward artificial neural network 1 unit between 1 and 10, and by trying different interval lengths of time series and fuzzy set numbers of error series.
- (vii) The test set forecast with the smallest RMSE value among these forecasts has been determined as the best result of the proposed method.

The best forecasts and forecasting performances obtained for the test set of the methods have been presented in Tables 3–8. Also the graphs of the best forecasts obtained for the test set via the proposed method are displayed in Figures 7–12.

The parameters during which the best results given in Table 3 were obtained are as follows:

- (i) when the number of fuzzy sets is 9 for the solution of Song and Chissom [1] method,
- (ii) when the interval length is 400 for the solution of Chen [5] method,
- (iii) when the interval length on 3rd degree model is 400 for the solution of Chen [14] method,
- (iv) when the interval length is 800 for Huarng's distribution-based approach [6],
- (v) when the interval length is 200 for Huarng's distribution-based approach [6],
- (vi) when the sample percentile is 0.5 for Huarng's ratio-based approach [8],

TABLE 3: Data set 1 forecasts and forecasting performances for the best results of the methods with the sample size 7 for the test data.

Date	IMKB	Song-Chissom method [1]	Chen method [5]	Chen method [14]	Huang distribution-based method [6]	Huang average-based method [6]	Huang ratio-based method [8]	Aladag et al. method [18]	Cheng et al. method [9]	Yolcu et al. method [25]	*Proposed method
23.12.2008	26294	26411,5	26400	26200	26666,7	26100	26708,8	26200	26390	26274	26200
24.12.2008	26055	26411,5	26400	26200	26666,7	26366,7	26708,8	26200	26390	26273	26200
25.12.2008	26059	26411,5	26400	26200	26666,7	26100	26708,8	26200	26390	26339	26200
26.12.2008	26499	26411,5	26400	26200	26666,7	26100	26708,8	26200	26390	26337	26600
29.12.2008	26424	26411,5	26600	26400	26666,7	26500	26708,8	26600	26390	26565	26600
30.12.2008	26411	26411,5	26600	26533,3	26666,7	26500	26708,8	26600	26390	26429	26600
31.12.2008	26864	26411,5	26600	26600	26666,7	26500	26708,8	26600	26390	26460	26600
RMSE		261,24	236,87	179,04	391,62	251,24	424,20	198,40	258,87	219,27	167,48
MAPE		0,0075	0,0083	0,0059	0,0134	0,0080	0,0145	0,0071	0,0076	0,0067	0,0060
DA		0,8333	0,5000	0,8333	0,5000	0,5000	0,5000	0,6667	0,6667	0,6667	0,6667

*The best result.

TABLE 4: Data set 1 forecasts and forecasting performances for the best results of the methods with the sample size 15 for the test data.

Date	IMKB	Song-Chissom method [1]	Chen method [5]	Chen method [14]	Huang distribution-based method [6]	Huang average-based method [6]	Huang ratio-based method [8]	Aladag et al. method [18]	Cheng et al. method [9]	Yolcu et al. method [25]	*Proposed method
05.12.2008	24035	24339,5	24350	24300	25200	24700	24534,7	24800	25541	24355	24600
12.12.2008	24937	24339,5	24350	24500	24000	24100	23901,5	24800	25406	24279	25000
15.12.2008	25598	25673	24350	24700	25200	26700	25366,7	24800	25406	25371	25000
16.12.2008	26396	25673	26450	25200	25200	24100	25416,9	26400	25406	25704	26200
17.12.2008	26765	26322,7	26450	26100	26666,7	26300	26787,7	26400	25406	26348	26600
18.12.2008	26396	26562	26450	26700	26666,7	26550	26787,7	26400	25406	26344	26600
19.12.2008	26205	26322,7	26450	26500	26666,7	26300	26787,7	26400	25406	26348	26600
22.12.2008	26199	26322,7	26450	26300	26666,7	26300	26787,7	26400	25406	26315	26200
23.12.2008	26294	26322,7	26450	26300	26666,7	26100	26787,7	26400	25406	26316	26200
24.12.2008	26055	26322,7	26450	26300	26666,7	26300	26787,7	26400	25406	26313	26200
25.12.2008	26059	26322,7	26450	26300	26666,7	26100	25416,9	26400	25406	26330	25800
26.12.2008	26499	26322,7	26450	26300	26666,7	26100	25416,9	26400	25406	26330	26200
29.12.2008	26424	26322,7	26450	26300	26666,7	26500	26787,7	26400	25406	26419	26600
30.12.2008	26411	26322,7	26450	26300	26666,7	26500	26787,7	26400	25406	26365	26600
31.12.2008	26864	26322,7	26450	26300	26666,7	26500	26787,7	26400	25406	26357	26600
RMSE		338,10	427,29	492,17	600,69	743,54	622,79	358,22	987,04	337,07	290,61
MAPE		0,0103	0,0117	0,0145	0,0193	0,0183	0,0208	0,0100	0,0354	0,0100	0,0093
DA		0,7143	0,6429	0,5000	0,5000	0,5000	0,2857	0,6429	0,5714	0,7143	0,7143

*The best result.

TABLE 5: Data set 2 forecasts and forecasting performances for the best results of the methods with the sample size 7 for the test data.

Date	IMKB	Song-Chissom method [1]	Chen method [5]	Chen method [14]	Huang distribution-based method [6]	Huang average-based method [6]	Huang ratio-based method [8]	Aladag et al. method [18]	Cheng et al. method [9]	Yolcu et al. method [25]	*Proposed method
23.12.2009	51162	51139	51950	51350	50400	51500	51617,3	51200	50872	51317	51000
24.12.2009	51461	51139	51050	51150	50400	50550	50749,8	51200	50763	51317	51400
25.12.2009	51661	51139	51050	51450	50400	51500	51617,3	51200	50763	51317	51800
28.12.2009	51619	51139	51950	51650	52000	51700	51617,3	51200	50763	51317	51400
29.12.2009	51786	51139	51950	51750	52000	51700	51617,3	52000	50763	51317	51800
30.12.2009	51668	51139	51950	51850	52000	51700	51617,3	51200	50763	51317	51400
31.12.2009	52825	51139	51950	51950	52000	51700	51617,3	52000	50763	51317	52600
RMSE		769,80	553,53	373,71	781,78	567,03	561,19	447,71	1084,21	640,41	177,24
MAPE		0,0115	0,0096	0,0050	0,0134	0,0075	0,0072	0,0074	0,0185	0,0090	0,0030
DA		0,3333	0,3333	0,5000	0,3333	0,6667	0,5000	0,8333	0,3333	0,5000	1,0000

*The best result.

TABLE 6: Data set 2 forecasts and forecasting performances for the best results of the methods with the sample size 15 for the test data.

Date	IMKB	Song-Chissom method [1]	Chen method [5]	Chen method [14]	Huang distribution-based method [6]	Huang average-based method [6]	Huang ratio-based method [8]	Aladag et al. method [18]	Cheng et al. method [9]	Yolcu et al. method [25]	*Proposed method
11.12.2009	49386	49271,7	48950	49250	49600	49100	49569,5	49300	49984	49516	49250
14.12.2009	50198	49791	48950	49250	49600	49300	49569,5	49100	49984	50064	50150
15.12.2009	50450	50139,3	50450	50150	49866,7	50500	50222,5	50500	49984	50942	50150
16.12.2009	50817	50139,3	50450	51050	49866,7	48900	49796	51300	49984	50217	50450
17.12.2009	49963	50171	50450	50150	50400	50900	50736,6	50500	49984	49641	50450
18.12.2009	50138	50068,7	50450	51050	49600	49900	50222,5	50100	49984	49619	50450
21.12.2009	51281	50068,7	50450	51050	49866,7	50500	50222,5	50500	49984	50932	50450
22.12.2009	51533	50333,9	51200	51050	50400	50966,7	50882,5	51100	49984	50817	51350
23.12.2009	51162	51121	51200	51350	50400	51500	50882,5	51500	49984	51100	51050
24.12.2009	51461	50333,9	50450	51350	50400	50550	50882,5	50500	49984	50646	51050
25.12.2009	51661	51121	51200	51200	50400	51500	50882,5	51500	49984	51073	51650
28.12.2009	51619	51121	51200	51500	52000	51700	51776,2	51900	49984	51117	51350
29.12.2009	51786	51121	51200	51800	52000	51700	51776,2	51900	49984	51114	51650
30.12.2009	51668	50741	51200	51950	52000	51700	51776,2	51900	49984	51119	51350
31.12.2009	52825	51121	51200	51950	52000	51700	51776,2	51900	49984	51117	51650
RMSE		804,45	713,35	471,49	803,72	760,77	623,27	553,84	1381,68	662,35	451,53
MAPE		0,0126	0,0112	0,0072	0,0140	0,0110	0,0099	0,0085	0,0225	0,0106	0,0066
DA		0,4286	0,5000	0,6429	0,3571	0,5000	0,6429	0,6429	0,4286	0,5000	0,7143

*The best result.

TABLE 7: Data set 3 forecasts and forecasting performances for the best results of the methods with the sample size 7 for the test data.

Date	IMKB	Song-Chissom method [1]	Chen method [5]	Chen method [14]	Huang distribution-based method [6]	Huang average-based method [6]	Huang ratio-based method [8]	Aladag et al. method [18]	Cheng et al. method [9]	Yolcu et al. method [25]	*Proposed method
15.12.2010	65499	65356	66550	65900,0	67400	66500	66784,7	67300	65992	65999	65350
16.12.2010	64429	65356	66250	65900,0	65400	66300	66178,0	64900	65992	64813	64450
17.12.2010	63524	65975	64450	64800,0	65900	64500	65878,7	64900	65992	64864	64450
20.12.2010	63502	64737	63550	64066,7	64900	63500	63514,3	64300	65992	64871	63550
21.12.2010	64820	64737	63550	63700,0	64900	63500	63514,3	63700	65992	64858	64450
22.12.2010	65440	65975	65800	64800,0	65900	65500	65878,7	65500	65992	65522	65350
23.12.2010	66219	65356	66250	65900,0	65400	66300	66178,0	66700	65992	64813	66250
RMSE		1161,91	1001,70	928,70	1365,14	1014,73	1317,77	1034,06	1544,90	929,82	383,29
MAPE		0,0139	0,0122	0,0128	0,0177	0,0118	0,0159	0,0135	0,0200	0,0113	0,0036
DA		0,50000	0,50000	0,33333	0,50000	0,50000	0,66667	0,66667	0,5000	0,5000	0,6667

*The best result.

TABLE 8: Data set 3 forecasts and forecasting performances for the best results of the methods with the sample size 15 for the test data.

Date	IMKB	Song-Chissom method [1]	Chen method [5]	Chen method [14]	Huang distribution-based method [6]	Huang average-based method [6]	Huang ratio-based method [8]	Aladag et al. method [18]	Cheng et al. method [9]	Yolcu et al. method [25]	*Proposed method
01.12.2010	66156	65695,5	66250	64833,3	65900,0	65300	66328,8	65650	65776	66421	66250
02.12.2010	66939	66438,2	65800	66700,0	65566,7	64100	66164,2	67150	65600	65817	67150
03.12.2010	66860	67088,0	67450	66700,0	67233,3	66700	67143,8	67150	65776	66776	67150
08.12.2010	67705	67088,0	67450	66700,0	67233,3	66700	67143,8	67150	65600	66752	67150
09.12.2010	65914	66252,5	66250	67633,3	65900,0	67700	66328,8	67150	65776	66783	67150
10.12.2010	64759	65695,5	65800	66233,3	65566,7	65900	65791,4	64150	65600	65023	65350
13.12.2010	66380	65695,5	65350	66700,0	65900,0	64700	65258,3	65650	65776	66005	65350
14.12.2010	66510	66438,2	65800	66700,0	65566,7	67100	66164,2	65650	65600	66774	67150
15.12.2010	65499	66438,2	65800	66700,0	67233,3	66500	66164,2	67150	65776	66775	66250
16.12.2010	64429	65695,5	66250	65766,7	65566,7	66300	66328,8	64150	65600	65064	64450
17.12.2010	63524	65695,5	65350	64366,7	65900,0	64500	65258,3	65650	65776	65019	65350
20.12.2010	63502	65138,5	63550	63900,0	64900,0	63500	63684,8	65650	65600	65037	62650
21.12.2010	64820	65138,5	63550	63900,0	64900,0	63500	63684,8	65650	65776	65035	65350
22.12.2010	65440	65695,5	65350	64833,3	65900,0	65500	65258,3	65650	65600	65599	65350
23.12.2010	66219	65695,5	66250	66700,0	65566,7	66300	66328,8	65650	65776	66416	66250
RMSE		919,47	925,42	954,17	1052,90	1283,18	896,96	1060,12	1197,78	816,92	761,91
MAPE		0,0112	0,0108	0,0125	0,0129	0,0156	0,0109	0,0131	0,0150	0,0099	0,0089
DA		0,7143	0,5714	0,6429	0,5000	0,5714	0,6429	0,6429	0,5714	0,6429	0,8571

*The best result.

FIGURE 7: The graph of 7-observation test set of data set 1 and the forecasts of the proposed methods.

FIGURE 9: The graph of 7-observation test set of data set 2 and the forecasts of the proposed methods.

FIGURE 8: The graph of 15-observation test set of data set 1 and the forecasts of the proposed methods.

FIGURE 10: The graph of 15-observation test set of data set 2 and the forecasts of the proposed methods.

- (vii) when the interval length is 400 on 2nd degree model and unit number of artificial neural network hidden layers is 1 for the solution of Aladag et al. [18] method,
- (viii) when the number of fuzzy sets is 5 for the solution of Cheng et al. [9] method,
- (ix) when the number of fuzzy sets is 11 and the unit number of artificial neural network hidden layers is 5 for the solution of Yolcu et al. [25] method,
- (x) when the interval length is 400 for time series ($Y(t)$), the number of fuzzy sets is 11 for error series ($e(t)$), and the unit number of artificial neural network hidden layers is 1 in the fuzzy ARMA(5, 5) model for the solution of the proposed method.

The parameters during which the best results given in Table 4 were obtained are as follows:

- (i) when the number of fuzzy sets is 15 for the solution of Song and Chissom [1] method,
- (ii) when the interval length is 2100 for the solution of Chen [5] method,
- (iii) when the interval length on 2nd degree model is 600 for the solution of Chen [14] method,

- (iv) when the interval length is 800 for Huarng's distribution-based approach [6],
- (v) when the interval length is 200 for Huarng's average-based approach [6],
- (vi) when the sample percentile is 0.5 for Huarng's ratio-based approach [8],
- (vii) when the interval length is 800 on 5th degree model and unit number of artificial neural network hidden layers is 2 for the solution of Aladag et al. [18] method,
- (viii) when the number of fuzzy sets is 9 for the solution of Cheng et al. [9] method,
- (ix) when the number of fuzzy sets is 12 and the unit number of artificial neural network hidden layers is 2 for the solution of Yolcu et al. [25] method,
- (x) when the interval length is 400 for $Y(t)$, the number of fuzzy sets is 19 for $e(t)$, and the unit number of artificial neural network hidden layers is 2 in the fuzzy ARMA(3, 5) model for the solution of the proposed method.

The parameters during which the best results given in Table 5 were obtained are as follows:

FIGURE 11: The graph of 7-observation test set of data set 3 and the forecasts of the proposed methods.

FIGURE 12: The graph of 15-observation test set of data set 3 and the forecasts of the proposed methods.

- (i) when the number of fuzzy sets is 9 for the solution of Song and Chissom [1] method,
- (ii) when the interval length is 900 for the solution of Chen [5] method,
- (iii) when the interval length on 5th degree model is 1500 for the solution of Chen [14] method,
- (iv) when the interval length is 800 for Huarng's distribution-based approach [6],
- (v) when the interval length is 200 for Huarng's average-based approach [6],
- (vi) when the sample percentile is 0.5 for Huarng's ratio-based approach [8],
- (vii) when the interval length is 800 on 4th degree model and unit number of artificial neural network hidden layers is 8 for the solution of Aladag et al. [18] method,
- (viii) when the number of fuzzy sets is 15 for the solution of Cheng et al. [9] method,
- (ix) when the number of fuzzy sets is 13 and the unit number of artificial neural network hidden layers is 7 for the solution of Yolcu et al. [25] method,

- (x) when the interval length is 400 for $Y(t)$, the number of fuzzy sets is 8 for $e(t)$, and the unit number of artificial neural network hidden layers is 6 in the fuzzy ARMA(4, 3) model for the solution of the proposed method.

The parameters during which the best results given in Table 6 were obtained are as follows:

- (i) when the number of fuzzy sets is 20 for the solution of Song and Chissom [1] method,
- (ii) when the interval length is 1500 for the solution of Chen [5] method,
- (iii) when the interval length on 3rd degree model is 900 for the solution of Chen [14] method,
- (iv) when the interval length is 800 for Huarng's distribution-based approach [6],
- (v) when the interval length is 200 for Huarng's average-based approach [6],
- (vi) when the sample percentile is 0.5 for Huarng's ratio-based approach [8],
- (vii) when the interval length is 200 on 2nd degree model and unit number of artificial neural network hidden layers is 5 for the solution of Aladag et al. [18] method,
- (viii) when the number of fuzzy sets is 6 for the solution of Cheng et al. [9] method,
- (ix) when the number of fuzzy sets is 7 and the unit number of artificial neural network hidden layers is 3 for the solution of Yolcu et al. [25] method,
- (x) when the interval length is 300 for $Y(t)$, the number of fuzzy sets is 9 for $e(t)$, and the unit number of artificial neural network hidden layers is 4 in the fuzzy ARMA(1, 2) model for the solution of the proposed method.

The parameters during which the best results given in Table 7 were obtained are as follows:

- (i) when the number of fuzzy sets is 9 for the solution of Song and Chissom [1] method,
- (ii) when the interval length is 300 for the solution of Chen [5] method,
- (iii) when the interval length on 3rd degree model is 2200 for the solution of Chen [14] method,
- (iv) when the interval length is 1000 for Huarng's distribution-based approach [6],
- (v) when the interval length is 200 for Huarng's average-based approach [6],
- (vi) when the sample percentile is 0.5 for Huarng's ratio-based approach [8],
- (vii) when the interval length is 600 on 2nd degree model and unit number of artificial neural network hidden layers is 5 for the solution of Aladag et al. [18] method,
- (viii) when the number of fuzzy sets is 9 for the solution of Cheng et al. [9] method,

- (ix) when the number of fuzzy sets is 7 and the unit number of artificial neural network hidden layers is 6 for the solution of Yolcu et al. [25] method,
- (x) when the interval length is 900 for $Y(t)$, the number of fuzzy sets is 15 for $e(t)$, and the unit number of artificial neural network hidden layers is 2 in the fuzzy ARMA(2, 2) model for the solution of the proposed method.

The parameters during which the best results given in Table 8 were obtained are as follows:

- (i) when the number of fuzzy sets is 20 for the solution of Song and Chissom [1] method,
- (ii) when the interval length is 900 for the solution of Chen [5] method,
- (iii) when the interval length on 2nd degree model is 1400 for the solution of Chen [14] method,
- (iv) when the interval length is 1000 for Huarng's distribution-based approach [6],
- (v) when the interval length is 200 for Huarng's average-based approach [6],
- (vi) when the sample percentile is 0.5 for Huarng's ratio-based approach [8],
- (vii) when the interval length is 1500 on 2nd degree model and unit number of artificial neural network hidden layers is 6 for the solution of Aladag et al. [18] method,
- (viii) when the number of fuzzy sets is 10 for the solution of Cheng et al. [9] method,
- (ix) when the number of fuzzy sets is 7 and the unit number of artificial neural network hidden layers is 7 for the solution of Yolcu et al. [25] method,
- (x) when the interval length is 900 for $Y(t)$, the number of fuzzy sets is 28 for $e(t)$, and the unit number of artificial neural network hidden layers is 10 in the fuzzy ARMA(1, 1) model for the solution of the proposed method.

In compliance with approach in which the forecasts that have the minimum RMSE value are the best forecasts, it has been determined that the proposed method has the best forecasting performance in comparison with 9 methods solved in this paper for each of 6 applications. It can also be seen clearly that the forecasts obtained through the proposed method are very close to the test set on the graphs presented in Figures 7–11.

The values of RMSE and MAPE obtained through the proposed method for the test set have significantly lower in comparison with other methods in all applications and direction accuracy is significantly higher compared to the other methods in the literature. These results show that fuzzy ARMA(p, q) model considerably can increase the forecasting performance. In addition, the fact of obtaining the RMSE value as 177.24 (the lowest RMSE value obtained in other methods is 373.21) and the direction accuracy as 100% for the solution of data set 2 (Table 5) draws attention from the point of display the effect of proposed method on increasing the forecasting performance.

6. Discussion and Conclusion

The actual point of emphasis in this paper is the consideration that it is more realistic to use one of the AR, MA, and ARMA models in accordance with the structure of the data in the manner that it is made in linear models of the classical time series. It is a deficiency that the fuzzy time series methods in the literature mostly consist of models using only AR variables. Not using MA variables could cause the model specification error in solutions of fuzzy time series. This deficiency has been shown clearly in the applications of this study, and so it is proven that forecasting performance towards future can be significantly increased with fuzzy ARMA model compared to the fuzzy AR models. The reasoning of this is that real lifetime series are affected not only by AR variables but also by MA variables.

The effect of the structure of the model in fuzzy time series approaches within the literature has been shown with the approach of being divided to two, namely, only first and high order models. Particularly significant progresses in fuzzy AR type have been executed by increasing the forecasting performance in the stage of identification of fuzzy relations via methods such as using membership values, identifying relations through artificial neural networks, and particle swarm optimisation. However, in the applications of the proposed method, it is clearly showed that the proposed method has better performance than those of many AR methods that use the membership values or optimisation methods. For this reason, selecting both the proper model degree (first order or high order) and the proper model type (AR, MA, or ARMA) is more realistic approach in solving of real life fuzzy time series.

By means of the proposed method, it has been clearly expressed how much using a high order fuzzy ARMA model based on artificial neural networks may significantly affect the forecasting performance. Therefore, it is considered that this study will constitute basis for new fuzzy ARMA type models in future because of the fact that the proposed method is the first study of high order fuzzy ARMA time series model.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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